Analytic computations of an effective lattice theory for heavy quarks

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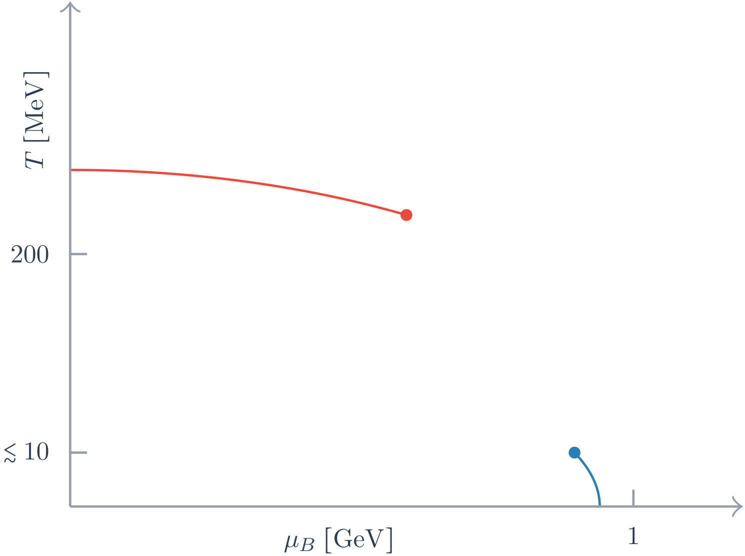
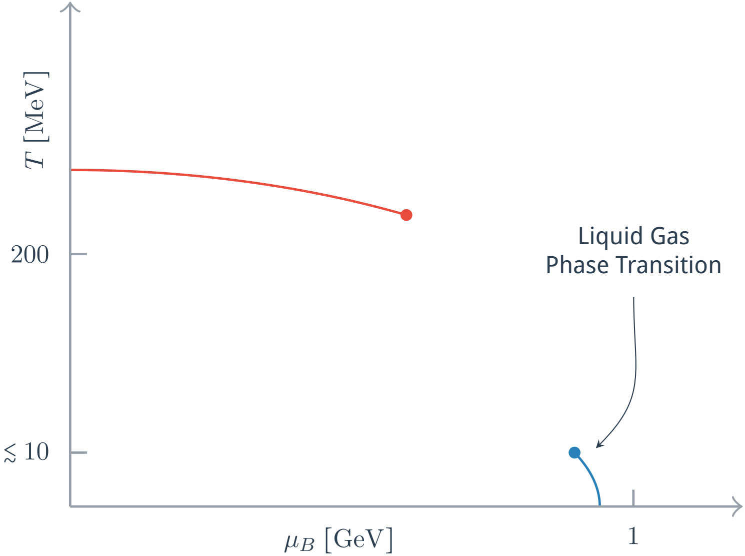
Heavy QCD Phase Diagram



Heavy QCD Phase Diagram



Advantages of the Effective Theory

- Dimensionally reduced theory
  - 4D $\rightarrow$ 3D
  - $U_\mu(x) \rightarrow L(x)$
- Very mild sign problem, most gauge fields integrated analytically
- Want to study the very dense limit, liquid gas transition

The Effective Theory

The Effective Lattice Theory

Effective Theory

- Integrate out all spatial gauge links

$$\mathcal{Z} = \int DU_\mu \exp\left\{-S_{\text{action}}\right\} = \int DU_0 \exp\left\{-S_{\text{effective action}}\right\}$$

Using:

- The strong coupling expansion
- The hopping parameter expansion

Effective Theory

$$\mathcal{Z} = \int \prod_x \mathrm{d}L(x)\, \exp\left\{-S_{\text{eff action}}\right\} \qquad (\dagger)$$

- Previous Talk: Monte Carlo simulations of (†)
- Current Talk: Analytic calculation of $\mathcal{Z}$

