The running coupling of the minimal sextet composite Higgs model

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in collaboration with

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The model

Composite Higgs

$SU(3)$ $\qquad N_f = 2$ $\qquad R = 2S$

massless, Dirac fermions

almost QCD but fundamental $\rightarrow$ sextet

Motivations

- ATLAS & CMS: possible new resonance in $2\,TeV$ range?
- Investigate sextet model from as many angles as possible
- This talk: running coupling
- Very important aspect: continuum limit
- Also: control all systematic errors

Motivations

In previous work: meson/baryon spectrum, chiral condensate, GMOR

Julius, Ricky, Santanu talks just before this one

see also Kieran's poster

These favor chiral symmetry breaking, light scalar and tower of new particles ($L = \infty$ and $T = 0$)

Running coupling with finite L is another tool in the lattice toolbox

May support or contradict the other findings

Good (in)consistency check

Plan

- Define gradient flow based running coupling scheme
- Lattice discretizations
- Continuum extrapolation
- Assess systematic effects
- Final result

By the way: first fully controlled non-perturbative continuum result on the model :)

Previous results

DeGrand-Shamir-Svetitsky, 0803.1707, 1006.0707 (no continuum)

Hasenfratz-Svetitsky: Nagoya 2015, LLNL 2015, USQCD 2015

Continuum running coupling scheme

Infinite volume gradient flow in a (really small) nutshell:

$$\frac{dA_\mu(t)}{dt} = -\frac{\delta S}{\delta A_\mu}\,, \qquad \langle t^2 E(t)\rangle\,, \qquad E = -\frac{1}{2}\mathrm{Tr}\,F_{\mu\nu}F_{\mu\nu}$$

Perturbatively:

$$\langle t^2 E(t)\rangle = g^2 \frac{3(N^2-1)}{128\pi^2}\left(1 + O(g^2)\right)$$

Continuum running coupling scheme

Hence, turning it around:

$$g^2(t) = \frac{128\pi^2}{3(N^2-1)}\langle t^2 E(t)\rangle$$

is a good scheme, $\mu = 1/\sqrt{8t}$.

But need $L = \infty$ or $L =$ large!

Continuum running coupling scheme

Finite volume, T^4

$$g^2(t,L) = \frac{128\pi^2}{3(N^2-1)\,(1+\delta(t,L))} \langle t^2 E(t) \rangle$$

Where $\delta(t,L)$ is calculable. Impose $c = \sqrt{8t}/L$ constant
$c = 7/20 = 0.35$

Single scale $\mu = 1/L$. Running with the volume.

Step scaling: finite change $L \to sL$ with $s = 3/2$.

Gauge fields periodic, fermions massless anti-periodic in all 4-directions

Lattice discretization

Fermions: $m = 0$, rooted staggered with stout

Gauge links: 3 ingredients

- Flow (Wilson and tree level Symanzik)
- Dynamical gauge action (tree level Symanzik)
- Observable E (clover)

Terminology: flow-action-observable: WSC and SSC.

Continuum should agree for both!

Wait, what???

Rooted staggered fermions with $m = 0$???

Rooting and $m = 0$

Golterman, Shamir, Sharpe, . . .:

Rooting is okay for $m > m_*$, where m_* depends on the lattice spacing, a decreases m_* decreases

But remember: above is for infinite volume!

In infinite volume, m is the only IR regulator

We have finite volume L which is itself an IR regulator

Rooting and $m = 0$

Modified Golterman, Shamir, Sharpe, . . .:

As long as we have a large enough IR regulator rooting is okay!

Key insight: rooting fails due to small Dirac eigenvalues

m fixes this, finite volume and anti-periodic fermions ditto
$\sim 1/L^{\alpha}$

Lower bound on m (HMC fails for too small m anyway)

Upper bound on L (HMC fails for too large L anyway)

Step scaling

$\frac{g^2(sL)-g^2(L)}{\log s^2}$ discrete β-function

$8 \to 12$, $12 \to 18$, $16 \to 24$, $20 \to 30$, $24 \to 36$

for many fixed β bare couplings

Plot discrete β-function as a function of $g^2(L)$

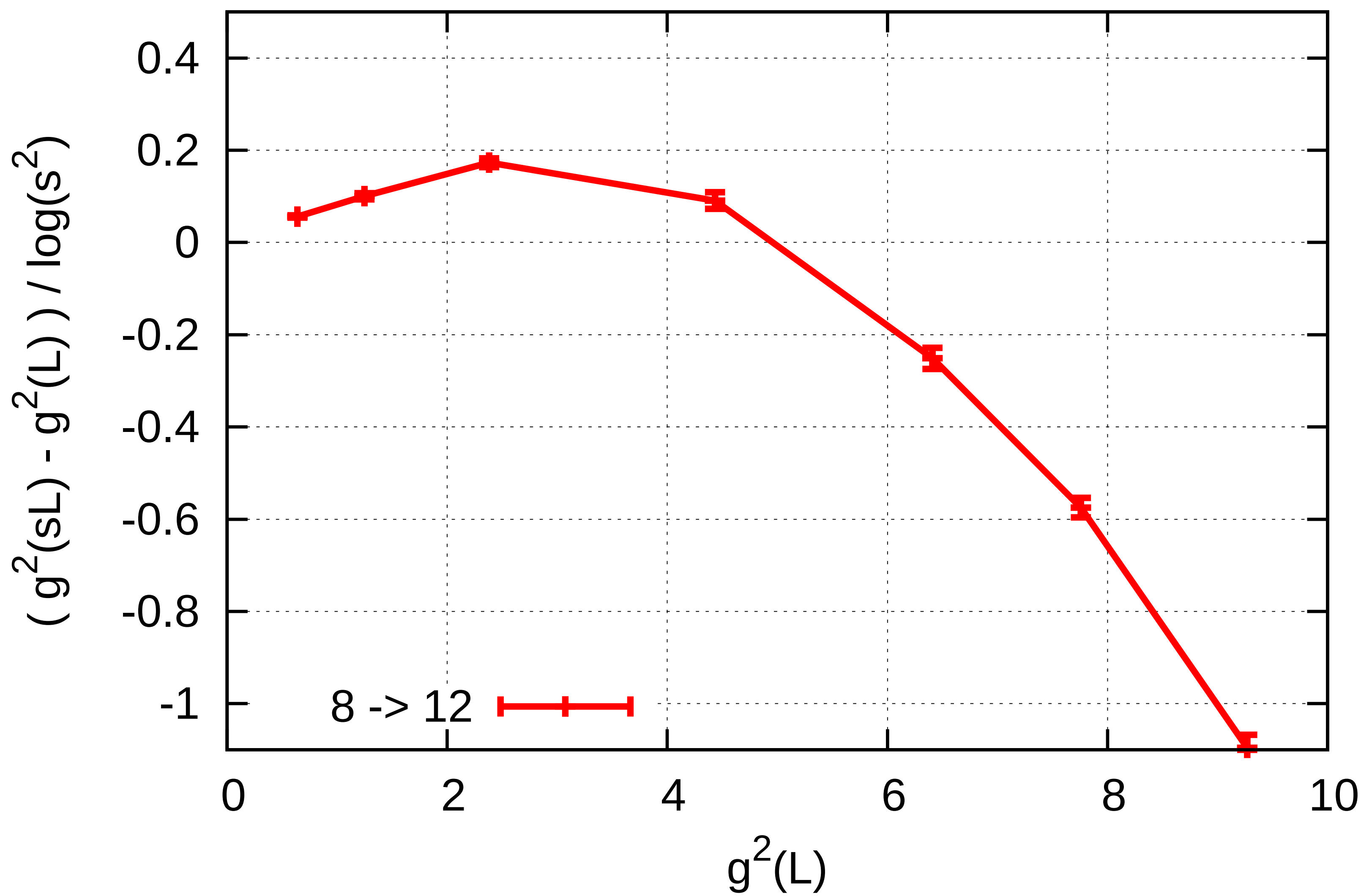
5 steps: 5 lattice spacings $\to$ can quantify systematic error from continuum extrapolation

