Thermodynamics and **reference scale** of SU(3) gauge theory from gradient flow on fine lattices

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for FlowQCD Collaboration
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FlowQCD, arXiv:1503.06516

LATTICE2015, 2015/Jul./15, Kobe, Japan

Outline

1. Yang-Mills Gradient flow

2. Reference scale and Lattice spacing

   FlowQCD, arXiv:1503.06516

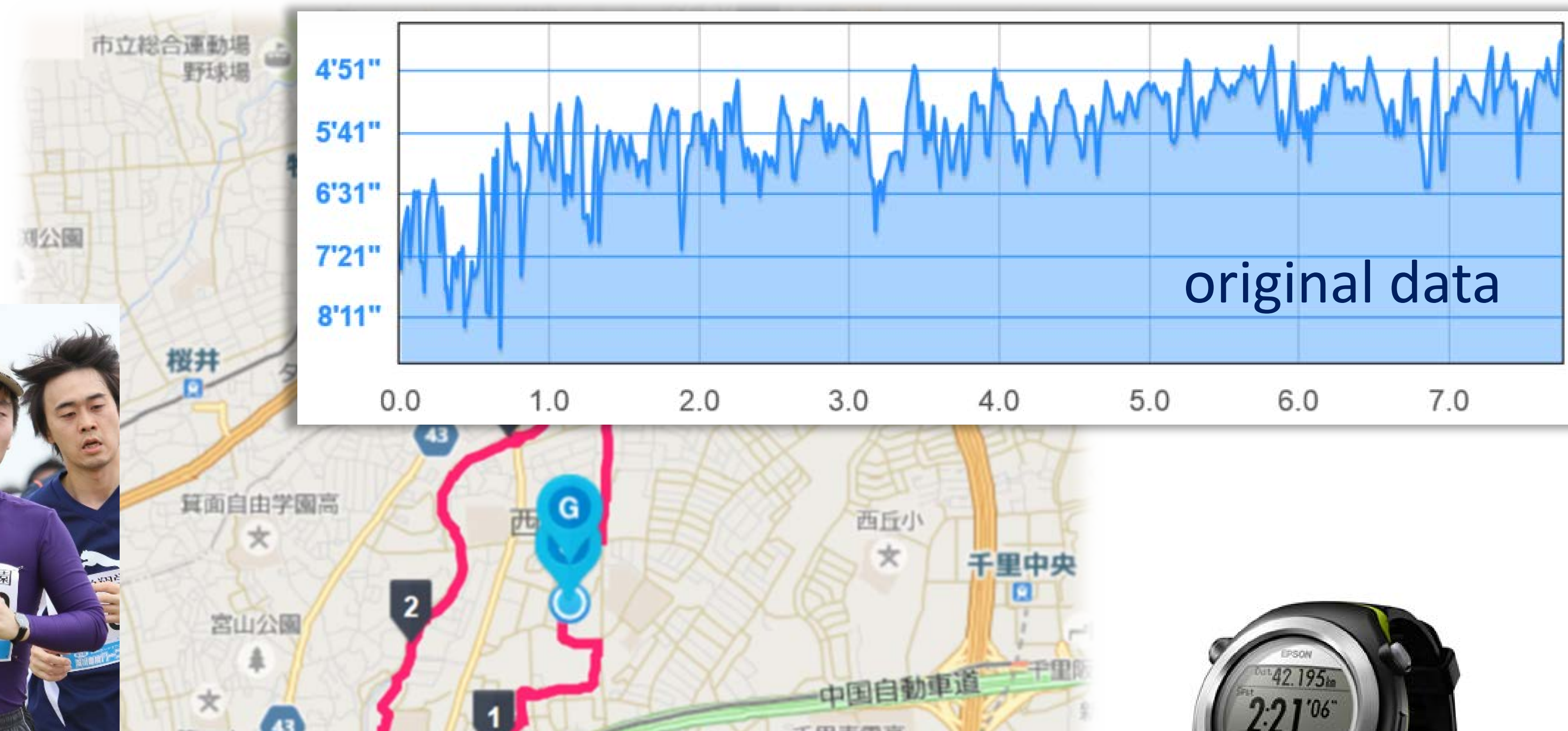
3. Thermodynamics

Yang-Mills Gradient Flow

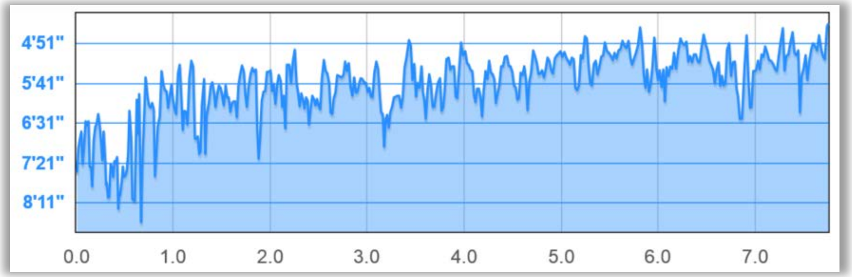
$$\partial_t A_\mu = D_\nu G_{\mu\nu}$$

Gradient Flow and Jogging

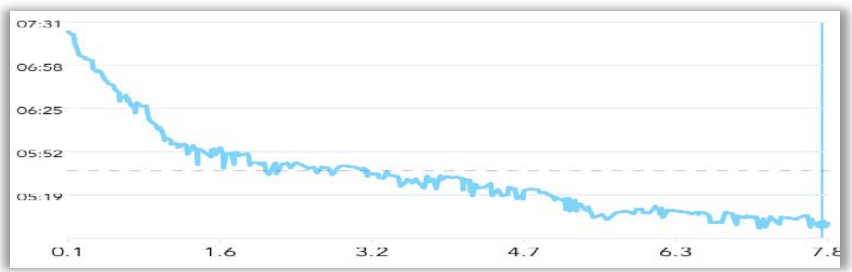
Gradient Flow and Jogging



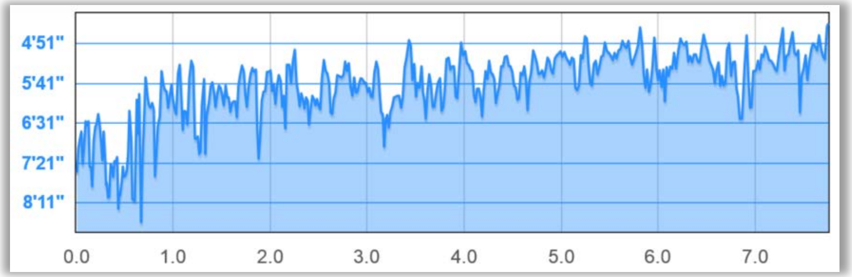
Gradient Flow and Jogging



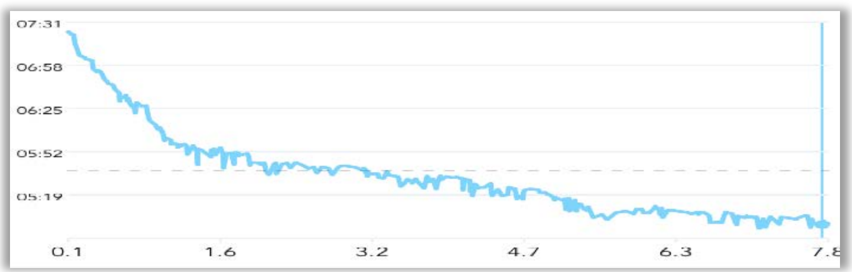
4'51"
5'41"
6'31"
7'21"
8'11"
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0

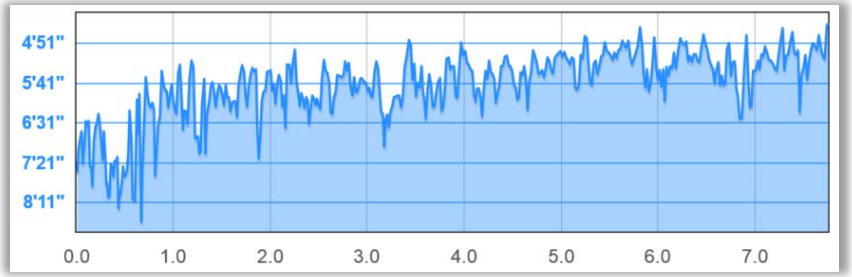


07:31
06:58
06:25
05:52
05:19
0.1
1.6
3.2
4.7
6.3



① $$x(t) \rightarrow x'(t) \sim \int dt' \exp\left[-\frac{(t-t')^2}{2\sigma^2}\right] x(t')$$