The Effective Theory Action

$$S_{\text{eff action}} = S_0\left[L\right] + S_I\left[L\right]$$

Where $S_I\left[L\right]$ is made up of interactions at varying distances

$$S_I\left[L\right] = \sum_{\text{terms}} \sum_{\text{dof}} v_i(1, 2, ..., n_i)\phi_1\left[L\right]\phi_2\left[L\right] \cdots \phi_{n_i}\left[L\right]$$

The Effective Theory Action

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Where $S_I[L]$ is made up of interactions at varying distances

$$S_I[L] = \sum_{\text{terms}} \sum_{\text{dof}} v_i(1, 2, \ldots, n_i)\phi_1[L]\phi_2[L] \cdots \phi_{n_i}[L]$$

Can be represented with connected graphs

The Effective Theory Action

$$S_I[L] = \sum_{\text{terms}} \sum_{\text{dof}} v_i(1, 2, ..., n_i)\phi_1[L]\phi_2[L] \cdots \phi_{n_i}[L]$$

In our theory:

- $v_i(1, 2, ...n_i) \rightarrow \{\lambda_i, h_i\} \times \text{geometry}$
- $\phi_i \rightarrow \{L_i, L_i^*, W_i\}$

Analytic Calculations

N-point Linked Cluster Expansion

Classical Linked Cluster Expansion

The action consists of two-point interactions which can be expanded in a set of connected graphs.

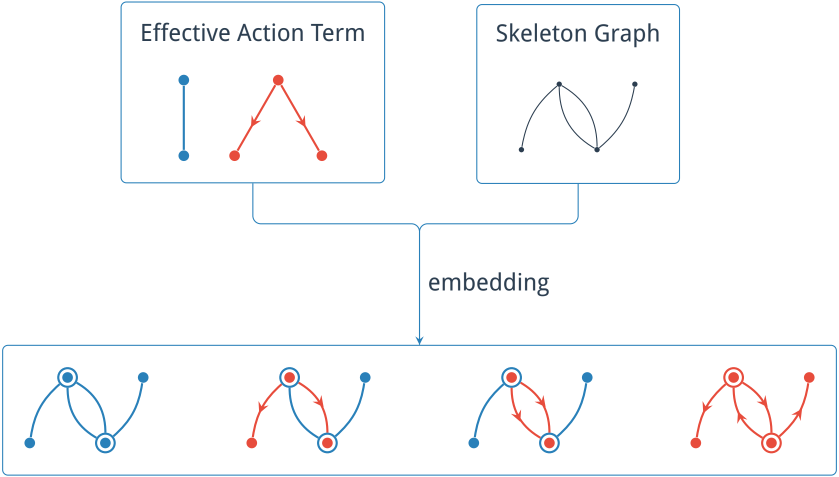
Our Problem

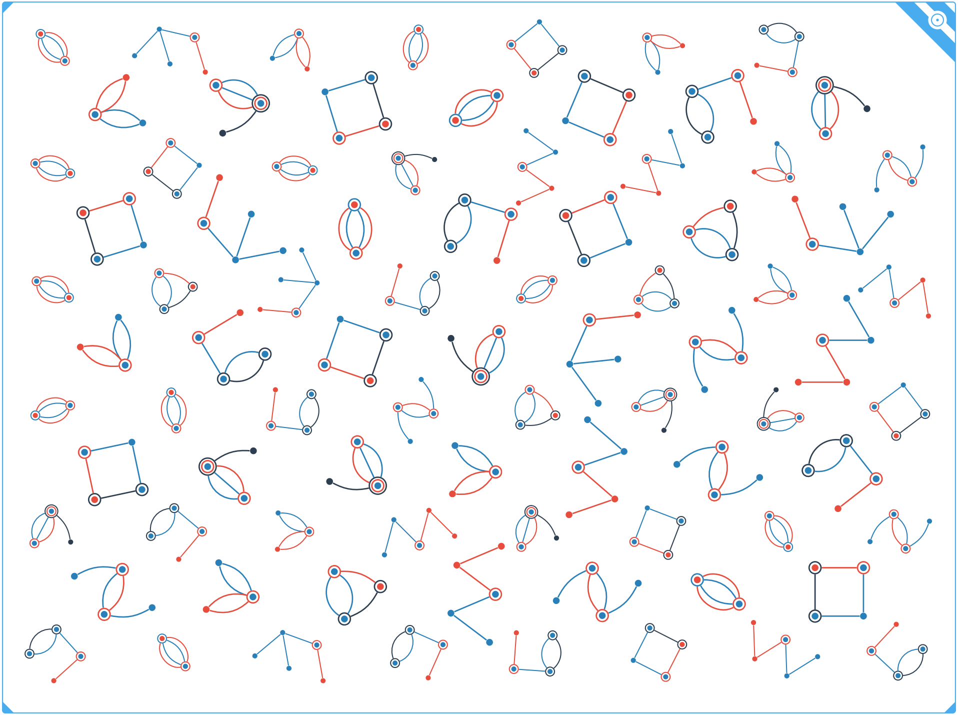
The action contains n-point interactions that we can embed on a set of connected graphs.

