

Encoding field theories into gravities

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LATTICE 2015

The 33rd International Symposium on Lattice Field Theory
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Collaborators

K. Kikuchi (YITP), T. Onogi (Osaka Univ.)

References

S. Aoki, K. Kikuchi, T. Onogi,
“Encoding field theories into gravities”
arXiv:1505.00131 [hep-th]

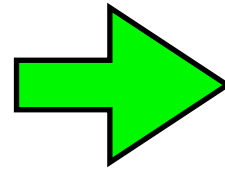
S. Aoki, K. Kikuchi, T. Onogi,
“Gradient Flow of $O(N)$ nonlinear sigma model at large N ”
JHEP 1504 (2015) 156 (arXiv:1412.8249[hep-th])

K. Kikuchi, Sat. 10:20-, Rm 402

Motivation

AdS/CFT correspondence

Maldacena 1997

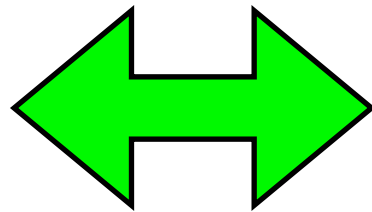


Gravity/Gauge

huge numbers of evidences but no proof

open string/closed string duality ?

d dimensional field theory



d+1 gravity

How can this be possible ?

Different approach

In this talk, we propose a method to encode information of a d dimensional field theory into a d+1 dimensional geometry, and explicitly apply it to the 2-dimensional $O(N)$ non-linear sigma model.

2. define the (d+1)-dimensional **induced metric** as

$$\hat{g}_{\mu\nu}(z) := g_{ab}(\phi(z)) h_{\alpha\beta} \partial_\mu \phi^{a,\alpha}(z) \partial_\nu \phi^{b,\beta}(z)$$

Induced metric from $\mathbb{R}^+ \times \mathbb{R}^d$ on a curved space in \mathbb{R}^N with the metric g_{ab}

any correlation functions can be calculated as

$$\langle \hat{g}_{\mu\nu}(z) \rangle := \langle \hat{g}_{\mu\nu}(z) \rangle_S,$$

$$\langle \hat{g}_{\mu_1\nu_1}(z_1) \hat{g}_{\mu_2\nu_2}(z_2) \rangle := \langle \hat{g}_{\mu_1\nu_1}(z_1) \hat{g}_{\mu_2\nu_2}(z_2) \rangle_S,$$

$$\langle \hat{g}_{\mu_1\nu_1}(z_1) \cdots \hat{g}_{\mu_n\nu_n}(z_n) \rangle := \langle \hat{g}_{\mu_1\nu_1}(z_1) \cdots \hat{g}_{\mu_n\nu_n}(z_n) \rangle_S,$$

$$\text{with } \langle \mathcal{O} \rangle_S := \frac{1}{Z} \int \mathcal{D}\varphi \mathcal{O}(\varphi) e^{-S}, \quad Z := \int \mathcal{D}\varphi e^{-S}$$

functional integral in d-dimensions

Key point 1

$g_{\mu\nu}(z) \sim \partial_\mu \phi(z) \partial_\nu \phi(z)$ is expected to be finite as long as $\tau \neq 0$

GF: a kind of RG transformation (heat kernel type smearing)

$\tau \rightarrow 0$ is UV while $\tau \rightarrow \infty$ is IR

cf. d dimensional induced metric $g_{\mu\nu}(x) \sim \partial_\mu \varphi(x) \partial_\nu \varphi(x)$ is badly divergent

Key point 2

the metric operator becomes **classical** in the large N limit

$$\langle \hat{g}_{\mu\nu}(z_1) \hat{g}_{\alpha\beta}(z_2) \rangle = \langle \hat{g}_{\mu\nu}(z_1) \rangle \langle \hat{g}_{\alpha\beta}(z_2) \rangle + O\left(\frac{1}{N}\right)$$

thanks to the large N factorization

Our proposal gives a correspondence between d -dimensional field theory and a $(d+1)$ -dimensional classical metric in the large N limit.

From the metric, we can determine the geometry of the $(d+1)$ -dimensional space.

