Perturbative renormalization of ΔS = 2 four-fermion operators with the chirally rotated Schrödinger functional

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Introduction

- Flavour physics play an important role in search of new physics, ΔF=2 transitions of particular relevance for constrains.

- Most general ΔF=2 weak effective Hamiltonian can be written in terms of

  Parity even: $Q_k^{\pm} \in \{Q^{\pm}_{VV+AA}, Q^{\pm}_{VV-AA}, Q^{\pm}_{SS-PP}, Q^{\pm}_{SS+PP}, Q^{\pm}_{TT}, \}$

  Parity odd: $\mathcal{Q}_k^{\pm} \in \left\{\mathcal{Q}^{\pm}_{VA+AV}, \mathcal{Q}^{\pm}_{VA-AV}, -\mathcal{Q}^{\pm}_{SP-PS}, \mathcal{Q}^{\pm}_{SP+PS}, \mathcal{Q}^{\pm}_{T\tilde{T}}, \right\}$

- Where, $O^{\pm}_{\Gamma_X \Gamma_Y} = \frac{1}{2}\left[(\bar{\psi}_1 \Gamma_X \psi_2)(\bar{\psi}_3 \Gamma_Y \psi_4) \pm (2 \leftrightarrow 4)\right]$

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BSM

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PE: $Q_1^{\pm}, Q_2^{\pm}, Q_3^{\pm}, Q_4^{\pm}, Q_5^{\pm}$ PO: $\mathcal{Q}_1^{\pm}, \mathcal{Q}_2^{\pm}, \mathcal{Q}_3^{\pm}, \mathcal{Q}_4^{\pm}, \mathcal{Q}_5^{\pm}$

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  - tmQCD at maximal twist
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In both cases, one only needs PO renormalization.

•Scale evolution has been studied in the SF

$\mathcal{Q}_1^{\pm}$ *[Palombi, Pena, Sint, '06]* *[Guagnelli et al., '06]*

$\mathcal{Q}_2^{\pm}, \mathcal{Q}_3^{\pm}, \mathcal{Q}_4^{\pm}, \mathcal{Q}_5^{\pm}$ *[Papinutto, Pena, Preti, '14]*

Introduction

- Here we want to renormalize the PO operators using the χSF.

$$\begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \\ \mathcal{Q}_3 \\ \mathcal{Q}_4 \\ \mathcal{Q}_5 \end{pmatrix}_{\rm R} = \begin{pmatrix} \mathcal{Z}_{11} & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Z}_{22} & \mathcal{Z}_{23} & 0 & 0 \\ 0 & \mathcal{Z}_{32} & \mathcal{Z}_{33} & 0 & 0 \\ 0 & 0 & 0 & \mathcal{Z}_{44} & \mathcal{Z}_{45} \\ 0 & 0 & 0 & \mathcal{Z}_{54} & \mathcal{Z}_{55} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \\ \mathcal{Q}_3 \\ \mathcal{Q}_4 \\ \mathcal{Q}_5 \end{pmatrix}$$

- The plan is:

  - Automatic O(a) improvement:
    No need for operator improvement.
  - Simpler correlation functions:
    3 point instead of 4 point.
  - Build schemes where LO and NLO evolution is closer.

Few words on the χSF

- Obtained through a chiral rotation of the fields in the SF

$$\psi \longrightarrow R(\alpha)\psi \qquad R(\alpha) = e^{i\gamma_5\tau^3\alpha/2} \qquad \alpha = \pi/2$$

[Sint, '06-'10]

$$\overline{\psi} \longrightarrow \overline{\psi}R(\alpha)$$

- χSF boundary conditions

$$\widetilde{Q}_+\psi\,|_{x_0=0} = \widetilde{Q}_-\psi\,|_{x_0=T} = 0 \qquad \widetilde{Q}_\pm = \frac{1}{2}\left(1 \pm i\gamma_0\gamma_5\tau_3\right)$$