$\hookrightarrow$ Two step embedding

Analytic Calculations

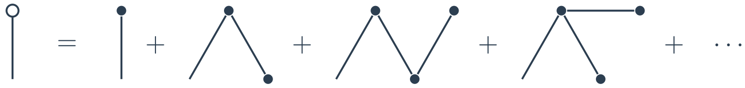
N-point Linked Cluster Expansion





The power of resummations

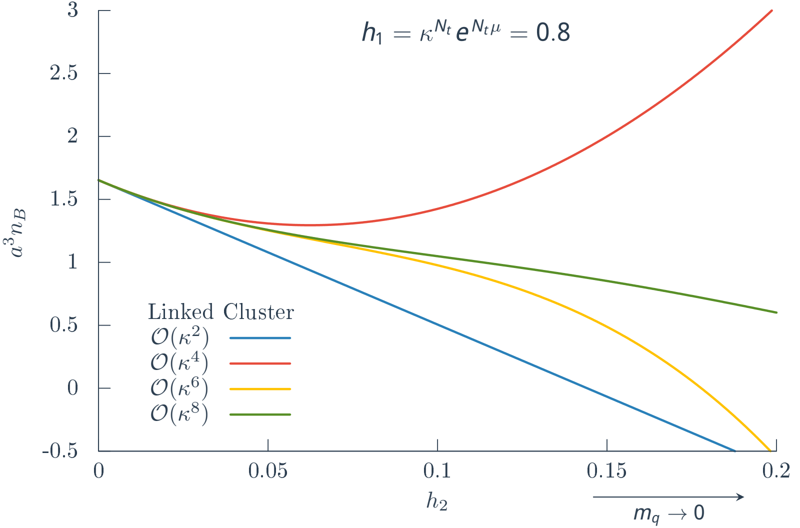
Using the resummed Linked Cluster Expansion as motivation



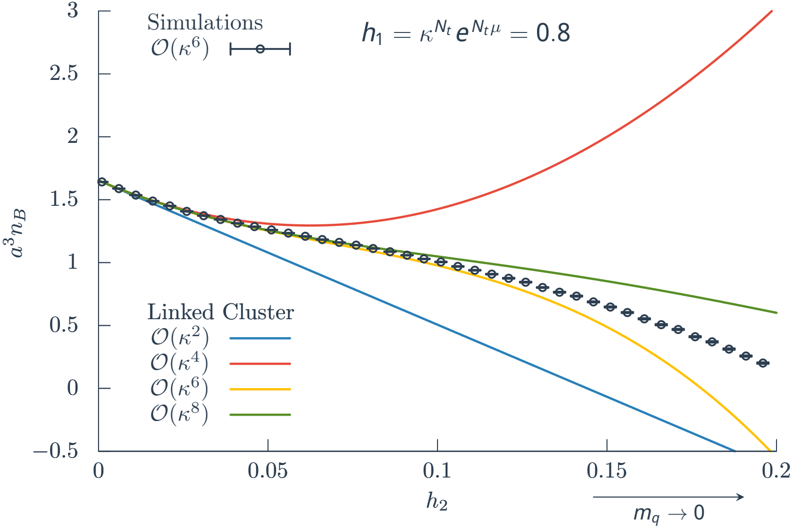
We can do the same resummation for the effective action itself, incorporating long-range effects