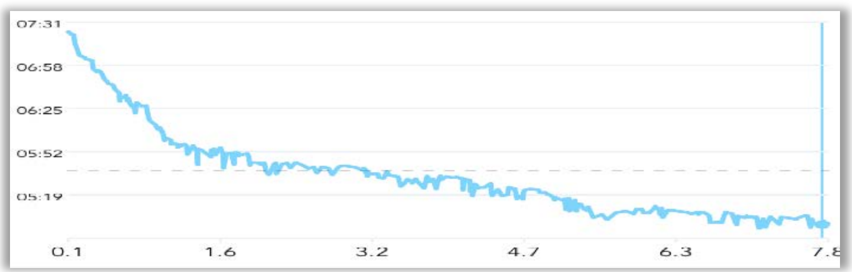


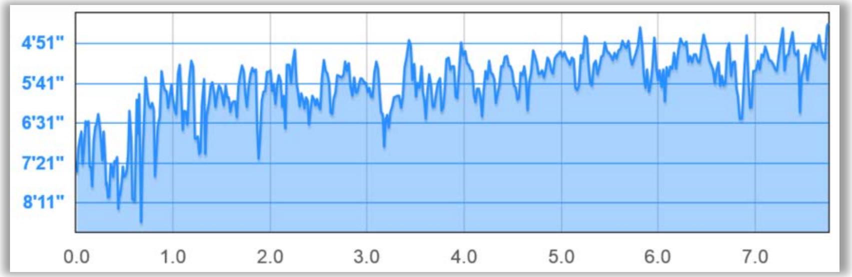


① $x(t) \to x'(t) \sim \int dt' \exp\left[-\frac{(t-t')^2}{2\sigma^2}\right] x(t')$

$\sigma = \sqrt{2s}$

② $\frac{d}{ds}x(t;s) = \frac{d^2}{dt^2}x(t,s) \qquad x(t;0) = x(t)$





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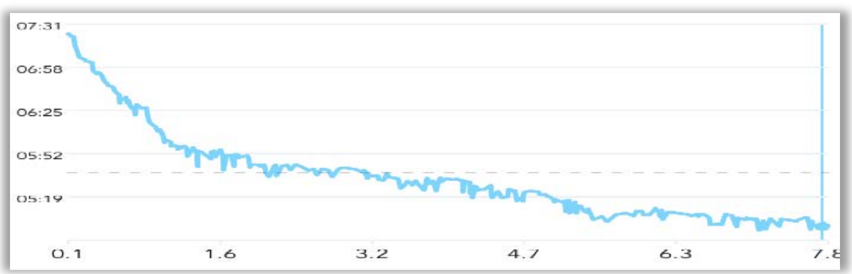


YM Gradient Flow

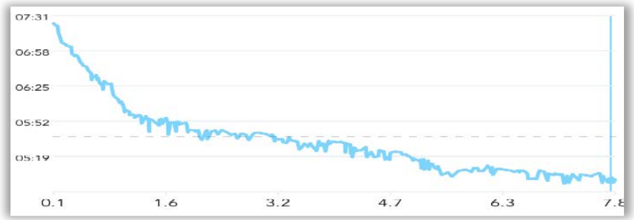
$$\partial_t A_\mu = D_\nu G_{\mu\nu}$$
$$= \partial_\nu \partial_\nu A_\mu + \cdots$$

- t: "flow time" dim: $[E^{-2}]$
- smearing length $r = \sqrt{8t}$

Use of Gradient Flow 1:

Observables at Nonzero Flow Time

$$\langle \mathcal{O} \rangle_t$$



- function of *t*
- independent of renormalization

Luescher, Weisz, 2011

independent of lattice spacing
(up to $O(a^2)$ corrections)

The *t* dep. can be used for the **scale setting** of the lattice.

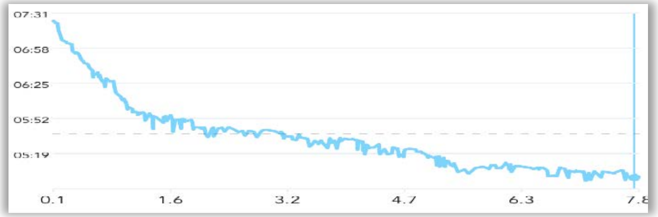
Use of Gradient Flow 2:

Small Flow Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

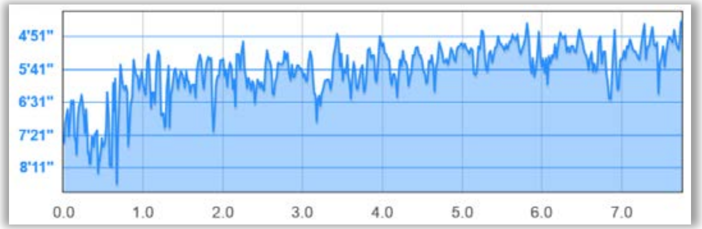
$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t \to 0]{} \sum_i c_i(t) \mathcal{O}_i^R(x)$$

An operator
in blurred world



less noisy, continuous

Renormalized operators
in original theory



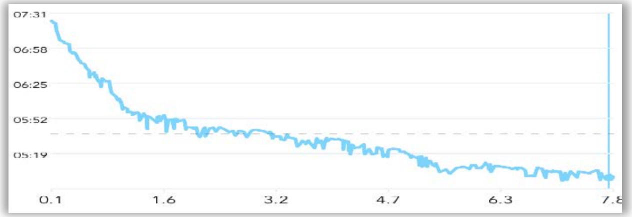
noisy, lattice discretized

Ex: energy momentum tensor, Suzuki, 2013; FlowQCD, 2014

Reference Scale and Lattice Spacing

Basic Idea

$$\langle \mathcal{O} \rangle_t$$



- function of t
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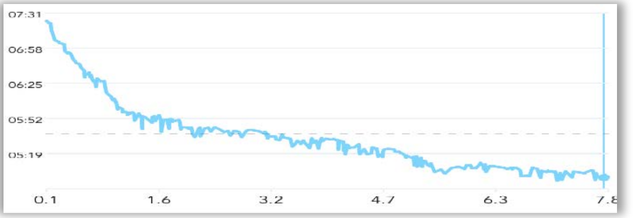
Basic Idea

$$\langle \mathcal{O} \rangle_t$$



- function of t
- independent of renormalization

Choice 1: $\mathcal{O} = t^2 E(t)$

Luescher, 2010

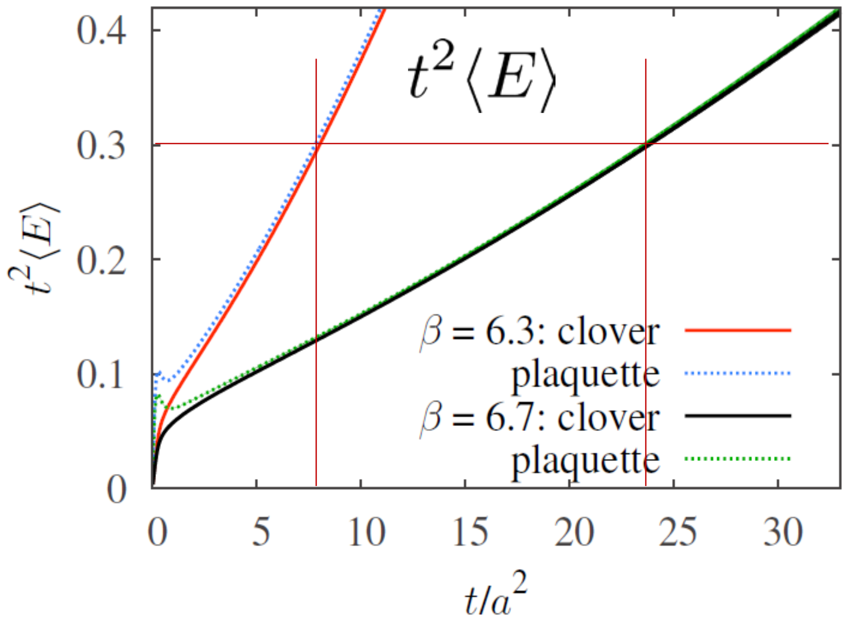
Choice 2: $\mathcal{O} = t \frac{d}{dt} t^2 E(t)$

