Encoding field theories into gravities

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The 33rd International Symposium on Lattice Field Theory

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Collaborators

K. Kikuchi (YITP), T. Onogi (Osaka Univ.)

References

S. Aoki, K. Kikuchi, T. Onogi,
"Encoding field theories into gravities"
arXiv:1505.00131[hep-th]

S. Aoki, K. Kikuchi, T. Onogi,
"Gradient Flow of O(N) nonlinear sigma model at large N"
JHEP 1504 (2015) 156 (arXiv:1412.8249[hep-th])

K. Kikuchi, Sat. 10:20-, Rm 402

Motivation

AdS/CFT correspondence Maldacena 1997 ⇒ **Gravity/Gauge**

huge numbers of evidences but no proof

open string/closed string duality ?

d dimensional field theory ⇔ d+1 gravity

How can this be possible ?

Different approach

In this talk, we propose a method to encode information of a d dimensional field theory into a d+1 dimensional geometry, and explicitly apply it to the 2-dimensional O(N) non-linear sigma model.

Our proposal

1. d-dimensional field -> (d+1)-dimensional one by gradient flow (GF) equation:

action for d-dim. theory

$$\frac{d}{dt}\phi^{a,\alpha}(t,x) = -g^{ab}(\phi(t,x)) \left.\frac{\delta S}{\delta\varphi^{b,\alpha}(x)}\right|_{\varphi\to\phi},$$

metric

(d+1)-dimensional field

d-dimensional field

initial condition (boundary value) $\phi^{a,\alpha}(0,x) = \varphi^{a,\alpha}(x)$

a : large N index, α: Lorentz index

$z = (\tau, x)$: $d+1$ dimensional coordinate on $\mathbb{R}^+ \times \mathbb{R}^d$ with $\tau = 2\sqrt{t} \geq 0$

2. define the (d+1)-dimensional induced metric as

$$\hat{g}_{\mu\nu}(z) \ := \ g_{ab}(\phi(z))h_{\alpha\beta}\partial_\mu\phi^{a,\alpha}(z)\partial_\nu\phi^{b,\beta}(z)$$

Induced metric from $\mathbb{R}^+ \times \mathbb{R}^d$ on a curved space in $\mathbb{R}^N$ with the metric g_{ab}

any correlation functions can be calculated as

$$\begin{aligned}
\langle \hat{g}_{\mu\nu}(z) \rangle \ &:= \ \langle \hat{g}_{\mu\nu}(z) \rangle_S, \\
\langle \hat{g}_{\mu_1\nu_1}(z_1)\hat{g}_{\mu_2\nu_2}(z_2) \rangle \ &:= \ \langle \hat{g}_{\mu_1\nu_1}(z_1)\hat{g}_{\mu_2\nu_2}(z_2) \rangle_S, \\
\langle \hat{g}_{\mu_1\nu_1}(z_1)\cdots\hat{g}_{\mu_n\nu_n}(z_n) \rangle \ &:= \ \langle \hat{g}_{\mu_1\nu_1}(z_1)\cdots\hat{g}_{\mu_n\nu_n}(z_n) \rangle_S,
\end{aligned}$$

with $\langle \mathcal{O} \rangle_S := \dfrac{1}{Z}\displaystyle\int \mathcal{D}\varphi \, \mathcal{O}(\varphi) \, e^{-S}, \quad Z := \int \mathcal{D}\varphi \, e^{-S}$

functional integral in d-dimensions

Key point 1

$g_{\mu\nu}(z) \sim \partial_\mu \phi(z) \partial_\nu \phi(z)$ is expected to be finite as long as $\tau \neq 0$

GF: a kind of RG transformation (heat kernel type smearing)

$\tau \to 0$ is UV while $\tau \to \infty$ is IR

cf. d dimensional induced metric $g_{\mu\nu}(x) \sim \partial_\mu \varphi(x) \partial_\nu \varphi(x)$ is badly divergent

Key point 2

the metric operator becomes classical in the large N limit

$$\langle \hat{g}_{\mu\nu}(z_1) \hat{g}_{\alpha\beta}(z_2) \rangle = \langle \hat{g}_{\mu\nu}(z_1) \rangle \langle \hat{g}_{\alpha\beta}(z_2) \rangle + O\left(\frac{1}{N}\right)$$

thanks to the large N factorization

Our proposal gives a correspondence between d-dimesnional field theory and a (d+1)-dimensional classical metric in the large N limit.

