Validity range of canonical approach to finite density QCD

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for Z_n **collaboration**

This talk is based on arXiv:1504.06351 .

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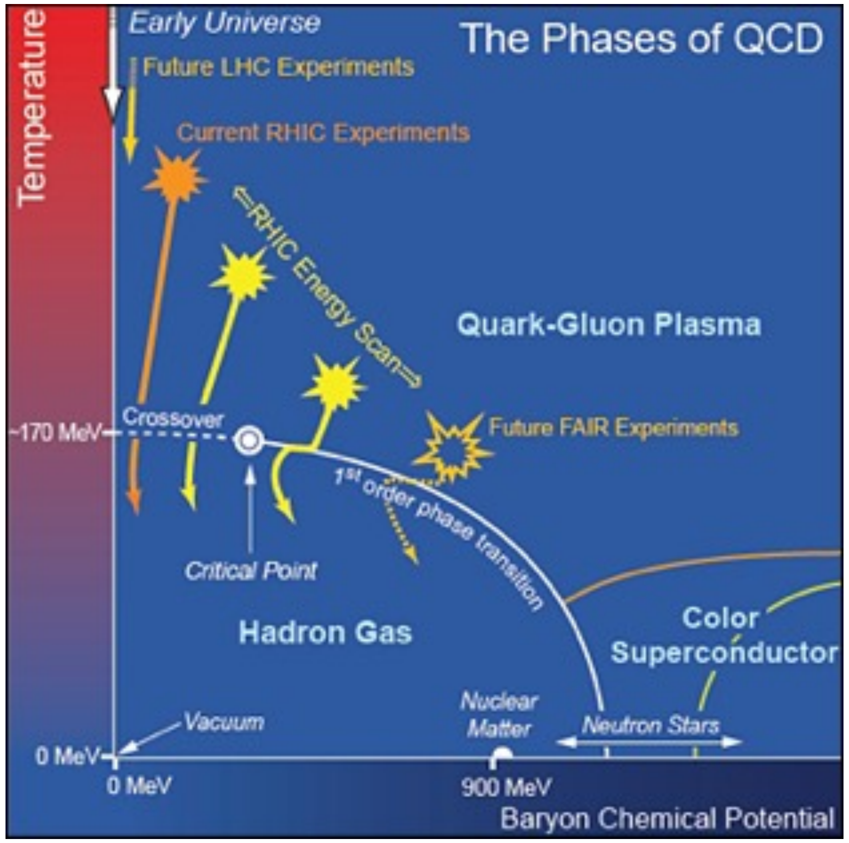
Our motivation



http://www.bnl.gov/physics/news/news.asp?a=1870&t=today

We adopt **canonical approach** to explore finite density QCD.

A. Hasenfratz and D. Toussaint, Nucl. Phys. B **371**, 539-549 (1992)

S. Kratochvila and Ph. de Forcrand, Nucl. Phys. B, Proc. Suppl. **140**, 514 (2005)

Purpose of this work

To calculate thermodynamic observables at finite density while comparing with results obtained by Multi parameter reweighing (MPR) method.

What is the canonical approach ?

Fugacity expansion of grand canonical partition function

$$Z_{GC}(\mu_B, T) = \sum_{n=-\infty}^{\infty} Z_n(T) \left(e^{\mu_B/T} \right)^n$$

μ_B : baryon chemical potential

n : net baryon number

How to get canonical partition function

$$Z_n(T) = \frac{1}{2\pi} \int_0^{2\pi} d\left(\frac{\mu_I}{T}\right) \underline{Z_{GC}(i\mu_I)}\, e^{-in\mu_I/T}$$

$\mu_I \in \mathbb{R}$

How to calculate $Z_n(T)$?

$$Z_{GC}(i\mu_I) = \int [dU] \left(\frac{\det \Delta(i\mu_I)}{\det \Delta(\mu_0)} \right) \underline{\det \Delta(\mu_0)\, e^{-S_g}} = Z_{GC}(\mu_0) \left\langle \frac{\det \Delta(i\mu_I)}{\det \Delta(\mu_0)} \right\rangle_{\mu_0}$$

