Non-perturbative Renormalization of bilinear operators with improved staggered quarks on Fine Lattice

Jangho Kim [1], Hwancheol Jeong [2], Weonjong Lee [2], Jeonghwan Pak [2], Sungwoo Park [2]

[1]Korea Institute of Science and Technology Information

[2]Seoul National University

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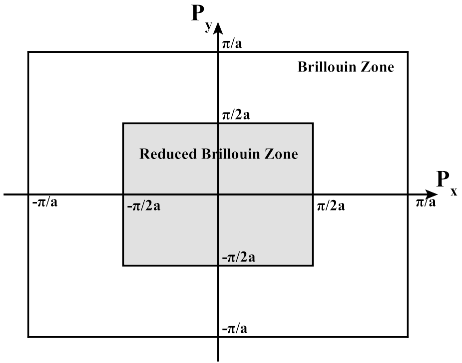
Introduction

- We present the preliminary results of the wave function renormalization factor Z_q and mass renormalization factor Z_m from the bilinear operators obtained using non-perturbative renormalization method(NPR) in the RI-MOM scheme with HYP improved staggered fermions on fine lattice.
- We use fine ensembles of MILC asqtad lattices ($N_f = 2 + 1$) with $28^3 \times 96$ geometry and ($a \approx 0.08 fm,\ am_\ell/am_s = 0.0062/0.031$).
- We also present the dependence of lattice spacing for Z_q and Z_m by comparing the results of coarse and fine lattices.

Momentum in Reduced Brillouin Zone



- $\tilde{p}$ is the momentum in reduced Brillouin zone.

$$p \in (-\frac{\pi}{a}, \frac{\pi}{a}]^4, \qquad \tilde{p} \in (-\frac{\pi}{2a}, \frac{\pi}{2a}]^4, \qquad p = \tilde{p} + \pi_B$$

  where $\pi_B (\equiv \frac{\pi}{a} B)$ is cut-off momentum in hypercube.

- B : vector in hypercube. Each element is 0 or 1. ex) $B = (0, 0, 1, 1)$

Green's function

Bilinear Operator

$$O_i(y) = \sum_{AB} \overline{\chi}_i(y_A)\overline{(\gamma_S \otimes \xi_F)}_{AB} U_{i;AB}(y)\chi_i(y_B)$$

Green's function

$$G_i(x_1, x_2, y) = \langle \chi_i(x_1) O_i(y) \overline{\chi}_i(x_2) \rangle$$

Green's function in momentum space

$$\begin{aligned}
&H(\widetilde{p} + \pi_A, \widetilde{q} + \pi_B, \widetilde{k} = \widetilde{p} - \widetilde{q}) \\
&= \frac{1}{N} \sum_{i}^{N} \sum_{y} \sum_{CD} e^{-i\widetilde{k}y} \hat{S}_i(\widetilde{p} + \pi_A, y_C) \overline{(\gamma_S \otimes \xi_F)}_{CD} U_{i;CD}(y) \hat{S}_i(y_D, \widetilde{q} + \pi_B) \\
&= V \widetilde{H}(\widetilde{p} + \pi_A, \widetilde{q} + \pi_B)
\end{aligned}$$

- i : gauge configuration index
- x_1, x_2 : coordinates on the lattice with its spacing a.
- y : hypercube coordinates on the lattice with its spacing $2a$, $y_A = 2y + A$

Amputated Green's function

Amputated Green's function

$$\widetilde{\Lambda}(\widetilde{p}+\pi_A,\widetilde{q}+\pi_B) = \sum_{CDEF} \widetilde{S}(\widetilde{p})^{-1}_{AC}\widetilde{H}(\widetilde{p}+\pi_C,\widetilde{q}+\pi_D)$$

$$\cdot \overline{\overline{(\gamma_5\otimes\gamma_5)}}_{DF}[\widetilde{S}(\widetilde{p})^{-1}]^{\dagger}_{FE}\overline{\overline{(\gamma_5\otimes\gamma_5)}}_{EB}$$