Budapest-Wuppertal, 2012

discretization effect suppressed

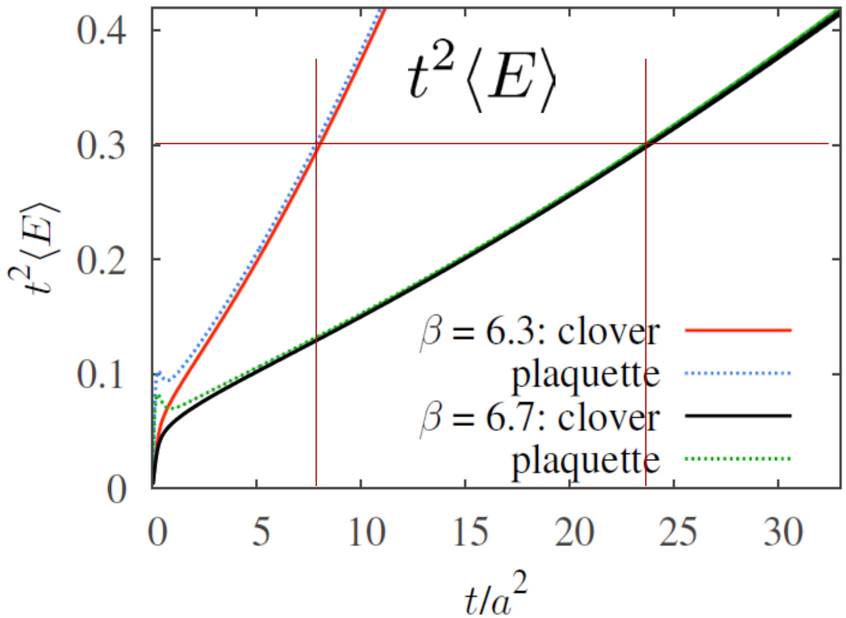
$$E = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

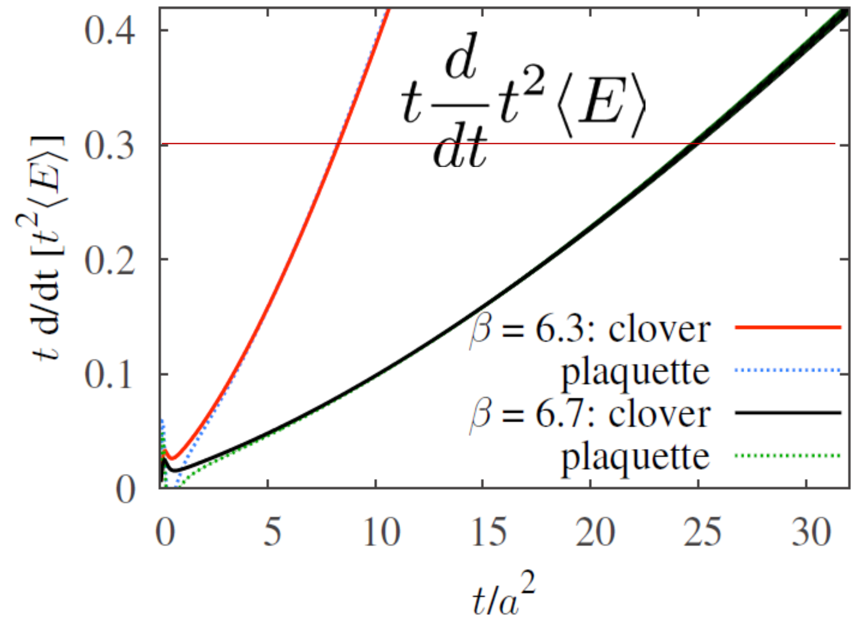
Behavior of t^2<E>



$$t_0\ :\ t^2\langle E\rangle|_{t=t_0} = 0.3$$

Behavior of t²<E>



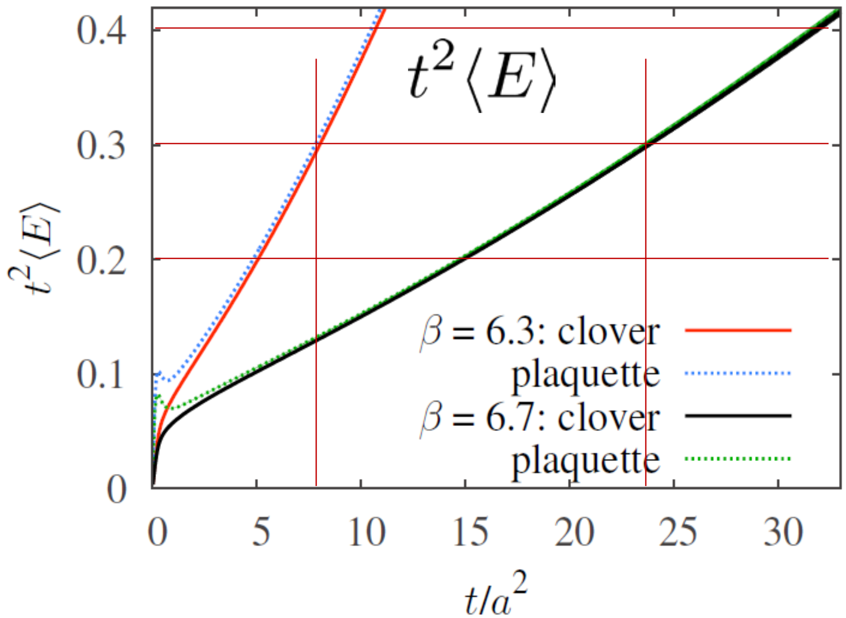


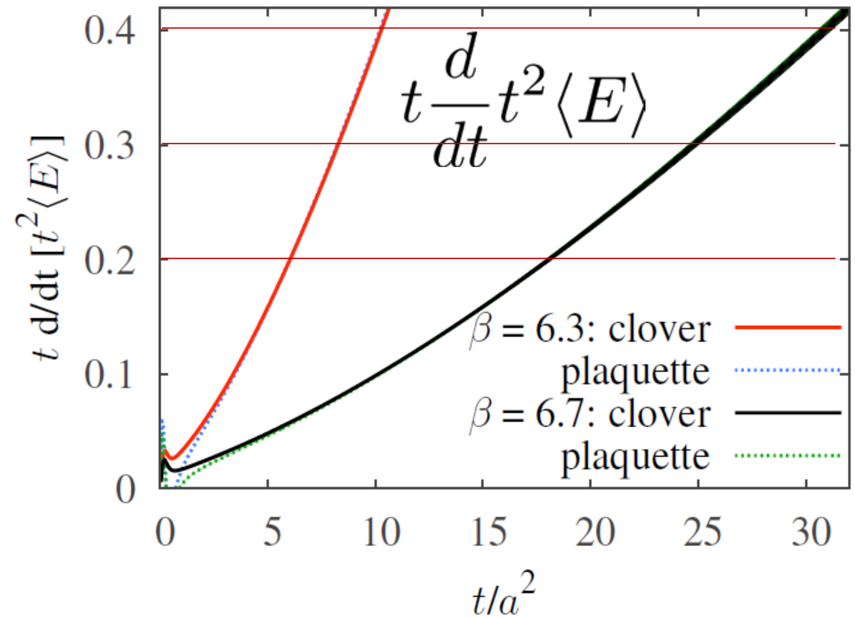
$$t_0 \; : \; t^2\langle E\rangle|_{t=t_0} = 0.3$$

$$w_0 \; : \; t\frac{d}{dt}t^2\langle E\rangle|_{t=w_0^2} = 0.3$$

Advangates:
- Less statistical errors
- No fit analyses
- No measurement of non-local obs.