Monte-Carlo weight

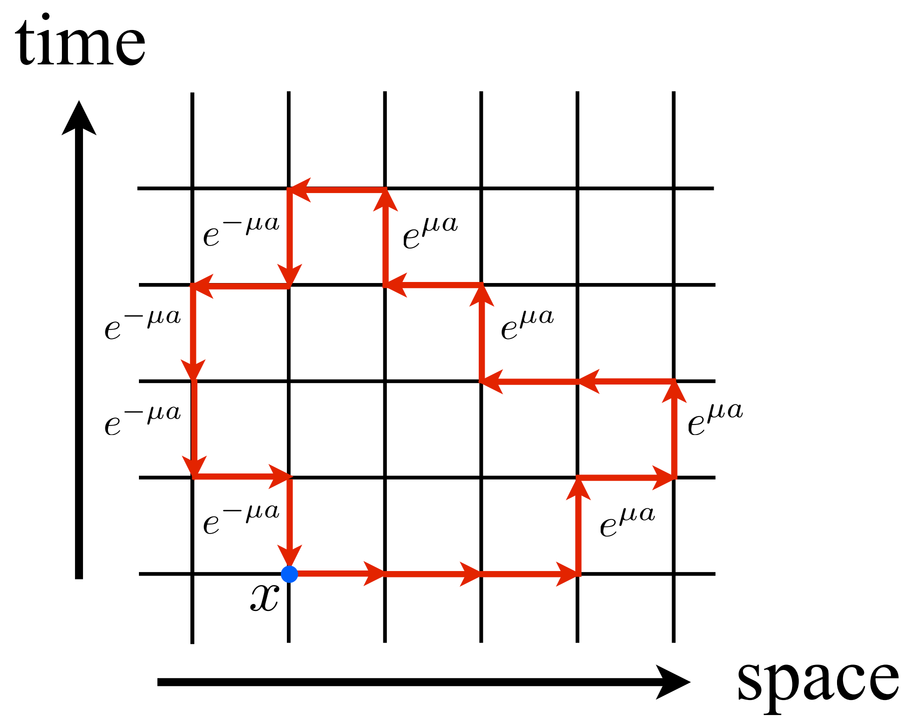
$$Z_n(T) = \underline{Z_{GC}(\mu_0)} \times \frac{1}{2\pi} \int_0^{2\pi} d\left(\frac{\mu_I}{T}\right) \left\langle \frac{\det \Delta(i\mu_I)}{\det \Delta(\mu_0)} \right\rangle_{\mu_0} e^{-in\mu_I/T}$$

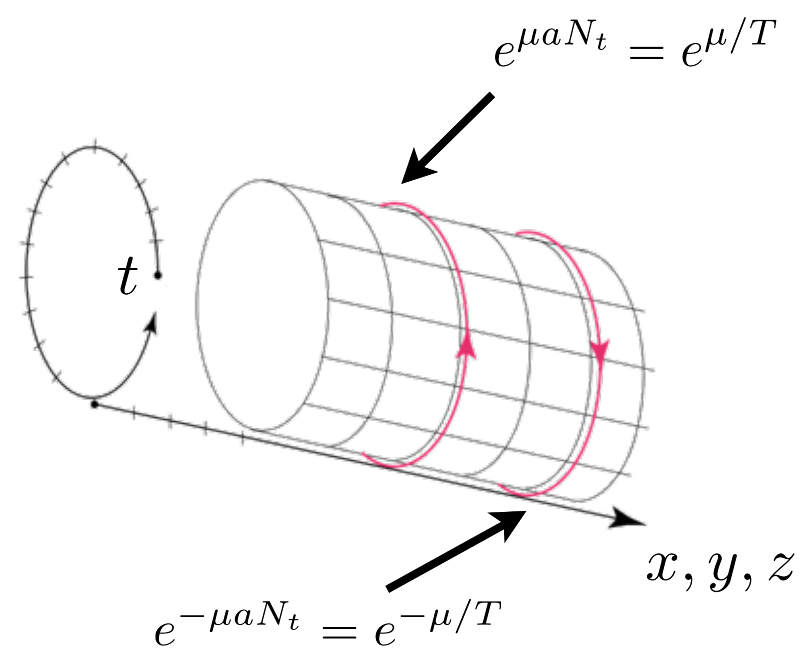
extra constant

To avoid this constant, we consider $\frac{Z_n(T)}{Z_0(T)}$.

We need to evaluate $\left\langle \frac{\det \Delta(i\mu_I)}{\det \Delta(\mu_0)} \right\rangle_{\mu_0}$ at various pure imaginary chemical potentials for discrete Fourier transformation (DFT).

Strategy for calculation of Wilson fermion determinant





$$\log \det \Delta(\mu, U) = \mathrm{Tr} \log \Big(1 - \kappa Q(\mu, U)\Big) \sim \sum_{n=1}^{N} \frac{\kappa^n}{n} \mathrm{Tr} Q^n(\mu, U)$$

$$= \cdots + W_{-1}(U) e^{-\mu/T} + W_0(U) + W_1(U) e^{\mu/T} + \cdots \equiv \sum_{n=-N/N_t}^{N/N_t} W_n(U) e^{n\mu/T}$$

① Generate gauge configuration U at $\mu_0 = 0$.

