Cluster expansions and chiral symmetry at large density in 2-color QCDC

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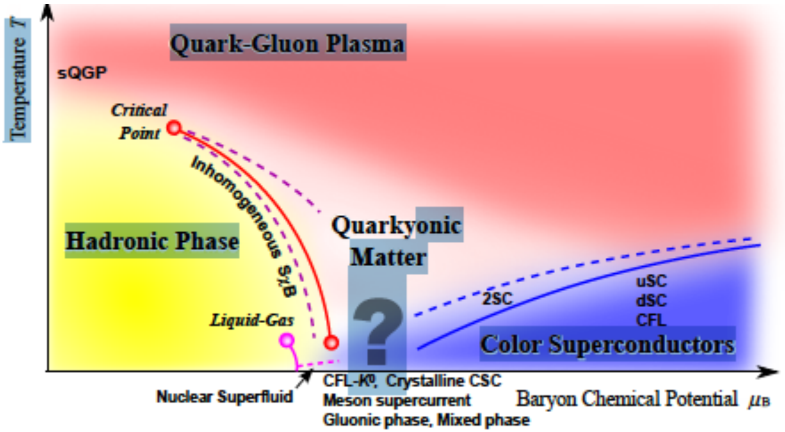
Typical conjectured QCD phase diagram



LGT strong coupling

- $\beta = 0$ - monomer-dimer-polymer (baryon loop) representations of PF $\Rightarrow$ alleviates sign problem - simulations possible: $SU(3), SU(2)$: (Dagoto, Moreo, Wolff; Karsch, Klaetke, Mütter; Chandrasekharan; Fromm, de Forcrand, Philipsen,Unger, ...)
- strong coupling hopping expansion (heavy quark) (Philipsen et al).
- $0 \leq$ small β - Truncated $1/d$ expansion and mean-field analysis: $SU(3), SU(2), SU(N)$: (Nishida, Kawamoto, Miura, Ohnishi, Ohmura, Nakano, ...)
- $0 \leq$ small β Complete cluster expansion for large μ(Tomboulis)

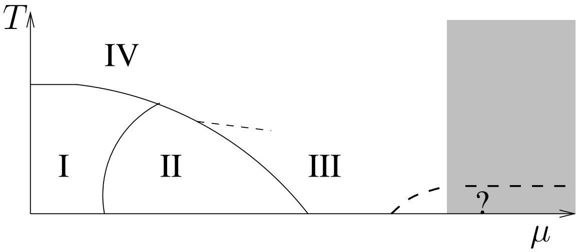
Any N_c: Phase-diagrams - large μ to chirally restored phase

Fundamental rep. staggered fermions $\Rightarrow$ Real determinant

Enhanced global symmetry at $\mu = 0$:

$$U(N_f) \times U(N_f) \longrightarrow U(2N_f) \xrightarrow{SB} O(2N_f).$$

- Strong-coupling: dimer, mean-field (Daggoto, Moreo, Wolff; Klaetke, Mütter; Chandrasekharan, Jiang; Nishida, Fukushima, Hatsuda, ...)
- Full scale simulations at weak(er) coupling (Hands, Kogut, ...; Cotter, Giudice, Hands, Skullerud, ...)



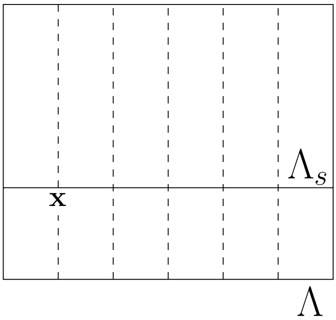
$$\mu = 0 : \quad N_f = 1, \qquad U(2) \xrightarrow{SB} U(1)$$

$SU(N_c)$: general set-up for small β, large μ expansions

A nonvanishing chemical potential μ (or T) introduces an anisotropy between the spacelike and timelike directions.

It can be exploited to set up expansion schemes for large μ:

Fermion spacelike hopping expansion in the measure provided by the timelike fermion action and gauge fields.



