# Calculation of Strange and Light Quark Condensate using Improved Staggered Fermions and Overlap Fermions

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## Introduction

- We calculate the chiral condensate, the strange quark condensate and the light quark condensate by using improved staggered fermions and overlap fermions, respectively.
- We report the mass dependence of the quark condensate.
- We use $N_f = 2+1$ MILC Asqtad gauge ensembles.

## Quark Condensate for Staggered Fermion

- staggered Dirac operator :

$$D_{x,y} = m\delta_{x,y} + \frac{1}{2}\sum_{\mu}\eta_\mu(x)\left[U_\mu(x)\delta_{x+\widehat{\mu},y} - U_\mu^\dagger(x-\widehat{\mu})\delta_{x-\widehat{\mu},y}\right] \quad (1)$$

where $\eta_\mu(x) = (-1)^{\sum_{\nu<\mu} x^\nu}$.

- quark condensate :

$$\langle\bar{\psi}\psi\rangle = \langle 0|\bar{\psi}_f\psi_f|0\rangle = -\frac{1}{V}\left\langle \mathrm{Tr}D(U)^{-1}\right\rangle_U \quad (2)$$

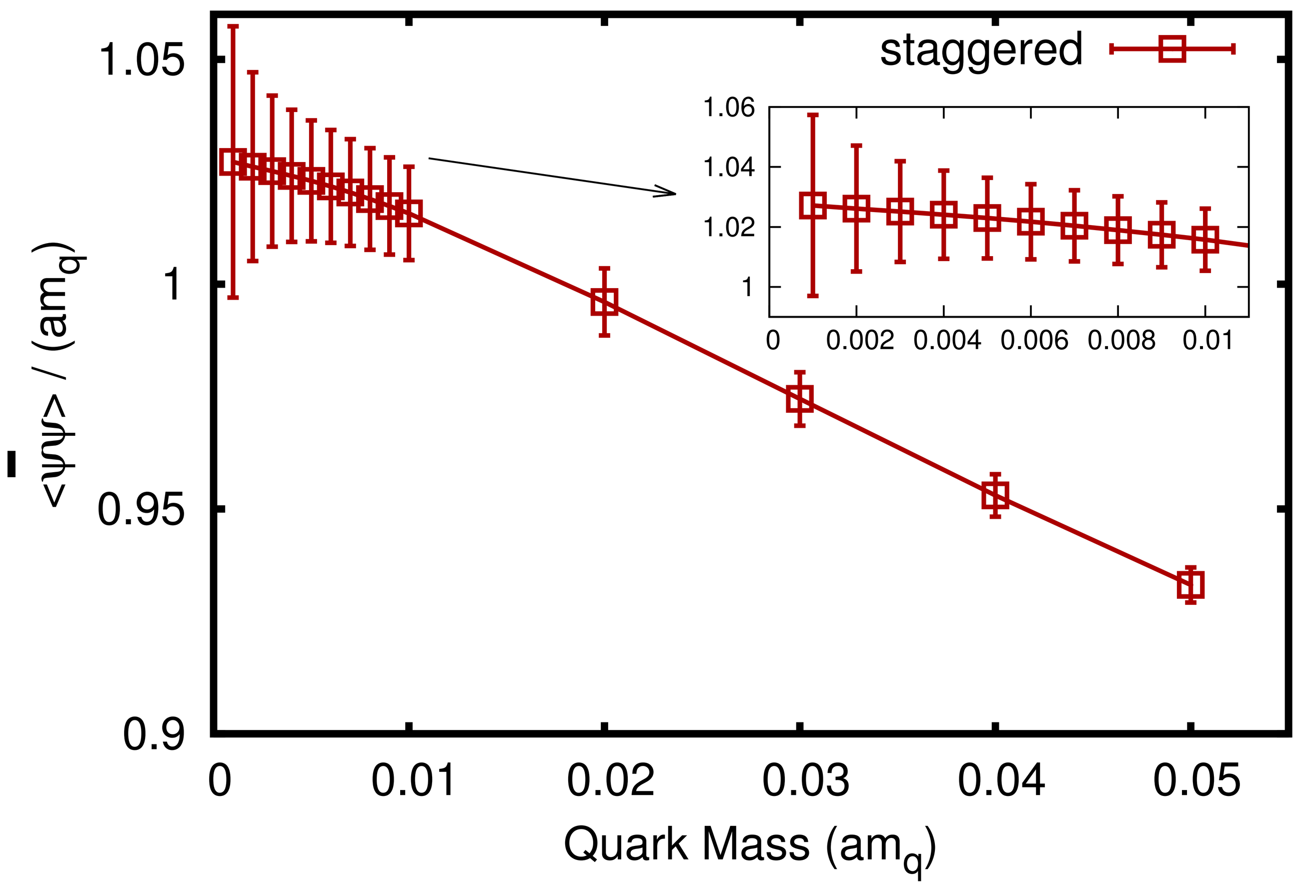
where $V$ : lattice volume, $U$ : HYP-smeared gauge field, and the trace is over spin, color and space-time points

