Lattice study for conformal windows of SU(2) and SU(3) gauge theories with fundamental (unimproved staggered) fermions

C.-J. David Lin
National Chiao-Tung University, Taiwan

Lattice 2015
Kobe, Japan

Motivation

- Controversy over the SU(3), 12-flavour theory.

➡ Control of the systematics.

- The SU(2), 6-flavour theory is chirally-broken.

➡ Next natural thing to study is the 8-flavour theory.

Running coupling and linearised beta function in SU(3) gauge theory with 12 flavours:

Gradient flow scheme with twisted BC and massless fermions

C.-J.D. Lin (NCTU)

K. Ogawa (NCTU → industry)

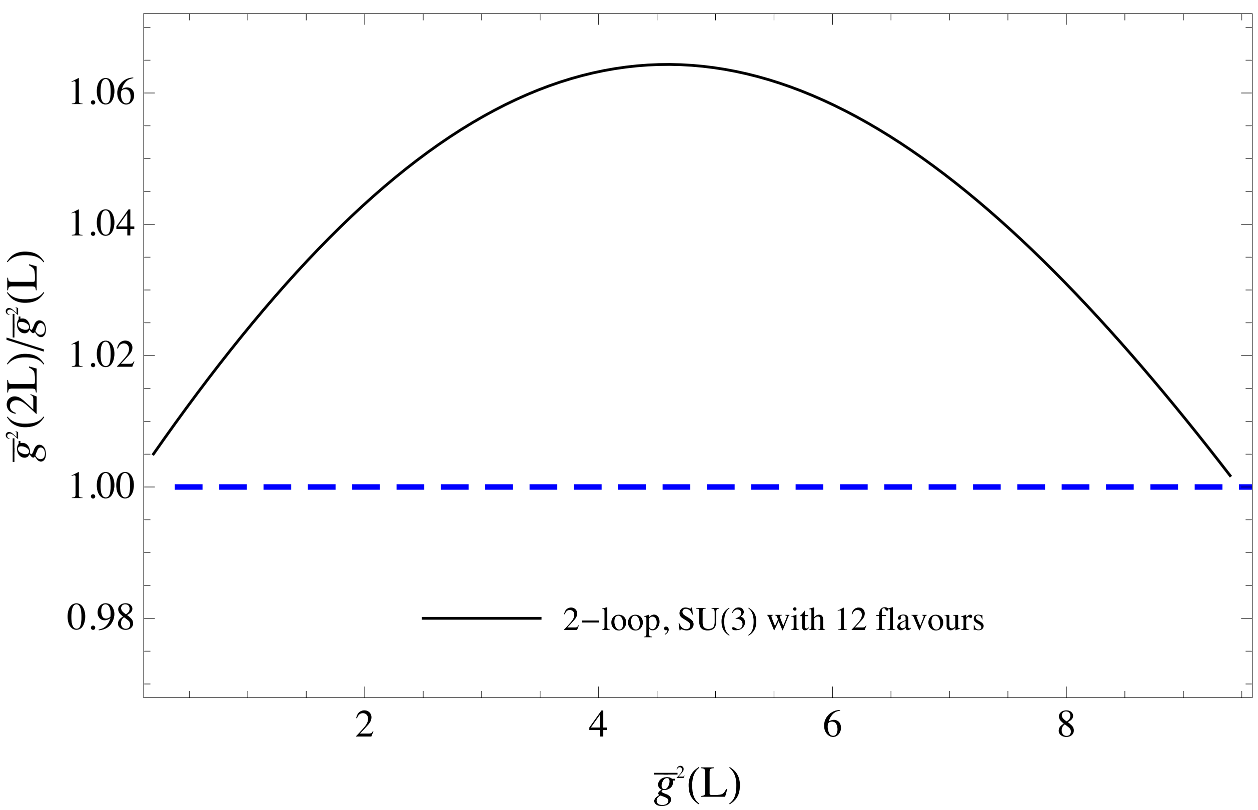
H. Ohki (KMI → RIKEN)

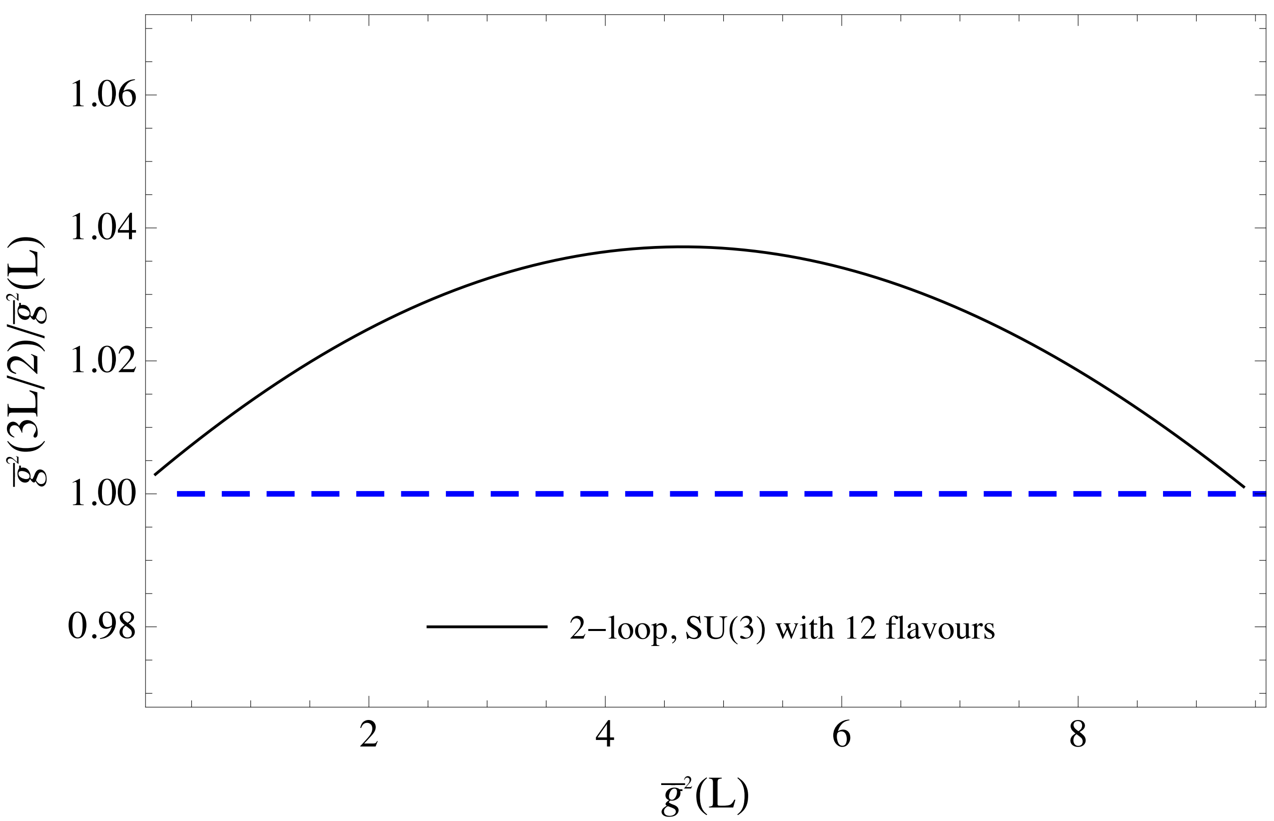
A. Ramos (CERN)

E. Shintani (Mainz U. → RIKEN)

How well do we have to control the errors?