From the metric, we can determine the geometry of the (d+1)-dimensional space.

Example

O(N) non-linear sigma model in 2 dimensions

$$S = \frac{1}{2g^2}\int d^2x \sum_{a,b=1}^{N-1} g_{ab}(\varphi)\sum_{k=1}^{2}\left(\partial_k\varphi^a(x)\partial^k\varphi^b(x)\right),$$

$$g_{ab}(\varphi) = \delta_{ab} + \frac{\varphi^a\varphi^b}{1-\varphi\cdot\varphi}, \quad g^{ab}(\varphi) = \delta_{ab} - \varphi^a\varphi^b$$

$$\varphi\cdot\varphi = \sum_{a=1}^{N-1}\varphi^a\varphi^a$$

$$\varphi^N = \pm\sqrt{1-\varphi\cdot\varphi},$$

gap equation

$$1 = \lambda\int\frac{d^2q}{(2\pi)^2}\frac{1}{q^2+m^2}$$

$\lambda = g^2N$ is the 't Hooft coupling constant

Solution to the GF equation in the large N

Aoki-Kikuchi-Onogi, JHEP1504(2015)156(arXiv:1412.8249[hep-th])

$$\phi^a(t,p) = f(t)e^{-p^2 t}\sum_{n=0}^{\infty} : X_{2n+1}(\varphi,p,t) :$$

X_{2n+1} only contains φ^{2n+1} terms and is $O(1/N^{2n+1})$.

$$X_1^a(\varphi,p,t) = \varphi^a(p)$$

$$f(t) = e^{-m^2 t}\sqrt{\frac{\log(1+\Lambda^2/m^2)}{\mathrm{Ei}\left(-2t(\Lambda^2+m^2)\right) - \mathrm{Ei}\left(-2tm^2\right)}},$$

$$\mathrm{Ei}(-x) = \int d\,x\, e^{-x}/x.$$

2-pt function

$$\langle \phi^a(t,x)\phi^b(s,y)\rangle_S = \int \frac{d^2q}{(2\pi)^2}\frac{e^{-q^2(t+s)}e^{iq(x-y)}}{q^2+m^2} \times \delta_{ab}\frac{\lambda}{N} f(t)f(s) + O(N^{-2}).$$

Induced metric

$\mathbb{R}^+ \times \mathbb{R}^2 \to S^{N-1}$ ($N-1$ dimensional sphere)

$$\hat{g}_{\mu\nu}(z) \ := \ g_{ab}(\phi(z))h_{\alpha\beta}\partial_\mu \phi^{a,\alpha}(z)\partial_\nu \phi^{b,\beta}(z)$$

VEV $$g_{\mu\nu}(z) := \langle \hat{g}_{\mu\nu}(z) \rangle$$

$$g_{\tau\tau}(z) = -\frac{\tau h}{4}\frac{d}{d\tau}\left(\frac{\dot{f}}{f}\right) + O\left(\frac{1}{N}\right) \qquad g_{ij}(z) = \delta_{ij}\frac{h}{2}\frac{\dot{f}}{f} + O\left(\frac{1}{N}\right)$$

$$g_{\mu\nu}(\tau) = \begin{pmatrix} B(\tau) & 0 & 0 \\ 0 & A(\tau) & 0 \\ 0 & 0 & A(\tau) \end{pmatrix}$$

$$2B(\tau) \ = \ -\tau dA(\tau)/d\tau$$

$$A(\tau) \ = \ \frac{h}{2}\left[-m^2 - \frac{e^{-\tau^2 m^2/2}}{\mathrm{E_i}(-\tau^2 m^2/2)\tau^2/2}\right]$$

finite as long as $\tau > 0$

Einstein equation and energy momentum tensor

$$\langle G_{\mu\nu}(\hat{g}_{\mu\nu})\rangle = G_{\mu\nu}(\langle \hat{g}_{\mu\nu}\rangle) \text{ in the large } N \text{ limit}$$

Assume the Einstein equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$T_{\mu\nu}(\tau) = \begin{pmatrix} T_{\tau\tau}(\tau) & 0 & 0 \\ 0 & T_{ss}(\tau) & 0 \\ 0 & 0 & T_{ss}(\tau) \end{pmatrix}$$

$$T_{\tau\tau}(\tau) = \frac{A_{,\tau}^2}{32\pi G A^2} \qquad A_{\tau}(\tau) := \frac{d}{d\tau}A(\tau)$$

$$T_{ss}(\tau) = \frac{1}{16\pi G}\left[\frac{1}{\tau^2} + \frac{A_{,\tau}}{\tau A} - \frac{A_{,\tau\tau}}{\tau A_{,\tau}}\right]$$