Results

Flow-Action-Observable = WSC

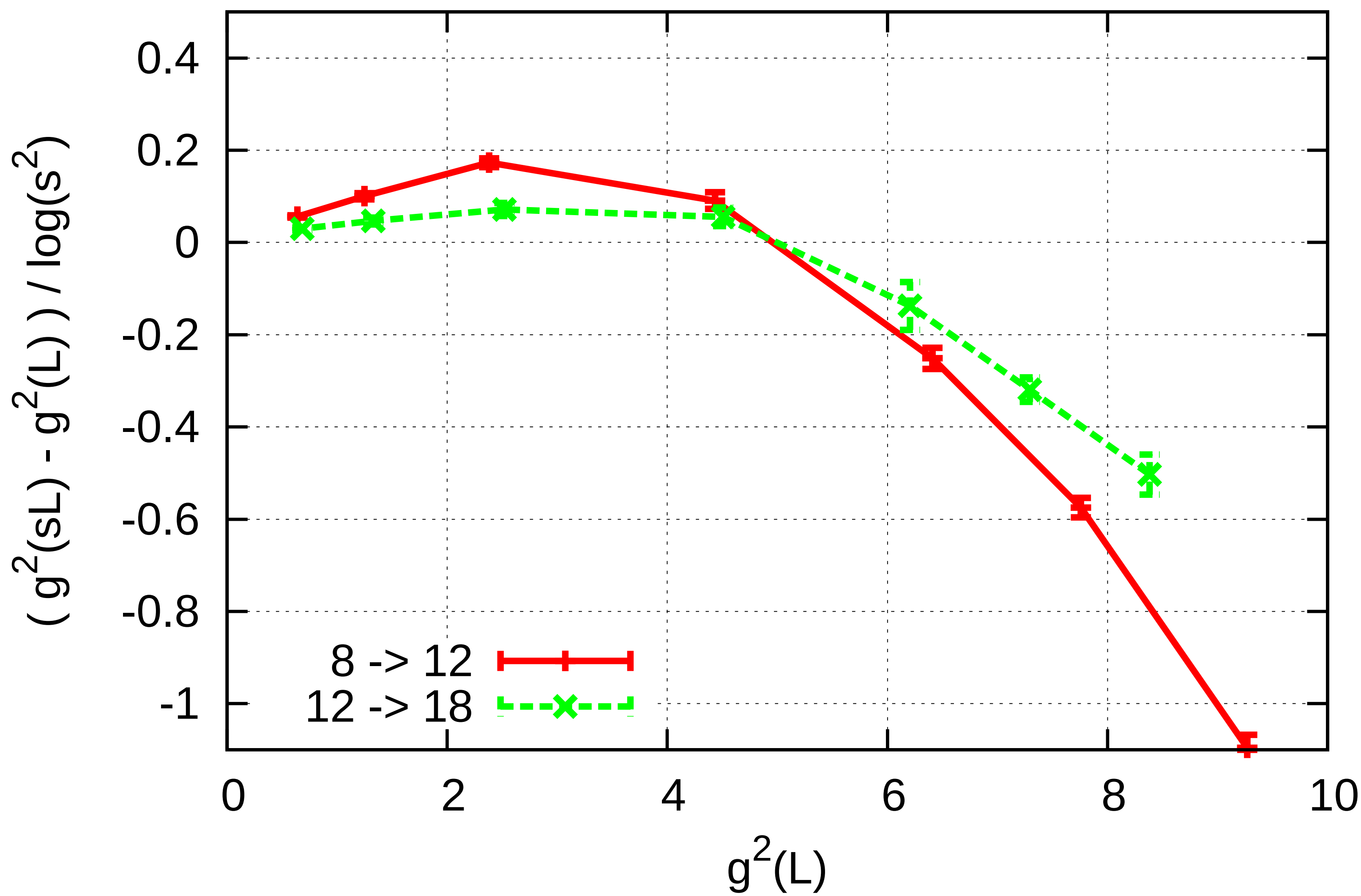
Results, WSC

$8 \to 12$,



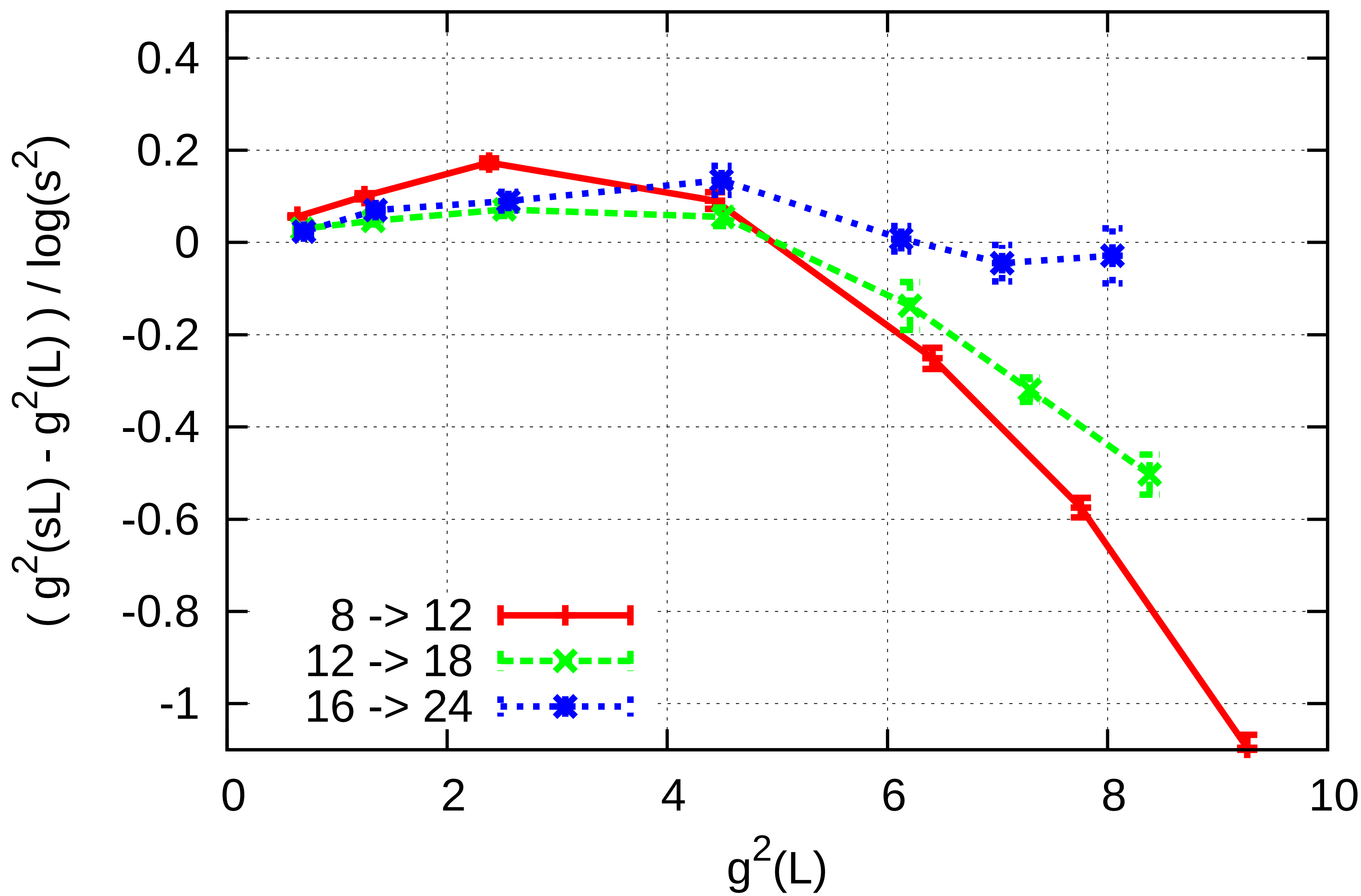
Results, WSC

$8 \to 12,\ 12 \to 18$



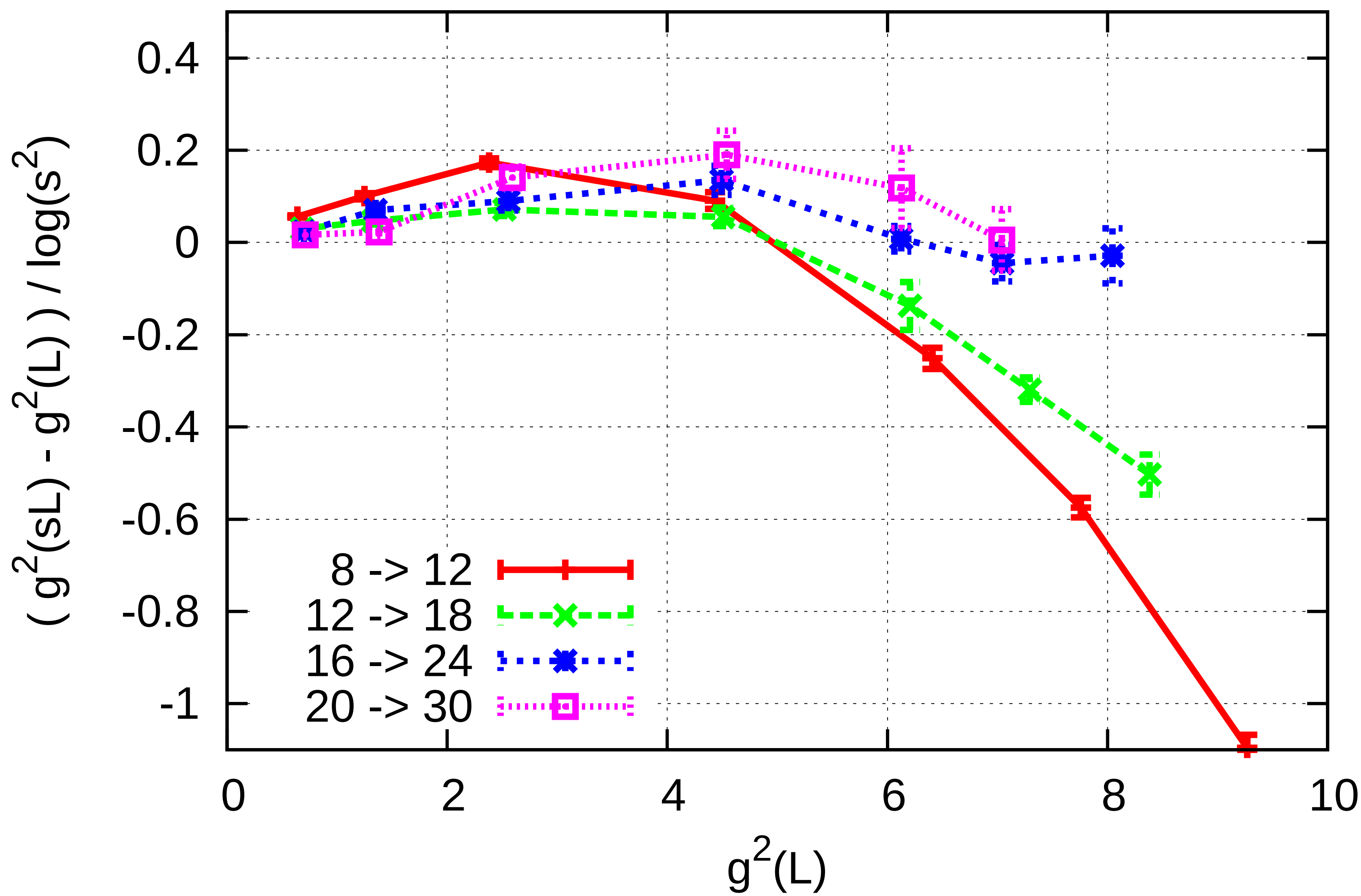
Results, WSC

$8 \rightarrow 12,\ 12 \rightarrow 18,\ 16 \rightarrow 24$