Results

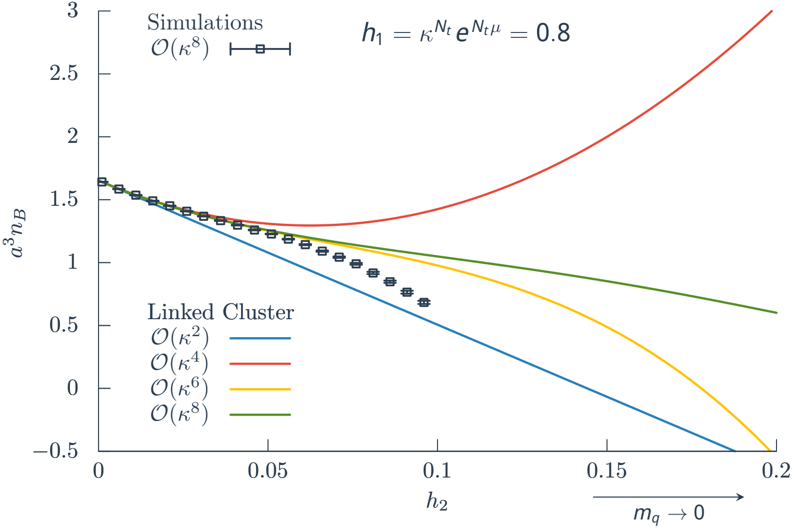
Convergence



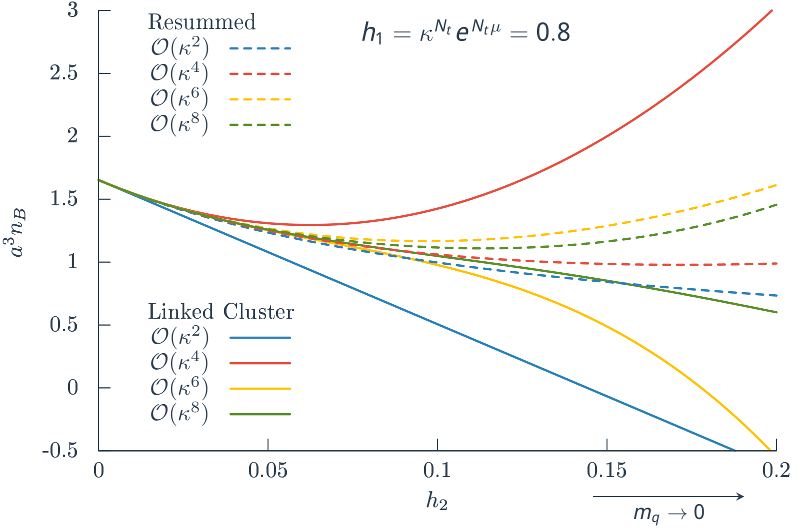
Convergence



Convergence



Effect of the resummations



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