Example

O(N) non-linear sigma model in 2 dimensions

$$S = \frac{1}{2g^2} \int d^2x \sum_{a,b=1}^{N-1} g_{ab}(\varphi) \sum_{k=1}^2 (\partial_k \varphi^a(x) \partial^k \varphi^b(x)),$$

$$g_{ab}(\varphi) = \delta_{ab} + \frac{\varphi^a \varphi^b}{1 - \varphi \cdot \varphi}, \quad g^{ab}(\varphi) = \delta_{ab} - \varphi^a \varphi^b$$

$$\varphi \cdot \varphi = \sum_{a=1}^{N-1} \varphi^a \varphi^a$$

$$\varphi^N = \pm \sqrt{1 - \varphi \cdot \varphi},$$

gap equation

$$1 = \lambda \int \frac{d^2q}{(2\pi)^2} \frac{1}{q^2 + m^2}$$

$\lambda = g^2 N$ is the 't Hooft coupling constant

Solution to the GF equation in the large N

Aoki-Kikuchi-Onogi, JHEP1504(2015)156(arXiv:1412.8249[hep-th])

$$\phi^a(t, p) = f(t) e^{-p^2 t} \sum_{n=0}^{\infty} : X_{2n+1}(\varphi, p, t) :$$

X_{2n+1} only contains φ^{2n+1} terms and is $O(1/N^{2n+1})$.

$$X_1^a(\varphi, p, t) = \varphi^a(p)$$

$$f(t) = e^{-m^2 t} \sqrt{\frac{\log(1 + \Lambda^2/m^2)}{\text{Ei}(-2t(\Lambda^2 + m^2)) - \text{Ei}(-2tm^2)}}$$

$$\text{Ei}(-x) = \int dx e^{-x}/x.$$

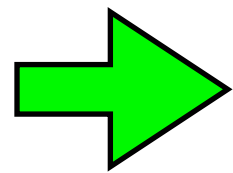
2-pt function

$$\begin{aligned} \langle \phi^a(t, x) \phi^b(s, y) \rangle_S &= \int \frac{d^2 q}{(2\pi)^2} \frac{e^{-q^2(t+s)} e^{iq(x-y)}}{q^2 + m^2} \\ &\times \delta_{ab} \frac{\lambda}{N} f(t) f(s) + O(N^{-2}). \end{aligned}$$

Induced metric $\mathbb{R}^+ \times \mathbb{R}^2 \rightarrow S^{N-1}$ ($N - 1$ dimensional sphere)

$$\hat{g}_{\mu\nu}(z) := g_{ab}(\phi(z)) h_{\alpha\beta} \partial_\mu \phi^{a,\alpha}(z) \partial_\nu \phi^{b,\beta}(z)$$

VEV $g_{\mu\nu}(z) := \langle \hat{g}_{\mu\nu}(z) \rangle$



$$g_{\tau\tau}(z) = -\frac{\tau h}{4} \frac{d}{d\tau} \left(\frac{\dot{f}}{f} \right) + O\left(\frac{1}{N}\right)$$

$$g_{ij}(z) = \delta_{ij} \frac{h}{2} \frac{\dot{f}}{f} + O\left(\frac{1}{N}\right)$$

$$g_{\mu\nu}(\tau) = \begin{pmatrix} B(\tau) & 0 & 0 \\ 0 & A(\tau) & 0 \\ 0 & 0 & A(\tau) \end{pmatrix}$$

$$2B(\tau) = -\tau dA(\tau)/d\tau$$

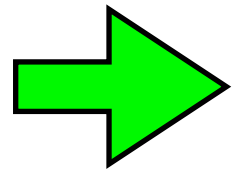
$$A(\tau) = \frac{h}{2} \left[-m^2 - \frac{e^{-\tau^2 m^2/2}}{\text{Ei}(-\tau^2 m^2/2) \tau^2/2} \right]$$

finite as long as $\tau > 0$

Einstein equation and energy momentum tensor

$$\langle G_{\mu\nu}(\hat{g}_{\mu\nu}) \rangle = G_{\mu\nu}(\langle \hat{g}_{\mu\nu} \rangle) \text{ in the large } N \text{ limit}$$

Assume the Einstein equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$



$$T_{\mu\nu}(\tau) = \begin{pmatrix} T_{\tau\tau}(\tau) & 0 & 0 \\ 0 & T_{ss}(\tau) & 0 \\ 0 & 0 & T_{ss}(\tau) \end{pmatrix}$$

$$T_{\tau\tau}(\tau) = \frac{A_{,\tau}^2}{32\pi G A^2}$$

$$A_{,\tau}(\tau) := \frac{d}{d\tau} A(\tau)$$

$$T_{ss}(\tau) = \frac{1}{16\pi G} \left[\frac{1}{\tau^2} + \frac{A_{,\tau}}{\tau A} - \frac{A_{,\tau\tau}}{\tau A_{,\tau}} \right]$$

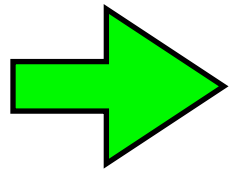
Massless limit and AdS space

$m \rightarrow 0$ (massless) limit

$$A = -\frac{h}{\tau^2 \log m^2}$$

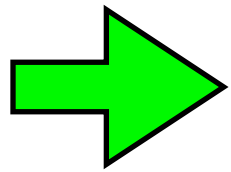
Take $h = -R_0^2 \log(m^2 R_0^2)$ R_0 : mass dimension -1 (length scale)

$$g_{\tau\tau} = \frac{R_0^2}{\tau^2}, \quad g_{ij} = \delta_{ij} \frac{R_0^2}{\tau^2}$$



$$ds^2 = \frac{R_0^2}{\tau^2} [d\tau^2 + (d\vec{x})^2]$$

metric of (Euclidean) AdS space



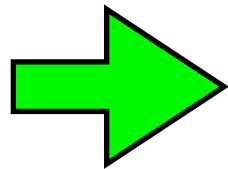
AdS space emerges in the massless limit (CFT) !