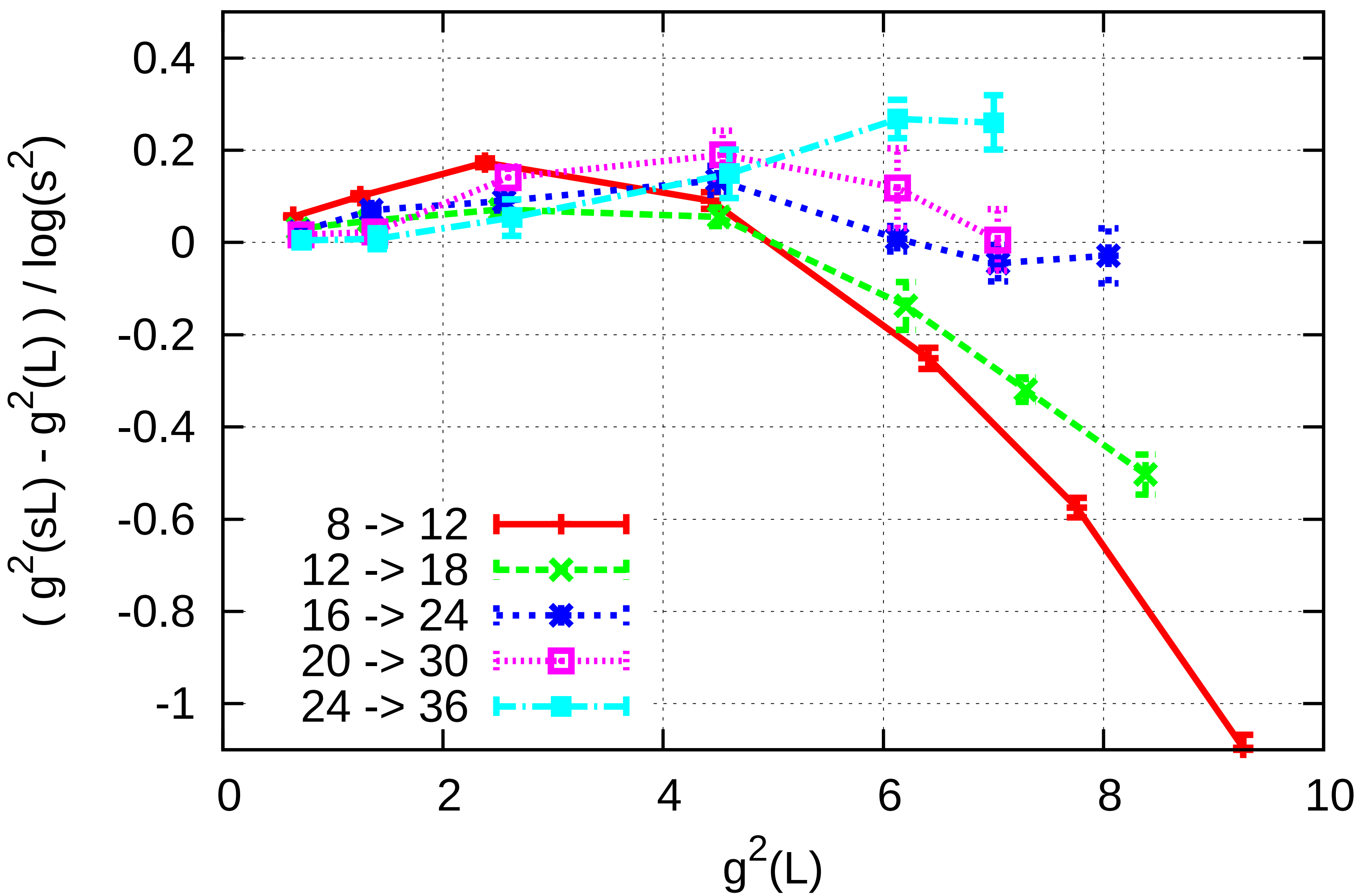
Results, WSC

$8 \rightarrow 12,\ 12 \rightarrow 18,\ 16 \rightarrow 24,\ 20 \rightarrow 30$



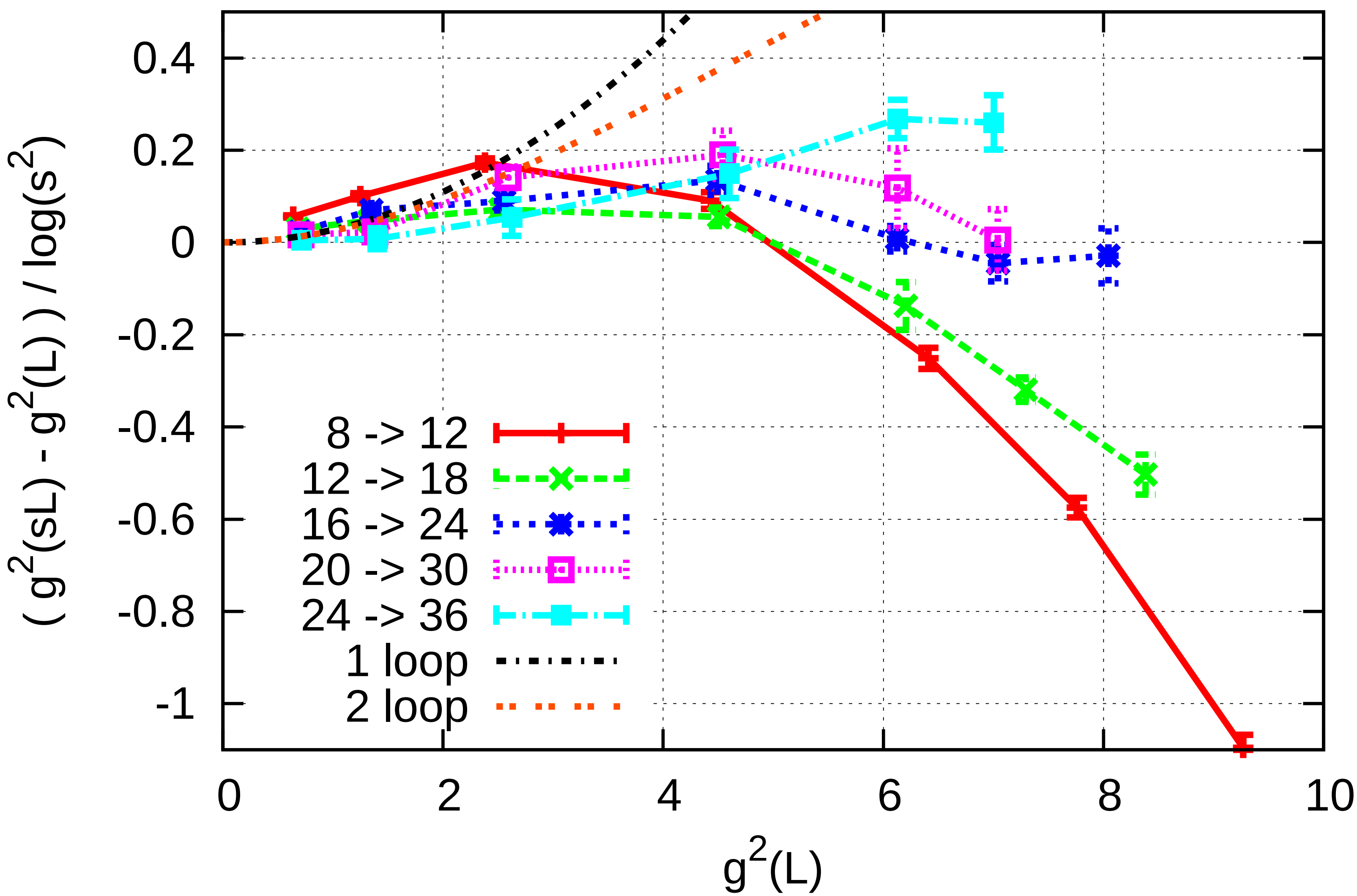
Results, WSC

$8 \rightarrow 12,\ 12 \rightarrow 18,\ 16 \rightarrow 24,\ 20 \rightarrow 30,\ 24 \rightarrow 36$



