Charmonia and bottomonia at finite temperature on large quenched lattice

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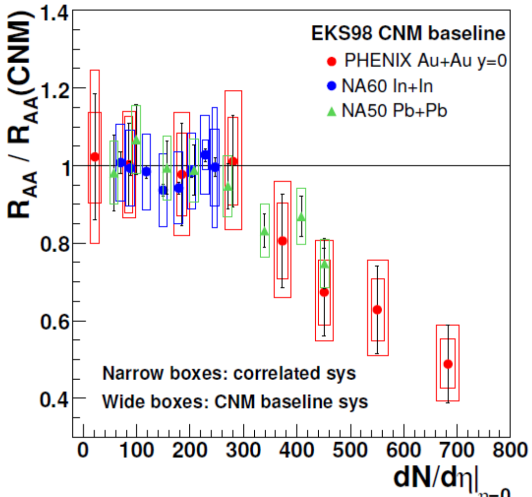
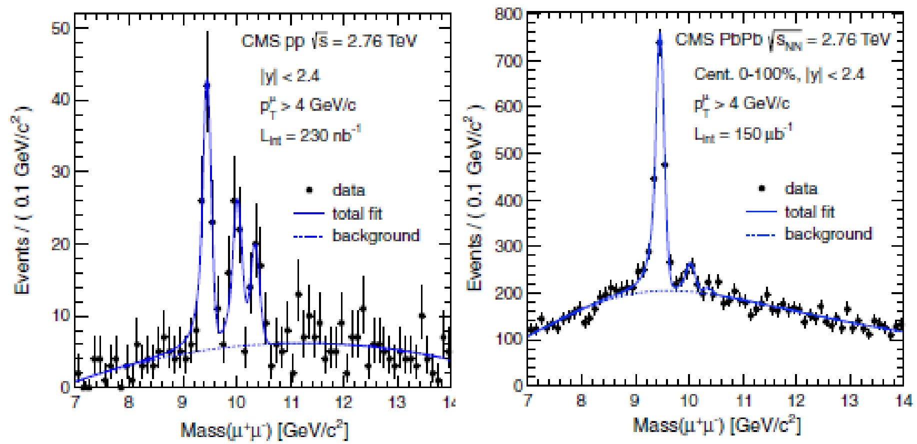
Quarkonia

- Bound states of heavy $q\bar{q}$
- At a certain temperature T_D, the dissociation should occur due to the color Debye screening
- An important probe of the quark-gluon plasma created in relativistic heavy ion collisions at RHIC, LHC

→ Theoretical investigation of in-medium properties of quarkonia plays an important role to understand experimental results.



N. Brambilla *et al.*, EPJ C71 (2011) 1534



S. Chatrchyan *et al.*, PRL 109 (2012) 222301

Quarkonium spectral function

Temporal Euclidian meson correlator

$$G_H(\tau,\vec{p}) \equiv \int d^3x e^{-i\vec{p}\cdot\vec{x}} \langle J_H(\tau,\vec{x}) J_H(0,\vec{0}) \rangle$$

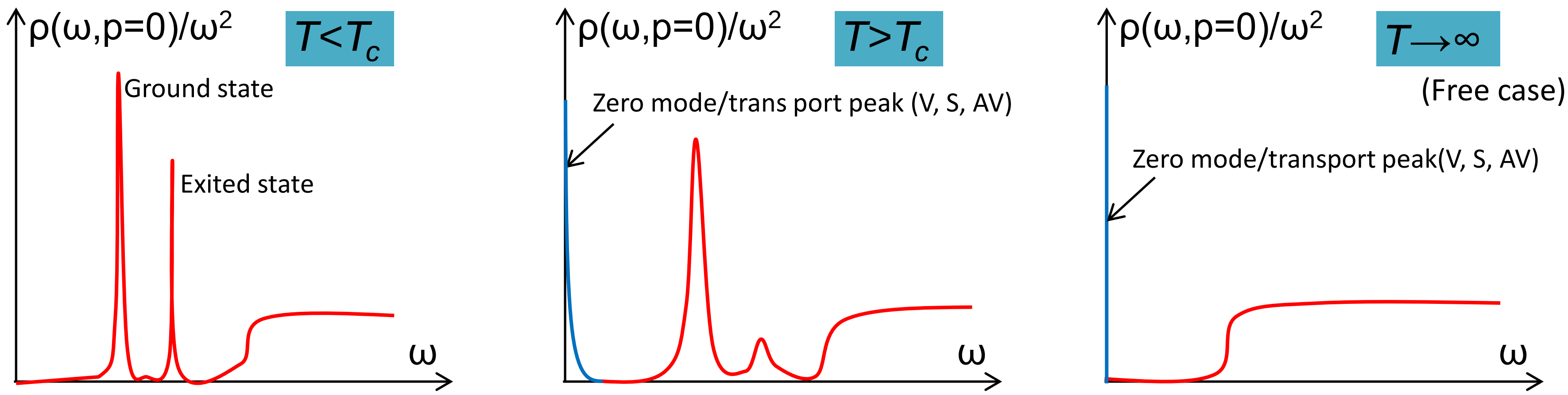
$$= \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega,\vec{p}) K(\omega,\tau)$$