In order to demonstrate the flow from the UV to the IR fixed points...





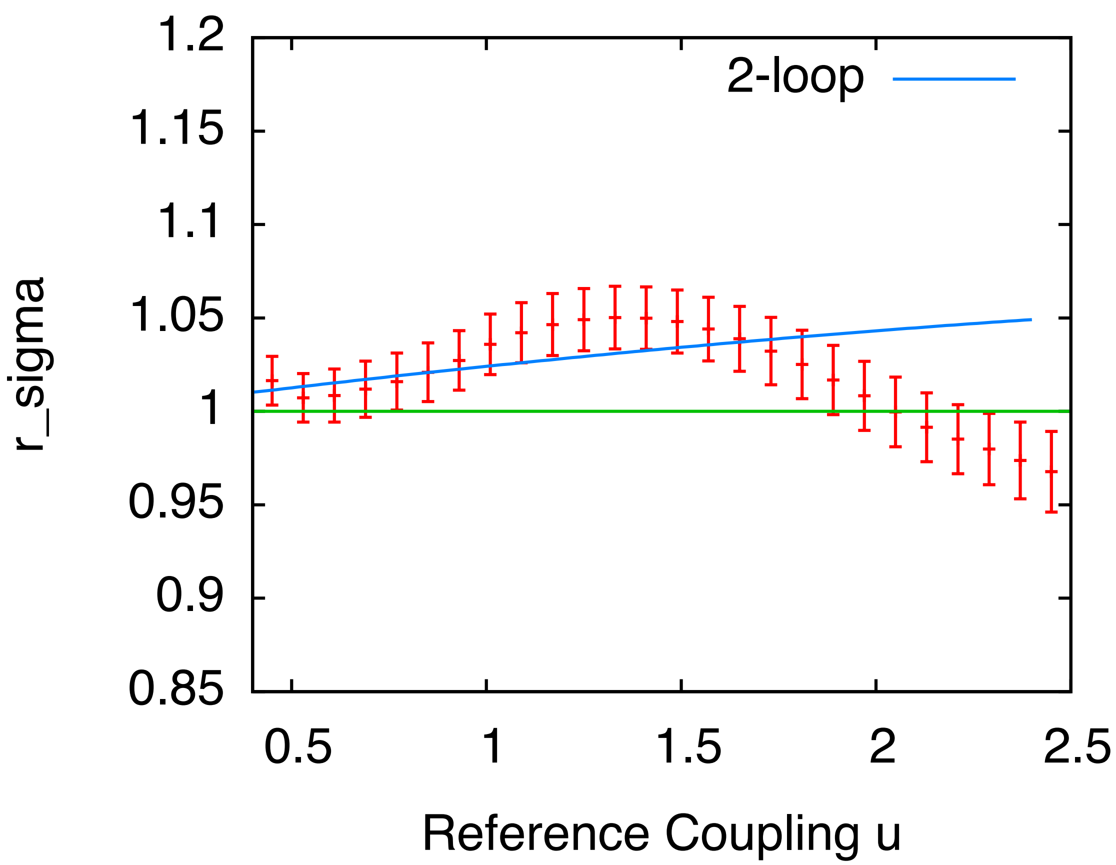
~1% ~0.6%

➡ It is important to choose good observables and strategies.

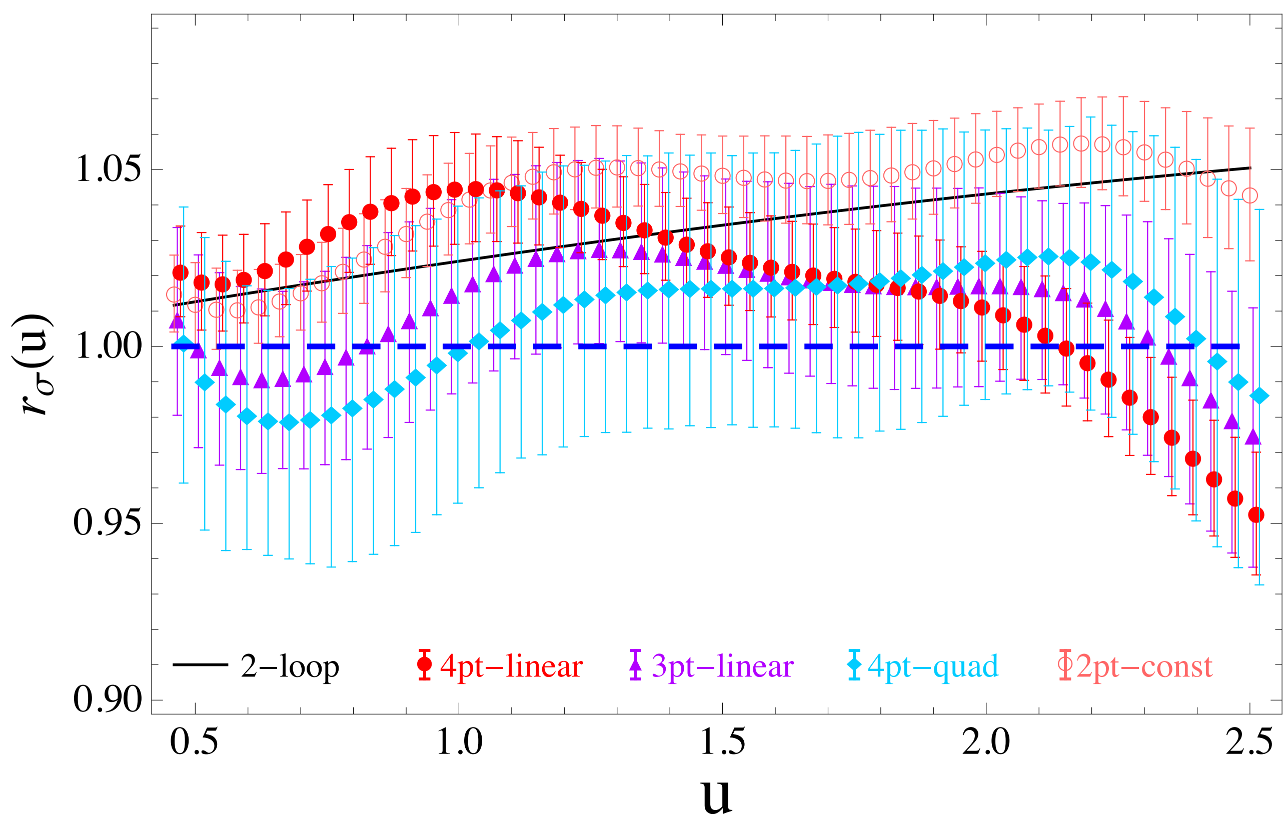
Example: The use of the Twisted Polyakov Loop scheme

C.-J.D.L., K.Ogawa, H.Ohki, E.Shintani, 2012

K.Ogawa, lattice 2013



without (L/a=12 ⟶ L/a = 24)
systematics severely underestimated...



with (L/a=12 ⟶ L/a=24)

It is challenging to draw any conclusion from such a "noisy scheme".

The Gradient Flow scheme

- The quantity, $\langle E(t)\rangle = \frac{1}{4}\langle G_{\mu\nu}(t)G_{\mu\nu}(t)\rangle$, is finite when expressed in terms of renormalised coupling at positive flow time.