Projected amputated Green's function

$$\Gamma^{\alpha\beta}(\widetilde{p},\widetilde{q}) = \sum_{AB}\sum_{c_1c_2}[\widetilde{\Lambda}^{\alpha}_{c_1c_2}(\widetilde{p}+\pi_A,\widetilde{q}+\pi_B)\hat{\mathbb{P}}^{\beta}_{BA;c_2c_1}]$$

The projection operator is

$$\hat{\mathbb{P}}^{\beta}_{BA;c_2c_1} = \frac{1}{48}\overline{\overline{(\gamma^{\dagger}_{S'}\otimes\xi^{\dagger}_{F'})}}_{BA}\delta_{c_2c_1}$$

- $\alpha = (\gamma_S\otimes\xi_F)$, $\beta = (\gamma_{S'}\otimes\xi_{F'})$
- A, B: hypercube index, c : color index
- $\overline{(\gamma_S\otimes\xi_F)}_{CD} = \dfrac{1}{4}\mathrm{tr}[\gamma^{\dagger}_C\gamma_S\gamma_D\gamma^{\dagger}_F]$, where $\gamma_D = \gamma_1^{D_1}\gamma_2^{D_2}\gamma_3^{D_3}\gamma_4^{D_4}$
- $\overline{\overline{(\gamma_S\otimes\xi_F)}}_{AB} = \dfrac{1}{16}\sum_{CD}(-1)^{A\cdot C}\overline{(\gamma_S\otimes\xi_F)}_{CD}(-1)^{D\cdot B}$

Renormalization of Projected Amputated Green's Function

The renormalization of projected amputated Green's function $\Gamma(\tilde{p}, \tilde{q})$ is

$$O_R^\alpha = Z^{\alpha\beta} O_B^\beta$$
$$\chi_R = Z_q^{1/2} \chi_B$$
$$\Gamma_R^{\alpha\sigma}(\tilde{p}, \tilde{q}) = \sum_\beta Z_q^{-1} Z^{\alpha\beta} \Gamma_B^{\beta\sigma}(\tilde{p}, \tilde{q})$$

As a matrix form,

$$\hat{\Gamma}_R(\tilde{p}, \tilde{q}) = Z_q^{-1} \hat{Z} \cdot \hat{\Gamma}_B(\tilde{p}, \tilde{q})$$

- α, β, σ: the indices to represent different operators.
- $\Gamma_{R(B)}$: renormalized(bare) projected amputated Green's function
- Z_q : the wave function renormalization factor for quark fields
- Z : the renormalization factor matrix.

RI-MOM scheme

The Regularization Independent Momentum-Subtraction (RI-MOM) scheme prescription is

$$\hat{\Gamma}_R(\tilde{p},\tilde{p}) = \hat{\Gamma}_{\text{tree}}(\tilde{p},\tilde{p}) = I \,,$$

where $\hat{\Gamma}_{\text{tree}}(\tilde{p},\tilde{p})$ is the projected amputated Green's function matrix calculated at the tree level.

Therefore, the renormalization factor is obtained from the following equation.

$$\hat{\Gamma}_R(\tilde{p},\tilde{p}) = Z_q^{-1}\hat{Z}\cdot\hat{\Gamma}_B(\tilde{p},\tilde{p}) = I$$
$$\hat{Z} = Z_q\cdot\hat{\Gamma}_B^{-1}(\tilde{p},\tilde{p})$$

Simulation Detail

- $28^3 \times 96$ MILC asqtad lattice ($a \approx 0.08 fm,\ am_\ell/am_s = 0.0062/0.031$).
- Landau gauge fixed, HYP smearing and Tadpole improvement.
- The number of configurations is 10.
- 5 valence quark masses (0.0062, 0.0124, 0.0186, 0.0248, 0.031)
- 9 external momenta in the units of $(\frac{2\pi}{L_s}, \frac{2\pi}{L_s}, \frac{2\pi}{L_s}, \frac{2\pi}{L_t})$.
- We use the jackknife resampling method to estimate statistical errors.