Spectral function (SPF) has all information about in-medium meson properties

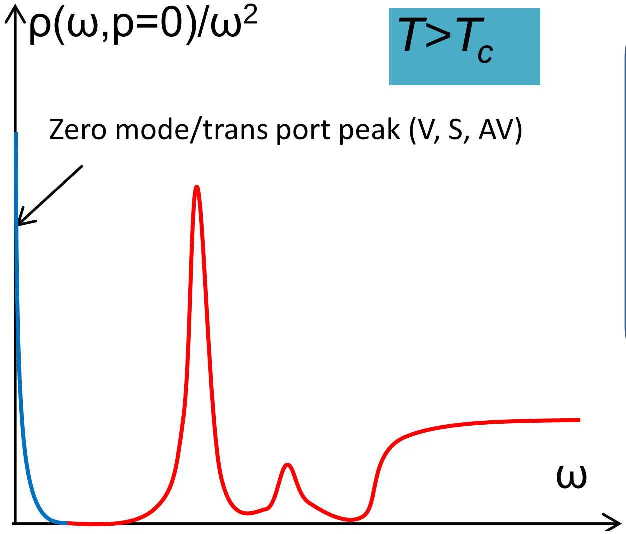
$$J_H(\tau,\vec{x}) \equiv \bar{\psi}(\tau,\vec{x}) \Gamma_H \psi(\tau,\vec{x})$$

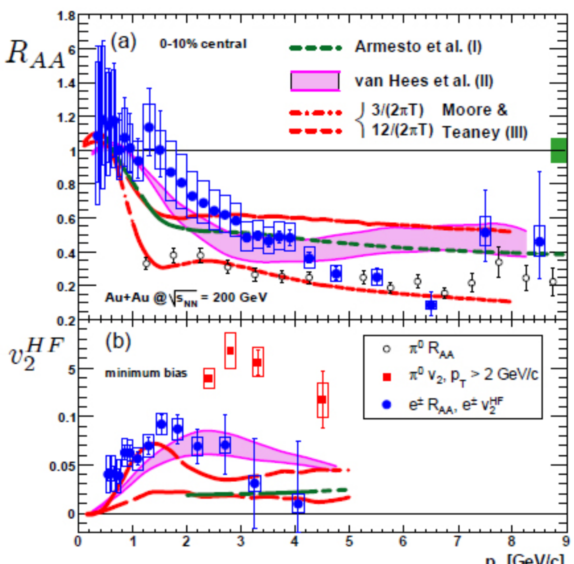
$$K(\omega,\tau) \equiv \frac{\cosh[\omega(\tau-1/2T)]}{\sinh(\omega/2T)}$$

Channel	Γ_H	$^{2S+1}L_J$	J^{PC}	Quarkonia
Pseudoscalar (PS)	γ_5	$^1S_0$	0^{-+}	η_c, η_b
Vector (V)	γ_i	$^3S_1$	1^{--}	J/ψ, Υ
Scalar (S)	$\mathbf{1}$	$^1P_0$	0^{++}	χ_{c0}, χ_{b0}
Axialvector (AV)	$\gamma_i\gamma_5$	$^3P_1$	1^{++}	χ_{c1}, χ_{b1}



Transport coefficients





Adare *et al*. [PHENIX Collaboration], PRL 98 (2007) 172301

Heavy quark diffusion constant

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^V(\omega, \mathbf{0})}{\omega}$$

$\rho_{ii}^V(\omega)$: spatial component of vector spectral function

χ_{00} : Quark number susceptibility

$\rho_{00}^V(\omega) = 2\pi\chi_{00}\omega\delta(\omega)$ ➡ $G_{00}^V(\tau) = T\chi_{00}$

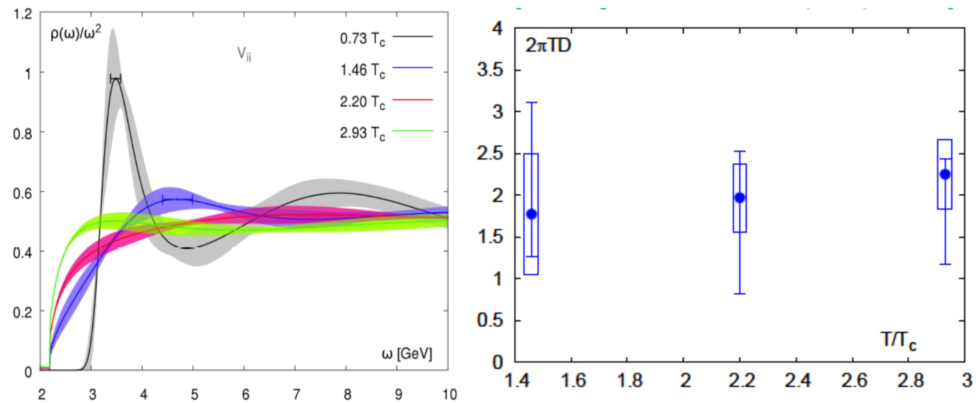
The evolution of the system in hydro models

→ **Transport coefficients are important.**

Determination by first principle calculations in QCD is needed.

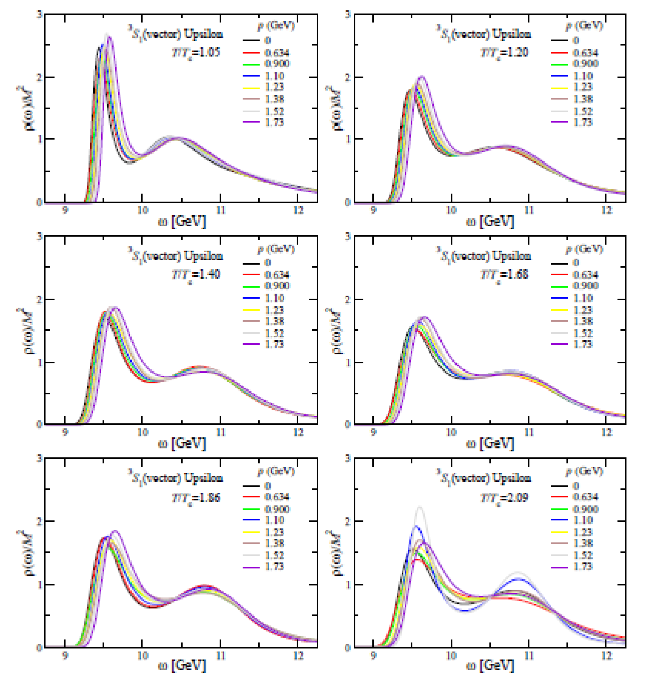
Recent lattice studies : spectral functions