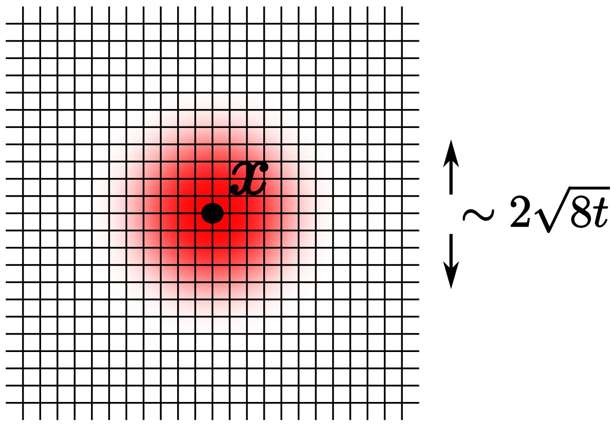
- In a colour-twisted box, can define,

$$\overline{g}^2_{\rm GF}(L) = \mathcal{N}^{-1}t^2\langle E(t)\rangle = \overline{g}^2_{\rm MS} + \mathcal{O}(\overline{g}^4_{\rm MS}),$$

  with tree-level improvement.

- Use the clover operator, as well as the plaquette, to extract $\langle E(t)\rangle$.

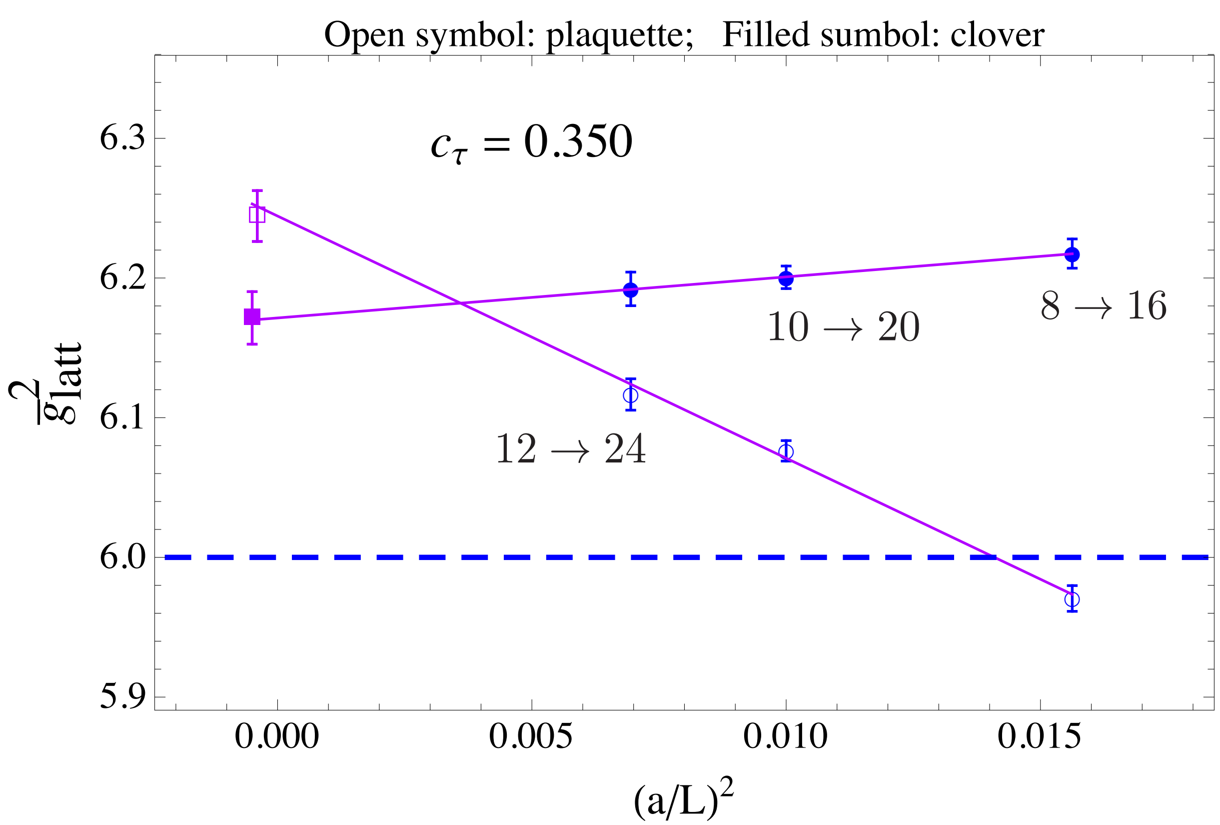
- Step scaling at fixed $c_\tau = \frac{\sqrt{8t}}{L}$.

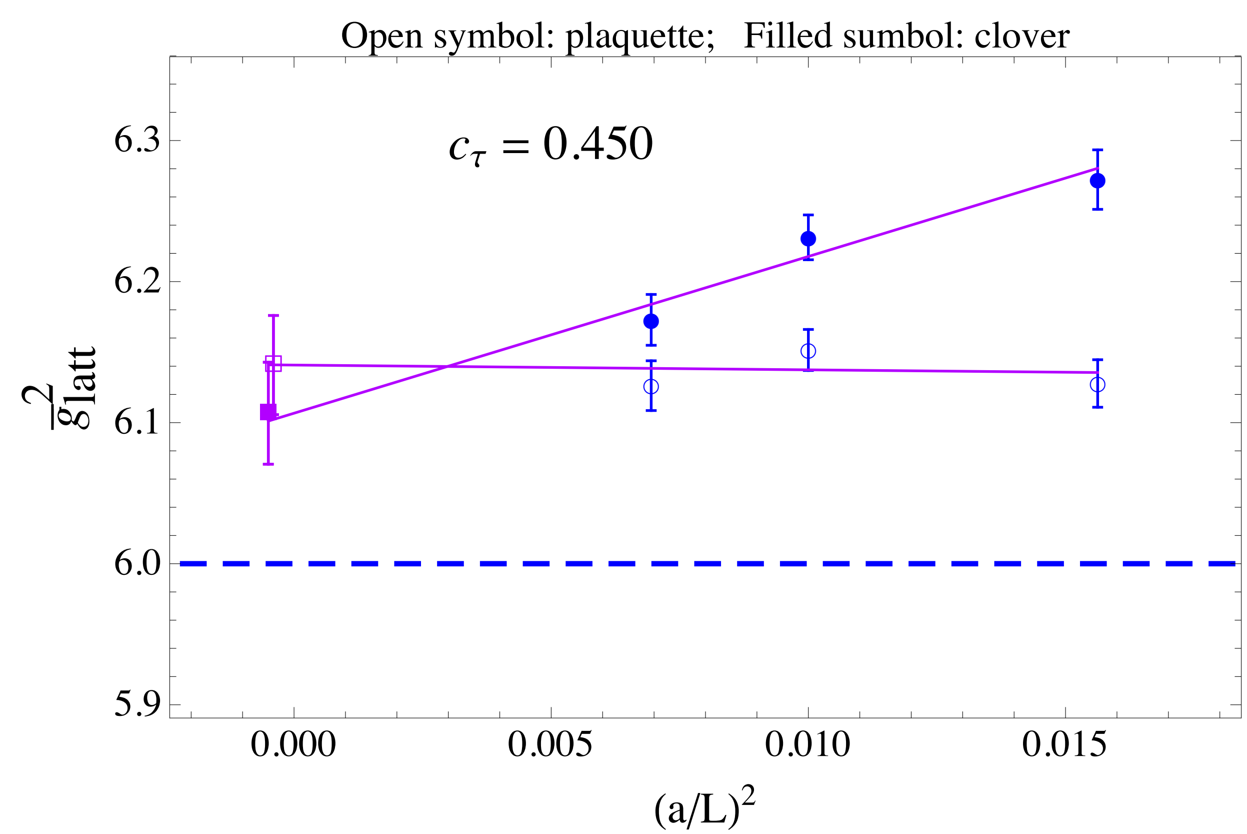


Caution for the continuum extrapolation

Lattice artefacts arise from the action, the observable, and the flow.