Masslessi limit and AdS space

$m \to 0$ (massless) limit

$$A = -\frac{h}{\tau^2 \log m^2}$$

Take $h = -R_0^2 \log(m^2 R_0^2)$ $\quad R_0$: mass dimension -1 (length scale)

$$g_{\tau\tau} = \frac{R_0^2}{\tau^2}, \quad g_{ij} = \delta_{ij} \frac{R_0^2}{\tau^2}$$

$$\Rightarrow \quad ds^2 = \frac{R_0^2}{\tau^2} \left[d\tau^2 + (d\vec{x})^2 \right]$$

metric of (Euclidean) AdS space

$\Rightarrow$ **AdS space emerges in the massless limit (CFT) !**

$$G_{\mu\nu} = -\Lambda_0 g_{\mu\nu}, \qquad \Lambda_0 = -\frac{1}{R_0^2} < 0$$

UV limit

$$m\tau \to 0 \text{ limit (UV limit)}$$

$$A \simeq -\frac{h}{\tau^2 \log(m^2\tau^2)}$$

UV singularity of 2-dim. original theory

$$ds^2 = \frac{R_0^2 \log(m^2 R_0^2)}{\tau^2 \log(m^2\tau^2)} \left[d\tau^2 + (d\vec{x})^2\right]$$

$$\Lambda_0^{\text{eff}} = -\frac{1}{R_0^2} \frac{\log(m^2\tau^2)}{\log(m^2 R_0^2)}$$

effective cosmological constant

IR limits

$$m\tau \to \infty \qquad A \simeq \frac{h}{\tau^2}$$

$$ds^2 = -\frac{R_0^2 \log(m^2 R_0^2)}{\tau^2} \left[d\tau^2 + (d\vec{x})^2\right]$$

$$\Lambda_0^{\text{eff}} = \frac{1}{R_0^2 \log(m^2 R_0^2)}$$

asymptotically AdS if $\log(m^2 R_0^2) < 0$

energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{8\pi G \tau^2}$$

in UV/IR/massless limits

Future studies and open issues

dictionary between geometry and field theory

GR -> FT

As $m\tau \to 0/\infty$, $\quad g_{\mu\nu} \sim \dfrac{1}{\tau^2 \log(m^2\tau^2)}$ ⇒ we can read off mass m and Z-factor

other quantities ?

FT-> GR interpretation of $T_{\mu\nu}^{\text{matter}}$?

$$T_{\mu\nu}^{\text{matter}} := T_{\mu\nu} + g_{\mu\nu}\frac{\Lambda_0}{8\pi G}$$

⇒ $T_{\mu\nu}^{\text{matter}} \to 0$ **in UV/IR/massless limits**

other examples

Our method can be applied to all large N models, and better if solvable.

Translational invariance of d dimensional theory → $g_{\mu\nu}$ depends only on τ.

Introduce boundaries/sources to create x-dependences

Finite T field theories => the black hole geometry ? (work in progress)

quantum fluctuations of the metric

Correlation functions can be calculated by the 1/N expansion.

ex. $$\langle g_{\mu_1\nu_1}(z_1) g_{\mu_2\nu_2}(z_2)\rangle_c = O\left(\frac{1}{N}\right)$$

renormalizable or even finite ?

All correlation functions

⇒ definition of "quantum theory" around the classical metric ?

effective action for "quantum theory" ?

choice for GF scheme

not unique

⇒ may give different gravity theories, but they are all equivalent.

new equivalent classes ?

Gauge theories

simple choices for the induced metric

$$g_{\mu\nu}(z) := h\sum_{i,j=1}^{d} \mathrm{Tr}\ D_\mu F_{ij}(z) D_\nu F^{ij}(z),$$

$$g_{\mu\nu}(z) := h\sum_{\alpha=0}^{d} \mathrm{Tr}\ F_{\mu\alpha}(z) F_\nu{}^\alpha(z),$$

$D_i \ (i = 1, \cdots, d)$

$D_\tau = \partial_\tau$

covariant derivative

$F_{\mu\nu} := [D_\mu, D_\nu]$

Field strength

invariant under τ-independent gauge transformation

large N gauge theory in 2-dim ('t Hooft model)