② Calculate a set of W_n using hopping parameter expansion (HPE).

Note : Sufficiently large N for HPE is needed .

$\mathrm{Tr} Q^n(\mu, U)$ is evaluated through noise method.

③ Calculate $\det \Delta(i\mu_I, U)$ at desired $\mu = i\mu_I$ using W_n .

Our strategy for discrete Fourier transformation

$$\frac{Z_n(T)}{Z_{GC}(\mu_0)} = \frac{1}{2\pi}\int_0^{2\pi} d\left(\frac{\mu_I}{T}\right)\left\langle \frac{\det\Delta(i\mu_I)}{\det\Delta(\mu_0)}\right\rangle_{\mu_0} e^{-in\mu_I/T}$$

$$\sim (\text{const.}) \times \sum_{k=0}^{M-1} Z_{GC}\left(i\frac{\mu_I}{T} = i\frac{2\pi k}{M}\right) e^{-i\frac{2\pi k}{M}n}$$

In case of large *n*, cancellation of significant digits is not negligible in DFT because of oscillatory integral.

We use multiple precision computation for DFT.

Numerical set up

Action

Fermion part : 2-flavor clover improved Wilson fermion action

Gauge part : Iwasaki gauge action

Parameters

(S. Ejiri et al, Phys. Rev. D82:014508, 2010)

$$N_s^3 \times N_t = 8^3 \times 4\,, \quad m_{\rm ps}/m_{\rm v} = 0.8$$

$$T/T_c = 1.35(7), 1.20(6), 1.08(5), 0.99(5), 0.93(5), 0.84(4)$$

Statistics : 400 configurations

Observables

$$\frac{\delta p(\mu_B/T)}{T^4} \equiv \frac{p(\mu_B/T) - p(0)}{T^4}\,, \quad \frac{n_B(\mu_B/T)}{T^3}\,, \quad \frac{\chi(\mu_B/T)}{T^2}$$

Truncation error estimation for fugacity expansion

$$Z_{GC}(\mu_B) = \sum_{n=-\infty}^{\infty} Z_n \, e^{n\mu_B/T} \sim \sum_{n=-N_{\max}}^{N_{\max}} Z_n \, e^{n\mu_B/T}$$

In numerical calculation, the fugacity expansion is finite series.

Truncation error cannot be negligible at large chemical potential.

Our analysis

Choose baryon number $N_{\max}$ and calculate observable $\left\langle O^{(1)} \right\rangle, \left\langle O^{(2)} \right\rangle$.

$$\left\langle O^{(1)}(\mu_B) \right\rangle = \sum_{n=-N_{\max}}^{N_{\max}} O_n Z_n \, e^{n\mu_B/T}$$

$$\left\langle O^{(2)}(\mu_B) \right\rangle = \sum_{n=-N_{\max}+1}^{N_{\max}-1} O_n Z_n e^{n\mu_B/T}$$

