Update on the Heavy-Meson Spectrum Tests of the Oktay–Kronfeld Action

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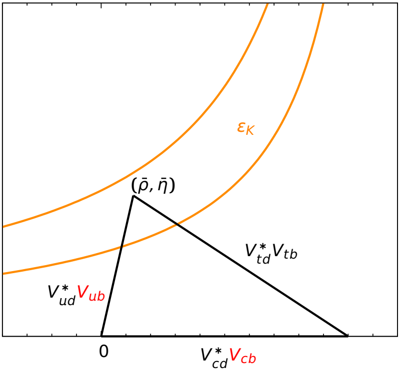
Unitarity Triangle, Standard Model Flavor Physics

ε_K, V_{ub}, V_{cb}

- Unitarity triangle (UT): $V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$
- V_{cb} normalizes the UT.
- V_{cb} enters to ε_K through $|V_{cb}|^4$.
- V_{ub} is directly related to the UT apex $(\bar{\rho}, \bar{\eta})$.
- CP violation in SM: $\bar{\eta} \neq 0$



- Differences
  $|\varepsilon_K|$: Experiment - SM $= 3.4\ \sigma$
  $|V_{cb}|$: Inclusive - Exclusive $= 2.9\ \sigma$
  $|V_{ub}|$: Inclusive - Exclusive $2 \sim 3\ \sigma$
- Problems with heavy quarks c, b

$$B \to \pi \ell \bar{\nu}_\ell \quad \Longrightarrow \quad V_{ub}$$
$$B \to D^{(*)} \ell \bar{\nu}_\ell \quad \Longrightarrow \quad V_{cb}$$

Lattice Calcuation with Heavy Quarks

- Combining HFAG average of experimental results and lattice form factor $\mathcal{F}$ calculation of the semi-leptonic decays, we can extract exclusive V_{cb}.

$$\bar{B} \to D^* l \nu_l \,,\ \bar{B} \to D l \nu_l$$

- $|V_{cb}| = 0.0390(5)_{\rm exp}(5)_{\rm lat}(2)_{\rm QED}$
  [Jon A. Bailey *et al.*, PRD **89**, 114504 (2014)]
- Because the dominant error for the form factor $\mathcal{F}$ calculation is heavy quark discretization error (1% / total 1.4%), we need an highly improved lattice action or finer lattice ensemble.
- Oktay–Kronfeld (OK) action was designed as an improved action.

Improvement for a Heavy Quark Action

$$\mathcal{L}_{\mathrm{LGT}} \doteq \bar{\mathcal{L}}_{\mathrm{QCD}} + \bar{\mathcal{L}}_I$$

$$\bar{\mathcal{L}}_{\mathrm{QCD}} = -\bar{Q}\left[\gamma_4 D_4 + m_1 + \sqrt{m_1/m_2}\,\boldsymbol{\gamma}\cdot\boldsymbol{D}\right]Q$$
$$\bar{\mathcal{L}}_I = \sum_i a^{\dim \bar{L}_i - 4}\bar{K}_i(m_2 a, g^2; c_j; \mu a)\bar{\mathcal{L}}_i \sim \sum_i \mathcal{O}(a\boldsymbol{p})^{\dim \bar{L}_i - 4}$$

[M. B. Oktay and A. S. Kronfeld, PRD **78**, 014504 (2008)]

- $\bar{\mathcal{L}}_i$ does not contain time derivative.
- All dependences on the (heavy) quark mass are isolated in the short-distance coefficients $\bar{K}_i(m_2 a, g^2; c_j; \mu a)$.
- $\bar{\mathcal{L}}_i$ can also be treated as a perturbation, if $a\boldsymbol{p} < 1$.
- $\bar{K}_i = 0\,;\ a\boldsymbol{p} < 1\,,\ m_2 a \geq 1$ yields non-relativistic interpretation of $\bar{\mathcal{L}}_{\mathrm{QCD}} \doteq \mathcal{L}_{\mathrm{HQET}}$ with a mistuned $m_1 \neq m_Q$, $(m_2 = m_Q)$.