→ Should improve all of these in principle.

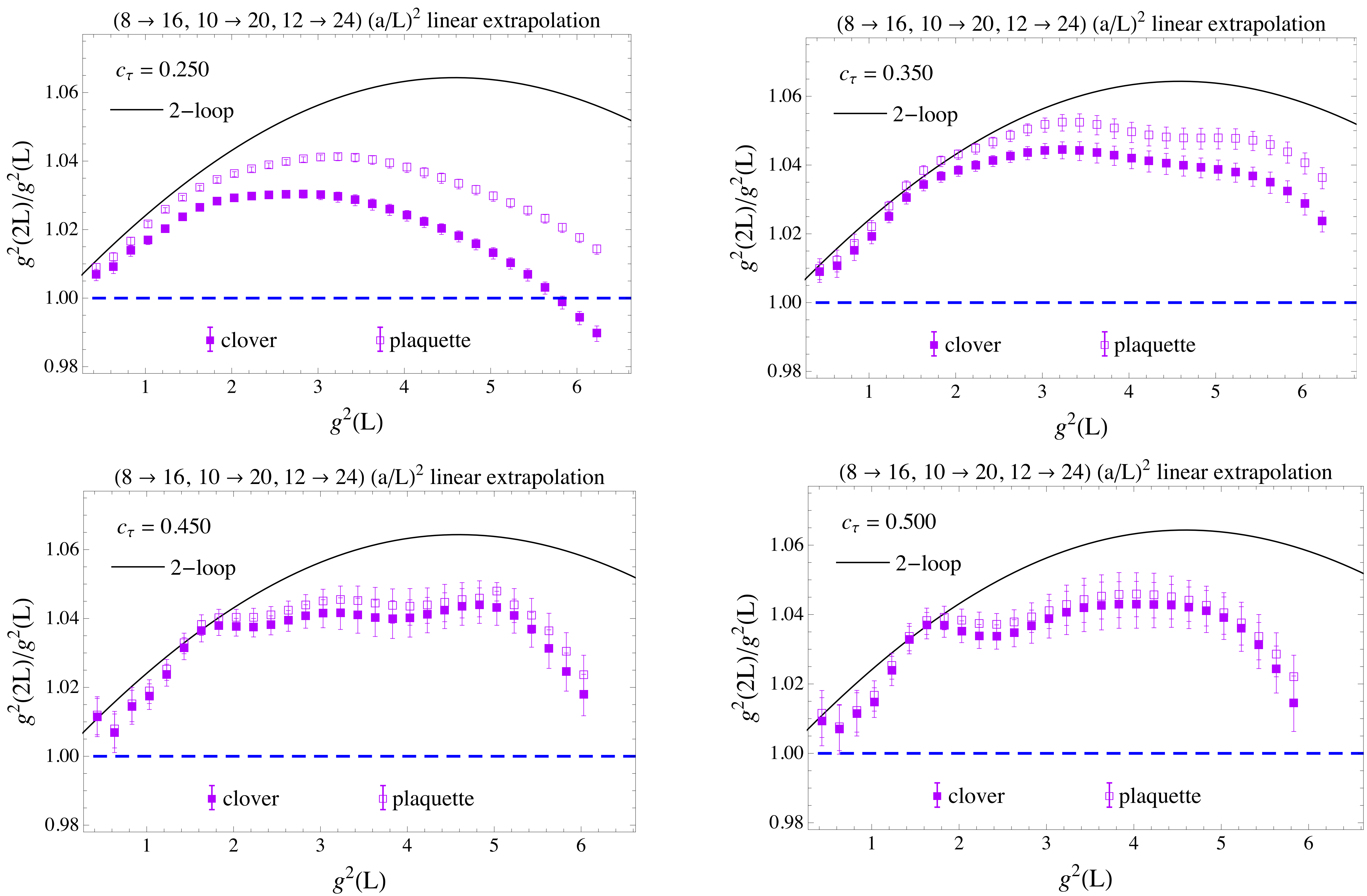




1-5% extrapolation, but...

→ It is important to use two discretisations and check the lattice artefacts.

The "conventional" continuum extrapolation



Require c_τ to be at least 0.45 to control the extrapolation.

Assuming we are close to an IRFP...

- The finite-size scaling behaviour is governed by the linearlised beta function,

$$\beta\left(g^2\right) \equiv -\hat{L}\frac{\mathrm{d}g^2}{\mathrm{d}\hat{L}} = \gamma\left(g^2 - g_*^2\right)$$

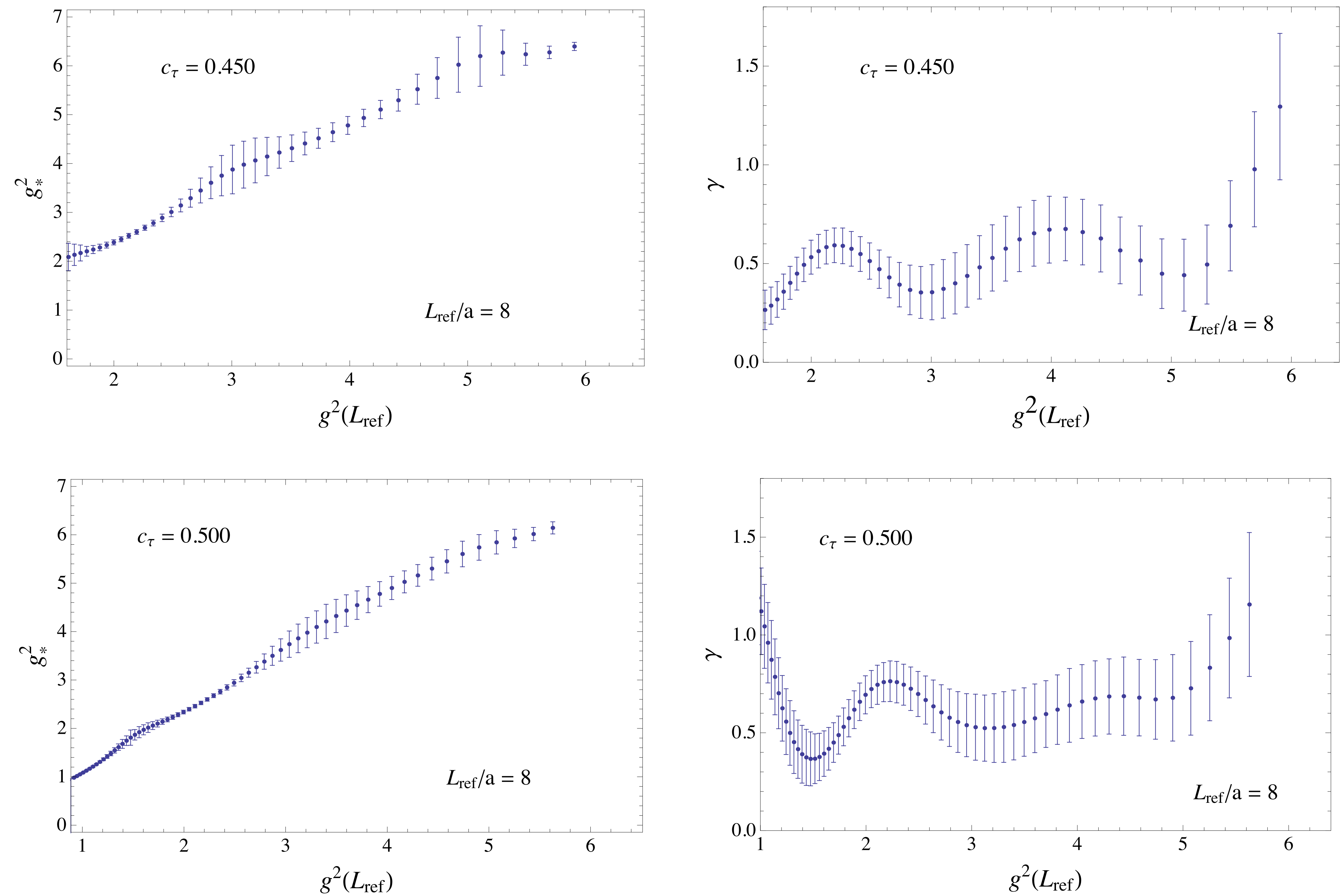
- Integrating from a reference volume, $\hat{L}_{\mathrm{ref}}$, at fixed lattice spacing,