- Raw condensate value can be obtained by

$$-a^3\langle\bar{\psi}\psi\rangle = (am_q)\sum_t C_\pi(t) \quad (3)$$

where $C_\pi(t) = \frac{1}{V}a^2\sum_{\mathbf{n}}\langle \mathrm{Tr}|D^{-1}_{n0}|^2\rangle$

- quark condensate $\langle\bar{\psi}\psi\rangle/(am_q)$ for staggered fermion (preliminary, not yet renormalized)




## Simulation Details

- $20^3 \times 64$ MILC asqtad coarse lattice at $a \cong 0.12$fm

| $\beta = 10/g^2$ | $am_l$ | $am_s$ | $(L/a)^3 \times (T/a)$ |
|---|---|---|---|
| 6.760 | 0.0100 | 0.0500 | $20^3 \times 64$ |

- simulation setup

| Fermion Type | # of Gauge Conf. | number of hits |
|---|---|---|
| staggered | 500 | 3 (color) |
| overlap | 50 | 12 (spin, color) |

- quark mass and inverter algorithm

| Fermion Type | Quark Mass ($am_q$) | Inverter |
|---|---|---|
| staggered | $\geq 0.001$ | CG |
| | $< 0.001$ | eigCG$^{(*)}$ |
| overlap | $\geq 0.01$ | GMRES, eigSUMR |
| | $< 0.01$ | multigrid$^{(\#)}$ |

( *: under construction , #: under construction, refer Nigel Cundy's poster)

## Quark Condensate for Overlap Fermion

- overlap Dirac operator :

$$D[\mu] = \frac{1}{2}(1+\mu+(1-\mu)\gamma_5\epsilon(K)) \quad (4)$$

  - $\mu$ : overlap mass input parameter satisfying $am_q = \frac{2\mu m_W}{1-\mu}$ where $m_W = 1.5$
  - $K = \gamma_5(D_W - m_W)$ where $D_W$ : Wilson Dirac operator
  - $\epsilon(A)$ : matrix sign function

- quark condensate for overlap fermion :

$$\langle\bar{\psi}\psi\rangle = -\frac{1}{V}\langle\mathrm{Tr}S\rangle \quad (5)$$