Fermilab Action

- Fermilab action is the Sheikholeslami-Wohlert "clover" action with a non-relativistic interpretation (Fermilab formulation).

$$S_{\mathrm{Fermilab}}(m_Q = m_2(m_0)) = S_0(\zeta = 1) + S_B + S_E$$

HQET power counting: $\lambda \sim a\Lambda_{\mathrm{QCD}},\ \Lambda_{\mathrm{QCD}}/m_Q$

$$S_0 = m_0 \sum_x \bar{\psi}(x)\psi(x) + \sum_x \bar{\psi}(x)\gamma_4 D_4 \psi(x) \qquad : \mathcal{O}(1)$$

$$+ \zeta \sum_x \bar{\psi}(x)\vec{\gamma}\cdot\vec{D}\psi(x) - \tfrac{1}{2}a \sum_x \bar{\psi}(x)\triangle_4\psi(x) - \tfrac{1}{2}r_s\zeta a \sum_x \bar{\psi}(x)\triangle^{(3)}\psi(x)$$

$$S_B = -\tfrac{1}{2}c_B\zeta a \sum_x \bar{\psi}(x) i\vec{\Sigma}\cdot\vec{B}\psi(x) \qquad : \mathcal{O}(\lambda)$$

$$S_E = -\tfrac{1}{2}c_E\zeta a \sum_x \bar{\psi}(x)\vec{\alpha}\cdot\vec{E}\psi(x) \qquad : \mathcal{O}(\lambda^2)$$

[A. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, PRD **55**, 3933 (1997)]

OK Action

- $c_B \neq c_E$ for the OK action.
- OK action includes dim-6 and -7 operators necessary for tree-level matching to QCD through order $\mathcal{O}(\lambda^3)$: $S_{\rm OK} = S_{\rm Fermilab} + S_{\rm new}$

$$
\begin{aligned}
S_{\rm new} = {} & c_1 a^2 \sum_x \bar\psi(x) \sum_i \gamma_i D_i \triangle_i \psi(x) + c_2 a^2 \sum_x \bar\psi(x) \{\vec\gamma \cdot \vec D, \triangle^{(3)}\} \psi(x) \\
& + c_3 a^2 \sum_x \bar\psi(x) \{\vec\gamma \cdot \vec D, i\vec\Sigma \cdot \vec B\} \psi(x) \\
& + c_{EE} a^2 \sum_x \bar\psi(x) \{\gamma_4 D_4, \vec\alpha \cdot \vec E\} \psi(x) \\
& + c_4 a^3 \sum_x \bar\psi(x) \sum_i \triangle_i{}^2 \psi(x) \\
& + c_5 a^3 \sum_x \bar\psi(x) \sum_i \sum_{j \neq i} \{ {\rm i}\Sigma_i B_i, \triangle_j\} \psi(x) \qquad : \mathcal{O}(\lambda^3)
\end{aligned}
$$

[M. B. Oktay and A. S. Kronfeld, PRD **78**, 014504 (2008)]

Meson Correlator

$$C^M(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}^{M\dagger}(t,\boldsymbol{x})\mathcal{O}^M(0,\boldsymbol{0})\rangle$$

- On the lattice, we calculate the 2-point correlator.
- 11 meson momenta $|\boldsymbol{p}a|(= 0, 0.099, \cdots, 1.26)$ for dispersion fit
- MILC asqtad $N_f = 2+1$ ensemble:

a(fm)	$N_L^3 \times N_T$	β	am'_l	am'_s	u_0	a^{-1}(GeV)	N_{conf}	$N_{t_{\text{src}}}$
0.12	$20^3 \times 64$	6.79	0.02	0.05	0.8688	1.683^{+43}_{-16}	500	6