Results, WSC

$8 \to 12,\ 12 \to 18,\ 16 \to 24,\ 20 \to 30,\ 24 \to 36$

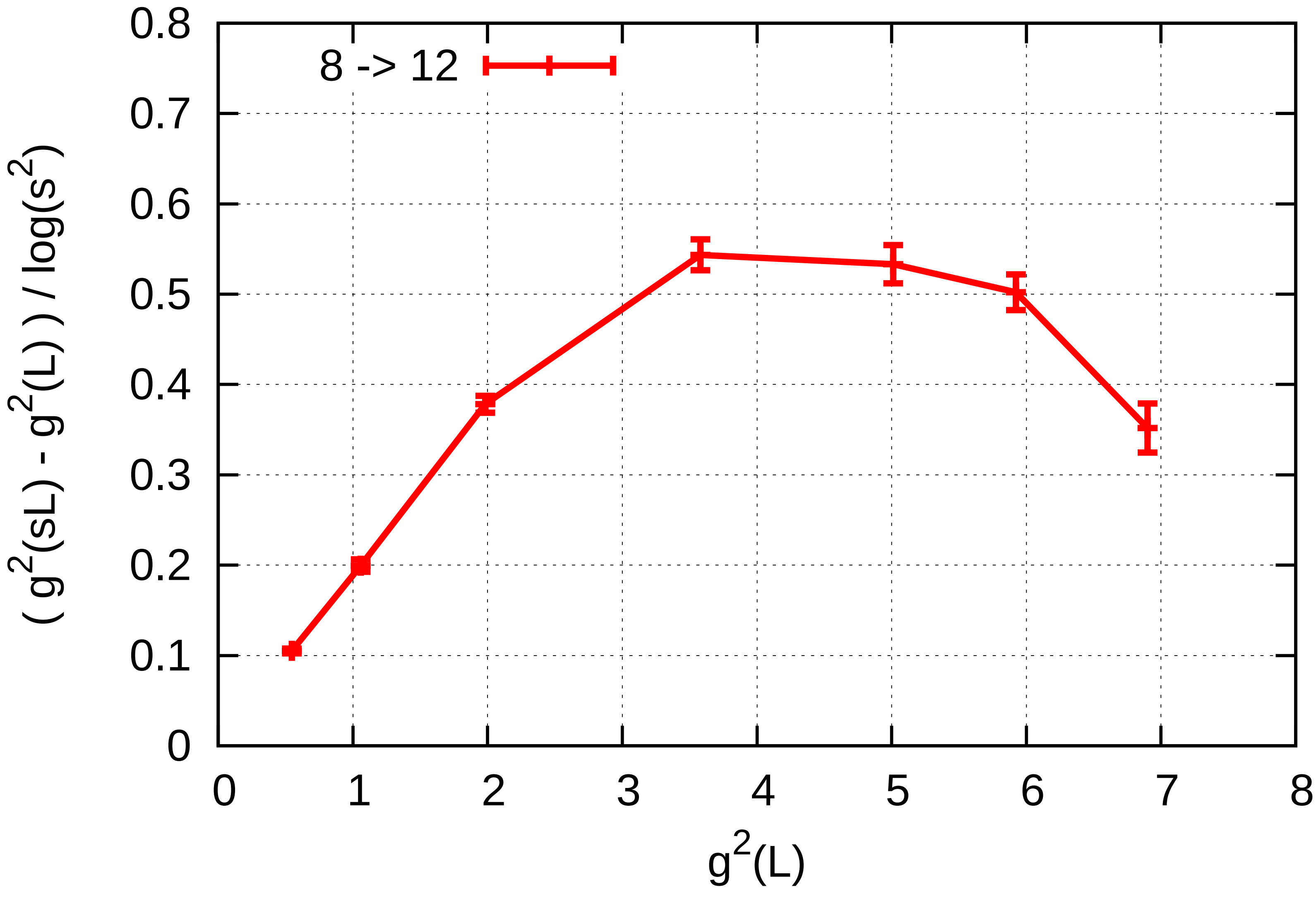


Results

Flow-Action-Observable = SSC

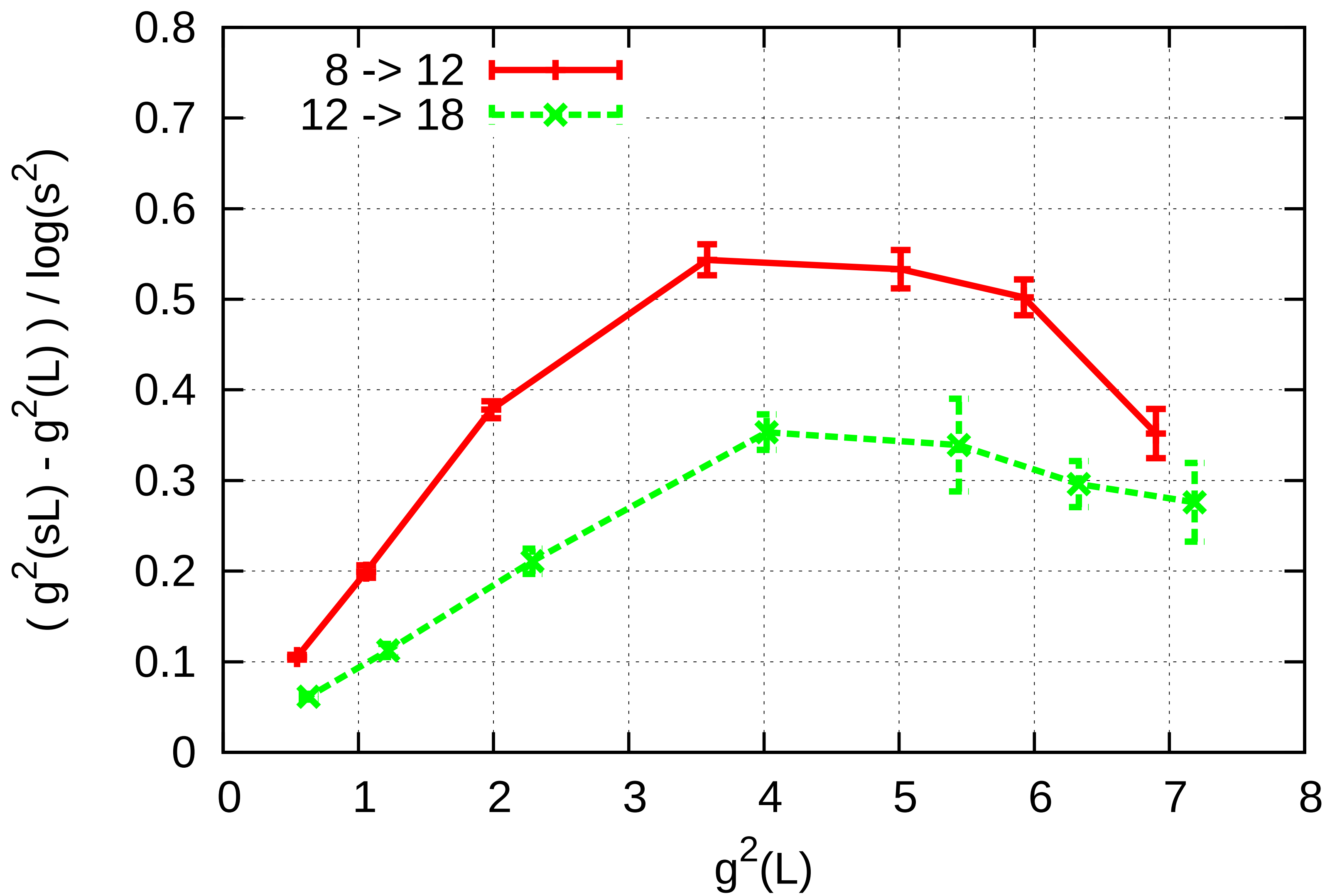
Results, SSC

$8 \rightarrow 12$,



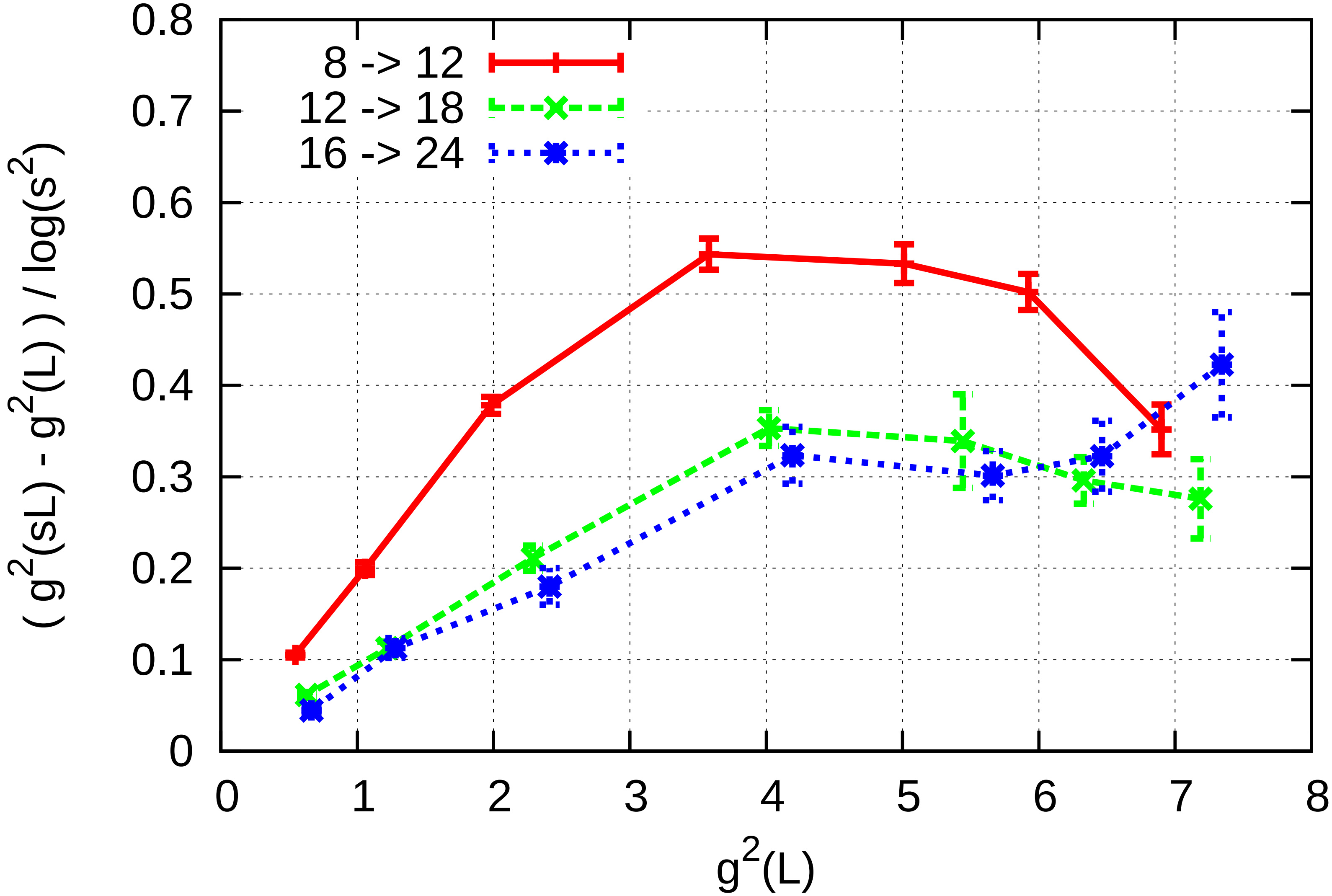
Results, SSC

$8 \rightarrow 12,\ 12 \rightarrow 18$



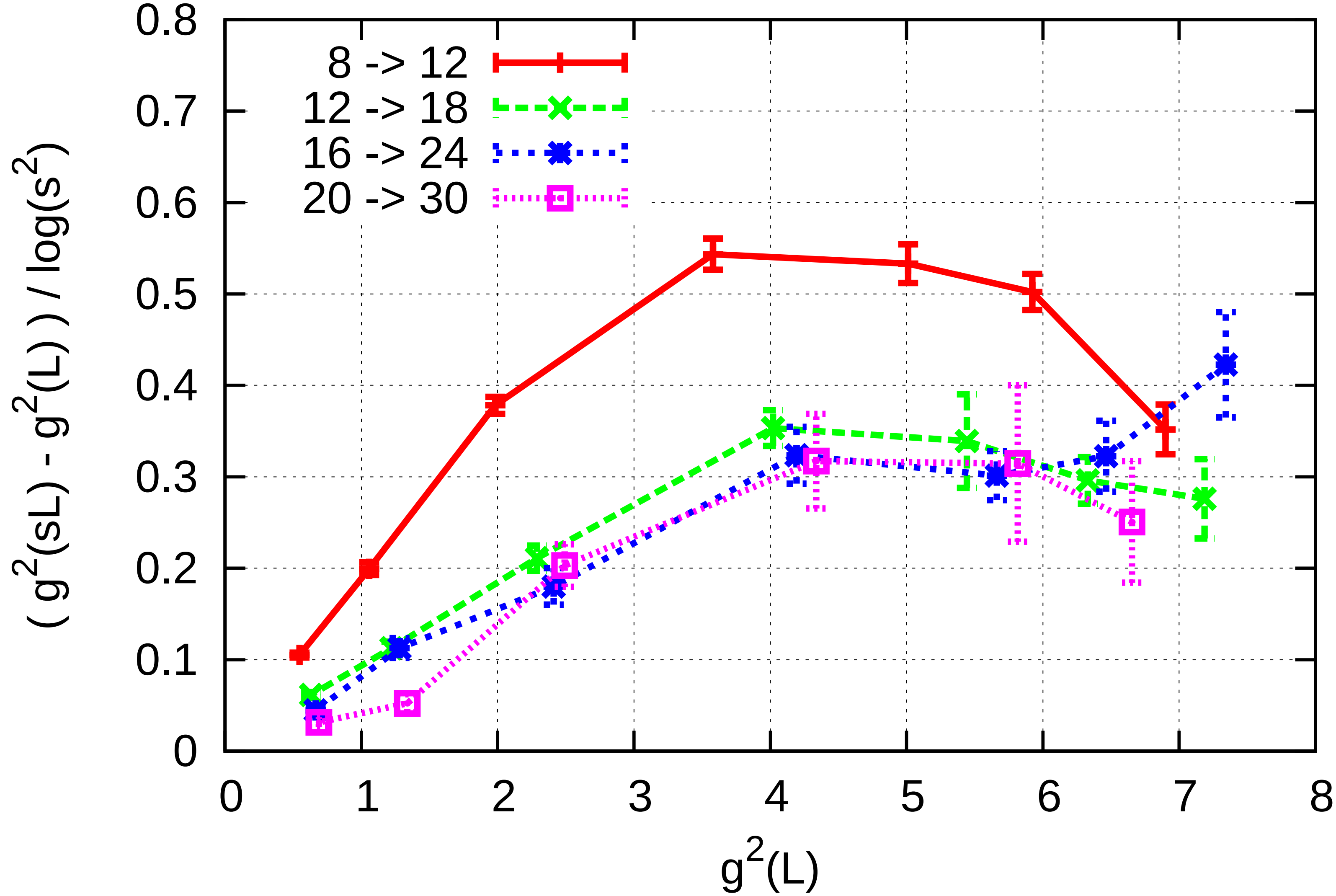
Results, SSC

$8 \to 12,\ 12 \to 18,\ 16 \to 24$



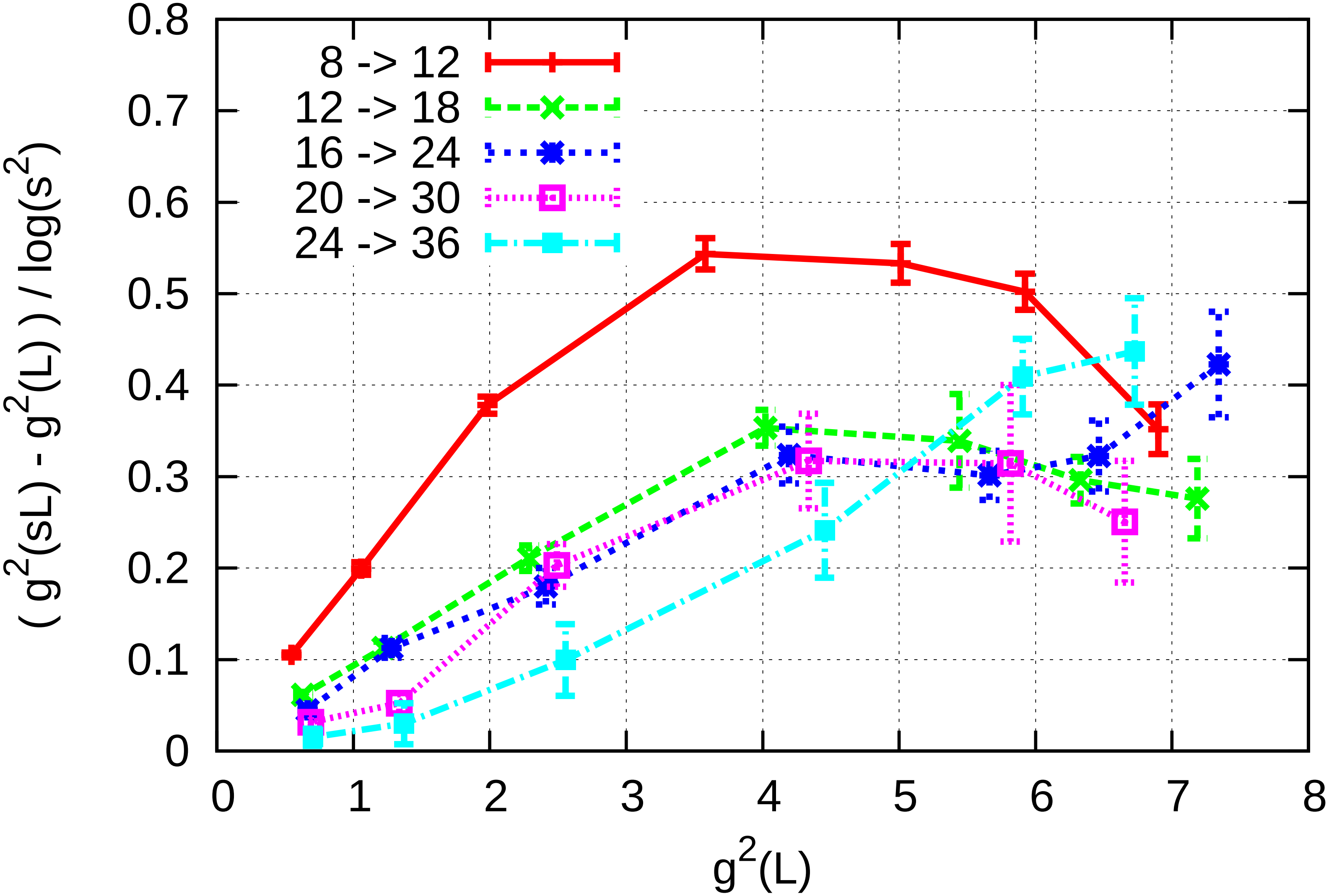
Results, SSC

$8 \rightarrow 12,\ 12 \rightarrow 18,\ 16 \rightarrow 24,\ 20 \rightarrow 30$



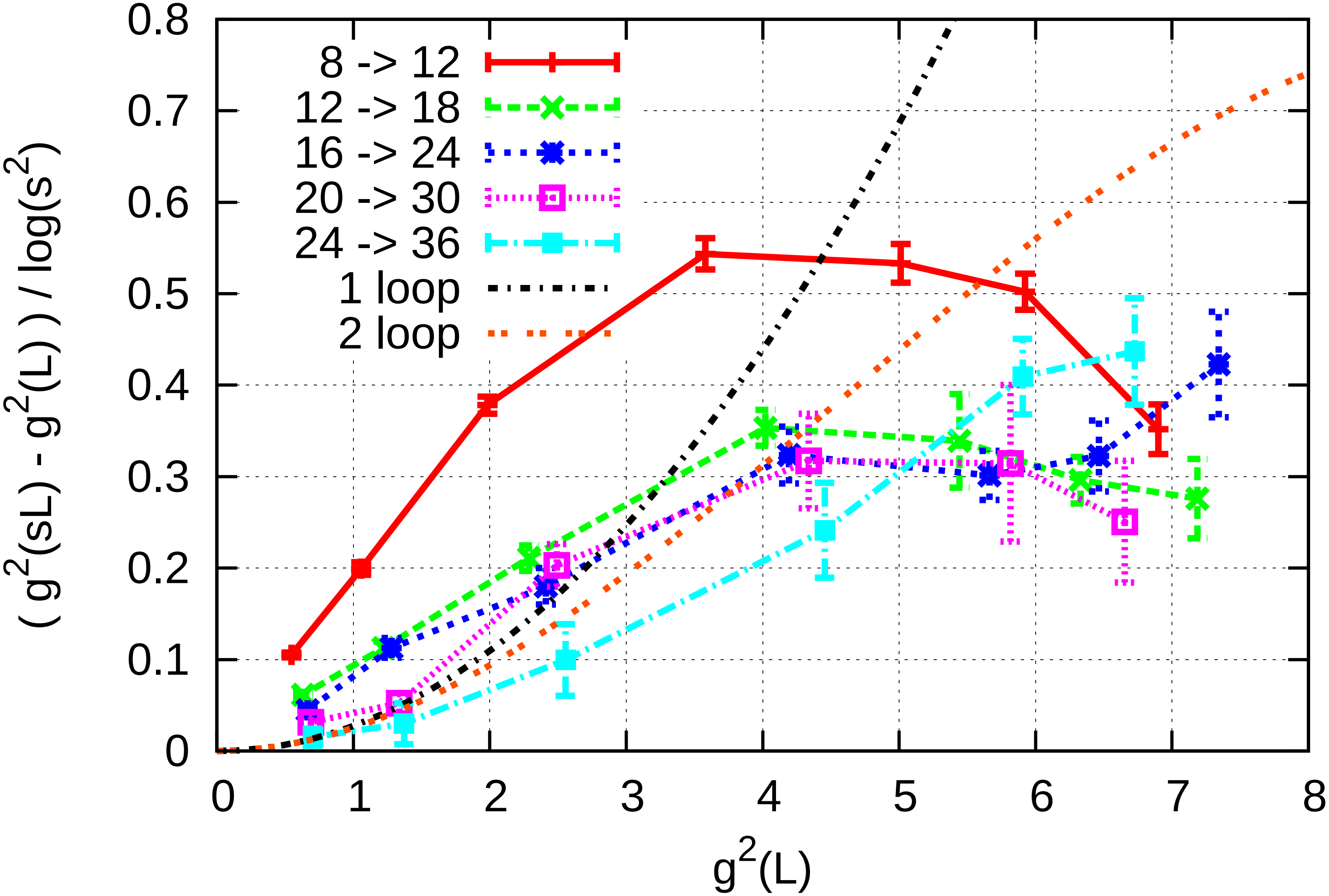
Results, SSC

$8 \rightarrow 12,\ 12 \rightarrow 18,\ 16 \rightarrow 24,\ 20 \rightarrow 30,\ 24 \rightarrow 36$



Results, SSC

$8 \to 12,\ 12 \to 18,\ 16 \to 24,\ 20 \to 30,\ 24 \to 36$



Continuum extrapolation

Interpolation $g^2(\beta)$ at fixed L/a

$$\frac{\beta}{6} - \frac{1}{g^2(\beta)} = \sum_{m=0}^{n} c_m \left(\frac{6}{\beta}\right)^m$$

Can now interpolate all data