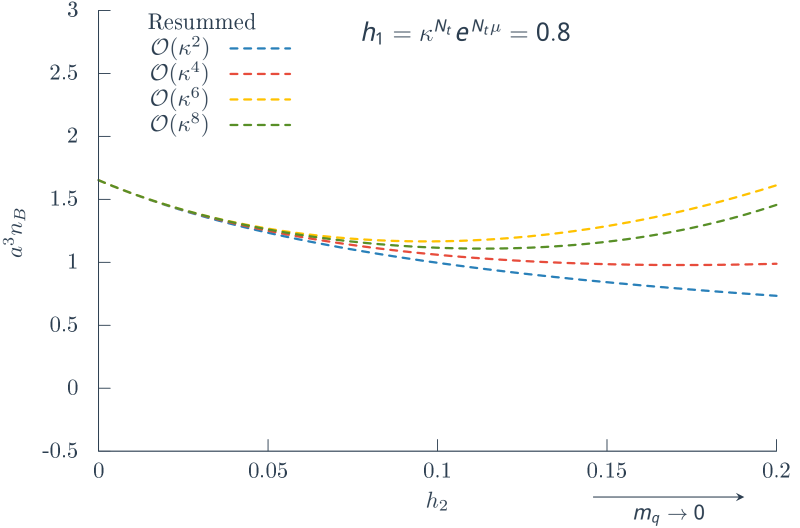

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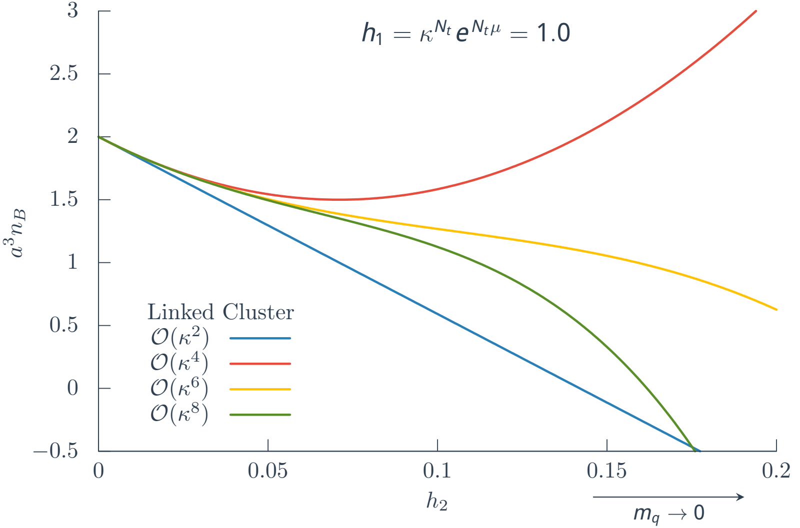

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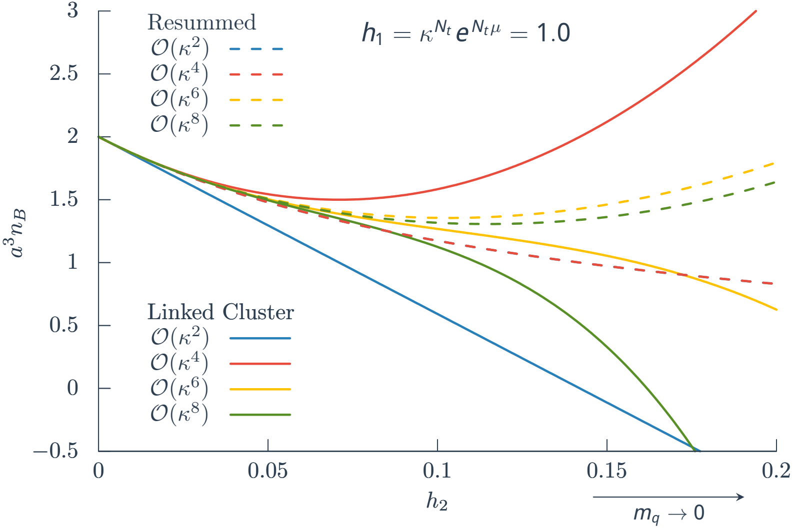