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- Implements in the SF the mechanism of auto O(a) improvement in terms of $\mathcal{P}_5$

$$P_5 : \begin{cases} \psi(x) \to i\gamma_0\gamma_5\tau^3\psi(\tilde{x}) \\ \bar{\psi}(x) \to -\bar{\psi}(\tilde{x})i\gamma_0\gamma_5\tau^3 \end{cases}$$

$$\tilde{x} = (x_0, -\mathbf{x})$$

$$O_{\text{even}} \longrightarrow O(1),\ O(a^2),\ \ldots$$

$$O_{\text{odd}} \longrightarrow O(a),\ O(a^3),\ \ldots$$

Few words on the χSF

- After renormalisation and O(a) improvement of the boundaries, $\mathcal{P}_5$ even observables are free from O(a) effects.

  renormalisation → m_{c} z_{f} O(a) improvement → d_{s} c_{t}

- Studied in PT *[Sint, Vilaseca '12-'14]*, quenched *[González López et al. '13]* *[Leader, Sint, '10]* and with 2 dynamical quarks *[Dalla Brida, Sint '14-'15]*

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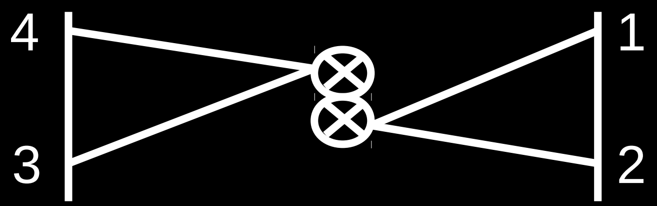
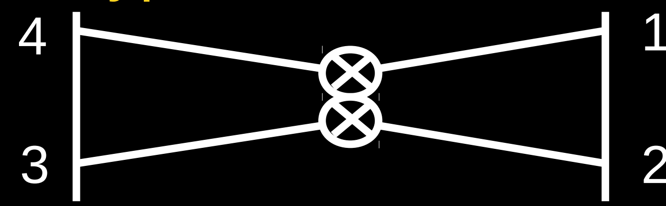
4-f correlation functions

- We start wih the matrix elements of the parity-even operators in the standard SF.
- In general, there are 2 types of correlations:

Type A:



Type B:



$$F_A(x_0) = \langle O_5'^{21} Q^{1234} O_5^{43} \rangle$$

$$K_A(x_0) = \frac{1}{3} \sum_k \langle O_k'^{21} Q^{1234} O_k^{43} \rangle$$

$$F_B(x_0) = \langle O_k'^{21} Q^{1432} O_k^{43} \rangle$$

$$K_B(x_0) = \frac{1}{3} \sum_k \langle O_k'^{21} Q^{1234} O_k^{43} \rangle$$

- Boundary operators:

$$O_5^{12} = \sum_{\mathbf{y},\mathbf{z}} \bar{\zeta}_1(\mathbf{y}) \gamma_5 P_- \zeta_2(\mathbf{z}) \qquad O_k^{12} = \sum_{\mathbf{y},\mathbf{z}} \bar{\zeta}_1(\mathbf{y}) \gamma_k P_- \zeta_2(\mathbf{z})$$

4-f correlation functions

- Next, we perform the chiral rotation to the χSF.
- We consider the flavour assignments: (not unique)

$$f_1 = s^-, \quad f_2 = d, \quad f_3 = s^+, \quad f_4 = d'.$$

Rotates like an up-type quark

The rotation

$$\psi \longrightarrow R(\pi/2)\psi \qquad \overline{\psi} \longrightarrow \overline{\psi}R(\pi/2) \qquad R(\alpha) = e^{i\gamma_5\tau^3\alpha/2}$$

Maps parity-even to parity odd:

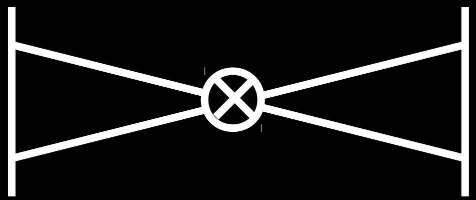
$$Q_1^\pm \longrightarrow -i\mathcal{Q}_1^{\pm,s^-ds^+d'}$$
$$Q_2^\pm \longrightarrow -i\mathcal{Q}_2^{\mp,s^-ds^+d'}$$
$$Q_3^\pm \longrightarrow -i\mathcal{Q}_3^{\mp,s^-ds^+d'}$$
$$Q_4^\pm \longrightarrow -i\mathcal{Q}_4^{\pm,s^-ds^+d'}$$
$$Q_5^\pm \longrightarrow -i\mathcal{Q}_5^{\pm,s^-ds^+d'}$$

4-f correlation functions

- Correlation functions in the χSF:

$$F_i(x_0) \longrightarrow G_i(x_0) = \langle \mathcal{O}'^{\,ds^-}_5\, \mathcal{Q}_i^{s^- ds^+ d'}\, \mathcal{O}_5^{d' s^+} \rangle$$

$$K_i(x_0) \longrightarrow L_i(x_0) = \frac{1}{3} \sum_k \langle \mathcal{O}'^{\,ds^-}_k\, \mathcal{Q}_i^{s^- ds^+ d'}\, \mathcal{O}_k^{d' s^+} \rangle$$



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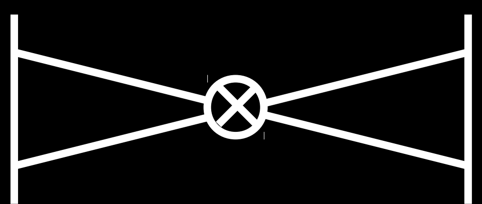

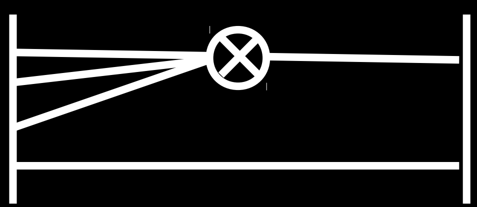

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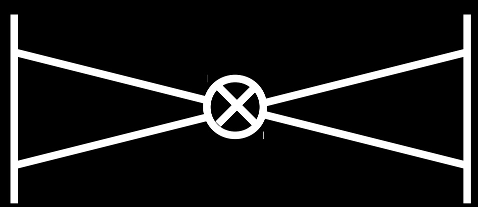

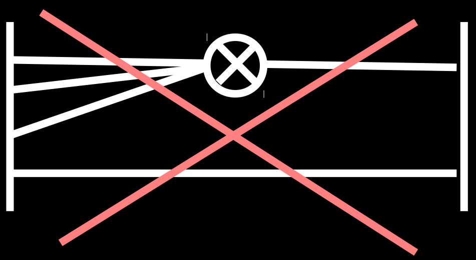

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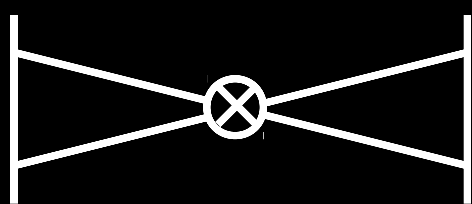
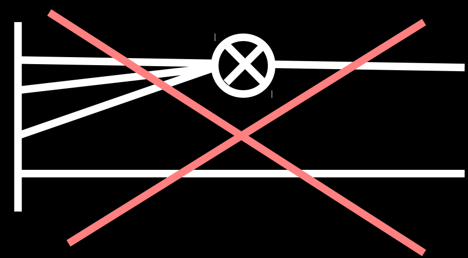




- Renormalized as:

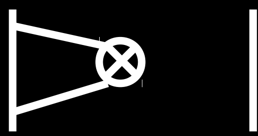
$$[G_i]_R = Z_i Z_\zeta^4 G_i$$

$$[L_i]_R = Z_i Z_\zeta^4 L_i$$

- Boundary field renormalization can be removed with:

$$[g_1]_R = Z_\zeta^4 g_1 \qquad [g_{\widetilde{V}}]_R = Z_\zeta^2 g_{\widetilde{V}}$$

$$[l_1]_R = Z_\zeta^4 l_1 \qquad [l_{\widetilde{V}}]_R = Z_\zeta^2 l_{\widetilde{V}}$$