n(x, y, z, t)	$(a\tilde{p})^2$	$\lvert a\tilde{p}\rvert$	GeV
$(2,3,2,9)$	1.0968	1.2030	2.5684
$(3,3,3,7)$	1.2528	1.5695	2.9337
$(3,3,3,8)$	1.2782	1.6337	2.9931
$(3,3,3,10)$	1.3371	1.7880	3.1312
$(3,4,3,9)$	1.4349	2.0591	3.3602
$(4,3,4,10)$	1.5789	2.4929	3.6973
$(4,4,4,10)$	1.6868	2.8454	3.9501
$(4,4,4,12)$	1.7418	3.0339	4.0788
$(4,4,4,14)$	1.8046	3.2566	4.2259

Wave Function Renormalization Factor

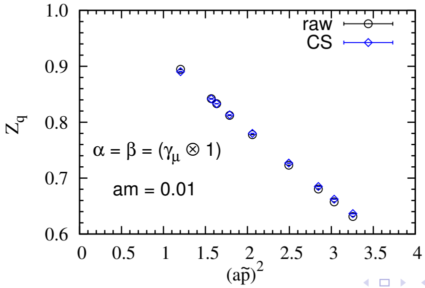
For the conserved vector current, the renormalization factor $Z_V = 1$.
Therefore

$$Z_V = Z_q^{\text{RI-MOM}} \cdot [\Gamma_B^{-1}(\tilde{p},\tilde{p})]^V = 1$$

$$Z_q^{\text{RI-MOM}} = \Gamma_B^V(\tilde{p},\tilde{p})\,,$$

We convert the raw data in the RI-MOM scheme from $\mu(=|\widetilde{p}|)$ to the common scale μ_0(=3GeV) using four-loop RG evolution factor $U_q(\mu_0,\mu)$.

$$Z_q^{\text{RI-MOM}}(\mu_0) = U_q(\mu_0,\mu) Z_q^{\text{RI-MOM}}(\mu), \qquad (\mu_0 = 3\text{GeV}, \quad \mu^2 = \widetilde{p}^2)$$

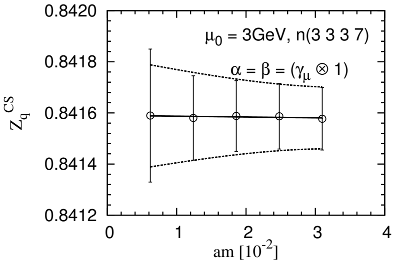


m-fit (fitting with respect to quark mass)

We fit the data with respect to quark mass for a fixed momentum to the following function f.

$$f(m, a, \tilde{p}) = c_1 + c_2 \cdot am,$$

where c_i is a function of $\widetilde{p}$. We call this m-fit. After m-fit, we take the $c_1(a, \widetilde{p})$ as chiral limit values.



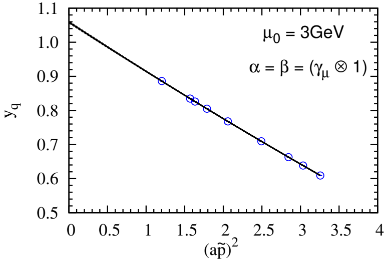
μ_0	c_1	c_2	χ^2/dof
3GeV	0.84159(22)	-0.0003(58)	0.006(16)

p-fit (fitting with respect to reduced momentum)

We fit $c_1(a, \widetilde{p})$ to the following fitting function.

$$g(a, \tilde{p}) = b_1 + b_2(a\tilde{p})^2 + b_3((a\tilde{p})^2)^2 + b_4(a\tilde{p})^4$$

To avoid non-perturbative effects at small $(a\tilde{p})^2$, we choose the momentum window as $(a\tilde{p})^2 > 1$. Because we assume that those terms of $\mathcal{O}((a\widetilde{p})^2)$ and higher order are pure lattice artifacts, we take the b_1 as Z_q in the RI-MOM scheme value. Here, $x_q = Z_q^{\text{RI-MOM}}(am = 0) - \langle b_4 \rangle (a\tilde{p})^4$



μ_0	b_1	b_2	b_3	b_4	χ^2/dof
3GeV	1.0589(14)	-0.1471(14)	0.00279(18)	0.0103(17)	0.26(13)

Systematic Error

We convert the result from RI-MOM scheme to $\overline{\mathrm{MS}}$ scheme using four-loop RG running formula. The systematic error of $Z_q^{\overline{\mathrm{MS}}}$ is estimated by two different ways.

