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Effects of higher dimension operators on the Standard Model Higgs sector.

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Electro-weak stability

- As the standard model couplings are run to very high energy the Higgs self-interaction λ turns negative at some point ($\sim 10^{10..12}$ GeV).
- New physics above the Electroweak scale can make the effective potential stable.
- In an Effective Field Theory language the new physics enters as higher dimension operators suppressed by their typical energy scale.
- The EFT approach captures general aspects of any UV completion of the SM.

Baryogenesis

- One important aspect where higher dimension operators can play a role is Electroweak Baryogenesis.
- To create a matter-anti-matter asymmetry a strong first order EW phase transition is needed.
- In the SM the phase transition is only first order for a Higgs mass below 72 GeV but higher dimension operators may raise the critical value of the Higgs mass above the measured 125 GeV.

Goal of the presentation:

- To determine the impact of higher dimension operators on the SM.
- We need a non-perturbative method because of the large Higgs-top Yukawa coupling and the finite temperature phase transition.
- Since a lattice formulation of the full SM is missing we consider separately two sectors of the SM:
  - The Higgs-Yukawa sector consisting of the Higgs field and all the fermions.
  - The Gauge-Higgs sector consisting of the $SU(2)$ gauge fields and the Higgs field.
- We consider the simplest operator $|\phi|^6$, which is suppressed by two powers of the scale of new physics M_{BSM}, as a proxy for UV-completion.

1) The Higgs-Yukawa model

- Poor man's version of the Standard Model which nonetheless captures the nonperturbative chiral Higgs-top interaction.
- The Higgs and Yukawa parts of the Lagrangian are given by:

$$\mathcal{L}_{\rm H} = |\partial_\mu \phi|^2 + m_0^2 |\phi|^2 + \lambda_4 |\phi|^4 + \lambda_6 M_{\rm BSM}^{-2} |\phi|^6$$

$$\mathcal{L}_{\rm tb} = \overline{\Psi}_t \partial\!\!\!/ \Psi_t + y_b \overline{\Psi}_{t,\rm L} \phi b_{\rm R} + y_t \overline{\Psi}_{t,\rm L} \widetilde{\phi} t_{\rm R} + \text{h.c.} \qquad \widetilde{\phi} = i\tau_2 \phi^\dagger$$

where $\Psi_t = (t, b)^\intercal = (t_{\rm L}, t_{\rm R}, b_{\rm L}, b_{\rm R})^\intercal$ and $\Psi_{t,\rm L} = (t_{\rm L}, b_{\rm L})^\intercal$.

Extended Mean Field Theory (EMFT)

OA, P. de Forcrand, P. Werner and A. Georges *Phys. Rev. D* 88 125006 (2013) [arXiv:1305.7136]

OA, P. de Forcrand, P. Werner and A. Georges *Phys. Rev. D* 90, 065008 (2014) [arXiv:1405.6613]

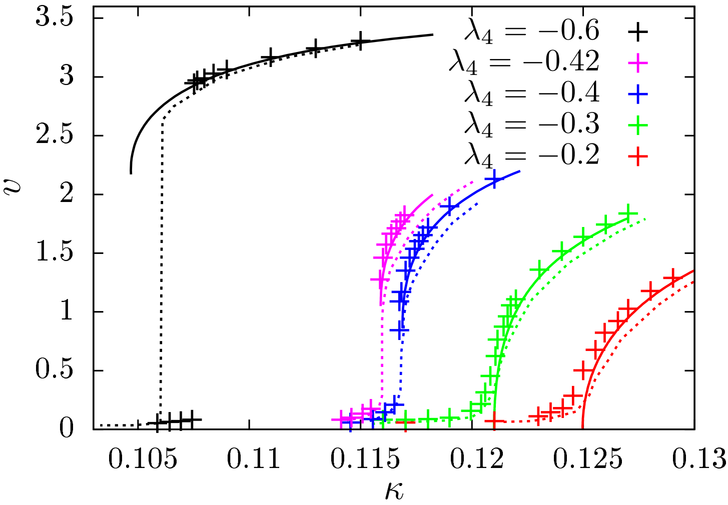
- We solve the model approximately by using an extended version of Mean Field Theory which takes also quadratic fluctuations into account.
- The original 4*d* problem reduces to a 0*d* problem with some self-consistency conditions which can be solved at a very low computational cost.
- EMFT can be formulated in any finite or infinite box which gives access to finite volume and finite temperature effects.
- EMFT yields an implicit action where expectation value and propagation (self-energy) need to be solved for self-consistently.