Renormalization conditions

- For imposing renormalization conditions, we consider matrix elements of the form

$$\mathcal{G}_i = \frac{G_i}{\mathcal{N}(\alpha,\beta,\gamma)} \qquad \mathcal{L}_i = \frac{L_i}{\mathcal{N}(\alpha,\beta,\gamma)} \qquad \alpha+\beta+\gamma<1$$

$$\mathcal{N}(\alpha,\beta,\gamma) = \left[\frac{g_1}{g_1^{(0)}}\right]^{\alpha}\left[\frac{l_1}{l_1^{(0)}}\right]^{\beta}\left[\frac{g_{\widetilde{V}}}{g_{\widetilde{V}}^{(0)}}\right]^{2\gamma}\left[\frac{l_{\widetilde{V}}}{l_{\widetilde{V}}^{(0)}}\right]^{2(1-\alpha-\beta-\gamma)}$$

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- We impose renormalization conditions:
  - Operator $\mathcal{Q}_1^{\pm}$
  - Operators $\mathcal{Q}_2^{\pm}, \mathcal{Q}_3^{\pm}, \mathcal{Q}_4^{\pm}, \mathcal{Q}_5^{\pm}$

$$Z_1\mathcal{G}_1 = \mathcal{G}_1^{(0)}$$

$$Z_1\mathcal{L}_1 = \mathcal{L}_1^{(0)}$$

$$Z\mathcal{A} = \mathcal{A}^{(0)}$$

$$Z = \begin{pmatrix} \mathcal{Z}_{22} & \mathcal{Z}_{23} \\ \mathcal{Z}_{32} & \mathcal{Z}_{33} \end{pmatrix} \qquad \mathcal{A} = \begin{pmatrix} \mathcal{G}_2 & \mathcal{L}_2 \\ \mathcal{G}_3 & \mathcal{L}_3 \end{pmatrix}$$

For a given scheme: fix all parameters !!!

Choosing a scheme

- A criteria to choose a renormalization scheme is to demand LO and NLO evolution to be as close as possible

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- In perturbation theory, renormalization conditions read

$$\mathcal{A} = \mathcal{A}^{(0)} + g_0^2 \mathcal{A}^{(1)} + O(g_0^4) \qquad Z^{(1)} = -\mathcal{A}^{(1)} \left[\mathcal{A}^{(0)}\right]^{-1}$$

$$Z = Z^{(0)} + g_0^2 Z^{(1)} + O(g_0^4)$$

- We can compute all Z factors as functions of the parameters that define a scheme.

  i.e.:

$$Z_1^{(1)} = f(\alpha,\beta,\gamma) - \frac{G_1^{(1)}}{G_1^{(0)}}$$

$$f(\alpha,\beta,\gamma) = \alpha \frac{g_1^{(1)}}{g_1^{(0)}} + \beta \frac{l_1^{(1)}}{l_1^{(0)}} + 2\gamma \frac{g_{\widetilde{V}}^{(1)}}{g_{\widetilde{V}}^{(0)}} + 2(1-\alpha-\beta-\gamma)\frac{l_{\widetilde{V}}^{(1)}}{l_{\widetilde{V}}^{(0)}}$$

- We obtain all universal leading order AD: $\frac{\gamma_0}{(4\pi)^2} \ln(a\mu)$