- Hopping paramters κ for heavy quarks:

$$\kappa = \tilde{\kappa}/u_0\,,\ 2\tilde{\kappa} = \left\{\tilde{m}_0 a + (1 + 3\tilde{r}_s\tilde{\zeta} + 18\tilde{c}_4)\right\}^{-1}$$

Q	b				c			
κ_{OK}	0.039	0.040	0.041	0.042	0.047	0.048	0.049	0.050
κ_{FL}			0.083	0.091		0.121	0.127	

Interpolating Operator $Q^M(x)$

- Quarkonium

$$\mathcal{O}^{\bar{Q}Q}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\psi_\beta(x)$$

- Spin structure

$$\Gamma = \begin{cases} \gamma_5 & \text{(pseudoscalar)} \\ \gamma_\mu & \text{(vector)} \end{cases}$$

- Heavy-light (strange $q = s$) meson

$$\mathcal{O}_{\color{red}t}{}^{\bar{Q}q}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\Omega_{\beta{\color{red}t}}(x)\chi(x)$$

- Taste for the staggered fermion

$$\Omega(x) \equiv \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}$$

- Valence heavy quark $\psi(x)$: (tadpole improved) OK action and Fermilab action
- Valence light quark $\chi(x)$: asqtad staggered action ($am_q = am'_s$)

[Wingate *et al.*, PRD **67**, 054505 (2003) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

Correlator Fit: Heavy-light $M = \overline{Q}q$

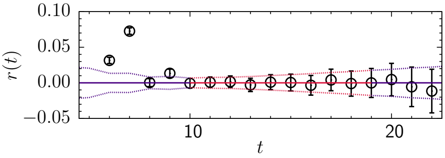
OK action, Pseudoscalar, $\kappa = 0.041$, $p = 0$

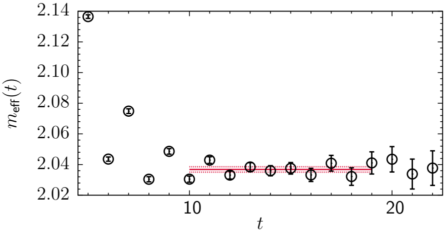
- fit function (T: time extent of lattice)

$$f(t) = A\left\{e^{-Et} + e^{-E(T-t)}\right\} + (-1)^t A^p \left\{e^{-E^p t} + e^{-E^p(T-t)}\right\}$$

- fit residual

$$r(t) = \frac{C^M(t) - f(t)}{|C^M(t)|}, \text{where } C^M(t) \text{ is data.}$$





Correlator Fit: Quarkonium $M = \overline{Q}Q$

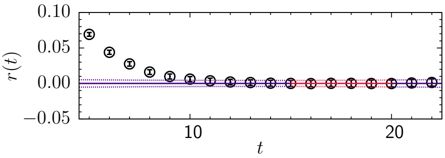
OK action, Pseudoscalar, $\kappa = 0.041$, $p = 0$

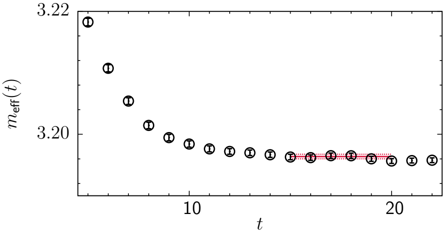
- fit function (T: time extent of lattice)