Behavior of t^2<E>





$$t_0 \ : \ t^2\langle E\rangle|_{t=t_0} = 0.3$$

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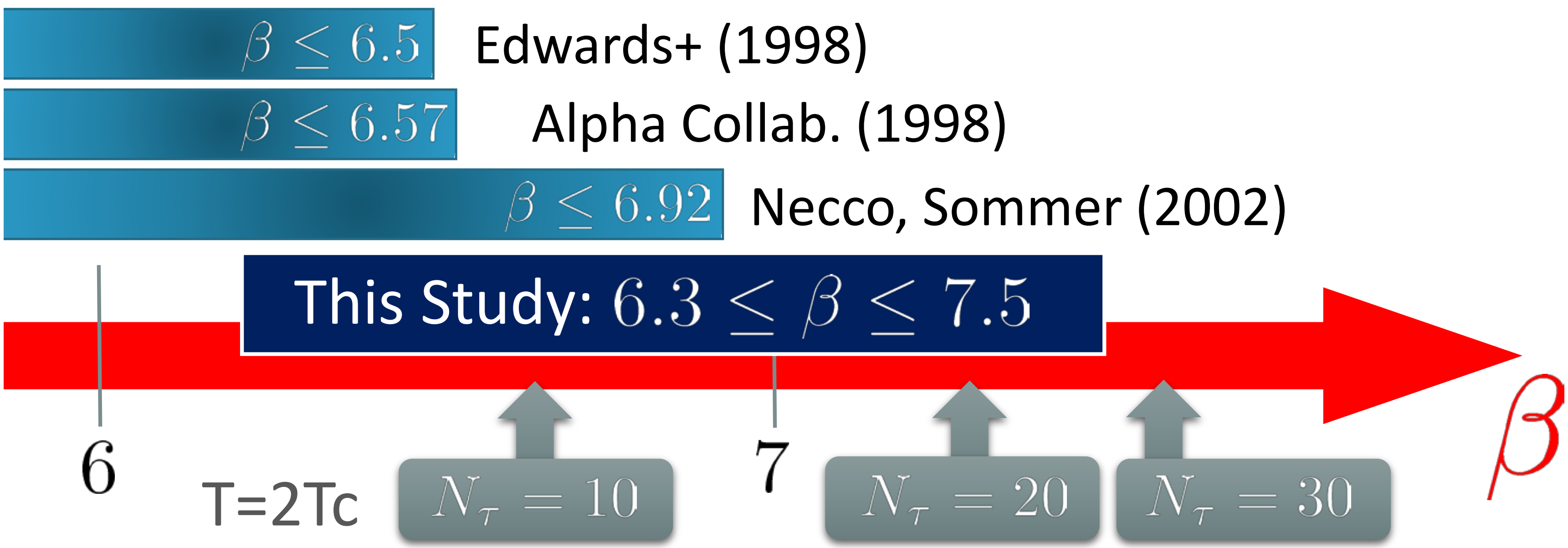
Our Choice

value in rhs is arbitrary

$$t_{0.2}, \ t_{0.3}, \ t_{0.4}$$

$$w_{0.2}, \ w_{0.3}, \ w_{0.4}$$

Lattice Spacing of SU(3) Wilson Action

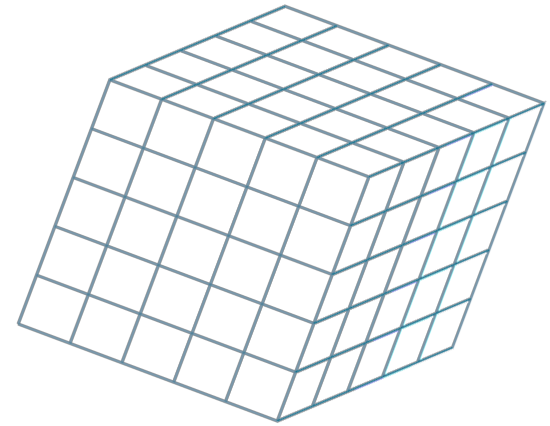


our parametrization for *a*

$$\log\left(\frac{w_{0.4}}{a}\right)(\beta) = \frac{4\pi^2}{33}\beta - 8.6853 + \frac{37.422}{\beta} - \frac{143.84}{\beta^2}$$

stat. err. < 0.4% / sys. err. < 0.7%

Numerical Setting



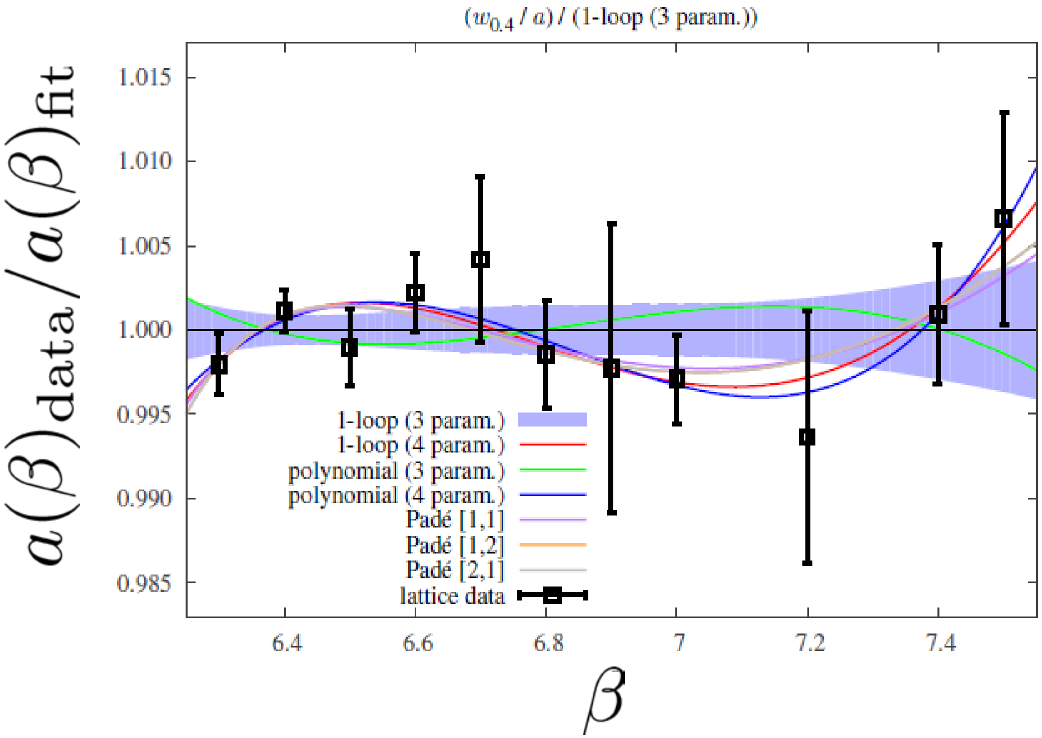
- SU(3) YM theory
- Wilson gauge action
- $w_{0.4}$ / $w_{0.2}$ scaling

b	size	N_{conf}	b	size	N_{conf}
6.3	64^4	30	6.9	64^4	30
6.4	64^4	100	7.0	96^4	60
6.5	64^4	49	7.2	96^4	53
6.6	64^4	100	7.4	128^4	40
6.7	64^4	30	7.5	128^4	60
6.8	64^4	100			

Each configuration is separated by 1000 updates (HB+OR^5)

BlueGene/Q @ KEK

Parametrization with $w_{0.4}$



- Use various fitting funcs
  → estimate sys. err.
- For β=7.4 and 7.5, $w_{0.4}$ is obtained from $w_{0.2}$.