Can pick fixed $g^2(L)$ and read off 5 discrete β-function values corresponding to 5 steps, i.e. 5 lattice spacings and continuum extrapolate linearly in a^2/L^2

Systematic uncertainty from continuum extrapolation

Twofold:

- Interpolation orders $n = 3, 4, 5$
- Number of points in continuum extrapolation: $4, 5$

Systematic uncertainty 1: interpolation orders

$L/a = 8, 12, 16, 18, 24$ we let $n = 3, 4, 5$

$L/a = 20, 30, 36$ we let $n = 4, 5$

Total: $3^5 \cdot 2^3 = 1944$ interpolations

Kolmogorov-Smirnoff test on the 1944 fits

Kolmogorov-Smirnoff: q-values are uniform, $P > 30\%$

SSC : 240 and WSC : 306

AIC weighted histograms: $\sim \exp(-\chi^2/2 - p)$

Systematic uncertainty 2: continuum extrapolation

Using all 5 or only 4 (dropping roughest) steps, i.e. lattice spacings

Previous page was for 5-point extrapolations

With 4-point extrapolations, drop $8 \to 12$:

Total: $3^4 \cdot 2^3 = 648$ interpolations

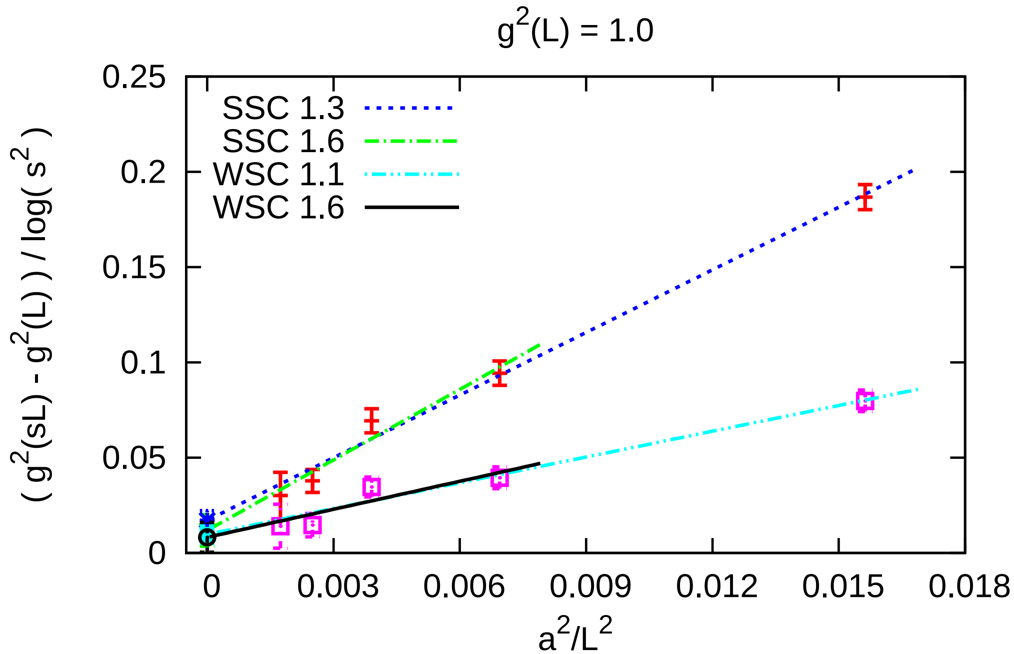
SSC : 240 and WSC : 249 with Kolmogorov-Smirnoff $> 30\%$