- The first systematic comes from truncated higher order of the four-loop RG running evolution factor(RI-MOM $\rightarrow \overline{\mathrm{MS}}$):$(O(\alpha_s^4))$.

$$E_t = Z_q^{\text{RI-MOM}} \cdot (\alpha_s)^4$$

- The second systematic error comes from the difference in $Z_q^{\overline{\mathrm{MS}}}$ from between the conserved vector and axial currents.

$$E_\Delta = |Z_q^{\overline{\mathrm{MS}}}(V \otimes S) - Z_q^{\overline{\mathrm{MS}}}(A \otimes P)|$$

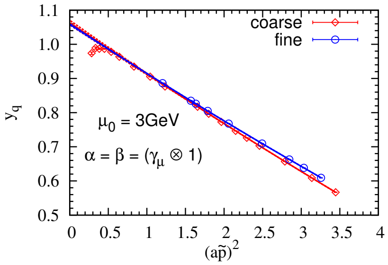
- The total systematic error is obtained adding two systematic errors in quadrature.

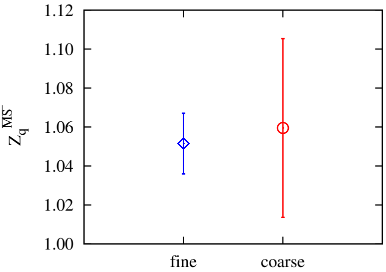
$$E_{tot} = \sqrt{E_t^2 + E_\Delta^2}$$

μ_0	$Z_q^{\overline{\mathrm{MS}}}$	E_t	E_Δ	E_{tot}
3GeV	1.0515(14)	0.0038	0.0155	0.0160

Comparison of Z_q between coarse and fine lattices

- coarse lattice : $20^3 \times 64,\ a \approx 0.12fm,\ am_\ell/am_s = 0.01/0.05$
- fine lattice : $28^3 \times 96,\ a \approx 0.08fm,\ am_\ell/am_s = 0.0062/0.031$





	coarse	fine
$Z_q^{\overline{\rm MS}}(3{\rm GeV})$	1.0595(84)(451)	1.0515(14)(160)

Mass Renormalization Factor

By the Ward identity, the mass renormalization factor is

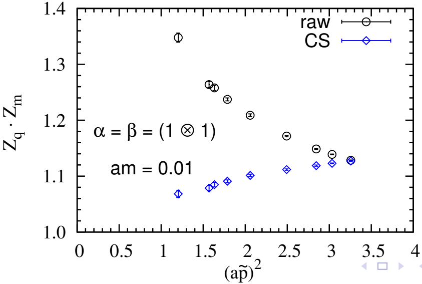
$$Z_m = \frac{1}{Z_{S\otimes S}}$$

where $Z_{S\otimes S}$ is a renormalization factor of scalar bilinear operator with scalar taste.

$$(Z_q \cdot Z_m)^{\text{RI-MOM}} = (\frac{Z_q}{Z_{S\otimes S}})^{\text{RI-MOM}} = \Gamma^B_{S\otimes S}(\tilde{p}, \tilde{p}),$$

We convert the raw data in the RI-MOM scheme from $\mu(= |\widetilde{p}|)$ to the common scale μ_0(3GeV) using four-loop RG evolution factor $U_m(\mu_0, \mu)$.

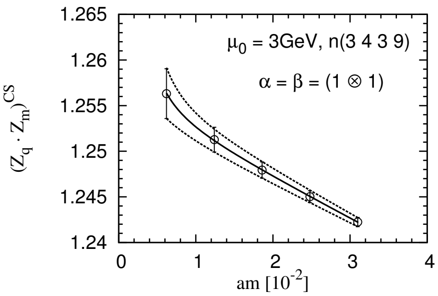
$$(Z_q \cdot Z_m)^{\text{RI-MOM}}(\mu_0) = U_m(\mu_0, \mu)(Z_q \cdot Z_m)^{\text{RI-MOM}}(\mu), \qquad (\mu_0 = 3\text{GeV}, \quad \mu^2 = \tilde{p}^2)$$



m-fit (fitting with respect to quark mass)

We use the following fitting function:

$$f_{Z_q \cdot Z_m}(m, a, \tilde{p}) = c_1 + c_2(am) + c_3 \frac{1}{(am)^2} \,,$$