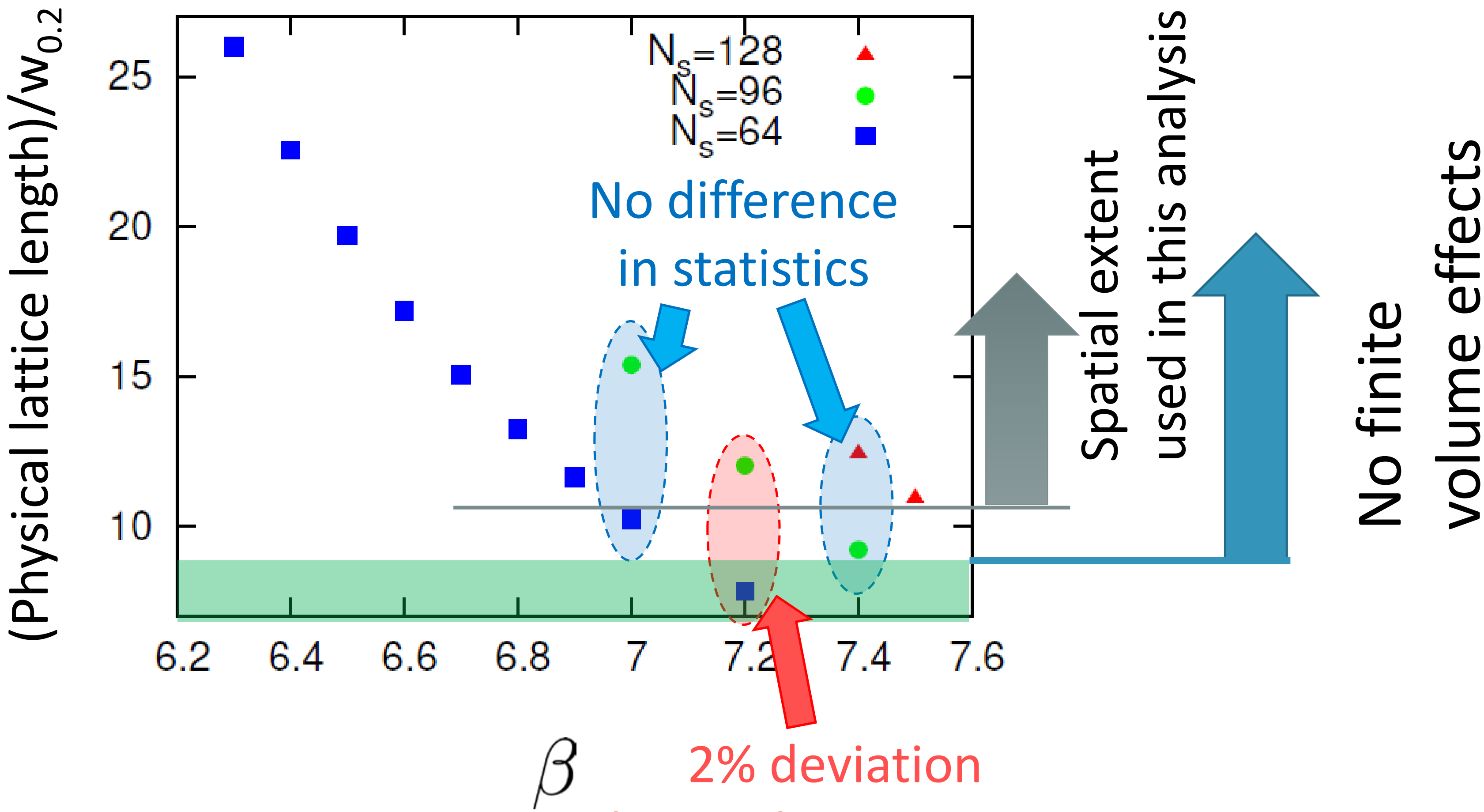
$$\log\left(\frac{w_{0.4}}{a}\right)(\beta) = \frac{4\pi^2}{33}\beta - 8.6853 + \frac{37.422}{\beta} - \frac{143.84}{\beta^2}$$

stat. err. < 0.4% / sys. err. < 0.7%

3 parameter fit

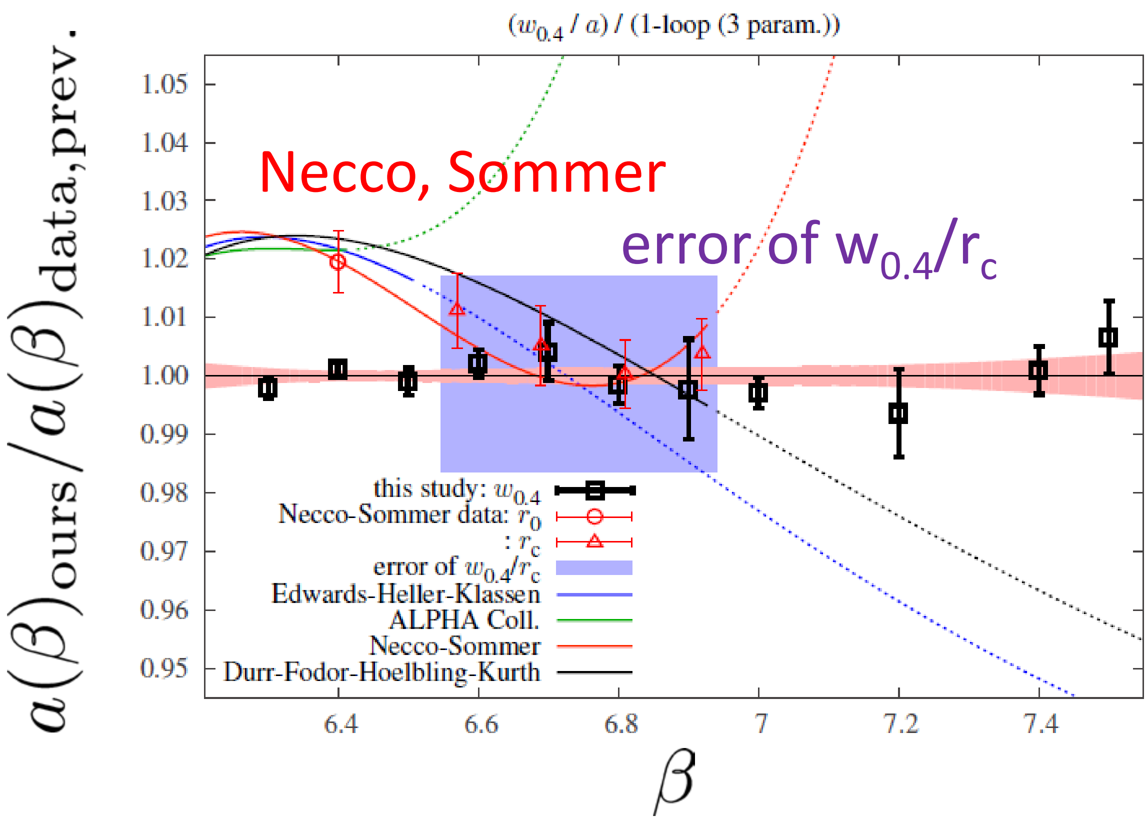
$\chi^2/\mathrm{dof} = 0.92$

Finite Volume Effects



No finite volume effects within statistics

Comparison with Previous Studies



Edwards, Heller, Klassen, 1998
Alpha-Collaboration, 1998
Necco, Sommer, 2002
Durr, Fodor, Hoelbling, 2007

our parametrization by $w_{0.4}$

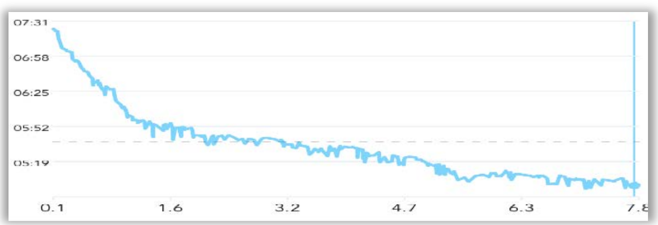
- Parametrizations agree with each other in available ranges.
- Errorbars of our data is smaller than previous ones.