- Charmonia
  - Several studies both in quenched QCD and with dynamical quarks
  - Dissociation temperatures are still not conclusive
  - A transport coefficient has been computed
  - **More precise determination of the SPFs on larger and finer lattice is needed**



H.-T. Ding *et al.*, PRD 86 (2012) 014509

- Bottomonia
  - NRQCD
  - **It is good to crosscheck without NRQCD**



G.Aarts *et al.*, PRL 106 (2011) 061602

G.Aarts *et al.*, JHEP 1303 (2012) 084

Our approach

- Computing correlators and spectral functions more precisely

  → **Using very large and fine lattices**

- Both charmonia and bottomonia are investigated

  → **Able to check quark mass dependence**

- Crosschecking previous results by using a different method to reconstruct spectral functions

  → **Estimating systematic uncertainties**

A stochastic construction of SPF: Stochastic Analytical Inference (SAI)

K.S.D. Beach, arXiv:cond-mat/0403055
S. Fuchs *et al.*, PRE81, 056701 (2010)

- Introducing a mapping $\phi : \mathbb{R} \mapsto [0,1]$

H.-T. Shu will talk about a different stochastic method on Fri @ 16:30

$$\phi(\omega) = \frac{1}{\mathcal{N}} \int_{-\infty}^{\omega} D(\nu) d\nu$$

Positive-definite
Same normalization to a spectral function

- Normalization of a spectral function

$$1 = \frac{1}{\mathcal{N}} \int d\omega A(\omega) = \int d\phi(\omega) \frac{A(\omega)}{D(\omega)} = \int_0^1 dx n(x) \qquad n(x) = \frac{A(\phi^{-1}(x))}{D(\phi^{-1}(x))}$$

Hamiltonian:

$$\frac{1}{2}\chi^2[A] = H[n(x)] = \int_0^{\beta} \frac{d\tau}{\sigma(\tau)^2} \left| \int_0^1 dx \hat{K}(\tau,x) n(x) - \bar{G}(\tau) \right|^2 \qquad \hat{K}(\tau,x) = K(\tau,\omega)$$

$$\langle n(x) \rangle = \frac{1}{Z} \int \mathcal{D}n \, n(x) e^{-H[n]/\alpha}$$

$$\int \mathcal{D}n = \int_0^{\infty} \left(\prod_x dn(x) \right) \Theta(n) \delta \left(\int_0^1 dx n(x) - 1 \right)$$

$\Rightarrow$

$$\langle A(\omega) \rangle = \langle n(\phi(\omega)) \rangle D(\omega)$$

Comparison with MEM (1)

Maximum entropy method (MEM)

Default model = prior information

– A standard technique for the SPF reconstruction
– Minimizing $Q[A] = \frac{1}{2}\chi^2[A] - \alpha S[A]$, where $S[A] = -\int d\omega A(\omega) \ln\left(\frac{A(\omega)}{D(\omega)}\right)$

- **A mean filed solution of SAI = A solution of MEM**
- (Mean filed) entropy SAI = Entropy of MEM

$$S[\bar{n}] = -\int_0^1 dx \bar{n}(x) \ln \bar{n}(x) = -\int d\omega \bar{A}(\omega) \ln\left(\frac{\bar{A}(\omega)}{D(\omega)}\right) = S[\bar{A}]$$

- (Mean filed) free energy of SAI = Q of MEM

$$F\mathcal{N} = H[\bar{n}] - \alpha S[\bar{n}] - \mu\mathcal{N} = \frac{1}{2}\chi^2[\bar{A}] - \alpha S[\bar{A}] - \mu\mathcal{N} = Q$$

Comparison with MEM (2)

- Bayesian inference

Likelihood function

Prior probability

$$P[A|\bar{G}] = \frac{P[\bar{G}|A]P[A]}{P[G]}$$

Posterior probability

Evidence

SAI

χ^2 term

$$P[\bar{G}|n] = \frac{1}{Z'} e^{-H[n]/\alpha} \qquad Z' = \int \mathcal{D}\bar{G} e^{-H[n]/\alpha}$$

Constraint of n filed

$$P[n] = \Theta(n)\delta\left(\int_0^1 dx n(x) - 1\right)$$

Probability = Weight of the path int.

$$P[n|\bar{G}] = \frac{P[\bar{G}|n]P[n]}{P[\bar{G}]} = \Theta(n)\delta\left(\int_0^1 dx n(x) - 1\right)\frac{1}{Z} e^{-H[n/\alpha]}$$

$$P[\bar{G}] = \int \mathcal{D}n \frac{1}{Z'} e^{-H[n]/\alpha} = \frac{Z}{Z'}$$

MEM

χ^2 term

$$P[\bar{G}|A] = \frac{1}{Z_1} \exp\left(-\frac{1}{2}\chi^2[A]\right) \quad Z_1 = \int \mathcal{D}\bar{G} e^{-\chi^2[A]/2}$$

Entropy term

$$P[A] = \frac{1}{Z_2} \exp\left(\alpha S[A]\right) \qquad Z_2 = \int \mathcal{D}A e^{\alpha S[A]}$$