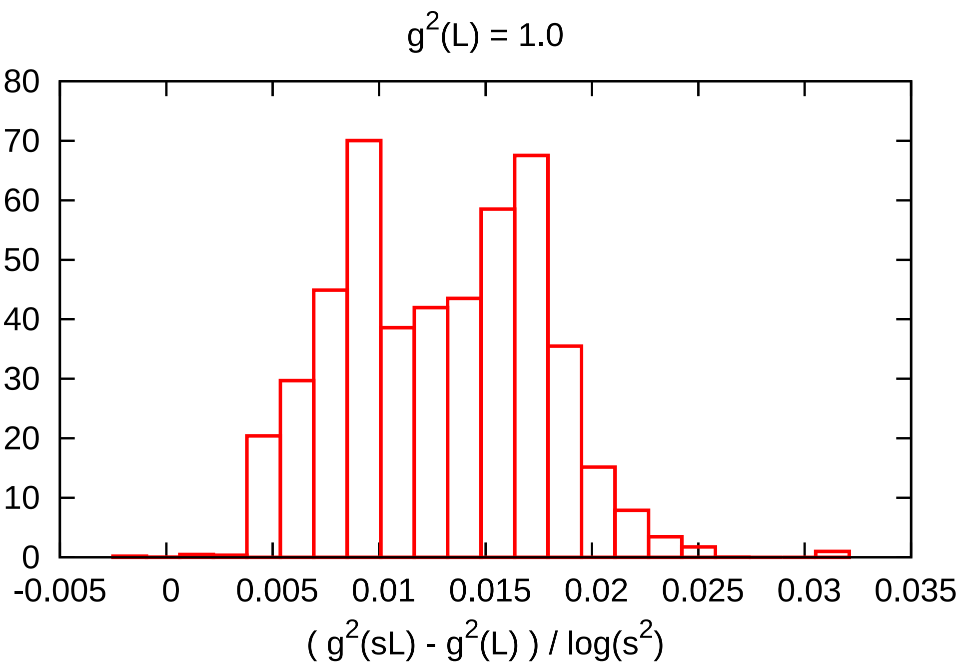
Include everything in AIC weighted histogram

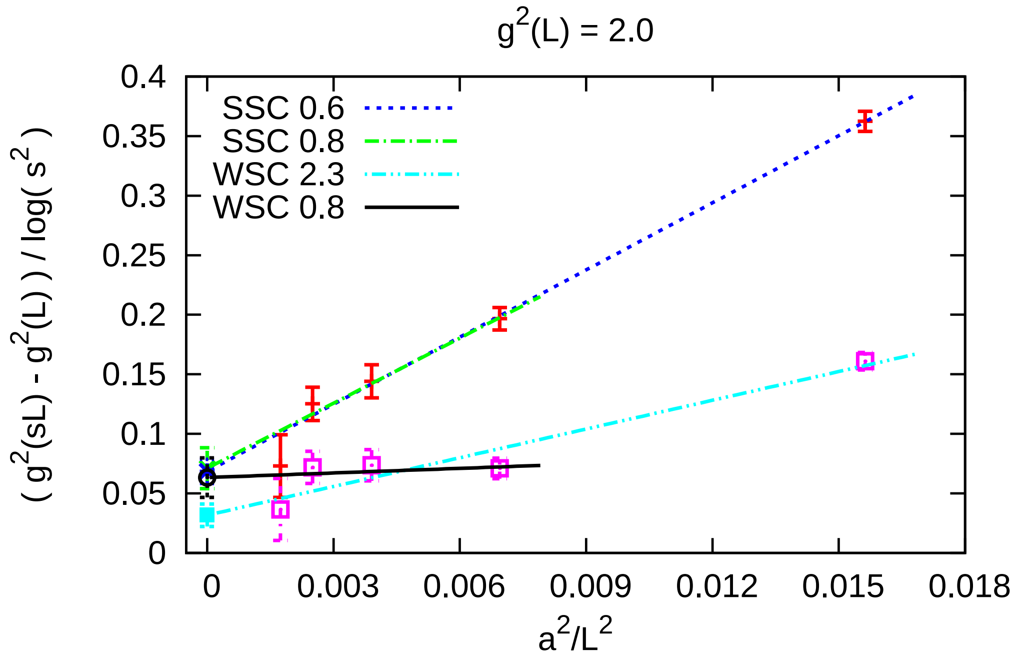
Repeat all this for each $g^2 = 1.0,\ 2.0,\ 3.0,\ 4.0,\ 5.0,\ 6.0$

Histograms and representative example of actual extrapolations for both WSC and SSC

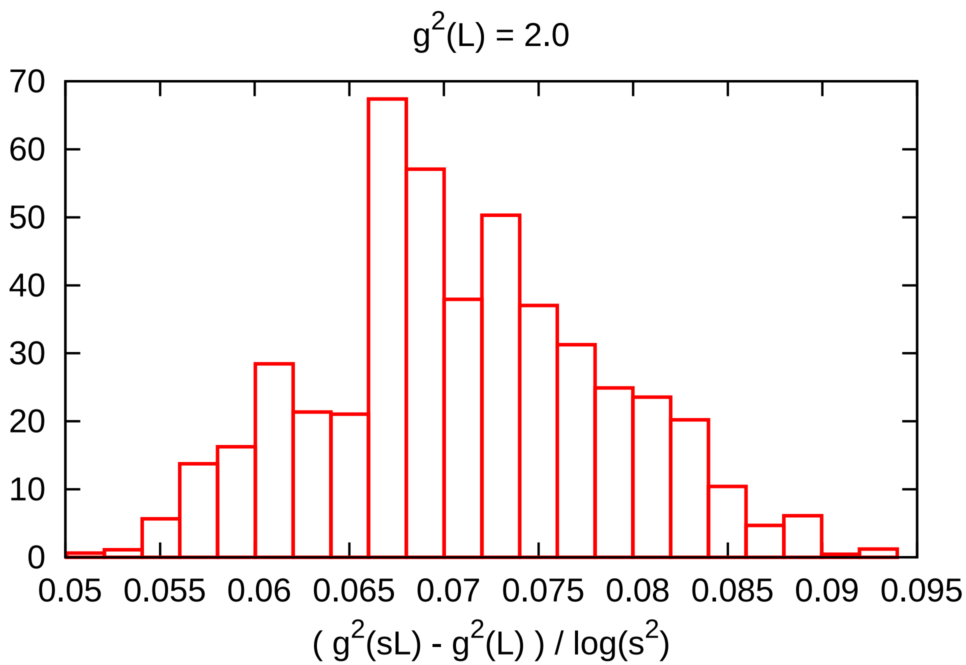
χ^2/dof of each fit in legend


g^2(L) = 1.0
SSC 1.3
SSC 1.6
WSC 1.1
WSC 1.6
(g^2(sL) - g^2(L)) / log(s^2)
a^2/L^2
0.25
0.2
0.15
0.1
0.05
0
0
0.003
0.006
0.009
0.012
0.015
0.018

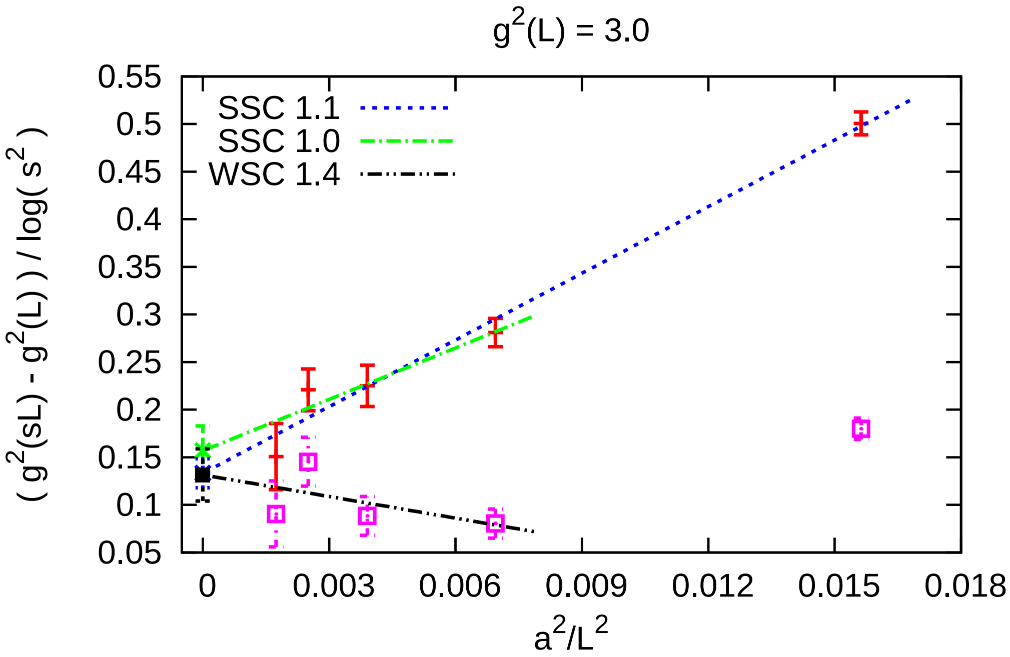

g^2(L) = 1.0
80
70
60
50
40
30
20
10
0
-0.005
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
(g^2(sL) - g^2(L)) / log(s^2)