Bench-marking the EMFT approximation

- EMFT has already proved to be very accurate on ϕ^4 models.
- To check that the fermions are treated correctly we compare to full Monte Carlo simulations of the Higgs-Yukawa model [1].
- The results are very encouraging (see next slide).
- Due to the large scale separation $M_{\mathrm{BSM}} \gg v$ and the Goldstone bosons, Monte Carlo simulations suffer from prohibitive finite size effects. With EMFT, infinite volume is available at no extra cost.
- Moreover, the Monte Carlo simulation suffers from a “sign problem” unless the fermions are mass-degenerate whereas EMFT can handle the physical case.

[1] P. Hegde, K. Jansen, C. -J. D. Lin and A. Nagy *PoS LATT13* [arXiv:1310.6260]

$\lambda_6 = 0.1$, "perturbative"



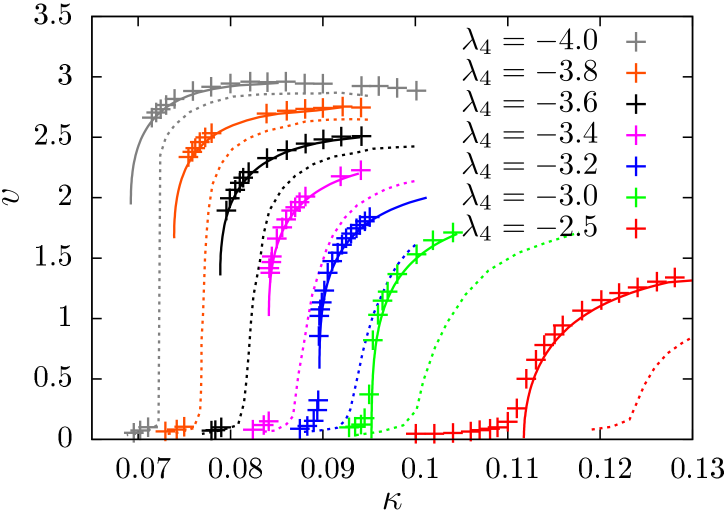
—: EMFT, this work

+ : Monte Carlo

– –: Constraint Effective Potential (1 loop)

Figure adapted from K. Jansen et al. (2013)

$\lambda_6 = 1$, “non-perturbative”



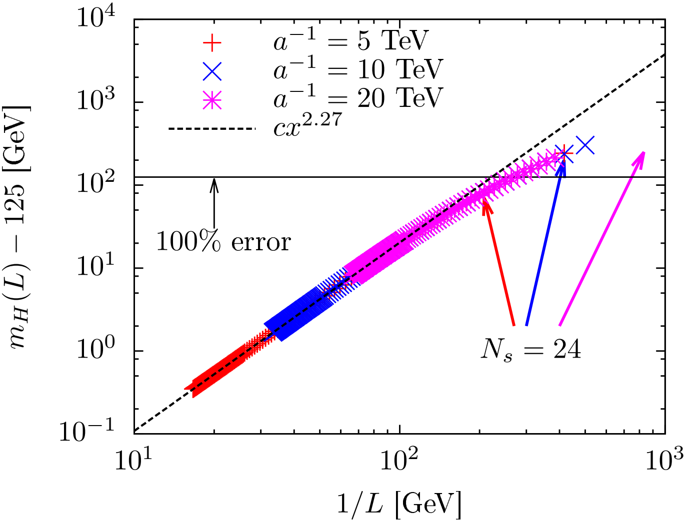
—: EMFT, this work

+ : Monte Carlo

– –: Constraint Effective Potential (1 loop)