Explicit evaluation of timelike propagator (Polyakov gauge) :

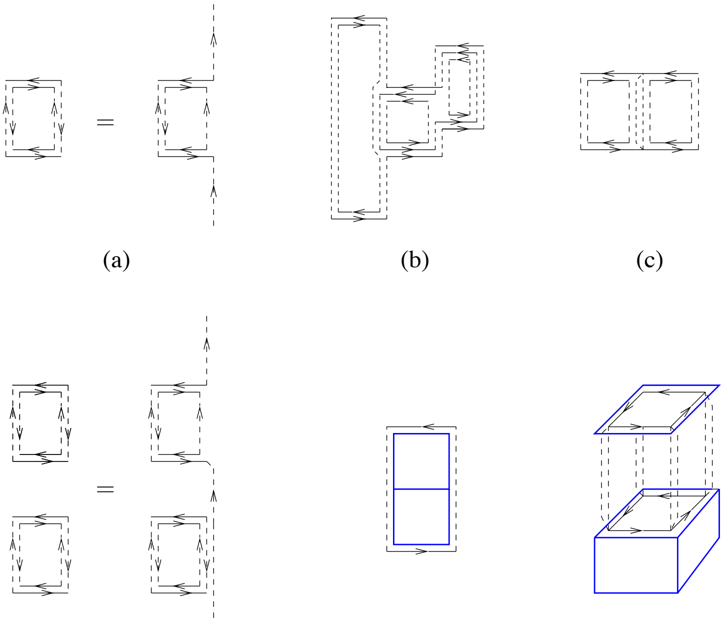
$$C(\tau-\tau',\Theta(\mathbf{x}))_{ai,bj} = \delta_{ab}\delta_{ij}[1-(-1)^{(\tau-\tau')}] \cdot \frac{e^{-i\theta_a(\mathbf{x})(\tau-\tau')/L}}{1+e^{-i\theta_a(\mathbf{x})}e^{-\mu L}}\, e^{-\mu(\tau-\tau')}$$

for $(\tau-\tau')>0, \quad \mu>0$

$$C(\tau-\tau',\Theta(\mathbf{x}))_{ai,bj} = -\delta_{ab}\delta_{ij}[1-(-1)^{|\tau-\tau'|}] \cdot \frac{e^{-i\theta_a(\mathbf{x})[1-|\tau-\tau'|/L]}}{1+e^{-i\theta_a(\mathbf{x})}e^{-\mu L}}\, e^{-\mu[L-|\tau-\tau'|]}$$

for $(\tau-\tau')<0, \quad \mu>0$.

$$U_0(\tau,\mathbf{x}) = \mathrm{diag}(e^{i\theta_1(\mathbf{x})/L}, e^{i\theta_2(\mathbf{x})/L}, \cdots, e^{i\theta_{N_c}(\mathbf{x})/L}) \equiv \exp(i\Theta(\mathbf{x})/L)$$



(a) (b) (c)

(d) (e) (f)

- Convergence for sufficiently large μ and sufficiently small β. – any quark mass.
  (TT, J. Math. Phys. 54,122301 (2013); review: Int. J. Mod. Phys. A, 29, 1445004 (2014))
- Convergence region in μ - κ (hopping parameter), i.e. smaller μ for heavier quark mass.

Consequence of convergence

Expectation of any local chirally non-invariant fermion operator $\mathscr{O}(x)$, e.g., $\bar{\psi}(x)\psi(x)$ or diquark operators, vanishes identically term by term in the expansion by the invariance of the measure.

Correlation functions $\langle \mathscr{O}(x)\mathscr{O}(y) \rangle$ then receive nonvanishing contributions arise only from polymers intersecting both sites x and y.

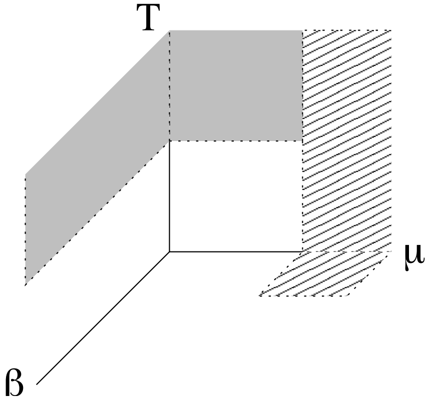
A straightforward consequence of this fact is that:

$$\left| \langle \mathscr{O}(x)\mathscr{O}(y) \rangle \right| < \mathrm{C}_0 \mathrm{C}^{-|x-y|} ,$$

where C_0, C are space dimension-dependent constants and $|x-y|$ is the minimum number of bonds connecting the two sites.

In other words, there is clustering of 2-point (and all higher) correlations: global $U(N_f) \times U(N_f)$ symmetry intact for sufficiently large μ, small β.

Resulting picture - Strong coupling regime



Picture essentially that of 'quarkyonic' matter with restored chiral symmetry at high μ (as surmised by large N_c arguments (Pisarsky and Mc Lerran)).

Arbitrary coupling

Can we extend these expansion setup to other value regimes of the gauge coupling?