where $S$ is the $O(a)$-improved quark propagator given by

$$S[\mu] \equiv (1 - aD[0])\frac{1}{D[\mu]} \quad (6)$$

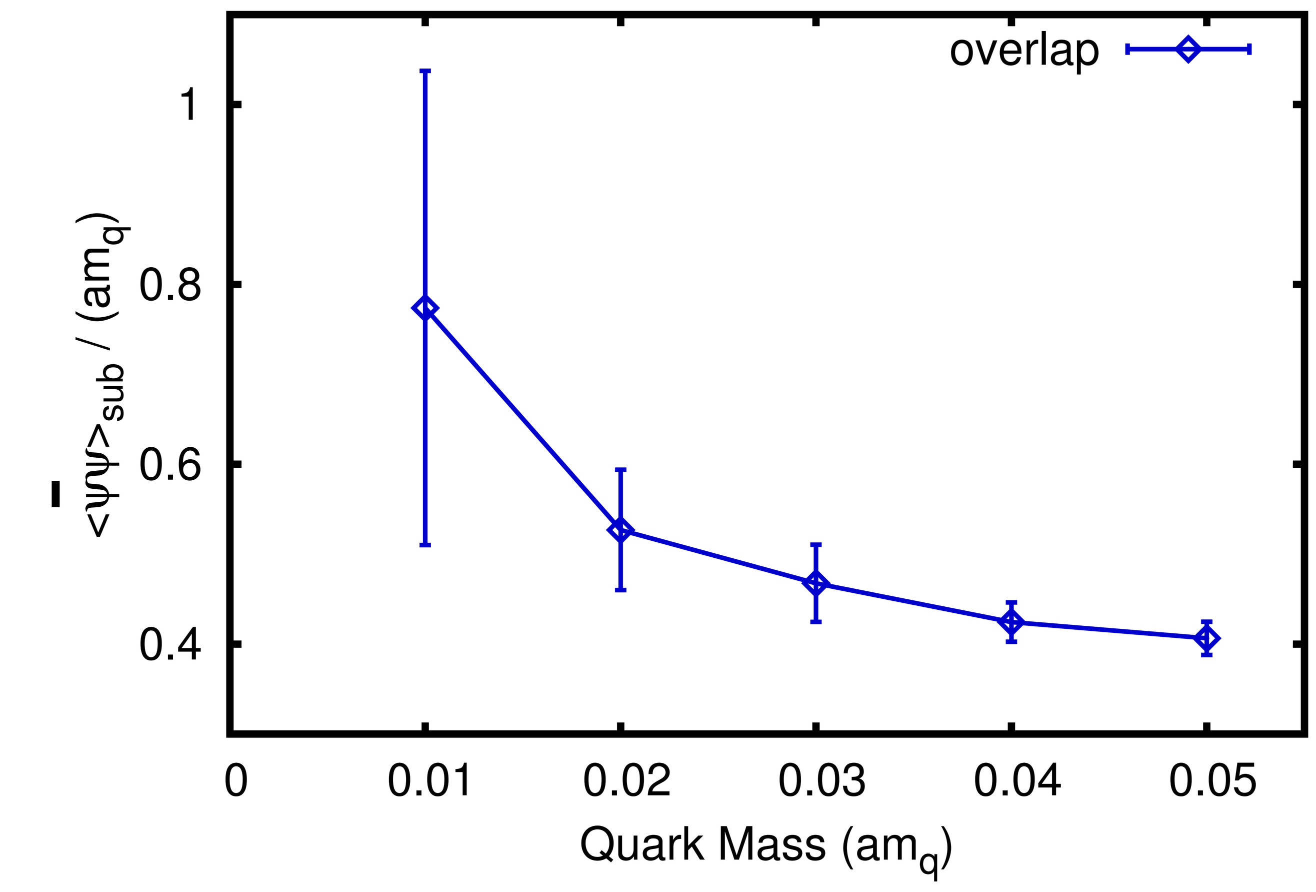
which satisfies $S + S^\dagger = 2\mu SS^\dagger$