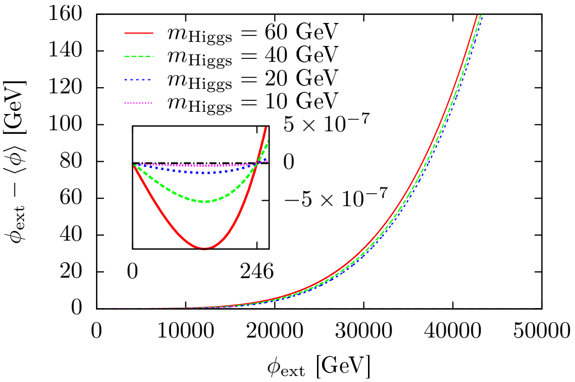
Figure adapted from K. Jansen et al. (2013)

Because of massless Goldstone bosons, the finite volume corrections are power like → major problem for Monte Carlo simulations.



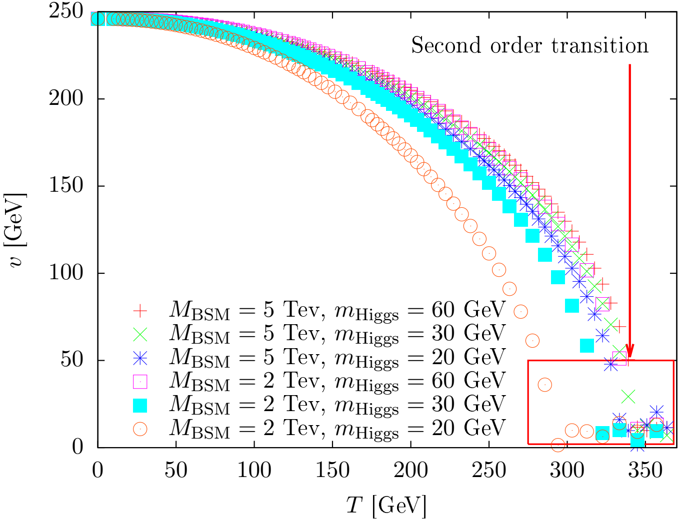
Lowering the Higgs mass bound

~Derivative of the effective potential, $M_{\mathrm{BSM}} = 50$ TeV



Even when M_{BSM} is as heavy as 50 TeV,
m_{Higgs} can be as small as 10 GeV

Finite temperature transition (no gauge fields)



2) Gauge-Higgs

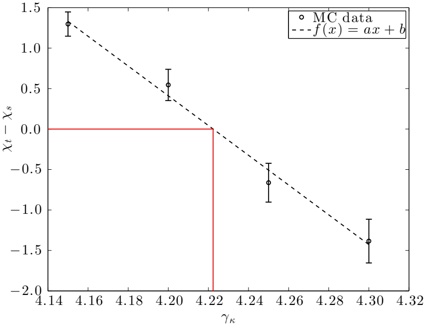
- Perturbatively, the coefficient of the ϕ^3 term which enables a first order transition gets important contributions from the gauge coupling.
- For simplicity the fermions are left out. Well justified in the light of the previous results.
- Lattice action:

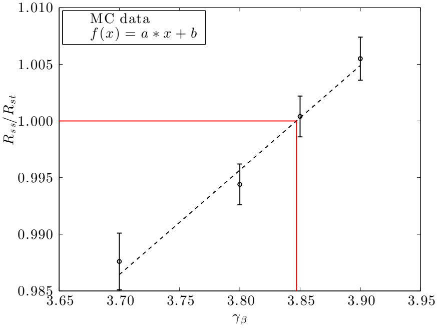
$$\mathcal{L}_{\mathrm{GH}} = -\sum_{\mu} \kappa_\mu \phi^\dagger(x) U_\mu(x) \phi(x+\hat{\mu}) + |\phi(x)|^2 + \lambda \left(|\phi(x)|^2 - 1 \right)^2 - \frac{1}{2} \sum_{\mu>\nu} \beta_\mu \mathrm{Re}\mathrm{Tr} P_{\mu\nu}(x).$$

Anisotropy: $\kappa_i = \kappa/\gamma_\kappa,\ \kappa_4 = \kappa\gamma_\kappa,\ \beta_i = \beta/\gamma_\beta,\ \beta_4 = \beta\gamma_\beta.$

Anisotropy

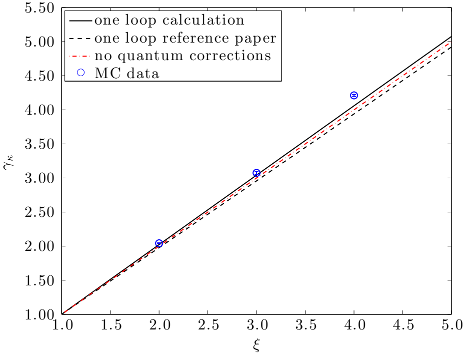
- In order to keep the number of spatial lattice sites small at high temperatures we used an anisotropic action.
- At tree level $\gamma_\kappa = \gamma_\beta = \xi \equiv a_s/a_t > 1$ but they get renormalized by quantum effects and have to be tuned.





Perturbation theory

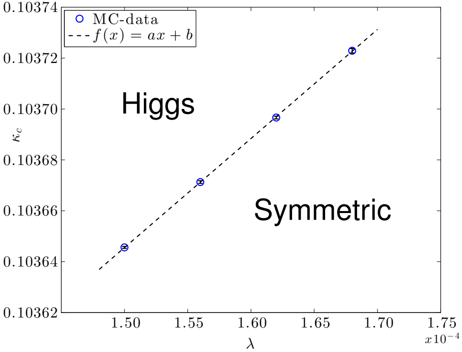
- As a cross check we compare the measured anisotropies to one loop perturbation theory.
- We find very good agreement as well as a typo in the literature [1].



[1] F. Csikor, Z. Fodor, and J. Heitger *Phys. Rev. D* **58** 094504 (1998)

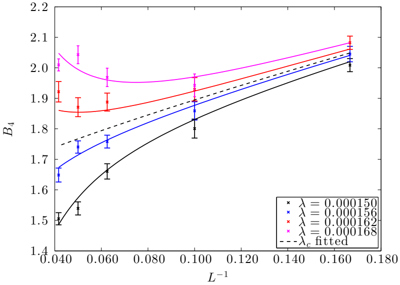
Locating the critical endpoint

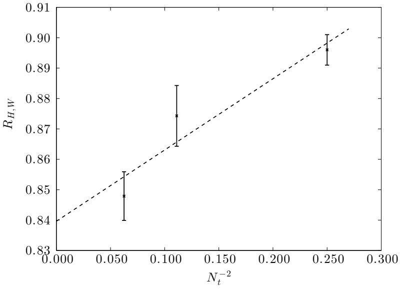
- Once the anisotropies have been tuned we determine the phase boundary in the (κ, λ)-plane.
- Somewhere on this line, in the direction of increasing λ, the transition turns from first order to crossover at a critical endpoint, which we wish to locate.