$$f(t) = A\left\{e^{-Et} + e^{-E(T-t)}\right\}$$

- effective mass

$$m_{\text{eff}}(t) = \frac{1}{2}\ln\left(\frac{C^M(t)}{C^M(t+2)}\right)$$



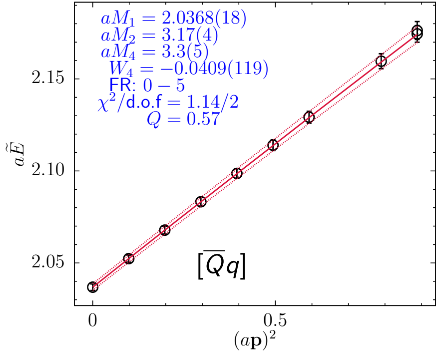


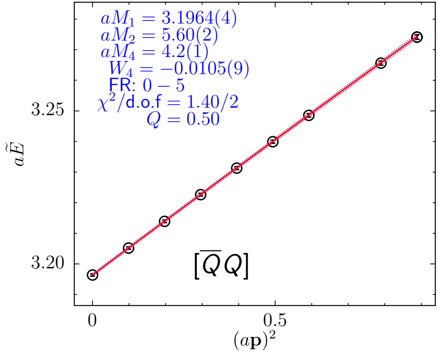
Dispersion Fit: extract the masses M_1 and M_2

OK action, Pseudoscalar, $\kappa = 0.041$

$$\text{fit: } E = M_1 + \frac{\boldsymbol{p}^2}{2M_2} - \frac{(\boldsymbol{p}^2)^2}{8M_4^3} - \frac{a^3 W_4}{6}\sum_i p_i^4 \;\Rightarrow\; \text{plot: } \widetilde{E} = E + \frac{a^3 W_4}{6}\sum_i p_i^4$$

- Two points with momentum $\boldsymbol{n} = (2,2,1)\,, (3,0,0)$ are distinguishable to the rotation symmetry breaking W_4 term.
- Fit the ground state energies for the lowest six momenta.





Improvement Test: Inconsistency Parameter

$$I \equiv \frac{2\delta M_{\overline{Q}q} - (\delta M_{\overline{Q}Q} + \delta M_{\overline{q}q})}{2M_{2\overline{Q}q}} = \frac{2\delta B_{\overline{Q}q} - (\delta B_{\overline{Q}Q} + \delta B_{\overline{q}q})}{2M_{2\overline{Q}q}}$$

$$M_{1\overline{Q}q} = m_{1\overline{Q}} + m_{1q} + B_{1\overline{Q}q} \qquad \delta M_{\overline{Q}q} = M_{2\overline{Q}q} - M_{1\overline{Q}q}$$

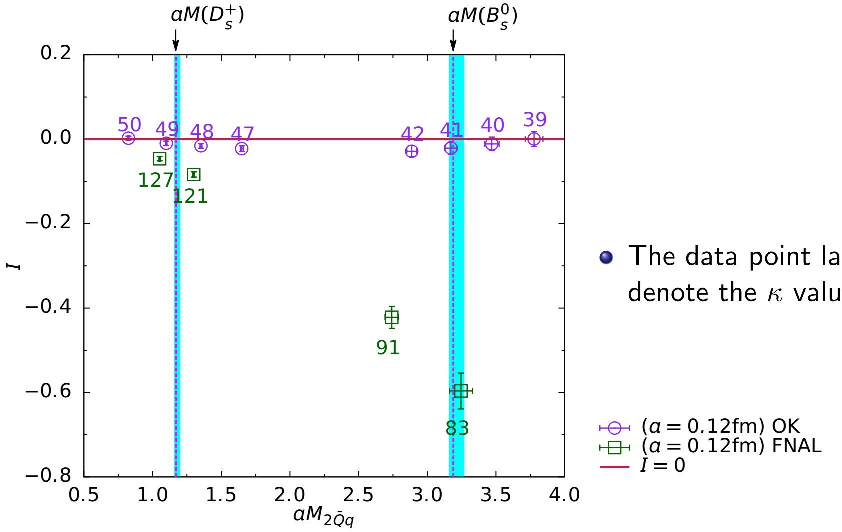
$$M_{2\overline{Q}q} = m_{2\overline{Q}} + m_{2q} + B_{2\overline{Q}q} \qquad \delta B_{\overline{Q}q} = B_{2\overline{Q}q} - B_{1\overline{Q}q}$$