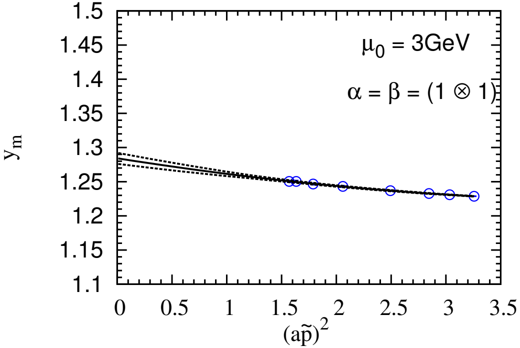
μ_0	c_1	c_2	c_3	χ^2/dof
3GeV	1.25627(89)	-0.251(21)	0.000000061(38)	0.0021(88)

p-fit (fitting with respect to reduced momentum)

We fit the $c_1(a, \widetilde{p})$ to the following fitting function.

$$f_{Z_q \cdot Z_m}(am = 0, a\tilde{p}) = d_1 + d_2(a\tilde{p})^2 + d_3((a\tilde{p})^2)^2 + d_4(a\tilde{p})^4$$

Here, $y_m = (Z_q \cdot Z_m)^{\text{RI-MOM}}(am = 0) - \langle d_4 \rangle (a\tilde{p})^4$



μ_0	d_1	d_2	d_3	d_4	χ^2/dof
3GeV	1.2843(80)	-0.0253(67)	0.0025(12)	0.0115(72)	0.11(15)

Systematic Error

We take the d_1 as $(Z_q \cdot Z_m)$ at 3GeV in RI-MOM scheme. Using Z_q obtained from conserved vector current$(V \otimes S)$, the mass renormalization factor is calculated. We convert the result from RI-MOM scheme to $\overline{\mathrm{MS}}$ scheme using four-loop RG running formula. The systematic error of $Z_m^{\overline{\mathrm{MS}}}$ is estimated in the same way before.

- The first systematic comes from truncated higher order of the four-loop RG running evolution factor(RI-MOM $\rightarrow \overline{\mathrm{MS}}$):$(O(\alpha_s^4))$.

$$E_t = Z_m^{\text{RI-MOM}} \cdot (\alpha_s)^4$$

- The second systematic error comes from the difference between the $Z_m^{\overline{\mathrm{MS}}}$ from $(S \otimes S)$ and $(P \otimes P)$ bilinear operators.

$$E_\Delta = |Z_m^{\overline{\mathrm{MS}}}(S \otimes S) - Z_m^{\overline{\mathrm{MS}}}(P \otimes P)|$$

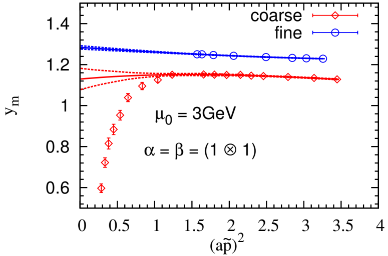
- The total systematic error is obtained adding two systematic errors in quadrature.

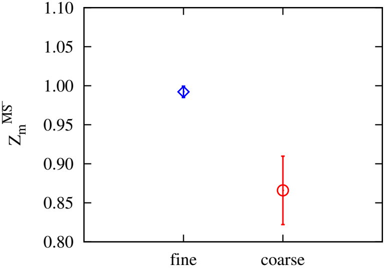
$$E_{tot} = \sqrt{E_t^2 + E_\Delta^2}$$

μ_0	$Z_m^{\overline{\mathrm{MS}}}(S \otimes S)$	E_t	E_Δ	E_{tot}
3GeV	0.9922(58)	0.0044	0.0007	0.0044

Comparison of Z_m between coarse and fine lattices

- coarse lattice : $20^3 \times 64,\ a \approx 0.12fm,\ am_\ell/am_s = 0.01/0.05$
- fine lattice : $28^3 \times 96,\ a \approx 0.08fm,\ am_\ell/am_s = 0.0062/0.031$





	coarse	fine
$Z_m^{\overline{\rm MS}}(3\text{GeV})$	0.866(36)(25)	0.9922(58)(44)