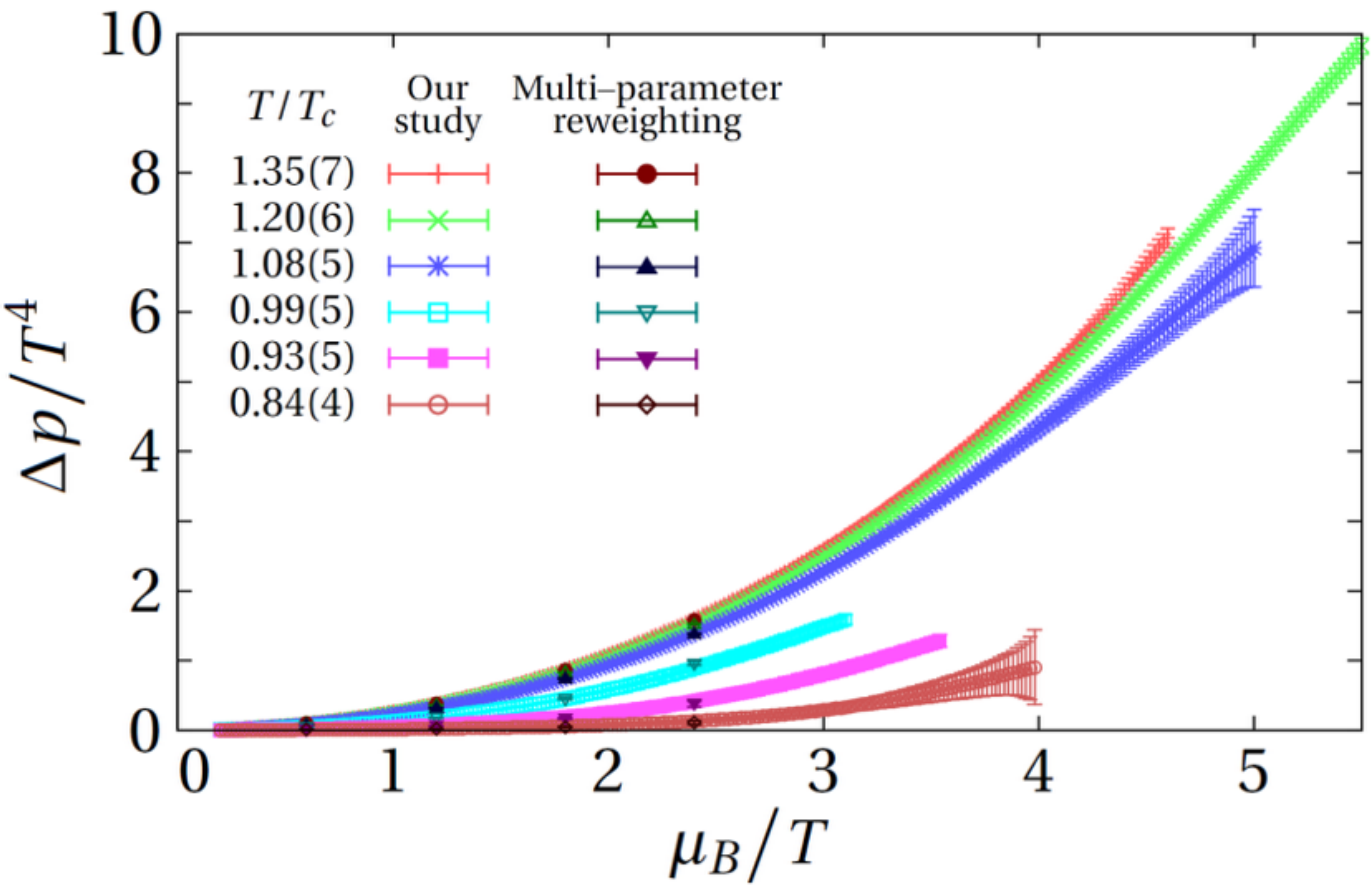
→

Consider chemical potential region where

$$1 - \frac{\left\langle O^{(1)}(\mu_B) \right\rangle}{\left\langle O^{(2)}(\mu_B) \right\rangle} < 10^{-3}$$

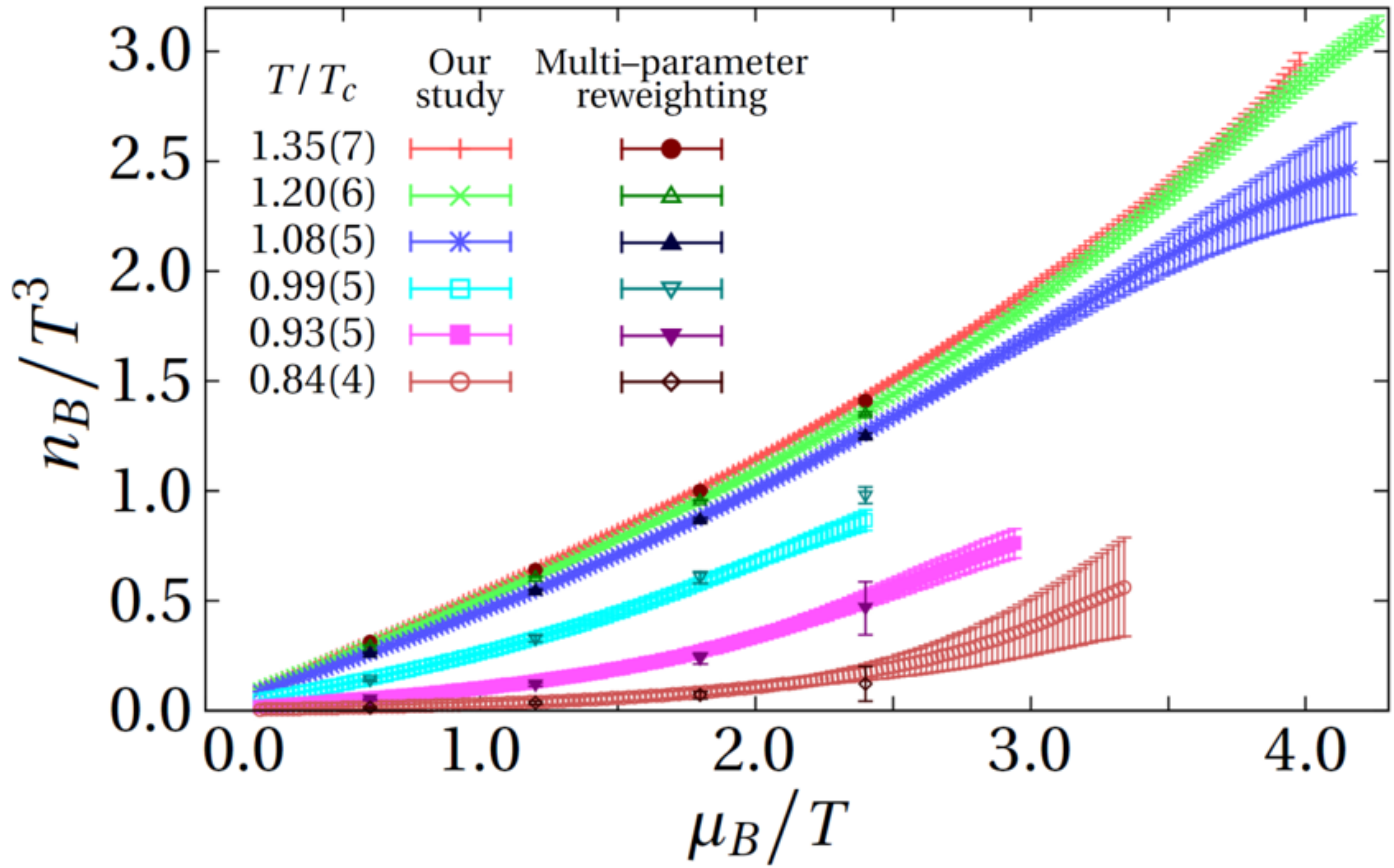
is fulfilled in our work.