Thermodynamics

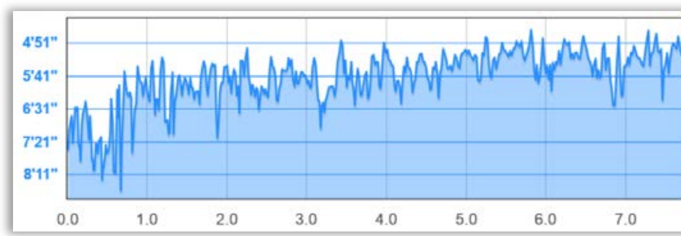
Basic Idea: Small Flow Time Expansion

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

An operator
in blurred world



Renormalized operators
in original theory



less noisy, continuous

noisy, lattice discretized

SFTE of Energy-Momentum Tensor

Suzuki, 2013

- gauge-invariant dimension 4 operators

$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

Remormalized EMT

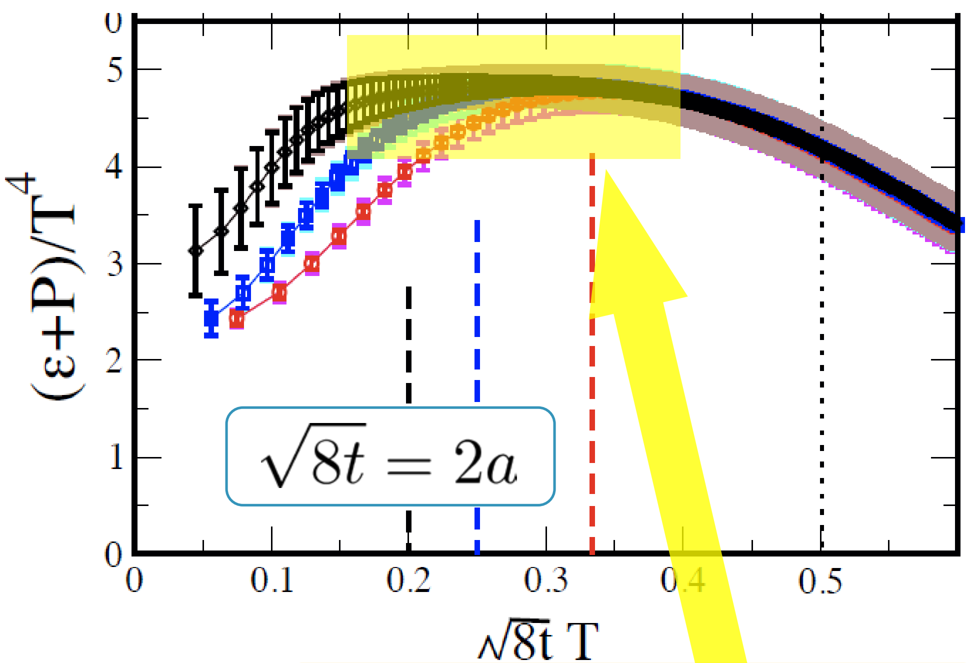
$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\mathrm{subt.}}\right]$$

Suzuki coeffs.
$$\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \frac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases} \qquad g = g(\sqrt{8t})$$

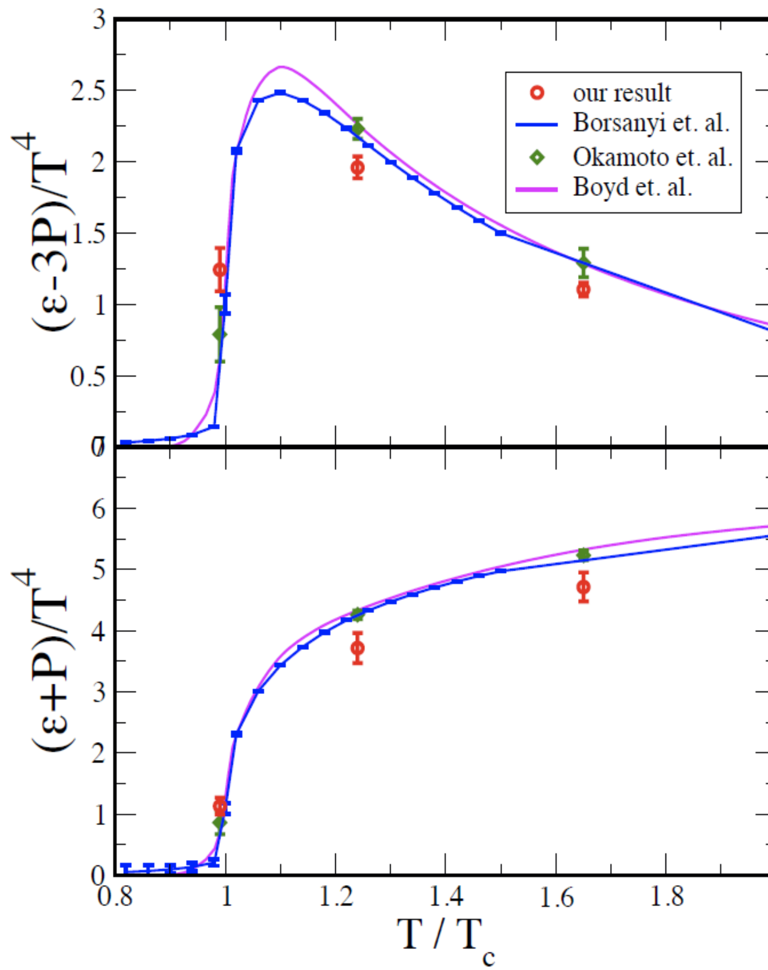
SU(3) Thermodynamics with Nt=6, 8, 10

FlowQCD, PRD90,011501 (2014)

Entropy density T=1.65Tc



Stable region for
$$2a < \sqrt{8t} < 0.4T^{-1}$$

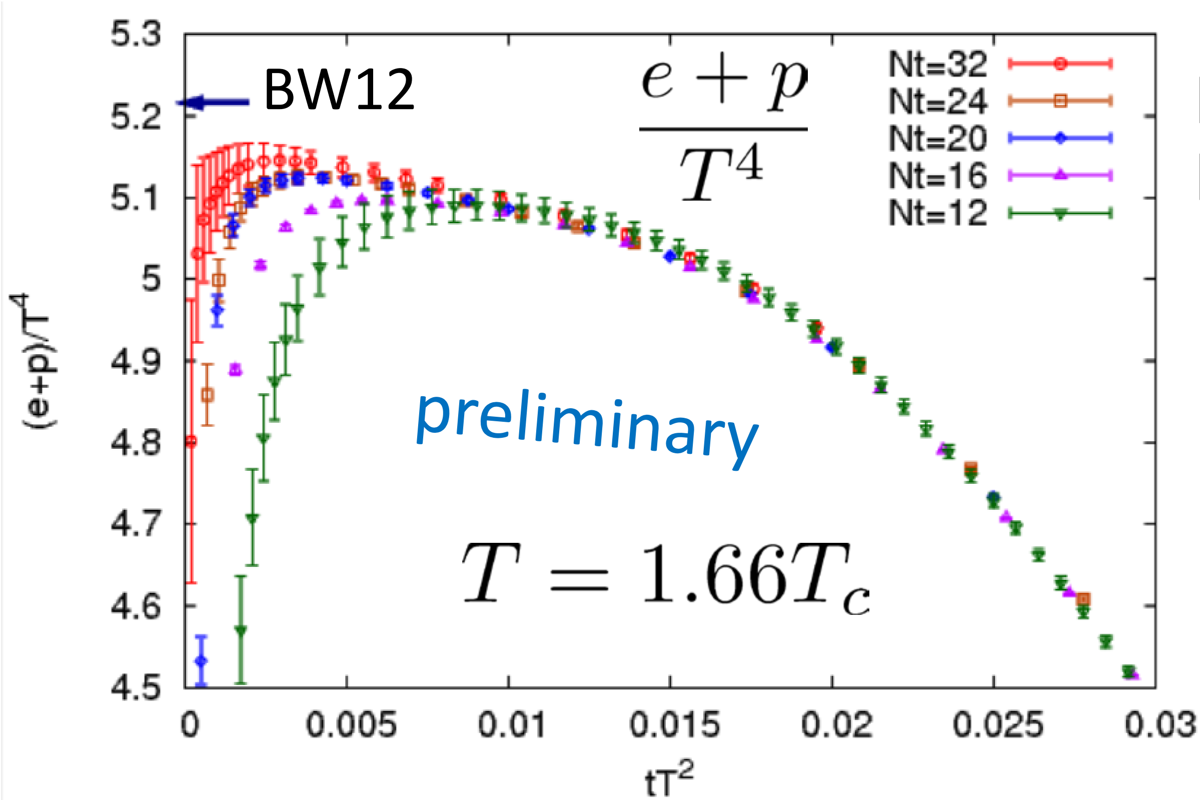


New Results: Thermodynamics (e+p)