Locating the critical endpoint

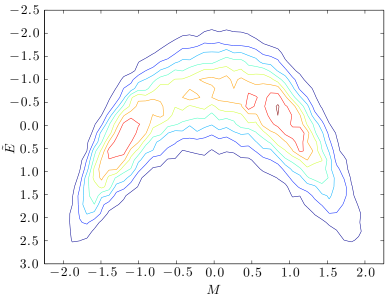
- At the critical endpoint the model is in the universality class of the $3d$ Ising model.
- We measure the 4th Binder cumulant of $\langle\phi\rangle$ along the critical line and look for the λ where it equals the $3d$ Ising value $B_{4,c} = 1.604$. At this point we measure the $T = 0$ Higgs-W boson mass ratio $R_{\mathrm{H,w}}$.



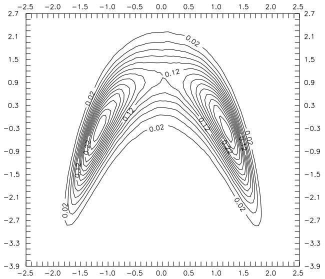


Locating the critical endpoint

- At the critical quartic coupling the distribution of the expectation value of the Higgs field contains two almost merged peaks.



SU(2)+Higgs

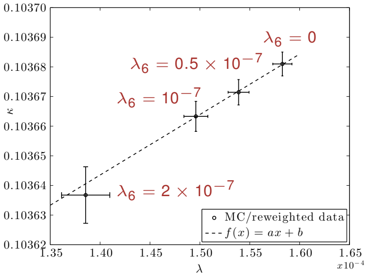


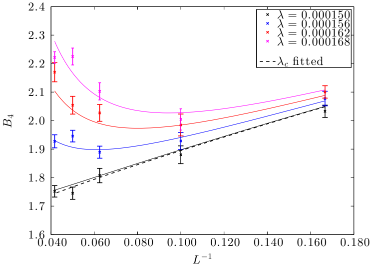
3*d* Ising [1]

[1] K. Rummukainen et al, *Nucl. Phys. B* **532** 283-314 (1998)

Adding the ϕ^6 operator

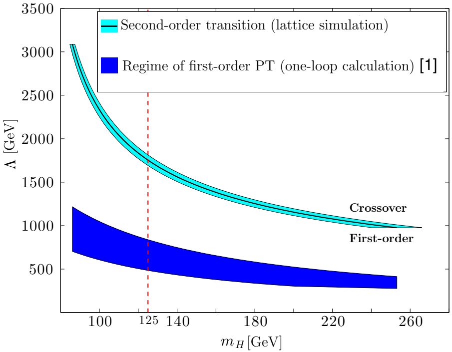
- The ϕ^6 operator is added by reweighing the configurations obtained at $\lambda_6 = 0$.
- Since the critical line $\kappa_c(\lambda)$ moves as a function of $\lambda_6 \equiv \Lambda^{-2}$ we have to reweigh in the three variables κ, λ and λ_6 simultaneously.
- We can then anew locate the critical endpoint and measure the $T = 0$ mass ratio.





Linear response

- Finally, we obtain the derivative of the mass ratio m_H/m_W with respect to the mass scale Λ of the ϕ^6 operator.



- We can then extrapolate to a mass ratio of $125/80$ to obtain a rough estimate of the scale of new physics needed to yield a first order transition at $m_H = 125$ GeV.

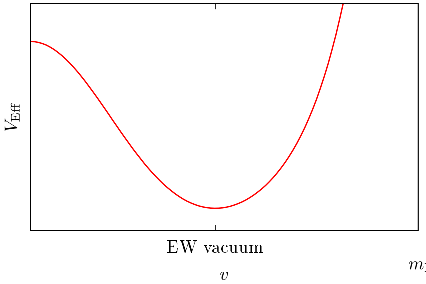
[1] C. Grojean, G. Servant, and J. D. Wells, *Phys. Rev. D* **71** 036001 (2005)