$$g^2(\hat{L}) = g_*^2 + \left[g^2(\hat{L}_{\mathrm{ref}}) - g_*^2\right]\left(\frac{\hat{L}}{\hat{L}_{\mathrm{ref}}}\right)^{-\gamma}.$$

- At fixed lattice spacing, fit γ and g_*, then examine whether plateaus against $g^2(\hat{L}_{\mathrm{ref}})$ are observed.

Results at two flow times

From the clover discretisation



Weak (if any) evidence for plateaus...

Conclusion for this theory

For the study of the running coupling...

- *Necessary* to control error to the sub-percentage level.
- *Necessary* to use two discretisations.
  To check the control of the continuum extrapolation.
- The coupling runs slower than the two-loop prediction.
- If there is an IRFP, $g_*^2 > 6$ in the gradient-flow scheme.
- However, we do not see an IRFP in our data.

Non-thermal phase structure of SU(2) gauge theory with 8 flavours

The chiral condensate via RMT
Fermion masses 0.005, 0.010, and 0.015

C.Y-H. Huang (NCTU)

I. Kanamori (NCTU → Hiroshima U.)

C.-J.D. Lin (NCTU)

K. Ogawa (NCTU → industry)

H. Ohki (NCTU → RIKEN)

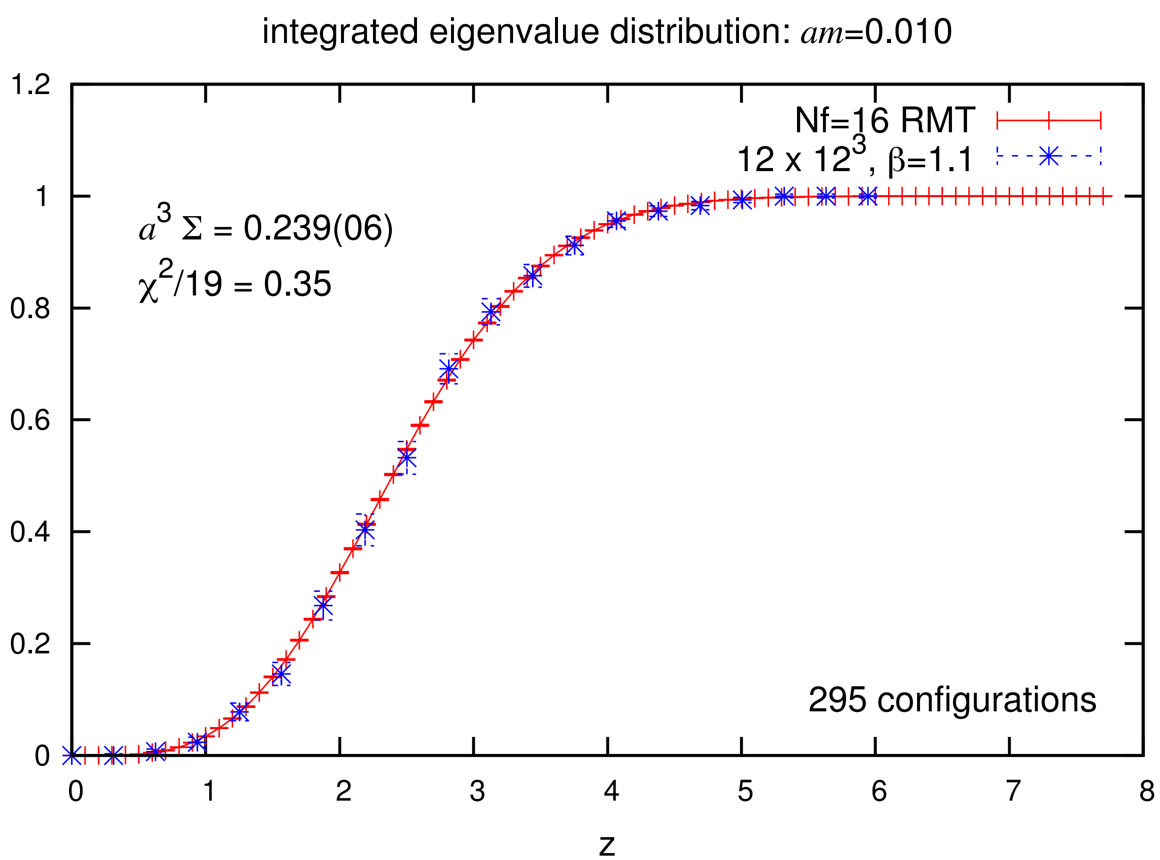
E. Rinaldi (LLNL)