Pressure



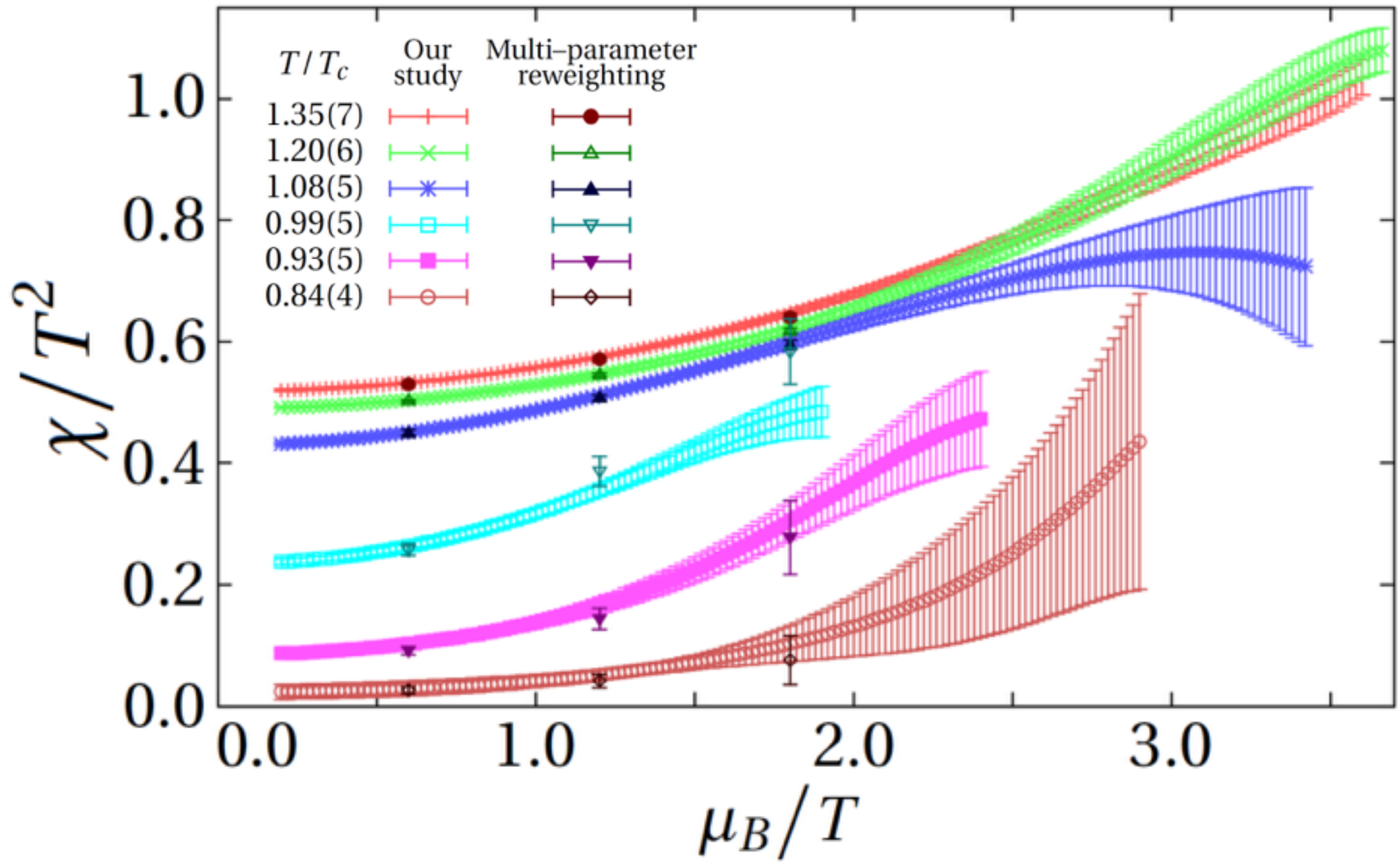
MPR results (statistics 400 conf.) : K.Nagata, A,Nakamura, 10.1007/JHEP04(2012)092