Maximum probability = Minimum Q

$$P[A|\bar{G}] = \frac{P[\bar{G}|A]P[A]}{P[\bar{G}]} = \frac{e^{-Q[A]}}{Z_1 Z_2 P[\bar{G}]}$$

$$P[\bar{G}] = \int \mathcal{D}A P[\bar{G}|A]P[A] = \frac{\int \mathcal{D}A e^{-Q[A]}}{Z_1 Z_2}$$

Monte Carlo evaluation of SAI

1. Generating a configuration as superposition of δ functions

$$n_C(x) = \sum_\gamma r_\gamma \delta(x - a_\gamma)$$

   - Update scheme:
     a. Shifting δ functions
     b. Changing residues of δ functions, keeping $\sum_\gamma r_\gamma = 1$
   - Updating with probability $P = \min\{1, e^{-\Delta H[n]/\alpha}\}$

2. Taking ensemble average at a certain α $\langle n(x)\rangle_\alpha = \frac{1}{N}\sum_C n_c(x)$

3. Converting to the SPF $\langle A(\omega)\rangle_\alpha = D(\omega)\langle n(\phi(\omega))\rangle_\alpha$

Repeat 1-3 for various αs

4. Integrating out α, similarly to MEM
   - A probability $P[\alpha|\bar{G}] \propto \int \mathcal{D}n e^{-H[n]/\alpha}$ needs to be calculated

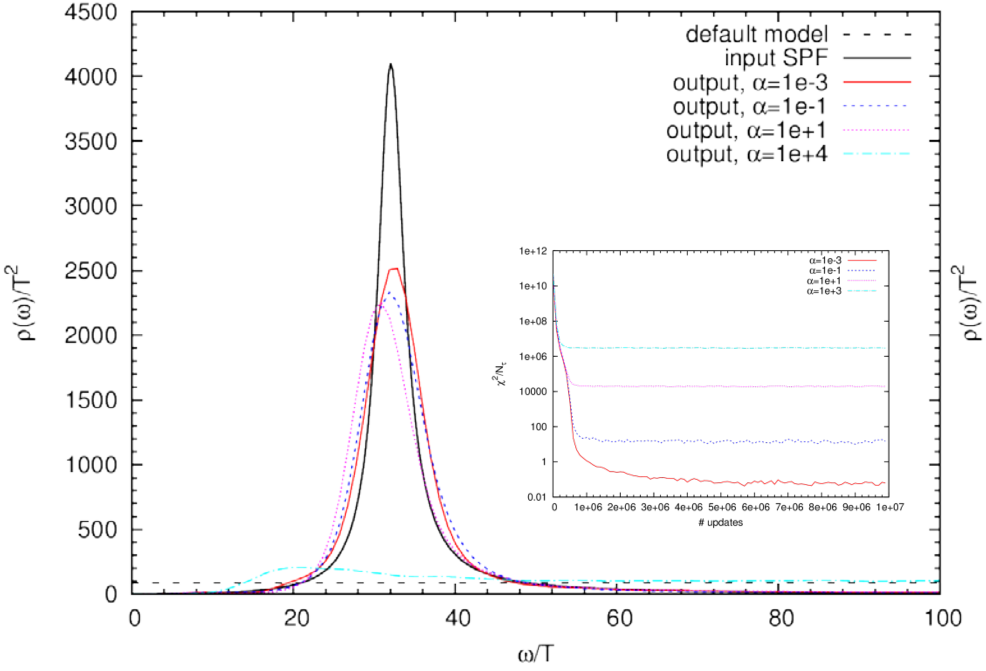
Mock data test

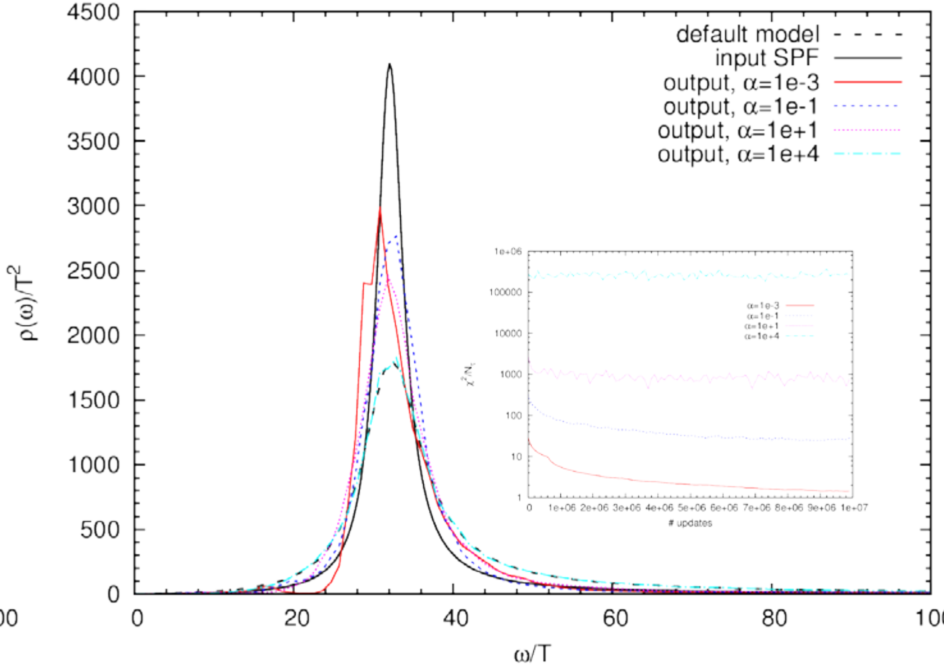
Input SPF:

$$\rho_{\rm in}(\omega) = \frac{0.002\omega}{(\omega-0.5)^2+0.001}$$

data points of the correlator:

$$N_\tau = 64$$





Simulation Setup

- Standard plauette gauge & O(a)-improved Wilson quark actions
- In quenched QCD
- On fine and large isotropic lattices
- $T = 0.73 - 2.2 T_c$
- Both charm & bottom
- Only show very preliminary V-channel at vanishing momentum results
- α has not integrated out yet