Write the expectation of general fermionic operator $O[\bar{\psi}, \psi]$ in the form:

$$\langle O \rangle = \int d\nu(U) \langle O \rangle_F(U) ,$$

where

$$\langle O \rangle_F(U) = \frac{1}{Z_F(U)} \int D\bar{\psi} D\psi \, e^{S_F(U)} O[\bar{\psi}, \psi] ,$$

$$Z = \int DU \, e^{S_g(U)} Z_F(U) , \qquad Z_F(U) = \int D\bar{\psi} D\psi \, e^{S_F(U)} = \mathrm{Det}\mathbf{M}(U)$$

$$\text{and} \qquad d\nu(U) \equiv \frac{dU}{Z} e^{S_g(U)} \, \mathrm{Det}\mathbf{M}(U)$$

is the (normalized) full effective gauge field measure at coupling β.

One may expand the fermion expectation $\langle O \rangle_F(U)$ in a given background gauge field configuration U in the same type of expansion as above.

Now this expansion for $\langle O \rangle_F(U)$, in generic gauge field background U and for operators O of bounded support and spatial dimension $d \geq 1$, *converges absolutely, and uniformly in the spatial lattice size, at any temperature T for sufficiently large μ*. Result holds for any choice of N_c, N_f.

The proof, and associated estimates, proceed as in the strong coupling case above except that it is actually *simpler* since no integration over the gauge field is involved engendering additional connectivity among diagrams.

Expansion of $\langle O \rangle_F(U)$ leads to expansion of

$$\langle O \rangle = \int d\nu(U) \langle O \rangle_F(U) \,.$$

Does this expansion converge?

Convergence of $\langle O \rangle_F(U)$ expansion implies

$$|\langle O \rangle_F(U)| < C_O \,,$$

C_O is an operator- and spacetime dimension-dependent constant, but independent of the background U and the spatial lattice volume.

So

$$|\langle O \rangle| = \int d\nu(U) \, |\langle O \rangle_F(U)| < C_O \,.$$

provided $d\nu(U)$ is real positive

Real fermion determinant theories

$d\nu(U)$ is real positive if the fermion determinant $\mathrm{Det}\mathbf{M}(U)$ is real positive.

The fermion determinant is known to be real positive for:

- $N_c = 2$ and fundamental rep. fermions
- any N_c, even N_f and adjoint fermions

Usual chiral condensate order parameter:
$O_{\bar{q}q} = \bar{\psi}(x)\psi(x)$.

$N_c = 2$ fundamental fermions diquark condensate:
$O_{qq} = \frac{1}{2}\left[\psi^T(x)\tau_2\psi(x) + \bar{\psi}(x)\tau_2\bar{\psi}^T(x)\right]$

$SU(N_c)$ Adjoint fermions diquark condensate:

$$O_{qq}^{\mathrm{ad}} = \frac{1}{2}\varepsilon^{ij\ldots kl}\left[\psi^{k\,T}(x)\psi^l(x) + \bar{\psi}^k(x)\bar{\psi}^{l\,T}(x)\right]$$

$U(N_f) \times U(N_f) \longrightarrow SU(2)$ isospin; in particular, condensate breaks $U_B(1)$. Such breaking of the global symmetries engenders superfluidity.

Color symmetry breaking condensate:

$$O_{qq}^a = \frac{1}{2}\left[\psi^T(x)t^a\psi(x) + \bar{\psi}(x)t^a\bar{\psi}^T(x)\right] .$$

Transforms in the adjoint representation of $SU(N_c)$, i.e. as a composite adjoint Higgs field. Breaks color symmetry, as well as global chiral symmetries $\longrightarrow$ color superconductivity.

But we just saw that expansion converges for sufficiently large μ.

$\Rightarrow$ Global $SU(N_f) \times U(N_f)$ symmetries preserved.

None of the above condensates form at large μ:
No Superfluidity/Superconductivity at large μ in theories with real fermion determinant

Is this a lattice artifact? Lattice saturation sets in at large μ (low T)

Possible remedy: partition spatial lattice into cubes of fixed size and "thin-out" quark μ by introducing antiquark $\mu_0 < 0$ on a fraction of the timeline bonds; adjust $\mu/|\mu_0|$ in large volume limit.

Conclusions

- Cluster expansions converging for large μ can be constructed in the strong coupling regime.

  In $SU(N_c)$ LGT with staggered fermion these expansions show recovery of the global $U(N_f) \times U(N_f)$ symmetries at large μ.

  This results in a 'quarkyonic' matter picture.

  If superfluidity/superconductivity involving breaking of these symmetries actually realized, the LGT must exhibit a phase transition as the gauge coupling is lowered.

- In theories with real fermion determinant the 'quarkyonic' matter picture can be extended to all coupling.
- Is this large μ picture a lattice artifact? - Lattice saturation at large μ (and low T) serious problem to be overcome.