Conclusions

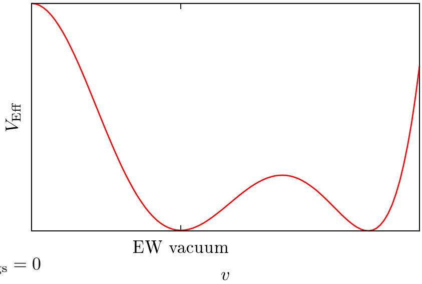
- Sizable deviations from 1-loop results, nonperturbative method crucial.
- Higher dimension operators can stabilize the Higgs potential even at negative quartic couplings.
- EMFT approximation accurately describes the nonperturbative physics.
- In the Higgs-Yukawa model the ϕ^6 operator is not enough to make the finite temperature phase transition first order.
- In the Gauge-Higgs model, on the other hand, it strengthens the transition and could make EW baryogenesis viable if the new physics scale is $1 - 2$ TeV.

Thank you for your attention!

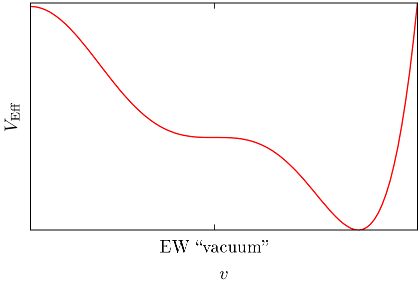
$m_{\text{Higgs}} > m_c$



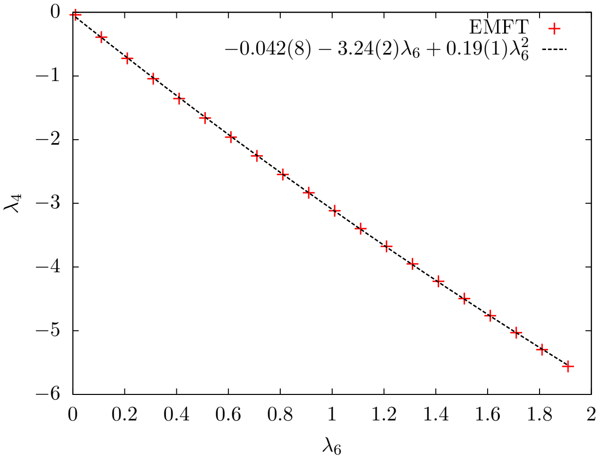
$m_{\text{Higgs}} = m_c$



$m_{\text{Higgs}} = 0$

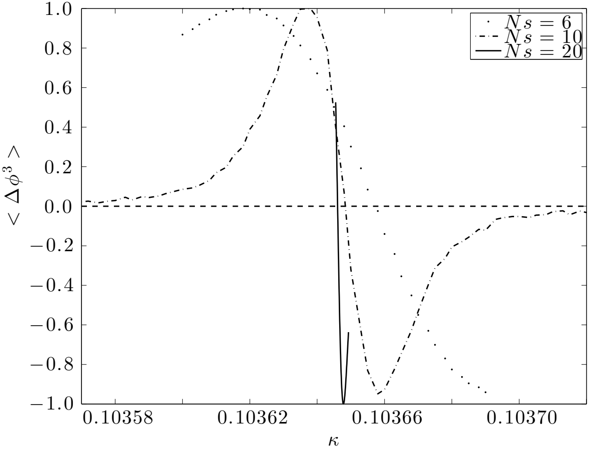


First order line in the $(\lambda_4, M_{\mathrm{BSM}})$-plane



3rd order cumulant

- We use the thrird order cumulant to determine the critical κ.



Extended Mean Field Theory (EMFT)

- We assume small fluctuations around the vacuum expectation value (vev).

$$\Phi_x = (\hat{h}_x, \hat{g}_{1,x}, \hat{g}_{2,x}, \hat{g}_{3,x})^\intercal + (\hat{v}, 0, 0, 0)^\intercal \equiv \delta\Phi_x^\intercal + \langle\Phi\rangle^\intercal ,$$

- The hopping term can be expressed as:

$$\Delta S = -2\kappa \sum_{\pm\mu} \delta\Phi_0^\intercal \delta\Phi_\mu - 4d\kappa\hat{v}\hat{h}_0 .$$

- We can integrate out all fields except Φ_0 at the cost of new couplings, $c_p \Phi_0^p,\ p \geq 2$.