β	N_σ	N_τ	T/T_c	# confs.
7.192	96	48	0.73	259
		32	1.1	476
		28	1.25	336
		24	1.5	336
		16	2.2	239

β	a [fm]	a^{-1} [GeV]	κ_{charm}	κ_{bottom}	$m_{J/\Psi}$ [GeV]	m_Υ [GeV]
7.192	0.0188	10.5	0.13194	0.12257	3.140(3)	9.574(3)

The scale has been set by r_0=0.49fm and with a formula for r_0/a in

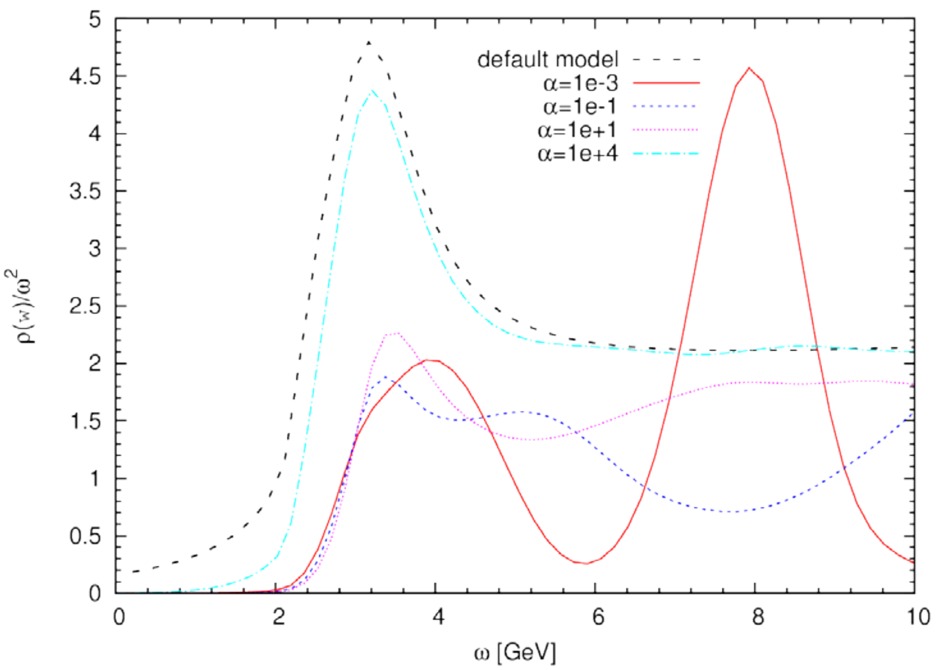
A. Francis, O. Kaczmarec, M. Laine, T. Neuhaus, HO, PRD 91 (2015) 9, 096002

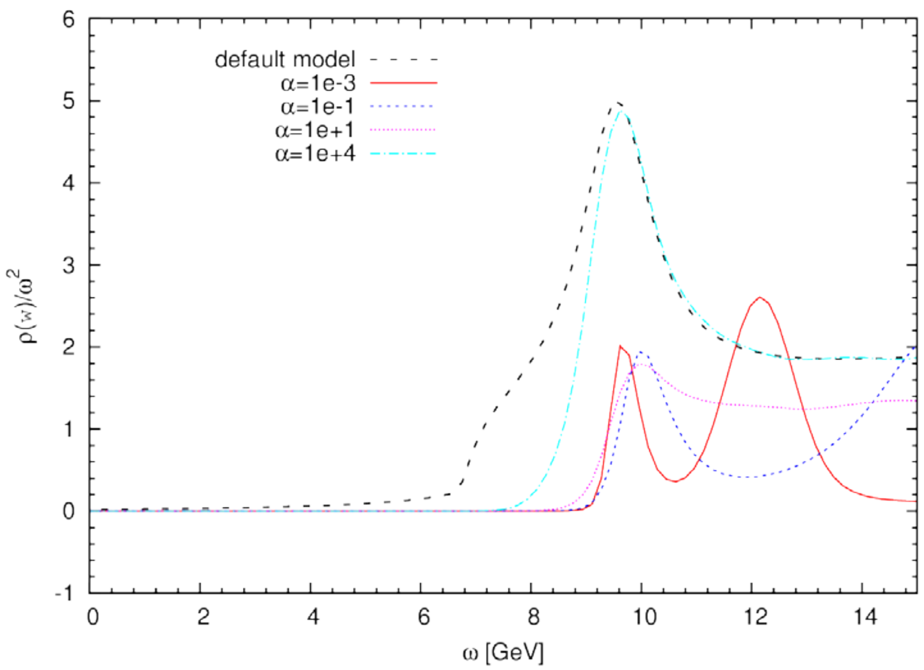
Experimental values: $m_{J/\psi}$ = 3.096.916(11) GeV, m_Υ = 9.46030(26) GeV

J. Beringer *et al*. [PDG], PRD 86 (2012) 010001

SPF for V channel at T = 0.73Tc (N_τ = 48)