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\mathrm{subt.}}$$

FlowQCD, in prep.

$$T^R_{\mu\nu} = \tilde{T}_{\mu\nu}(t) + O(t)$$



- ☐ Existence of O(t) effect
- ☐ Linear behavior for

$$tT^2 < 0.015$$
$$(\sqrt{8t} < 0.35T^{-1})$$

- t→0 limit is necessary

BW12:Budapest-Wuppertal, 2012

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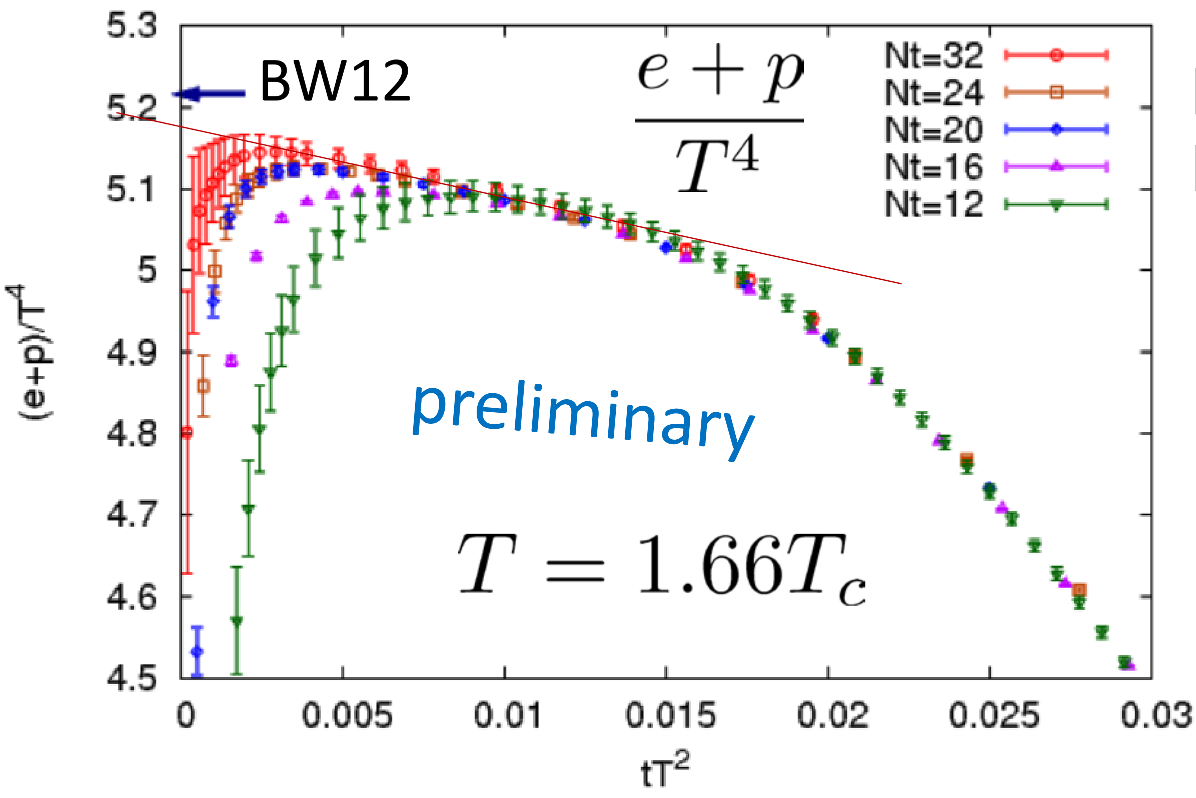

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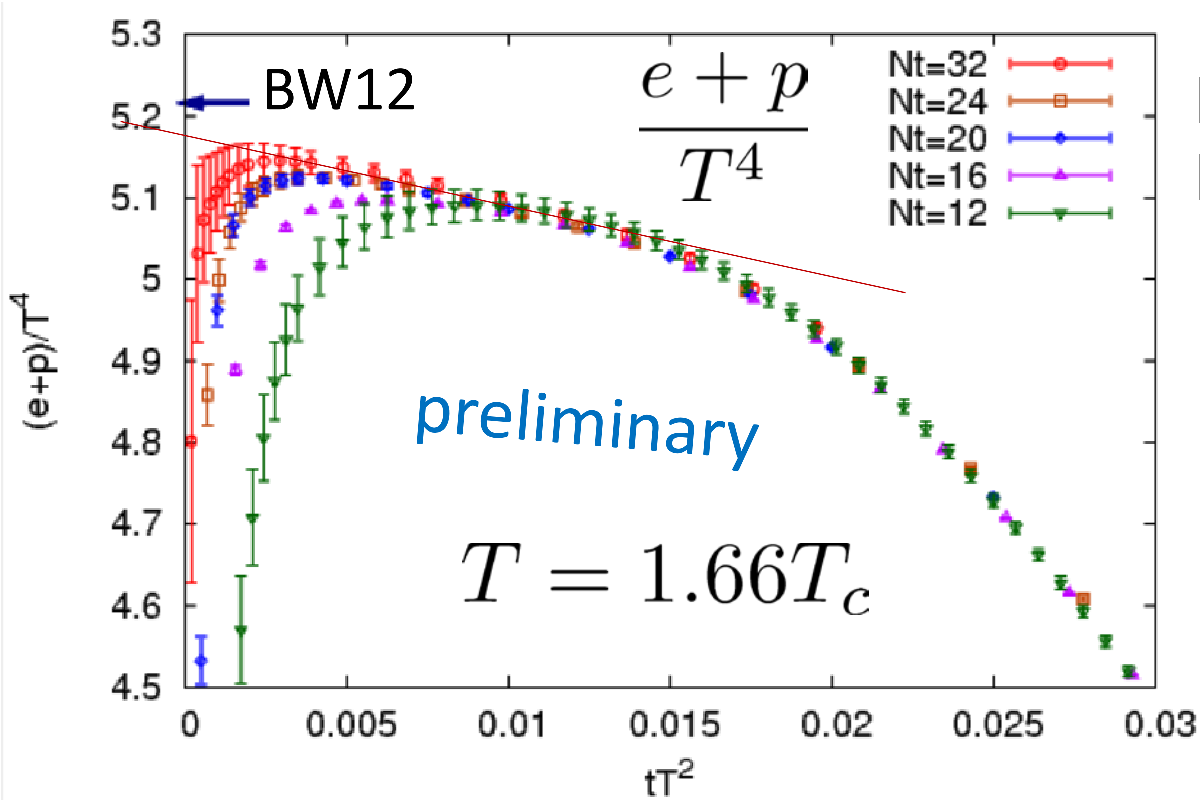