Baryon number density



MPR results (statistics 400 conf.) : K.Nagata, A,Nakamura, 10.1007/JHEP04(2012)092

Baryon susceptibility



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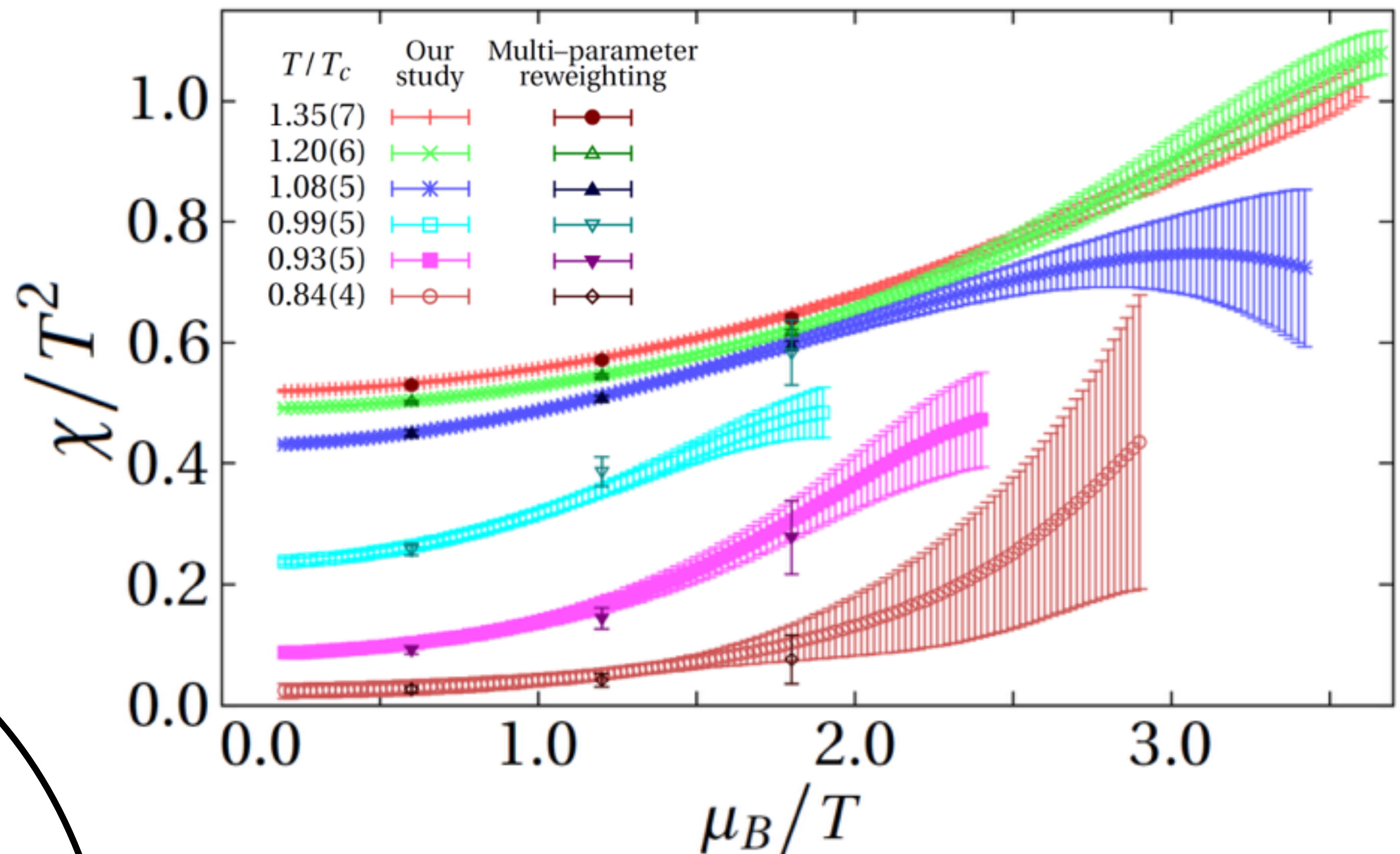
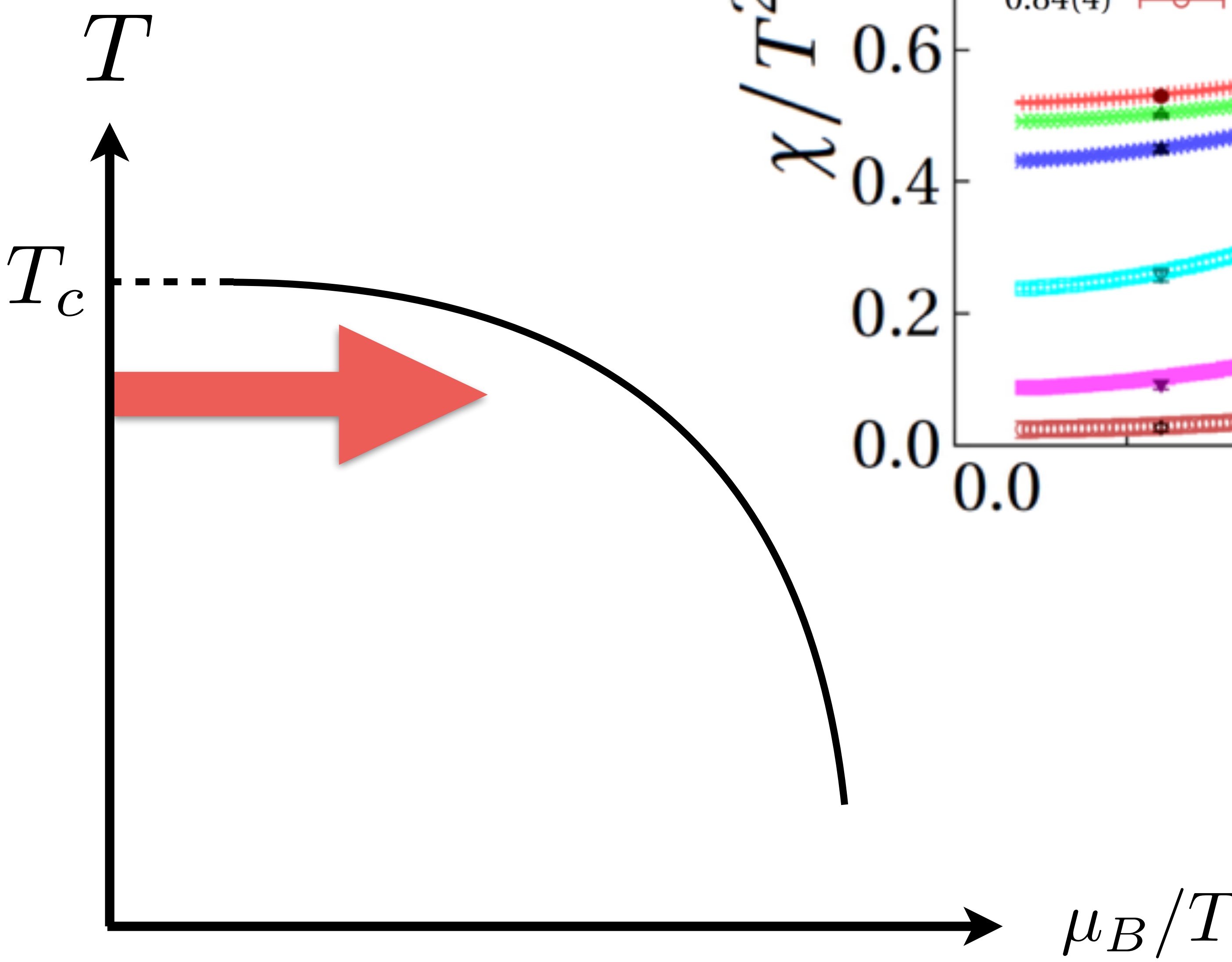




MPR results (statistics 400 conf.) : K.Nagata, A,Nakamura, 10.1007/JHEP04(2012)092

Summary

We checked that the canonical approach can provide consistent results with multi-parameter reweighting method in low chemical potential region.