Charm **Preliminary** Bottom



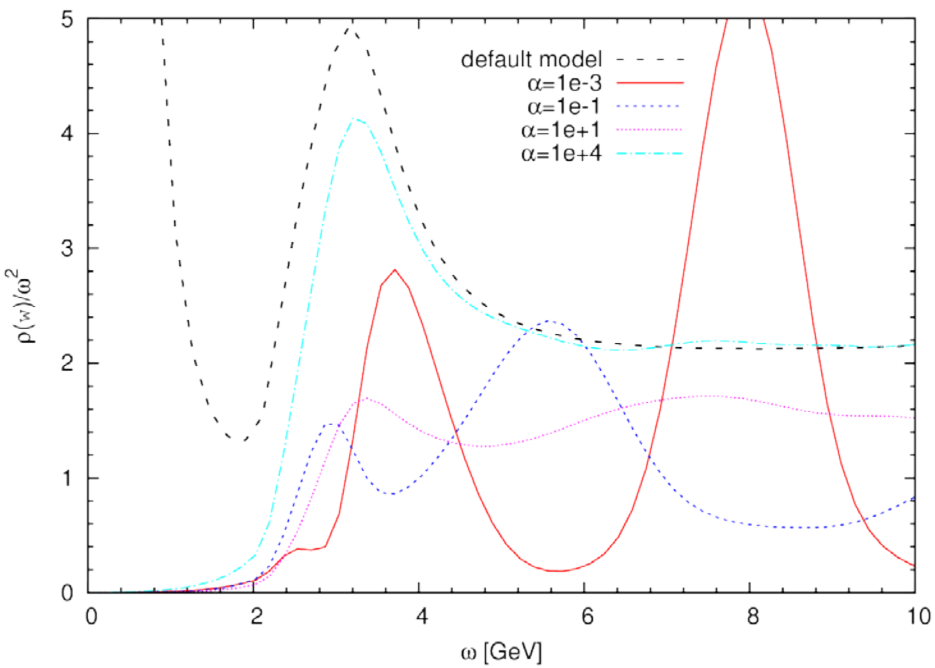


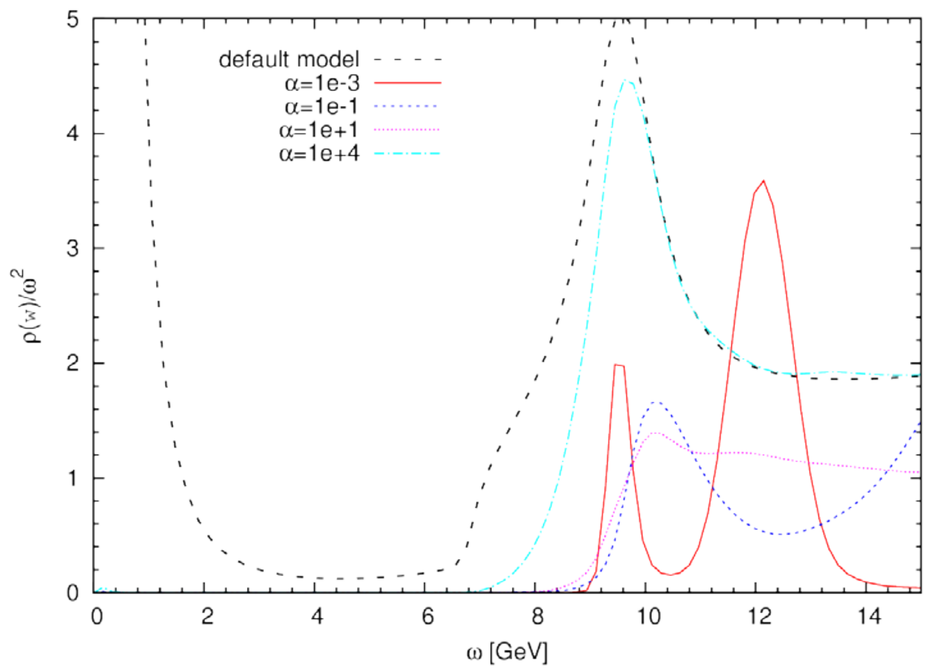
Default model = Wilson free SPF + Resonance peak

There is large α dependence

SPF for V channel at T = 1.1Tc (N_τ = 32)

Charm **Preliminary** Bottom





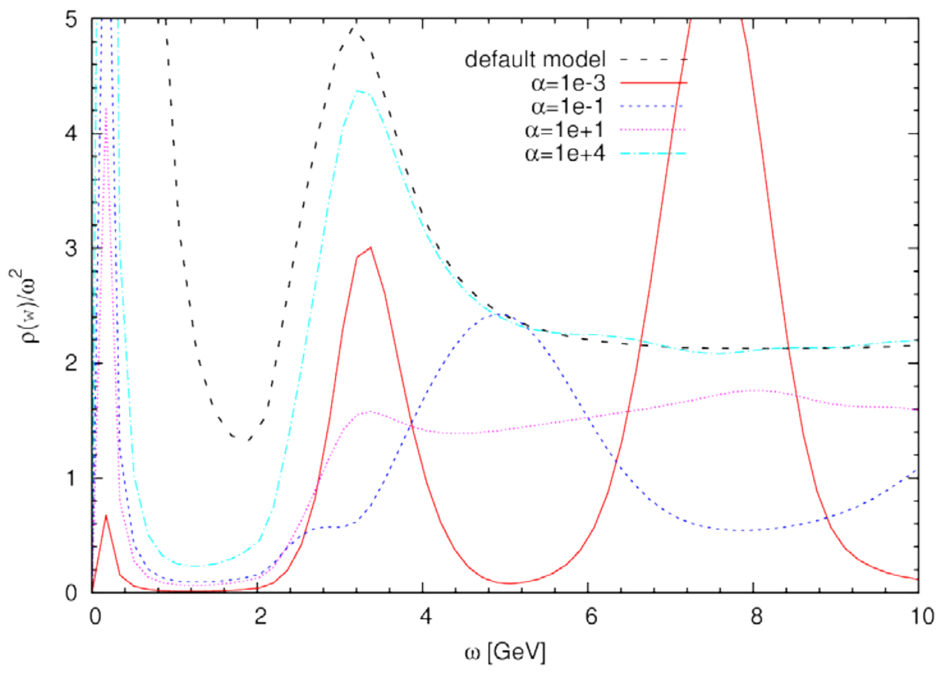
Default model = Wilson free SPF + Resonance peak + Transport peak