- ☐ Existence of O(t) effect
- ☐ Linear behavior for

  $tT^2 < 0.015$

  $(\sqrt{8t} < 0.35T^{-1})$

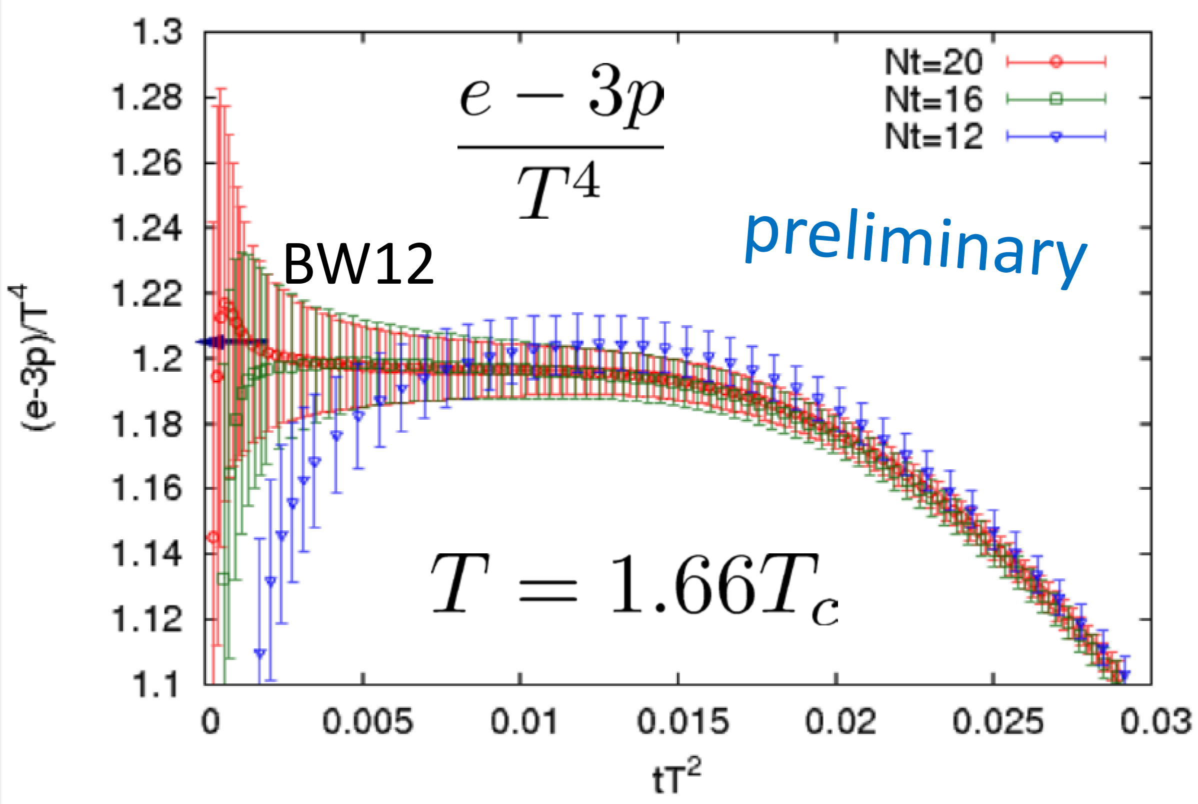
- t→0 limit is necessary

BW12:Budapest-Wuppertal, 2012

New Results: Thermodynamics (e-3p)

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\text{subt.}}$$

FlowQCD, in prep.



- Good agreement with previous study
- Stable plateau for

$$tT^2 < 0.015$$
$$(\sqrt{8t} < 0.35T^{-1})$$

BW12: Budapest-Wuppertal, 2012

Summary

Accurate lattice spacing of Wilson gauge action for 6.3<β<7.5 is now available.

$$\log\left(\frac{w_{0.4}}{a}\right)(\beta) = \frac{4\pi^2}{33}\beta - 8.6853 + \frac{37.704}{\beta} - \frac{144.77}{\beta^2}$$

The EMT defined with gradient flow is nicely applied to the measurement of thermodynamics.
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- taking t→0 limit is needed.
- application to full QCD → E. Itou, 18th, room 403

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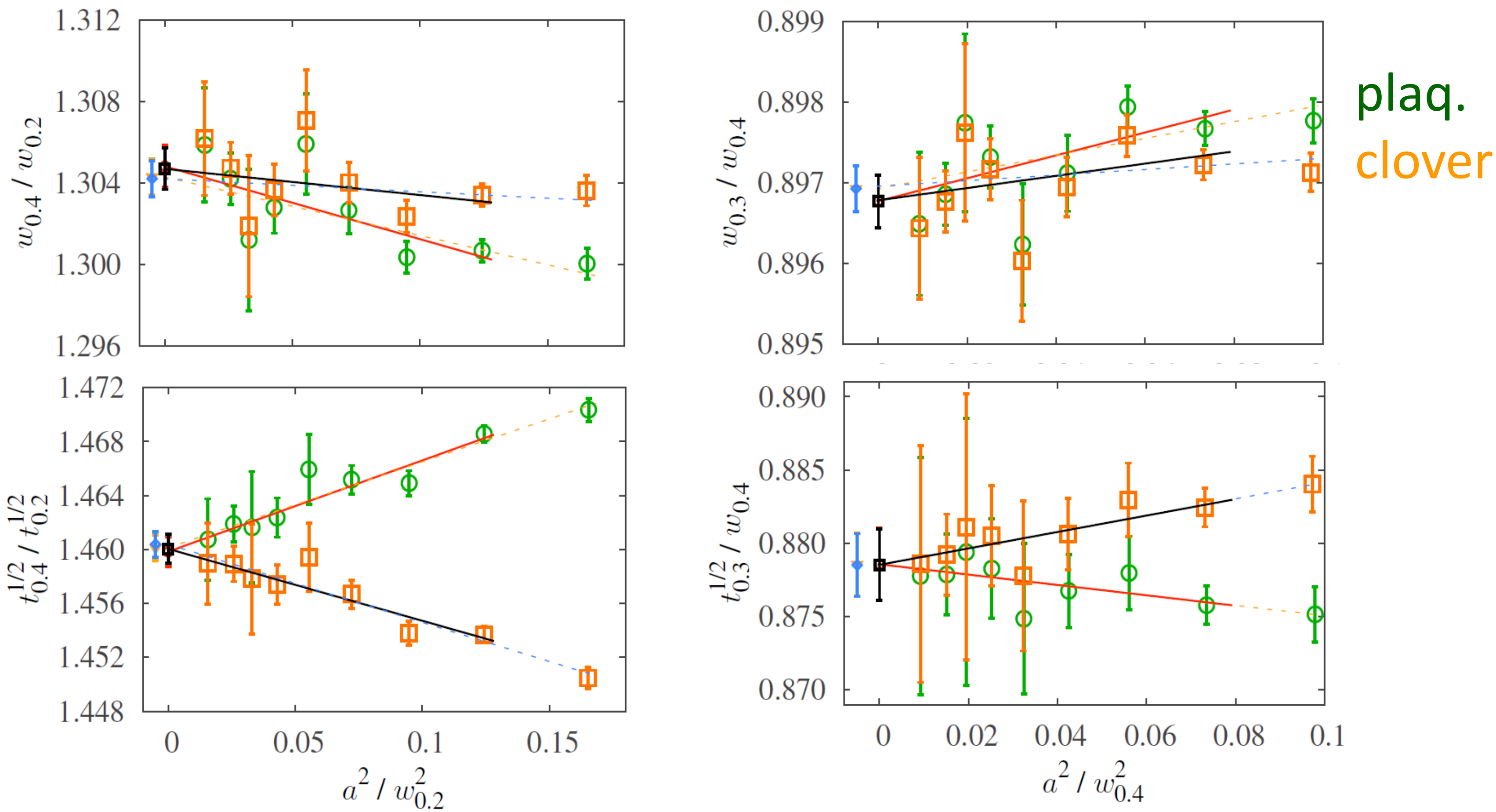
Backup

Various Reference Scales



$\sqrt{t_{0.4}}/w_{0.4}$	$\sqrt{t_{0.3}}/w_{0.4}$	$\sqrt{t_{0.2}}/w_{0.4}$	$w_{0.3}/w_{0.4}$	$w_{0.2}/w_{0.4}$
1.0164(32)(3)	0.8785(24)(0)	0.6952(18)(2)	0.8968(3)(2)	0.7665(6)(2)

$r_c/w_{0.4}$	$r_0/w_{0.4}$	$\sqrt{\sigma}w_{0.4}$	$T_c w_{0.4}$	$w_{0.4}\Lambda_{\overline{\rm MS}}$
1.328(21)(7)	2.587(45)	0.455(8)	0.285(5)	0.233(19)