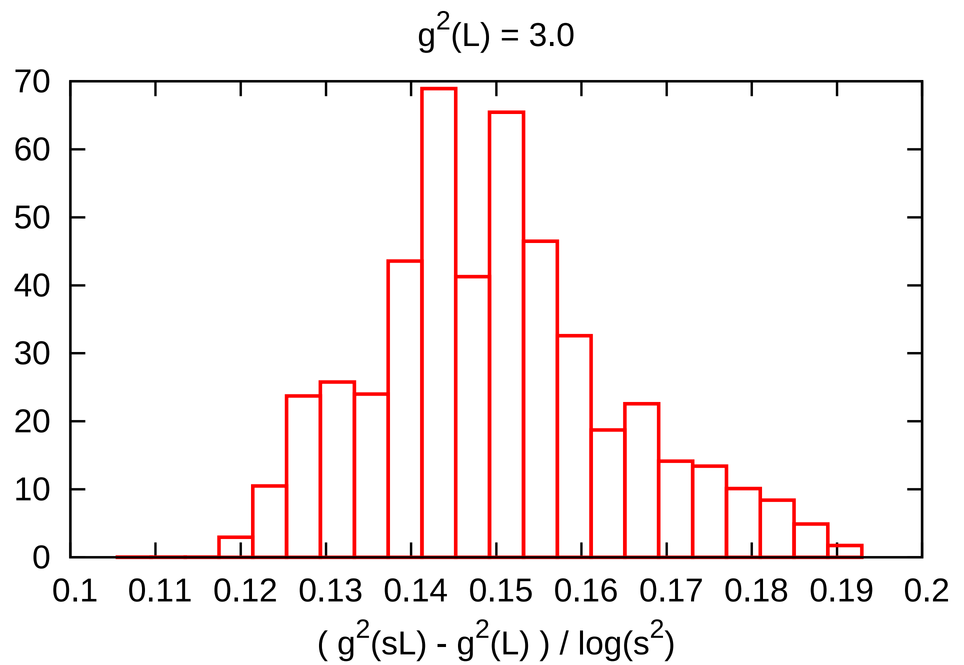


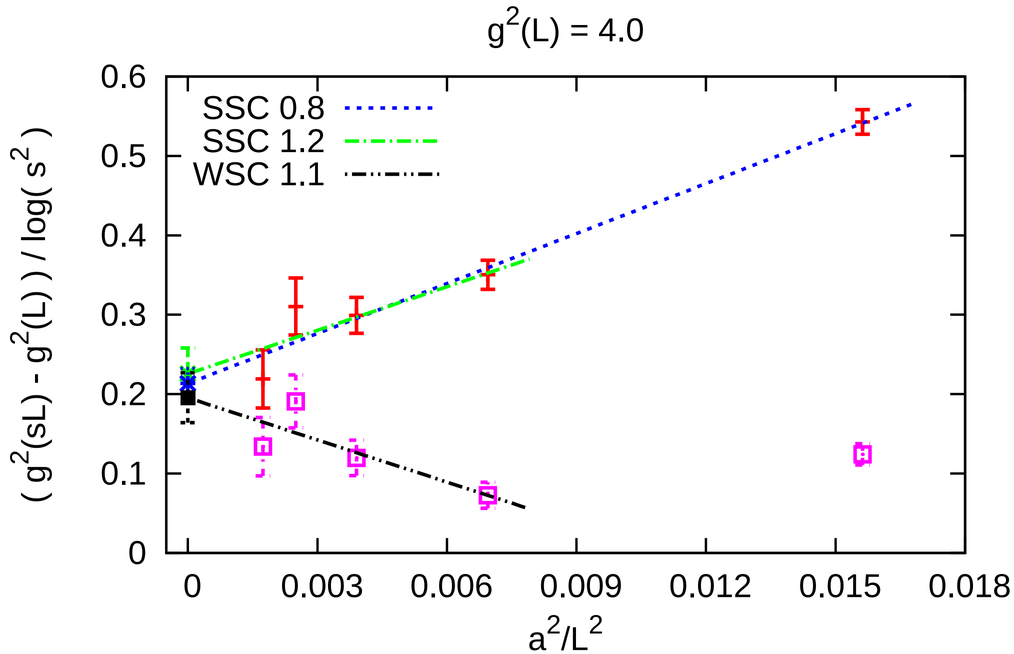
g²(L) = 2.0
SSC 0.6
SSC 0.8
WSC 2.3
WSC 0.8
(g²(sL) - g²(L)) / log(s²)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.003
0.006
0.009
0.012
0.015
0.018
a²/L²



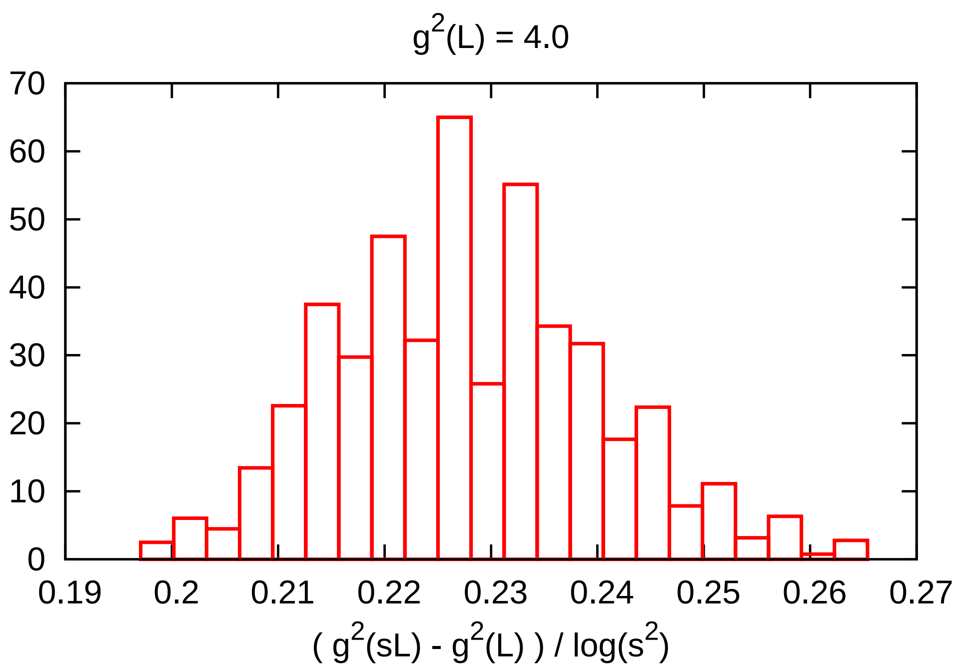
g²(L) = 2.0
70
60
50
40
30
20
10
0
0.05
0.055
0.06
0.065
0.07
0.075
0.08
0.085
0.09
0.095
(g²(sL) - g²(L)) / log(s²)


g2(L) = 3.0
SSC 1.1
SSC 1.0
WSC 1.4
(g2(sL) - g2(L)) / log(s2)
a2/L2
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.003
0.006
0.009
0.012
0.015
0.018

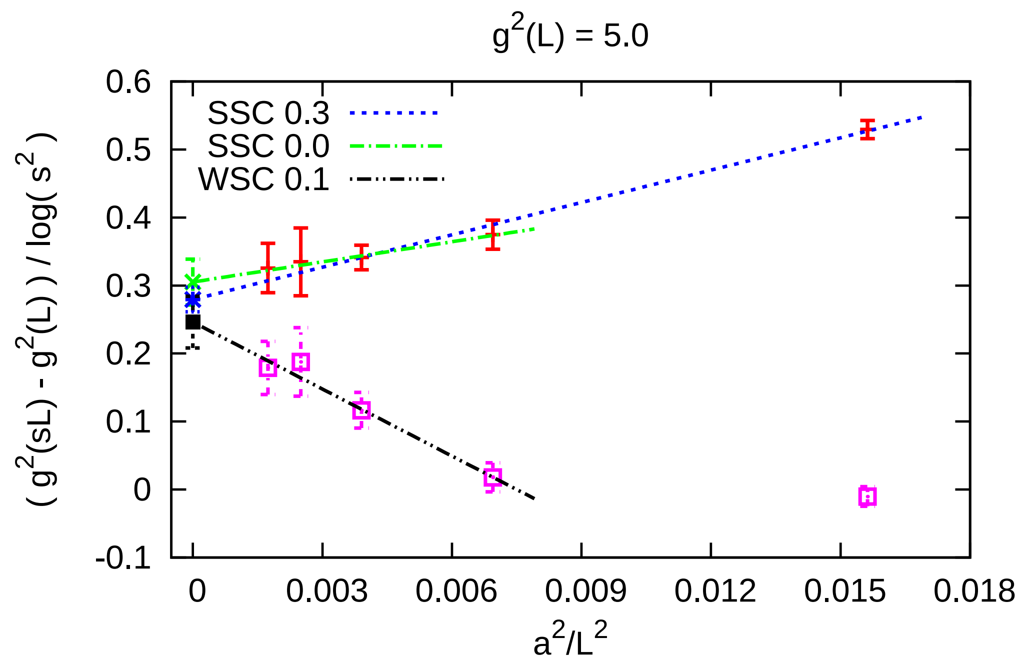

g2(L) = 3.0
70
60
50
40
30
20
10
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
(g2(sL) - g2(L)) / log(s2)



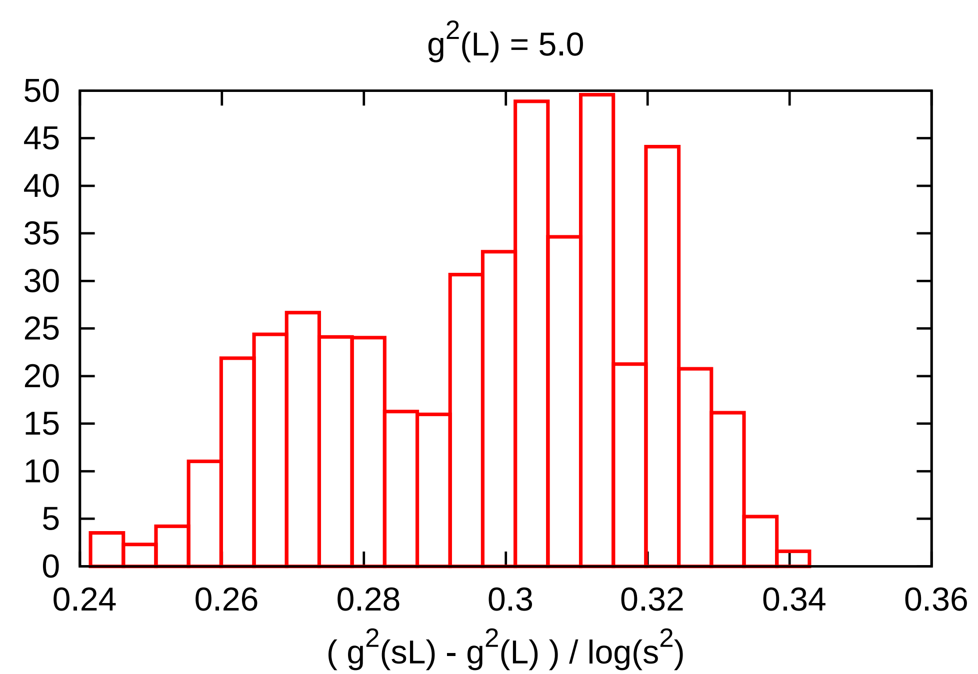
g^2(L) = 4.0
SSC 0.8
SSC 1.2
WSC 1.1
(g^2(sL) - g^2(L)) / log(s^2)
a^2/L^2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.003
0.006
0.009
0.012
0.015
0.018