$$G_{\mu\nu} = -\Lambda_0 g_{\mu\nu}, \quad \Lambda_0 = -\frac{1}{R_0^2} < 0$$

UV limit $m\tau \rightarrow 0$ limit (UV limit)

$$A \simeq -\frac{h}{\tau^2 \log(m^2 \tau^2)}$$

UV singularity of 2-dim. original theory

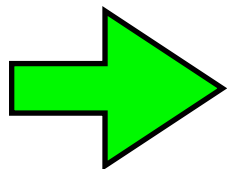


$$ds^2 = \frac{R_0^2 \log(m^2 R_0^2)}{\tau^2 \log(m^2 \tau^2)} [d\tau^2 + (d\vec{x})^2]$$

$$\Lambda_0^{\text{eff}} = -\frac{1}{R_0^2} \frac{\log(m^2 \tau^2)}{\log(m^2 R_0^2)}$$

effective cosmological constant**IR limits** $m\tau \rightarrow \infty$

$$A \simeq \frac{h}{\tau^2}$$



$$ds^2 = -\frac{R_0^2 \log(m^2 R_0^2)}{\tau^2} [d\tau^2 + (d\vec{x})^2]$$

$$\Lambda_0^{\text{eff}} = \frac{1}{R_0^2 \log(m^2 R_0^2)}$$

asymptotically AdS if

$$\log(m^2 R_0^2) < 0$$

energy-momentum tensor

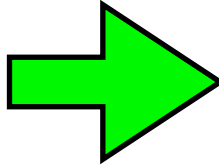
$$T_{\mu\nu} = \frac{1}{8\pi G\tau^2}$$

in UV/IR/massless limits

Future studies and open issues

dictionary between geometry and field theory

GR \rightarrow FT

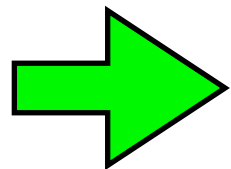
As $m\tau \rightarrow 0/\infty$, $g_{\mu\nu} \sim \frac{1}{\tau^2 \log(m^2\tau^2)}$  we can read off mass m and Z -factor

other quantities ?

FT \rightarrow GR

interpretation of $T_{\mu\nu}^{\text{matter}}$?

$$T_{\mu\nu}^{\text{matter}} := T_{\mu\nu} + g_{\mu\nu} \frac{\Lambda_0}{8\pi G}$$



$$T_{\mu\nu}^{\text{matter}} \rightarrow 0$$

in UV/IR/massless limits

other examples

Our method can be applied to all large N models, and better if solvable.

Translational invariance of d dimensional theory  $g_{\mu\nu}$ depends only on τ .

Introduce boundaries/sources to create x-dependences

Finite T field theories \Rightarrow the black hole geometry ? (work in progress)

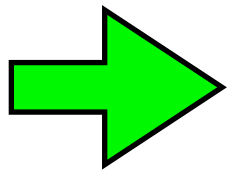
quantum fluctuations of the metric

Correlation functions can be calculated by the $1/N$ expansion.

$$\text{ex. } \langle g_{\mu_1\nu_1}(z_1)g_{\mu_2\nu_2}(z_2) \rangle_c = O\left(\frac{1}{N}\right)$$

renormalizable or even finite ?

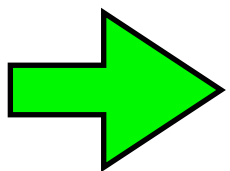
All correlation functions



definition of “quantum theory” around the classical metric ?
effective action for “quantum theory” ?

choice for GF scheme

not unique



may give different gravity theories, but they are all equivalent.

new equivalent classes ?

Gauge theories

simple choices for the induced metric

$$g_{\mu\nu}(z) := h \sum_{i,j=1}^d \text{Tr} D_\mu F_{ij}(z) D_\nu F^{ij}(z),$$

$$D_i \quad (i = 1, \dots, d)$$

$$D_\tau = \partial_\tau$$

covariant derivative

$$g_{\mu\nu}(z) := h \sum_{\alpha=0}^d \text{Tr} F_{\mu\alpha}(z) F_\nu^\alpha(z),$$

$$F_{\mu\nu} := [D_\mu, D_\nu]$$

Field strength

invariant under τ -independent gauge transformation

large N gauge theory in 2-dim ('t Hooft model)