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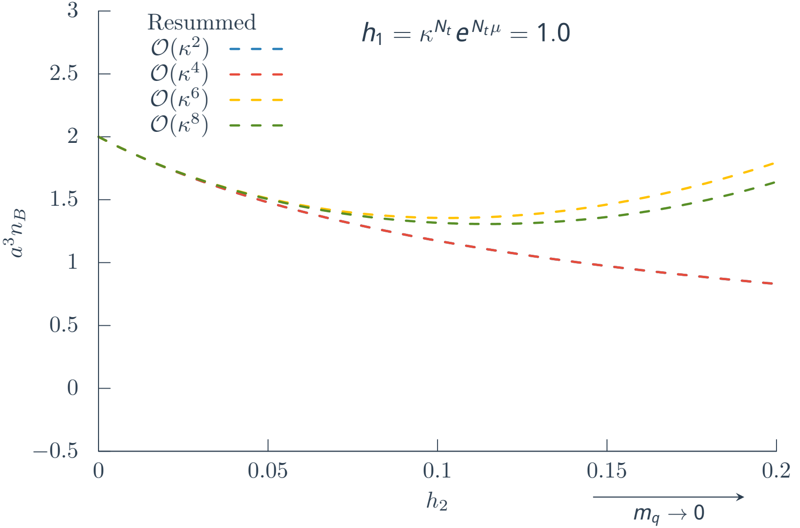
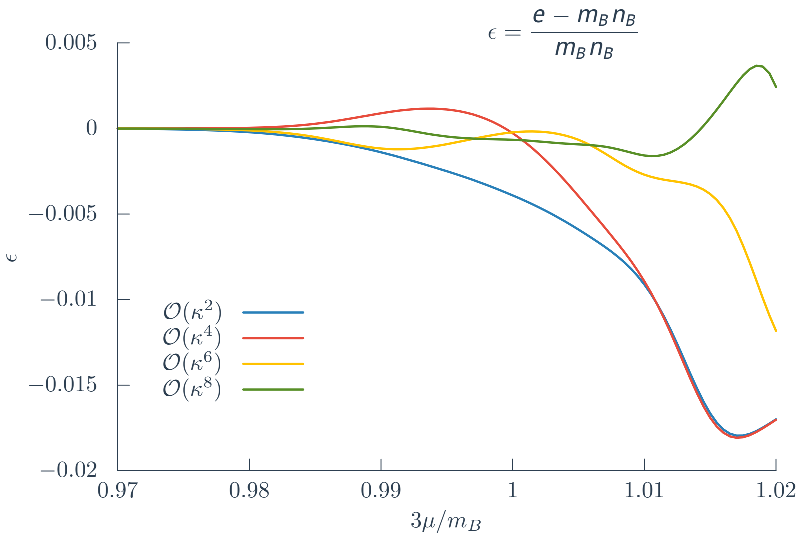
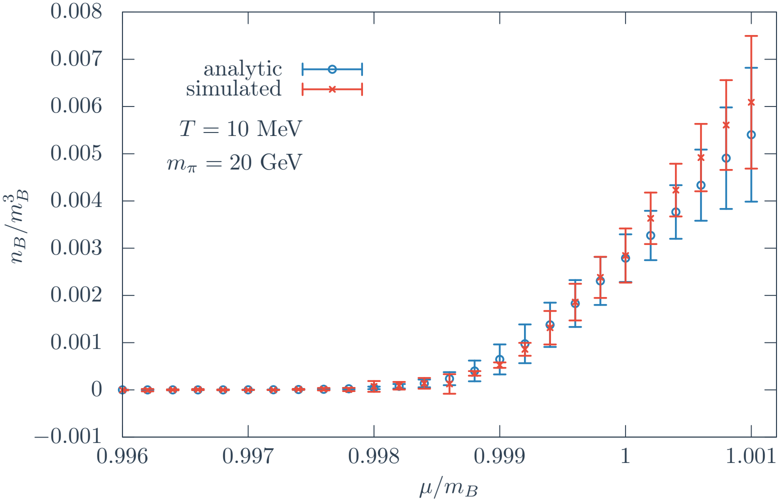