In many cases, the canonical approach works not only up to $\mu_B/T = 3$ but also for $\mu_B/T > 3$.

There is a possibility that the **canonical approach is a candidate to overcome the "sign problem" .**

Thank you for your attention.

old slides

Is canonical approach useful ?

Absolutely, yes !!

③ We can get thermodynamic observables directly.

$$\frac{p(\mu,T)}{T^4} = \frac{1}{V_s T^3}\log Z_{GC}(\mu,T) = \frac{1}{V_s T^3}\log\left(\sum_n Z_n(T)e^{n\mu/T}\right)$$

$$\frac{n(\mu,T)}{T^3} = \frac{\partial}{\partial(\mu/T)}\frac{p(\mu,T)}{T^4} \quad , \qquad \frac{\chi(\mu,T)}{T^2} = \frac{\partial^2}{\partial(\mu/T)^2}\frac{p(\mu,T)}{T^4}$$

Hopping parameter expansion

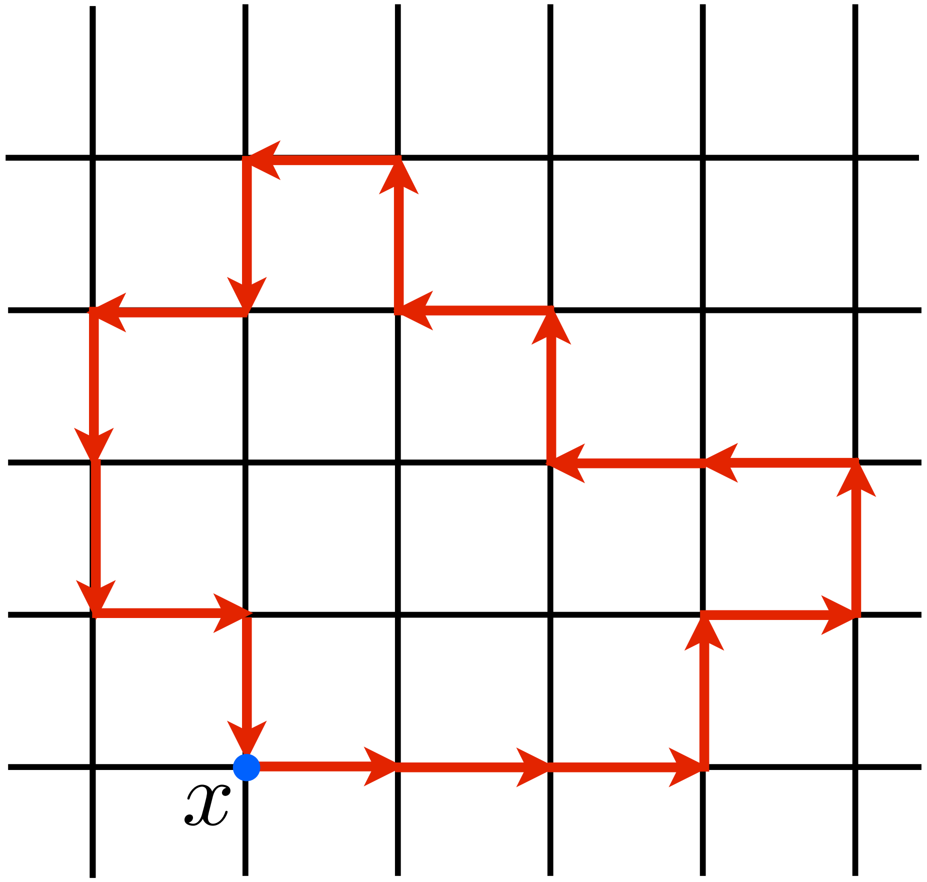
Wilson fermion matrix : $\Delta(\mu) = 1 - \kappa Q(\mu)$

Key identity : $\det \Delta = e^{\mathrm{Tr} \log \Delta} = e^{\mathrm{Tr} \log (1-\kappa Q)}$

$$\mathrm{Tr} \log(1 - \kappa Q(\mu)) = \sum_{n=1}^{\infty} \frac{\kappa^n}{n} \boxed{\mathrm{Tr} Q^n(\mu)}$$

$$\sum_{x,\alpha,a} \langle x, \alpha, a | \, Q^n(\mu) \, | x, \alpha, a \rangle$$

x : space-time α : spinor a : color



Non-zero contribution comes from **closed loops**.

Cancelled significant digits in F.T.