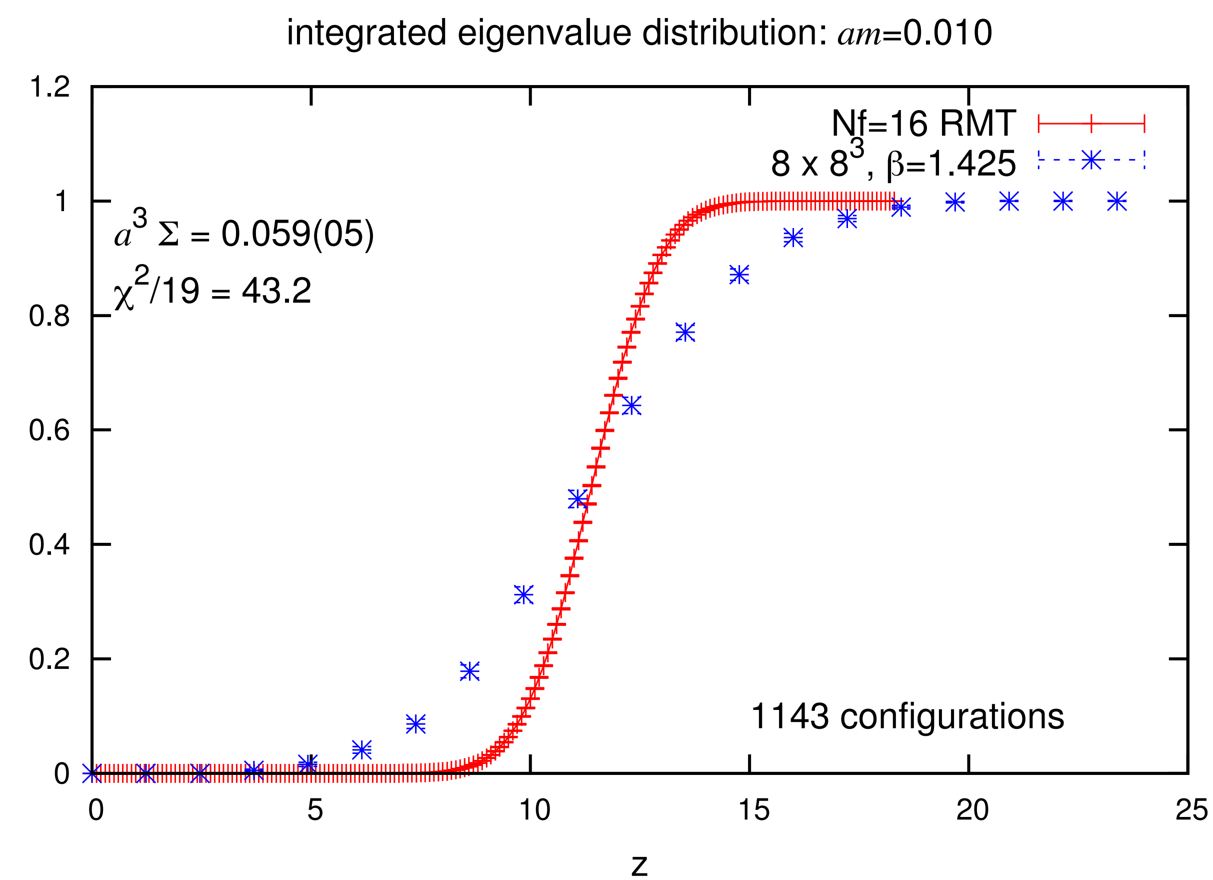
Analysis details

- Study the distribution of the lowest eigenvalue of the Dirac operator at zero topological charge.
- Compare to Random Matrix Theory.
  ➡ Dyson index = 4.
- Two staggered flavours.
  ➡ Eight flavours.
- Two-fold degeneracy from the SU(2) gauge group.
  ➡ Compare to Nf=16 RMT.
- Strong taste breaking effects.
  ➡ Compare to Nf=4 and 8 RMT.