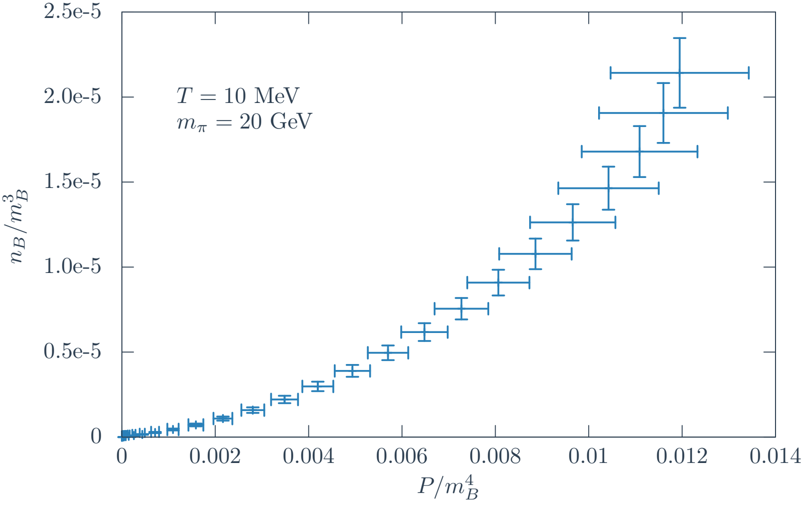
Binding energy



Continuum comparison



Continuum Equation of State



Conclusion

Summary & Outlook

Summary

- Introduced the effective dimensionally reduced lattice theory
- Looked at how a consistent analytic calculation could be carried out
- Demonstrated convergence and comparisons with numerics

Summary & Outlook

Outlook

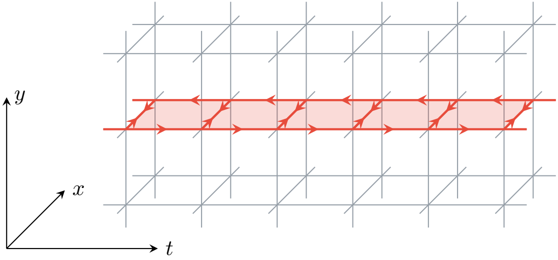
- Use the analytic results as a tool to study the characteristics of the effective theory
- Find analytic resummation schemes to incorporate long-range effects

Thank you!

Backup slides

The Effective Lattice Theory

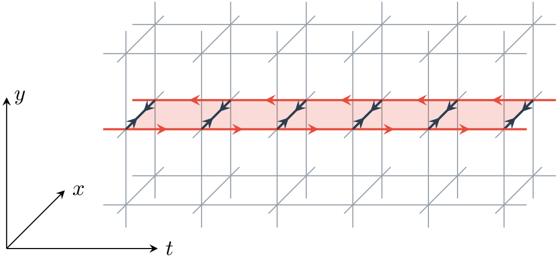
Pure gluon contributions



Put a line of plaquettes in the time direction

The Effective Lattice Theory

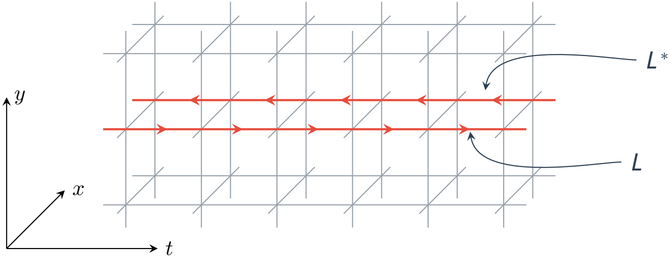
Pure gluon contributions



Integrate over all spatial gauge links

The Effective Lattice Theory

Pure gluon contributions



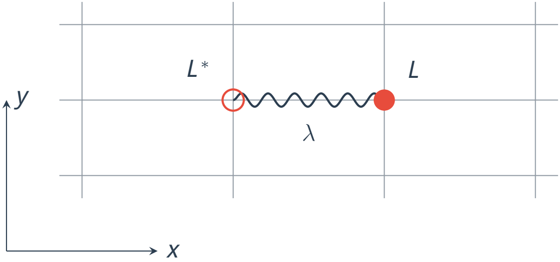
What remains is an interaction between Polyakov Loops