- Truncating at second order is enough to capture most of the dynamics, cf. mass renormalization.

- The effective action becomes:

$$
\begin{aligned}
S_{\mathrm{EMFT}} = & \Phi_0^{\mathrm{T}} (\boldsymbol{I}_4 - \boldsymbol{\Delta}) \Phi_0 + \hat{\lambda} \left(\|\Phi_0\|^2 - 1 \right)^2 - 2\hat{v}(\hat{v} + \hat{h})(2d\kappa - \Delta_1) \\
& + \mathrm{TrLog} \left(M \left(\|\Phi_0\| \right) \right) + V_{\mathrm{BSM}} \left(\|\Phi_0\| \right)
\end{aligned}
$$

- Where $\boldsymbol{\Delta} = \mathrm{diag}\left(\Delta_1, \Delta_2, \Delta_2, \Delta_2\right)$ emulates propagation in the effective bath.

Self-consistency equations

- We have introduced three unknowns in the action so we need three self-consistency conditions:

1. $$\left\langle \Phi_0^{\mathsf{T}} \right\rangle = (\hat{v}, 0, 0, 0)^{\mathsf{T}}$$

2. $$2\left\langle \hat{h}_0^2 \right\rangle_c = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{\frac{1}{2\left\langle \hat{h}_0^2 \right\rangle_c} + \Delta_1 - 2\kappa \sum_\mu \cos(p_\mu)}$$

3. $$2\left\langle \hat{g}_{i,0}^2 \right\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{\frac{1}{2\left\langle \hat{g}_{i,0}^2 \right\rangle} + \Delta_2 - 2\kappa \sum_\mu \cos(p_\mu)}$$

The fermion determinant

- In the EMFT approximation the fermions see the uniform field $\|\Phi_0\| = \sqrt{(\hat{v}+\hat{h}_0)^2 + \hat{g}_{1,0}^2 + \hat{g}_{2,0}^2 + \hat{g}_{3,0}^2} \Rightarrow$ the fermion matrix is diagonal in Fourier space.
- We can choose a basis for the determinant where the different flavors decouple:

$$M\left(\|\Phi_0\|\right)_{ff'} \to \left(\partial\!\!\!/ + y_f\sqrt{2\kappa}\|\Phi_0\|\right)\delta_{ff'}$$

- Fermions discretized using the Neuberger overlap operator which respects chiral symmetry up to $\mathcal{O}(a^2)$ corrections.

- The fermion matrix becomes:

$$M_f^{(\text{ov})} = D^{(\text{ov})} + y_f\sqrt{2\kappa}\|\Phi\|\left(I_4 - \frac{1}{2}D^{(\text{ov})}\right).$$

- Since $\|\Phi\|$ is constant we can calculate the TrLog efficiently in Fourier space:

$$\mathrm{TrLog}\left(M_f^{(\text{ov})}\right) = 2\int\frac{\mathrm{d}^4 p}{(2\pi)^4}\log\left|\nu(p) + y_f\sqrt{2\kappa}\|\Phi_0\|\left(1 - \frac{\nu(p)}{2}\right)\right|^2$$

$$\nu(p) = 1 + \frac{i\sqrt{\tilde{p}^2} + \frac{1}{2}\hat{p}^2 - 1}{\sqrt{\tilde{p}^2 + \left(\frac{1}{2}\hat{p}^2 - 1\right)^2}}$$

$$\tilde{p}^2 = \sum_\mu \sin^2(p_\mu),\ \hat{p}^2 = 4\sum_\mu \sin^2\left(\frac{p_\mu}{2}\right)$$