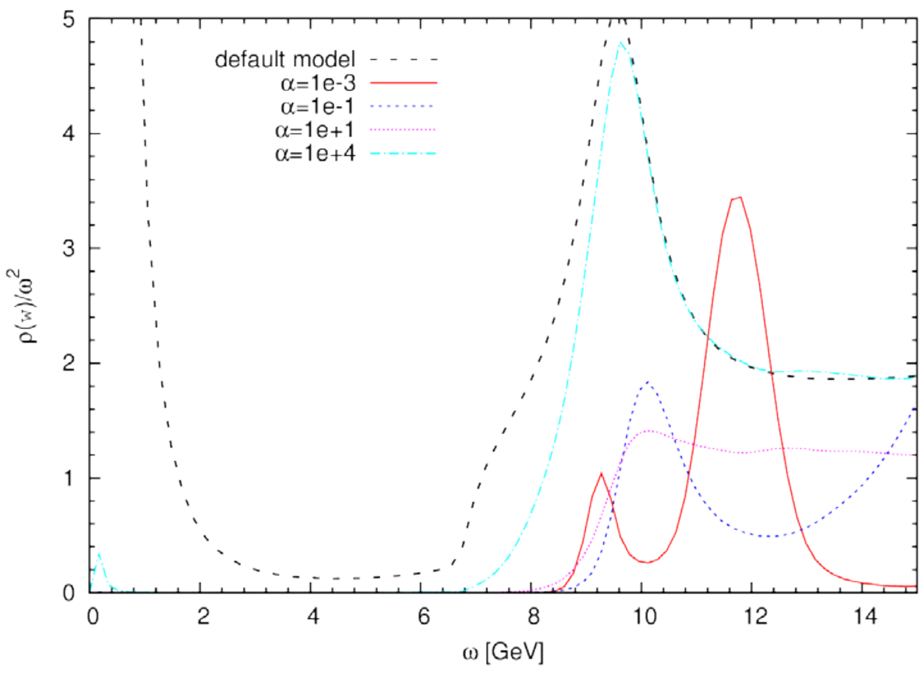
There is large α dependence

There is no transport peak both charm and bottom

SPF for V channel at T = 1.25Tc (N_τ = 28)

Charm **Preliminary** Bottom





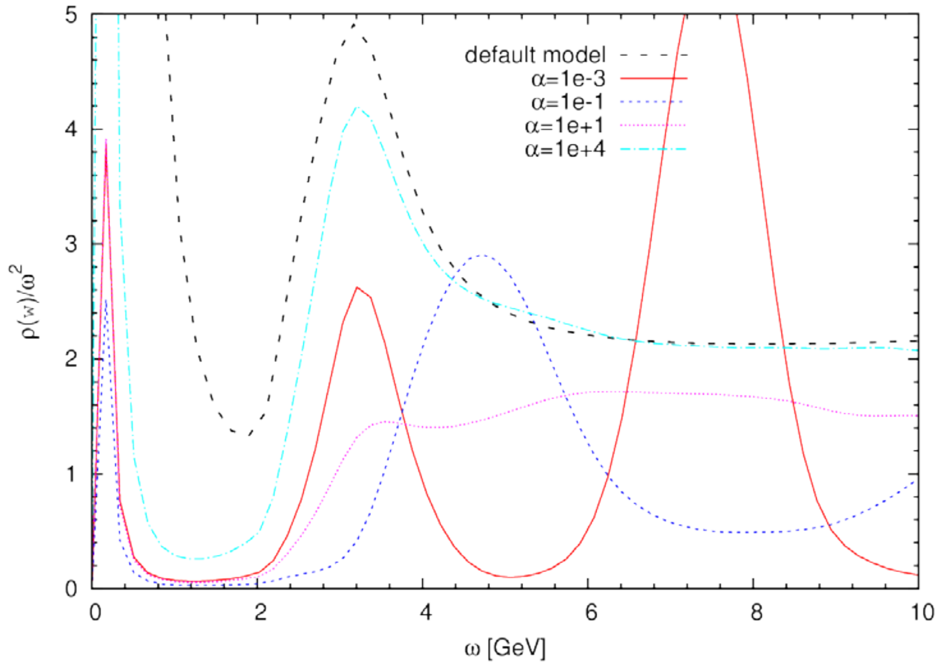
Default model = Wilson free SPF + Resonance peak + Transport peak