[S. Collins *et al.*, NPB **47**, 455 (1996) , A. S. Kronfeld, NPB **53**, 401 (1997)]

- By design, the inconsistency parameter I can examine the action improvements by $\mathcal{O}(\boldsymbol{p}^4)$ terms. I isolate the δB of $\mathcal{O}(\boldsymbol{p}^2)$ effect.
- In the continuum limit, $B_1 = B_2$ and I vanishes.
- By including up to $\mathcal{O}(\boldsymbol{p}^4)$, the OK action is closer to the renormalized trajectory S_{RT} than the Fermilab action.
- We expect I is close to 0.

Improvement Test: Inconsistency Parameter

- Near B_s^0 mass, the coarse ($a = 0.12$fm) ensemble data shows significant improvement compared to the Fermilab action.



- The data point labels denote the κ values.

Improvement Test: Hyperfine Splitting Δ

- The difference in hyperfine splittings $\Delta_2 - \Delta_1$ also can be used to examine the improvement from $\mathcal{O}(\boldsymbol{p}^4)$ terms in the action.

$$\Delta_1 = M_1^* - M_1\,,\ \Delta_2 = M_2^* - M_2$$

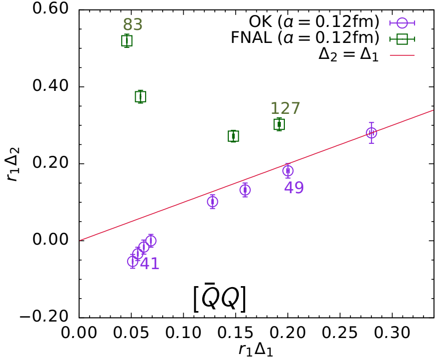
$$M^{(*)}_{1\overline{Q}q} = m_{1\overline{Q}} + m_{1q} + B^{(*)}_{1\overline{Q}q}$$
$$M^{(*)}_{2\overline{Q}q} = m_{2\overline{Q}} + m_{2q} + B^{(*)}_{2\overline{Q}q}$$
$$\delta B^{(*)} = B_2^{(*)} - B_1^{(*)}$$

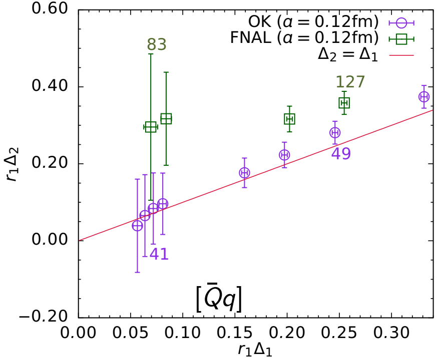
$$\Delta_2 = \Delta_1 + \delta B^* - \delta B$$

- In the continuum limit $\Delta_2 = \Delta_1$.

Improvement Test: Hyperfine Splitting Δ

- Hyperfine splitting shows that the OK action clearly improves the higher dimension magnetic effects for the quarkonium.
- The heavy-light results also shows an improvement near charm region; for bottom region, it is not clear to say an improvement. However, we expect that OK action gives an improvement in the bottom region ($r_1\Delta_1$).





Conclusions and Outlook

- Inconsistency parameter shows that the OK action clearly improves $\mathcal{O}(\boldsymbol{p}^4)$ terms.
- Smaller error on the kinetic mass M_2 implies that we can tune κ more precisely.
- Hyperfine splitting shows that the OK action clearly improves the higher dimension magnetic effects for the quarkonium.
- Hyperfine splitting for the heavy-light system shows a clear improvement near D_s.

- We plan to calculate form factor of the decay $\bar{B} \to D^* l\nu$ by using OK action. Then, we can determine exclusive V_{cb} with higher precision.
- Lattice current improvement to the same order of the OK action is under way.
- We plan on the 1-loop improvement for the coefficients c_B and c_E in the OK action.