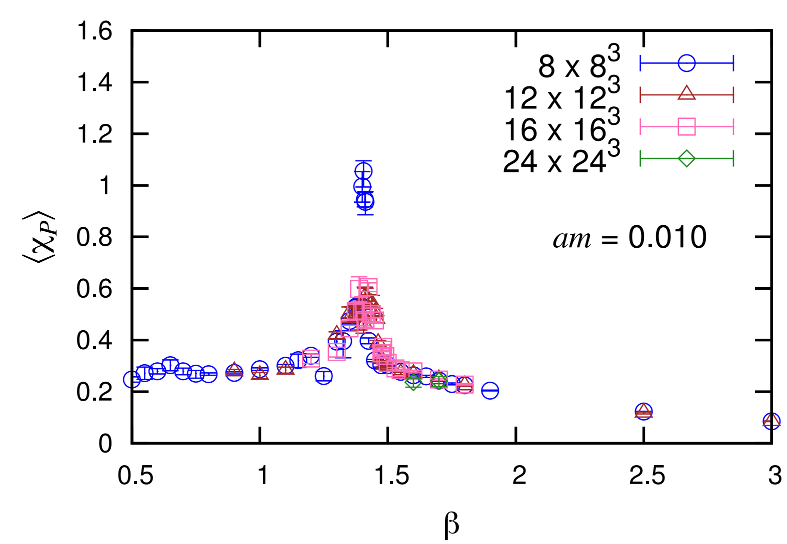
Fits to the RMT predictions

$$z = V\Sigma\sqrt{\lambda_1}$$



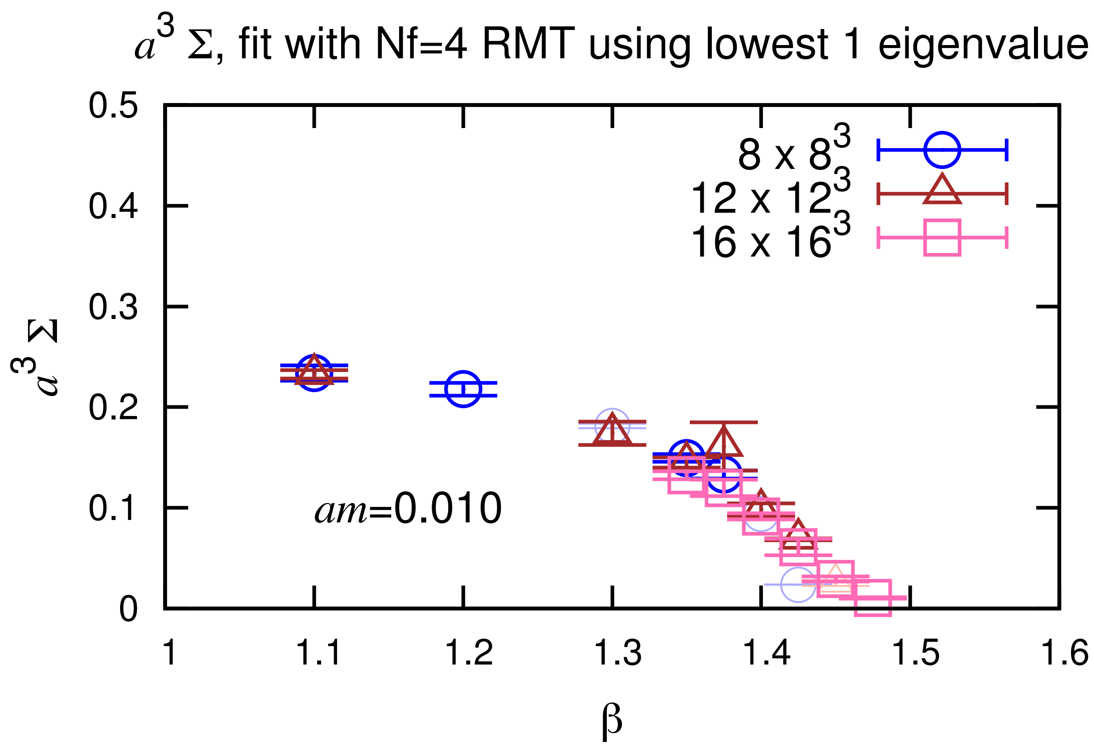


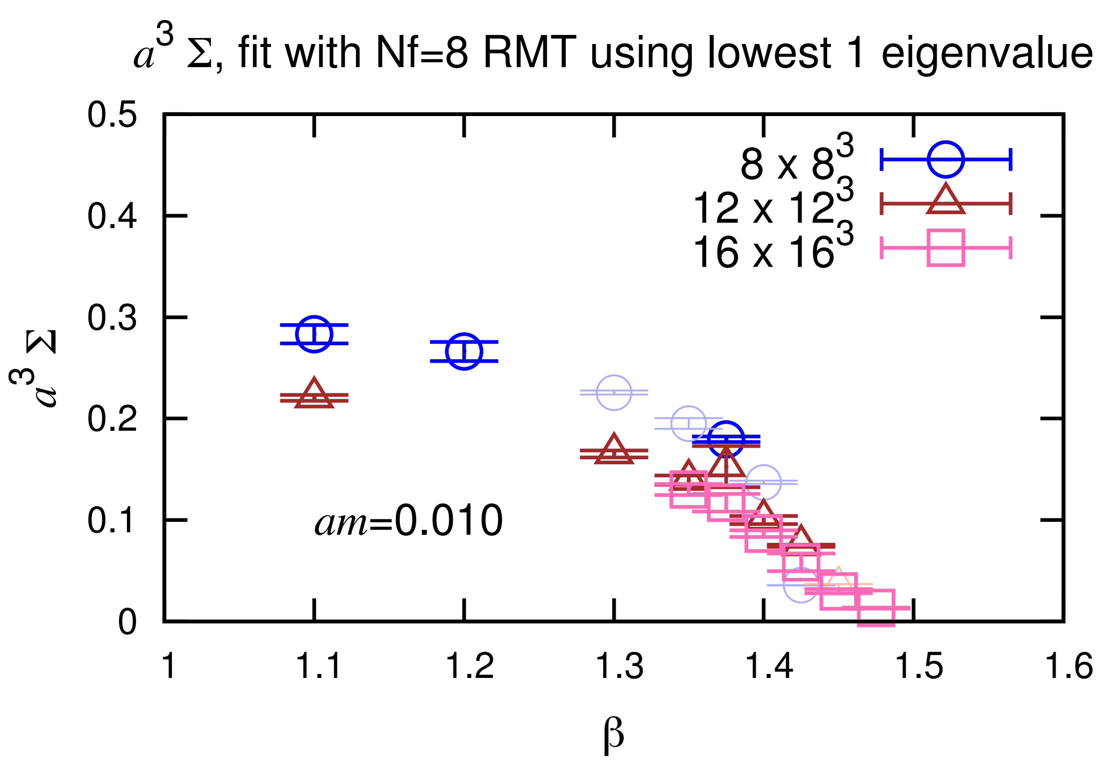
a good fit

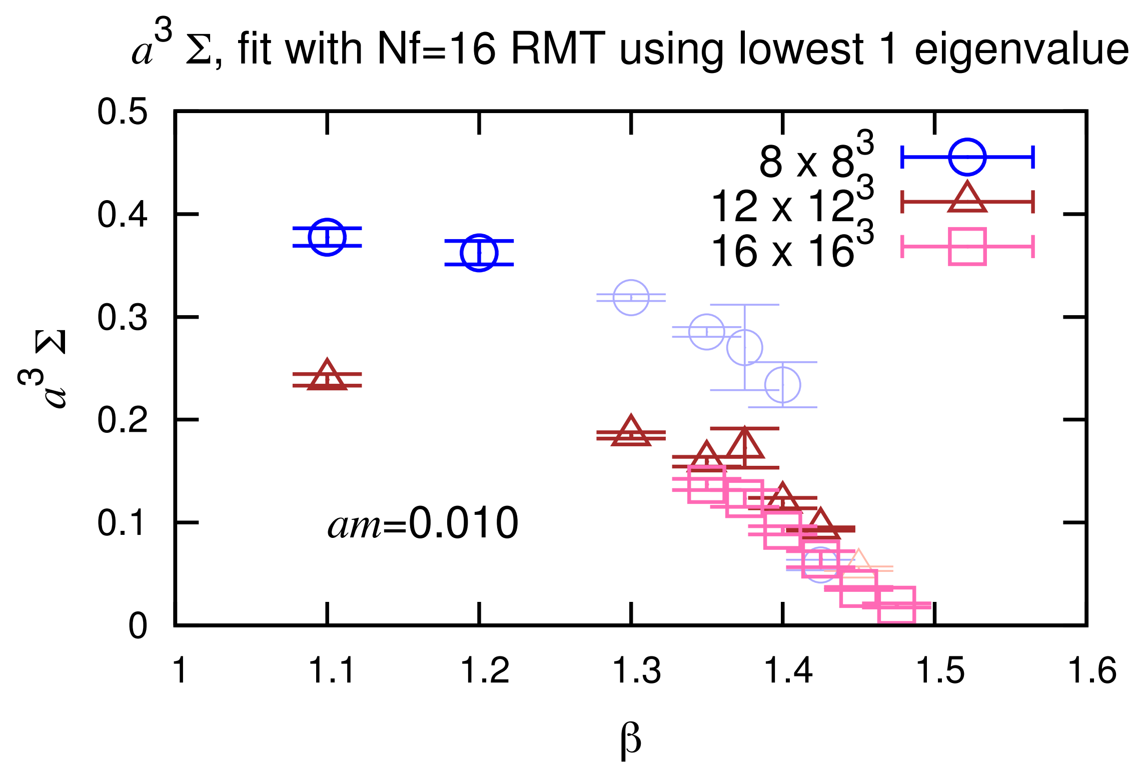


a bad fit
(chiral symmetry restoration?)