The Effective Lattice Theory

Pure gluon contributions

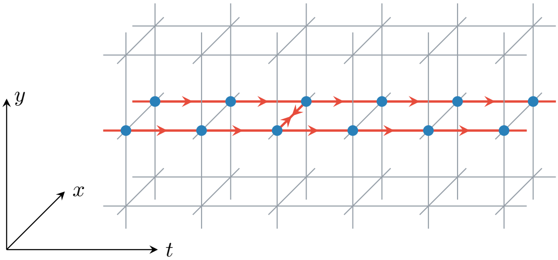
Effective Gluon Interactions

$$S_{\text{eff gluon}} \sim \lambda \sum_{\langle x,y \rangle} L(x) L^*(y)$$



The Effective Lattice Theory

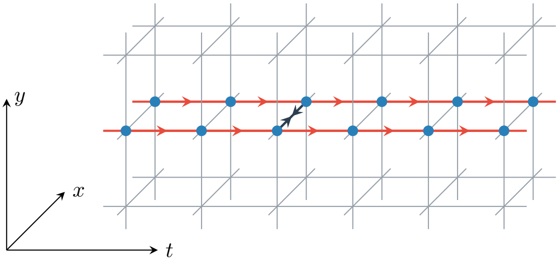
Pure quark contributions



Can produce a closed quark loop with multiple temporal windings

The Effective Lattice Theory

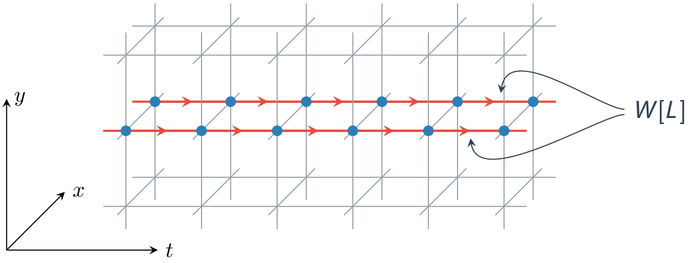
Pure quark contributions



Once again integrate out spatial links

The Effective Lattice Theory

Pure quark contributions



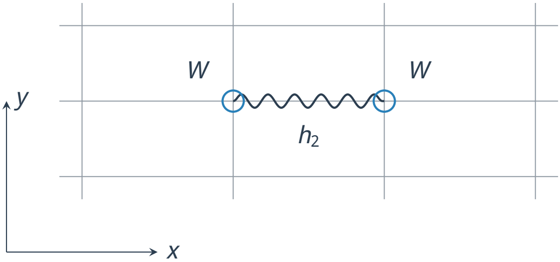
Producing an interaction between the *W* objects

The Effective Lattice Theory

Pure quark contributions

Effective Quark Interactions

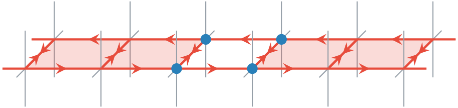
$$S_{\text{eff quarks}} \sim h_2 \sum_{\langle x,y \rangle} W(x)W(y)$$



The Effective Lattice Theory

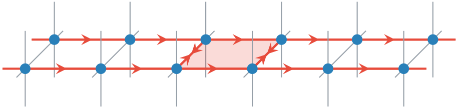
Mixed contributions

Correction to λ



- Rescales λ
- $\lambda \to \lambda(\kappa)$

Correction to h_2



- Rescales h_2
- $h_2 \to h_2(\beta)$

EoS in lattice units