Choosing a scheme

- The NLO anomalous dimensions can be obtained matching to a reference scheme

$$\gamma_1^{\chi SF} = \gamma_1^{\mathrm{ref}} + [\chi_{\chi SF,\mathrm{ref}}^{(1)}, \gamma_0] + 2\beta_0 \chi_{\chi SF}^{(1)} + \beta_0^{\lambda} \lambda \frac{\partial}{\partial \lambda} \chi_{\chi SF,\mathrm{ref}}^{(1)} - \gamma_0 \chi_g^{(1)}$$

$$\overline{g}^2_{\chi SF} = \chi_g(g_{\mathrm{ref}}) \overline{g}^2_{\mathrm{ref}} \qquad O_R^{\chi SF} = \chi_{\chi SF,\mathrm{ref}}(g_{\mathrm{ref}}) O_R^{\mathrm{ref}}$$

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- RG-evolution formally obtained by *[Papinutto, Pena, Preti, '06]*

$$U(\mu_2, \mu_1) = \mathrm{T} \exp \left\{ \int_{\overline{g}(\mu_1)}^{\overline{g}(\mu_2)} \frac{\gamma(g)}{\beta(g)} dg \right\}$$

$$U(\mu_2, \mu_1) = [W(\mu_2)]^{-1} U(\mu_2, \mu_1)_{LO} W(\mu_1)$$

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$$W(\mu) = 1 + \bar{g}^2(\mu) \underline{J(\mu)} + O(\bar{g}^4)$$

$$J - \left[\frac{\gamma_0}{2\beta_0}, J\right] = \frac{\beta_1}{2\beta_0^2}\gamma_0 - \frac{1}{2\beta_0}\gamma_1$$

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$$W(\mu) = 1 + \bar{g}^2(\mu) J(\mu) + O(\bar{g}^4)$$

$$J - \left[\frac{\gamma_0}{2\beta_0}, J\right] = \frac{\beta_1}{2\beta_0^2}\gamma_0 - \frac{1}{2\beta_0}\gamma_1$$

Choose (α, β, γ) such that the norm of J is small

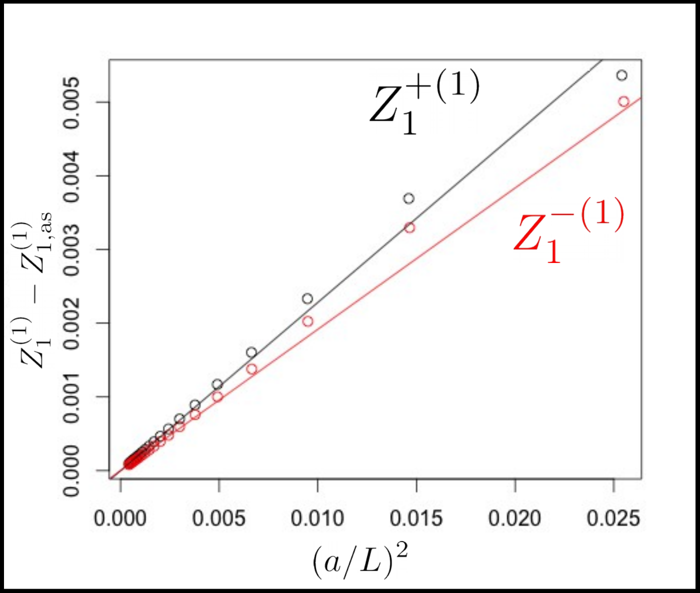
Results

- For $\mathcal{Q}_1^{\pm}$, we find schemes for which J = 0:

$$Z_1 \mathcal{L}_1^+ = \mathcal{L}_1^{+,(0)} \longrightarrow \alpha = \beta = 0.16378, \quad \gamma = 0$$

$$Z_1 \mathcal{L}_1^- = \mathcal{L}_1^{-,(0)} \longrightarrow \alpha = \beta = 0, \quad \gamma = 0.42828$$

- Convergence to the asymptotic value:



Results

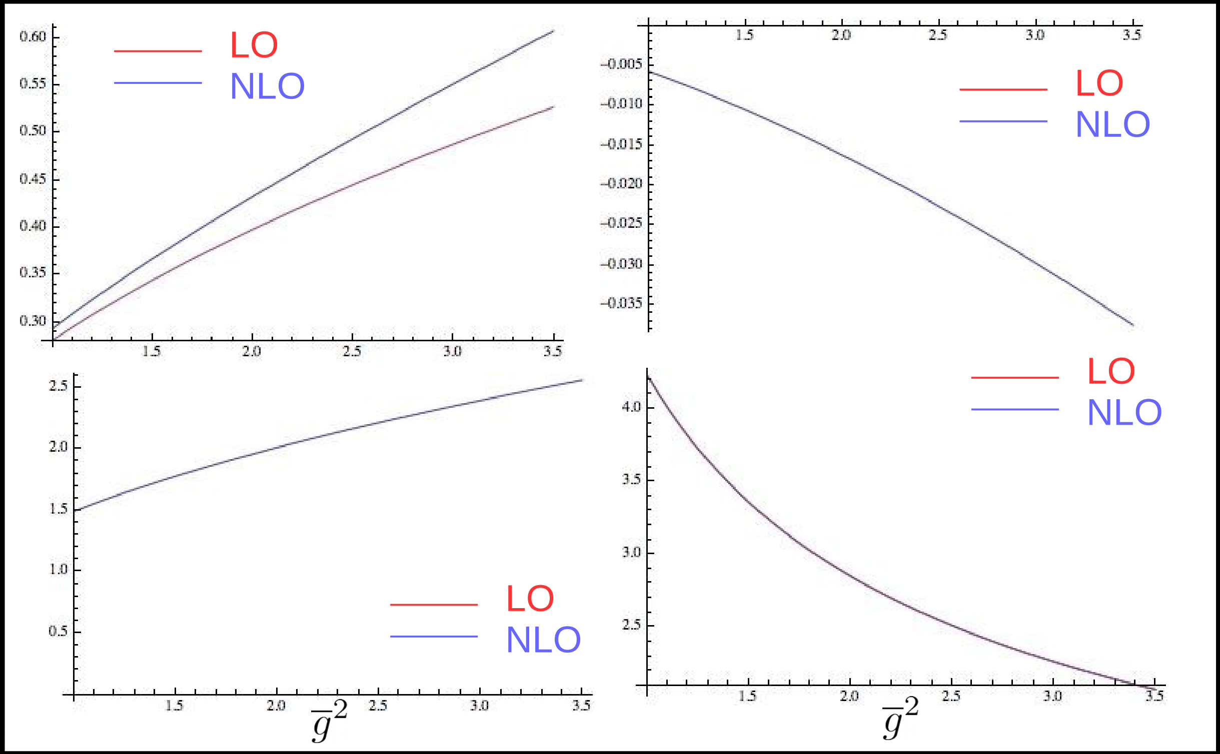
- For $\mathcal{Q}_2^{\pm}, \mathcal{Q}_3^{\pm}, \mathcal{Q}_4^{\pm}, \mathcal{Q}_5^{\pm}$, we are scanning parameters, looking for appealing choices and discarding ugly schemes

  i.e.:

$$\begin{pmatrix} \mathcal{G}_4^{+}(\alpha_1,\beta_1,\gamma_1) & \mathcal{L}_4^{+}(\alpha_2,\beta_2,\gamma_2) \\ \mathcal{G}_5^{+}(\alpha_1,\beta_1,\gamma_1) & \mathcal{L}_5^{+}(\alpha_2,\beta_2,\gamma_2) \end{pmatrix}_{\theta=0.5}$$

$$(\alpha_1,\beta_1,\gamma_1) = (0.014, 0.264, 0.657)$$
$$(\alpha_2,\beta_2,\gamma_2) = (0.95, 0.05, 0)$$

- Running in this scheme:



Conclusions

- We have prepared the χSF for the study of 4-quark ops.

- We wanted:

    - Automatic O(a) improvement
    - Simpler correlation functions
    - Schemes with small NLO anomalous dimensions

- The overall strategy seems to work. We want to study further the different possibilities for building schemes.

- Non-perturbative study ready to go (see Dalla Brida's talk)

Oh well, that was the story !!!

Thank you very much !!!