Results: volume dependence

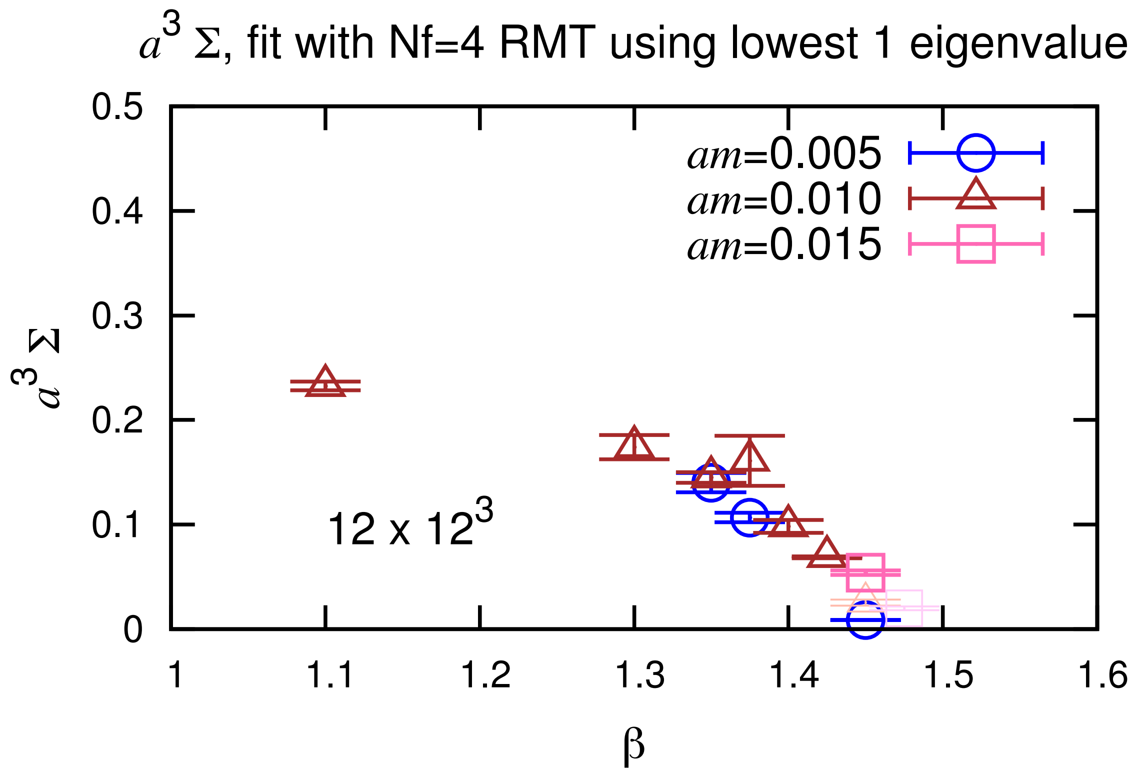


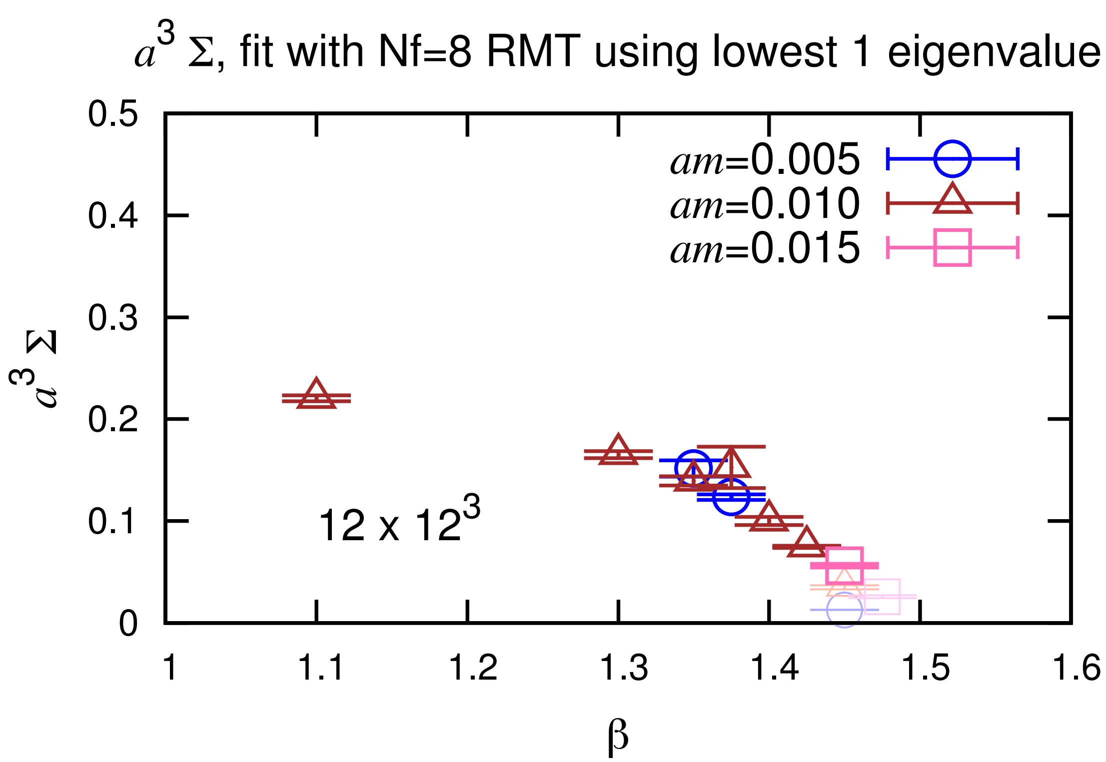


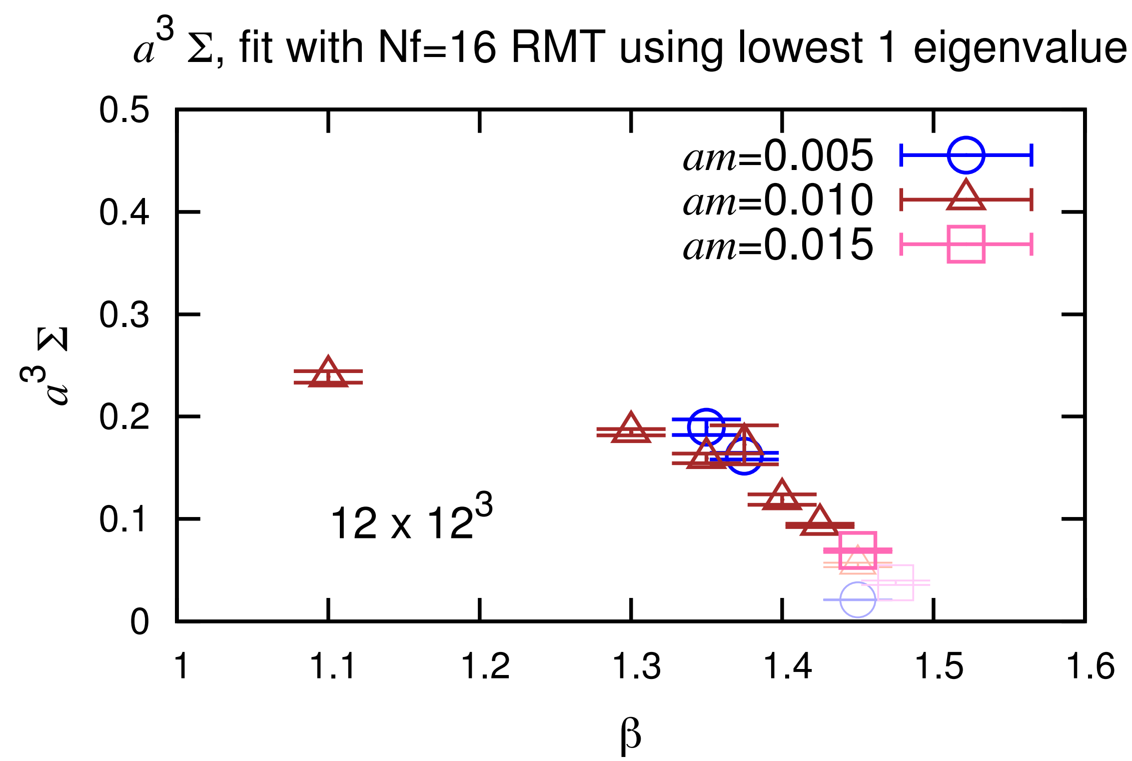


"Thin" symbols indicate bad RMT fits.

Results: mass dependence







“Thin” symbols indicate bad RMT fits.

Discussion

- For $L = 8$, RMT analysis indicates taste breaking effects seem to be large.
- For $L \geq 12$, RMT analyses at Nf=4, 8, 12,16 lead to consistent results.
- The chiral phase transition occurs at $\beta \sim 1.47$.
  ➡ No visible mass and volume dependences.
  ➡ Bulk phase transition.
- Strong taste breaking effects at β as large as 2.
  ➡ Seems to be irrelevant for the chiral transition.
- The chiral transition does not look like a first order phase transition.