- Here we subtract the contribution from the exact zero mode of $D_{overlap}$

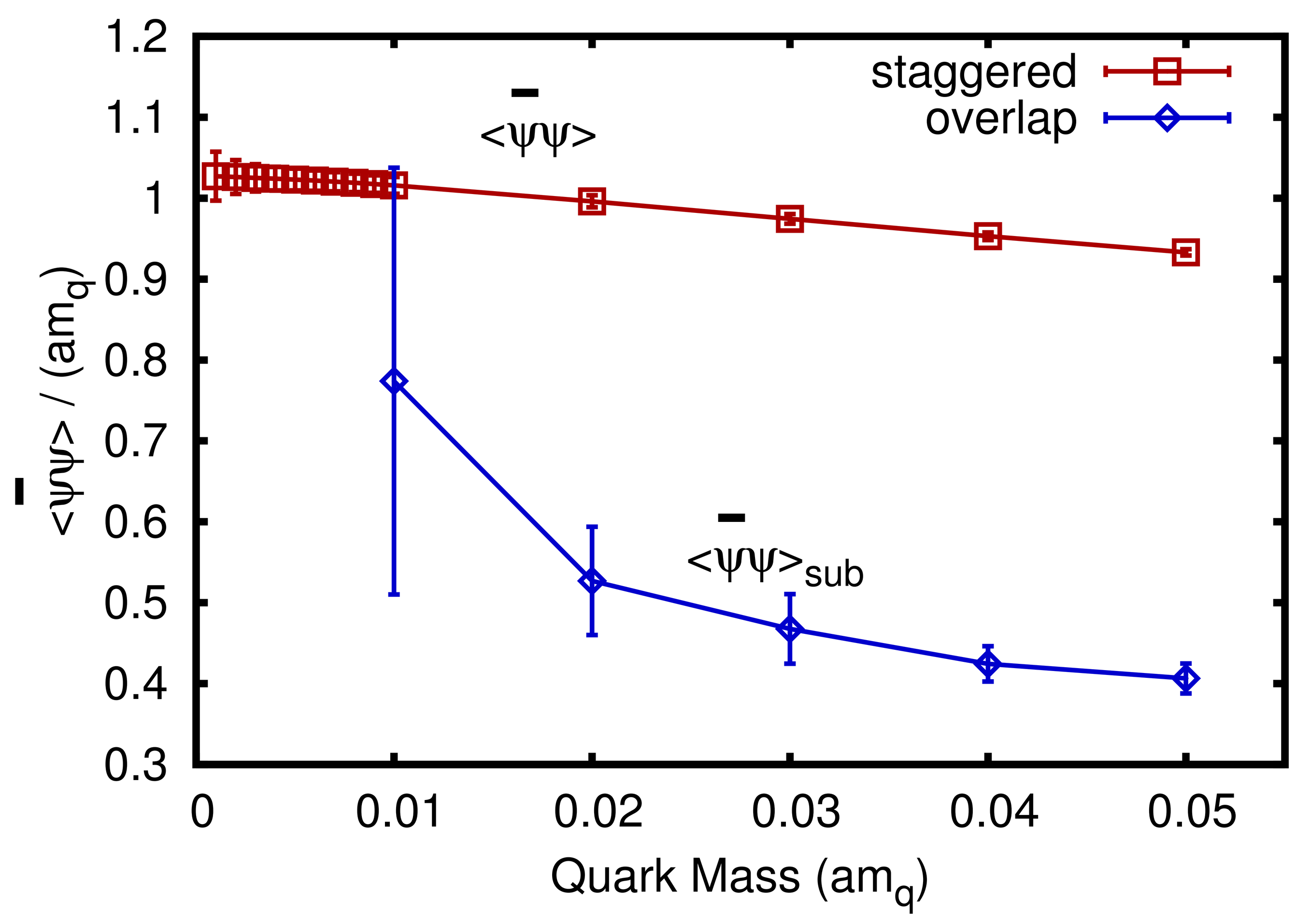
$$\langle\bar{\psi}\psi\rangle_{sub} = -\frac{1}{V}\langle\mathrm{Tr}S\rangle + \frac{Q_t}{mV} \quad (7)$$

where $Q_t = \nu =$ (winding number) $=$ (topological charge)

- quark condensate $\langle\bar{\psi}\psi\rangle_{sub}/(am_q)$ for overlap fermion (preliminary, not yet renormalized)




## Staggered vs Overlap




## Summary and Plan

- We calculated $\langle\bar{\psi}\psi\rangle$ for different quark masses by using staggered fermion and overlap fermion, respectively. We are going to increase the statistics.
- We will calculate the quark condensates for more lighter masses by adapting multi-mass method with eigCG and multigrid algorithms.
- We will find the topological charges of each gauge configuration exactly from the overlap fermion, so that we subtract the zero mode topological charge contribution from the staggered result, too. Also, we will calculate the quark condensates for each topological sector.
- We will also subtract the $O(a^{-3})$-contribution from the overlap quark condensate. Then we will fit the result to a polynomial function of $ma + ma^2 + \Sigma$
- We plan to use $N_f = 2+1+1$ MILC HISQ(highly improved staggered quark) gauge ensembles to calculate the quark condensates.