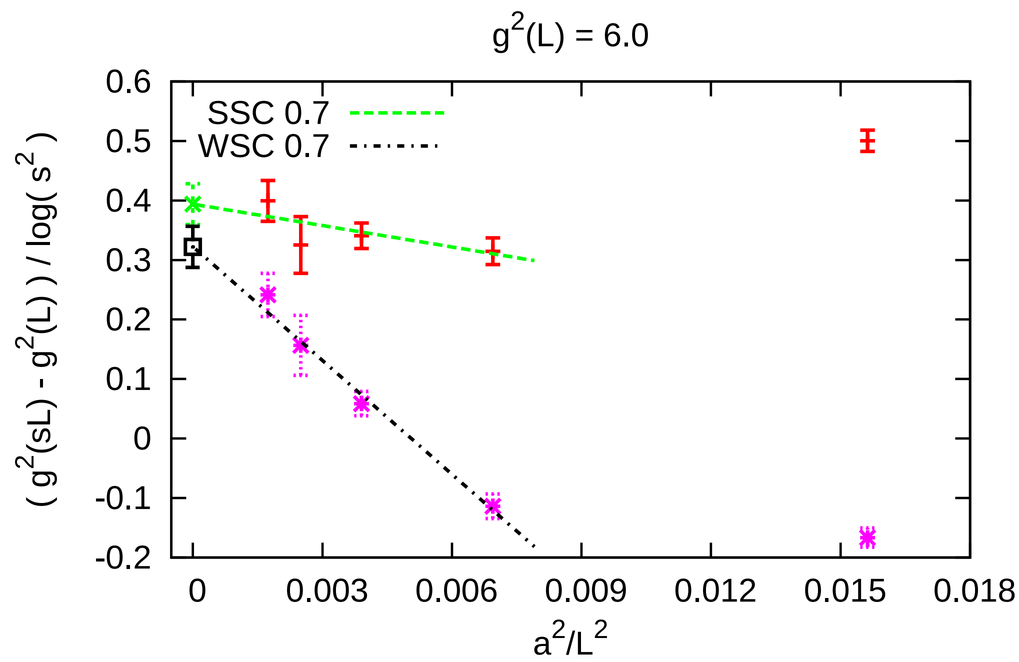
g^2(L) = 4.0
0
10
20
30
40
50
60
70
0.19
0.2
0.21
0.22
0.23
0.24
0.25
0.26
0.27
(g^2(sL) - g^2(L)) / log(s^2)



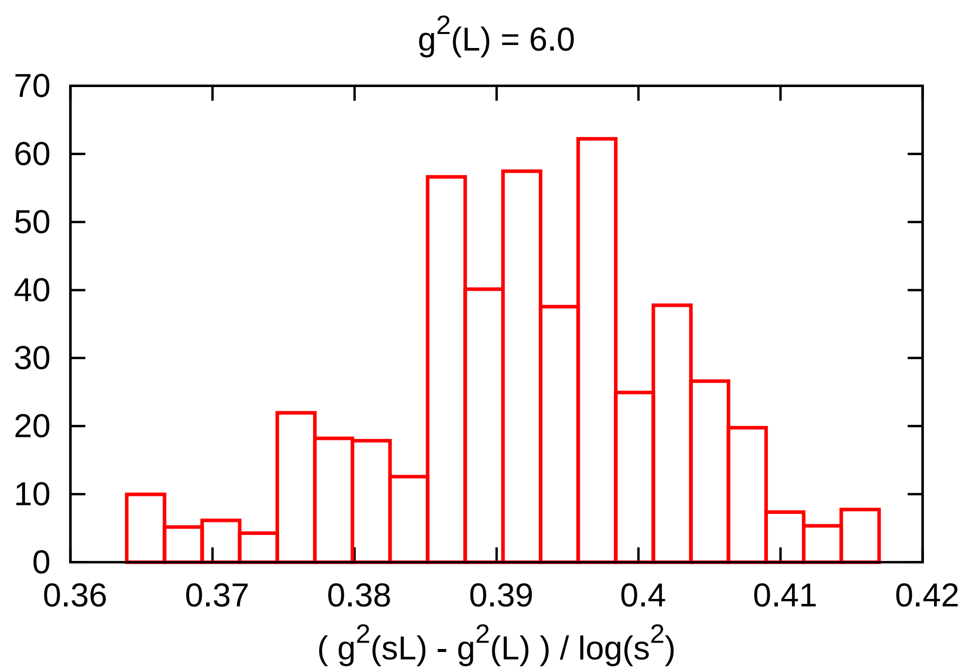
g^2(L) = 5.0
SSC 0.3
SSC 0.0
WSC 0.1
(g^2(sL) - g^2(L)) / log(s^2)
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.003
0.006
0.009
0.012
0.015
0.018
a^2/L^2



g^2(L) = 5.0
50
45
40
35
30
25
20
15
10
5
0
0.24
0.26
0.28
0.3
0.32
0.34
0.36
(g^2(sL) - g^2(L)) / log(s^2)



g^2(L) = 6.0
SSC 0.7
WSC 0.7
(g^2(sL) - g^2(L)) / log(s^2)
a^2/L^2
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
0.003
0.006
0.009
0.012
0.015
0.018



g^2(L) = 6.0
(g^2(sL) - g^2(L)) / log(s^2)
70
60
50
40
30
20
10
0
0.36
0.37
0.38
0.39
0.4
0.41
0.42

Notes

Agreement between continuum WSC and SSC, consistency check

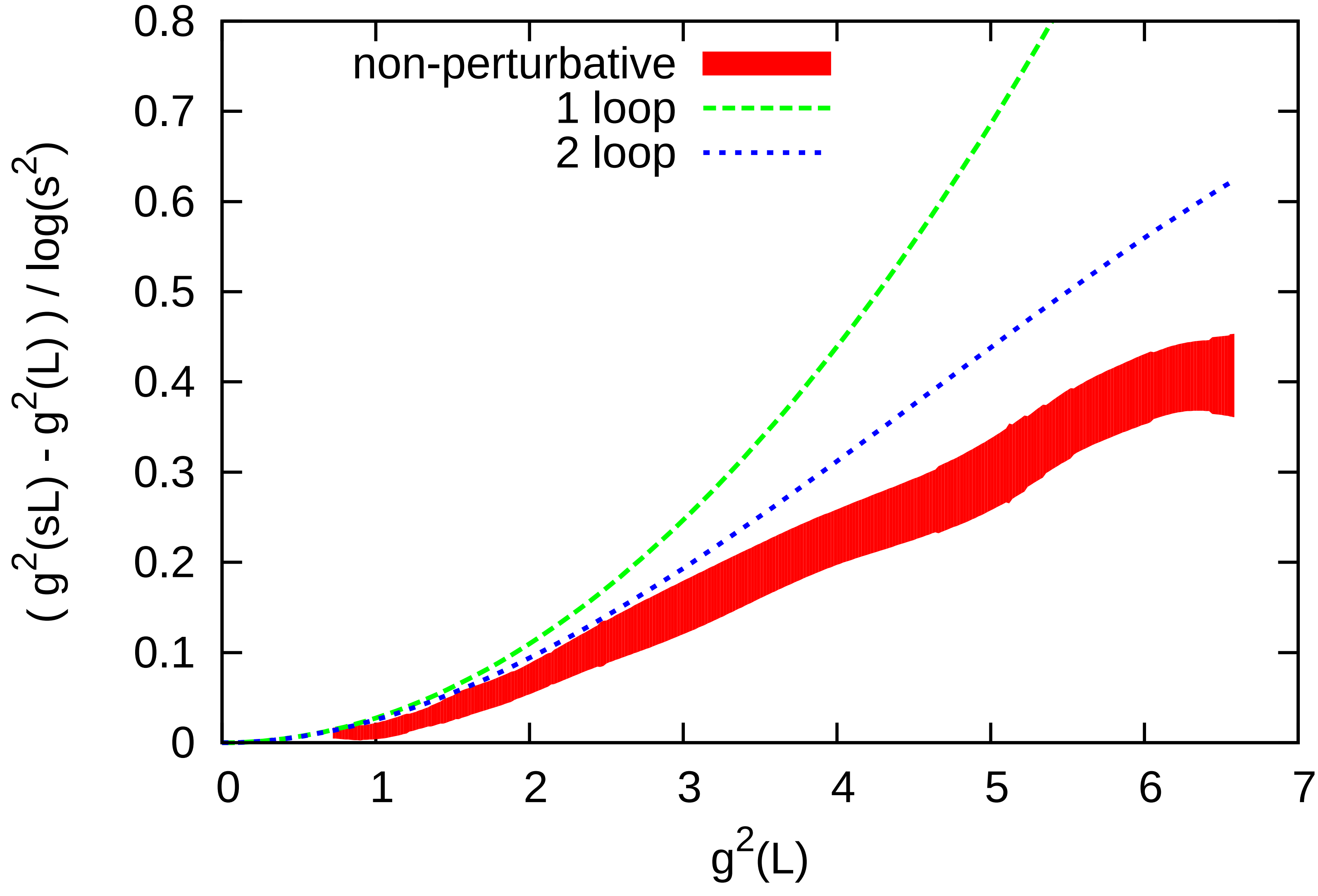
5 points in scaling region: $g^2_{WSC} < 2.5$ and $g^2_{SSC} < 5.5$

SSC scales better, final result for SSC

What happened to the WSC “fix points” at finite lattice volumes?

They didn’t survive the continuum limit $\rightarrow$ lattice artifacts

Final result from SSC



Conclusions and lessons learned

- In the range $0 < g^2 < 6.5$ no sign of β-function turning back

- This range includes 3-loop and 4-loop MSbar fixed points $g_*^2 = 6.28$ and $g_*^2 = 5.73$

- Probably they are perturbative artifacts, similarly to large 2-loop fixed point, $g_*^2 = 10.58$

- Agreement with Schwinger-Dyson resummation (chiral symmetry breaking happens before reaching would-be fixed point)

- Consistency with our previous work on chiral dynamics and mass spectrum

Conclusions and lessons learned

- Continuum limit extremely important and control of related uncertainty
- May lead to qualitative change in behavior (finite lattice volume “fix point” disappears in continuum)
- Extremely important to consider large volumes
- Extremely important to consider several discretizations

The model is in good shape ...

... and we look forward to seeing what happens to the approx. 3σ and 2σ exesses at ATLAS and CMS!

Thank you for your attention!

Backup slides