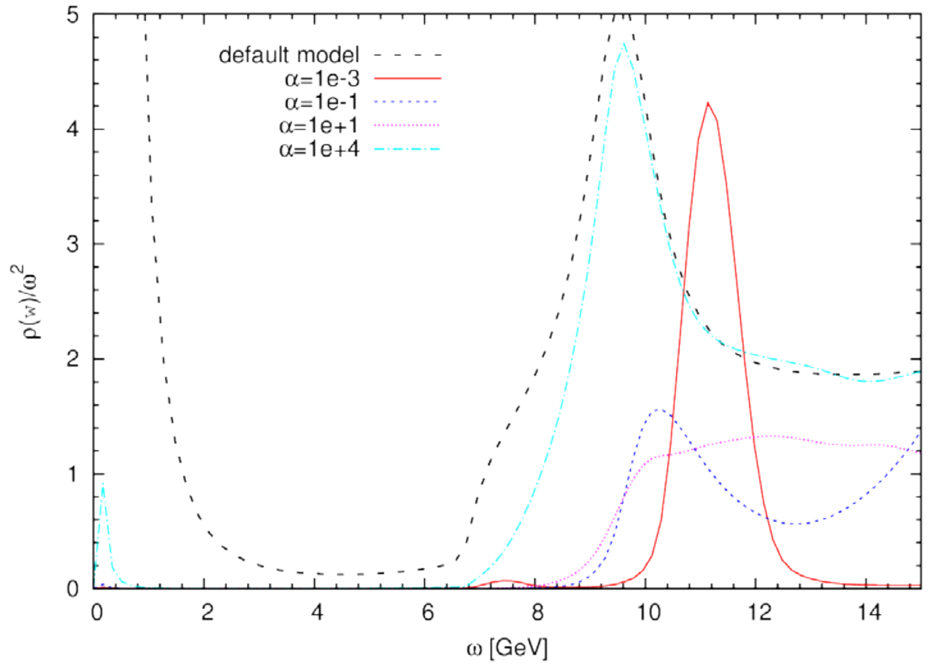
There is large α dependence

A transport peak clearly appears for charm

SPF for V channel at T = 1.5Tc (N_τ = 24)

Charm Preliminary Bottom



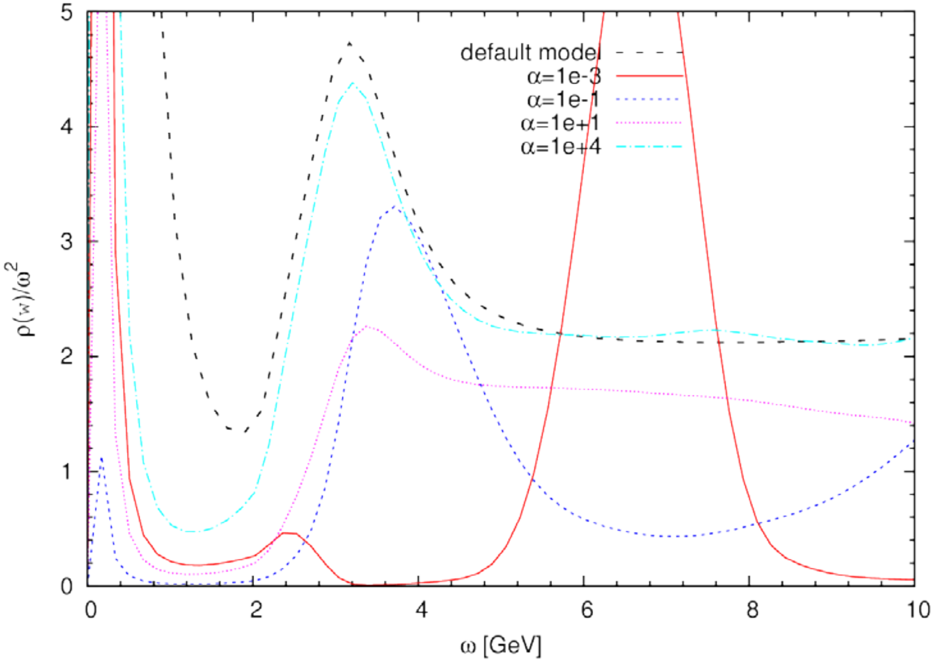


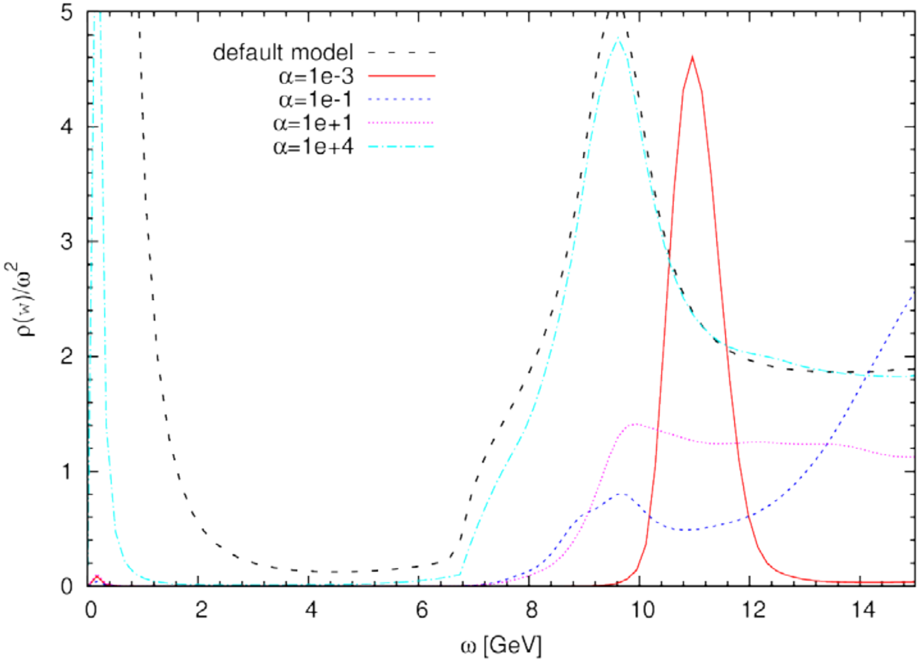
Default model = Wilson free SPF + Resonance peak + Transport peak

There is large α dependence
A transport peak clearly appears for charm

SPF for V channel at T = 2.2Tc (N_τ = 16)

Charm **Preliminary** Bottom





Default model = Wilson free SPF + Resonance peak + Transport peak

There is large α dependence

A transport peak clearly appears for charm

Conclusions & outlook

- We calculate quarkonium spectral functions
  - On fine and large isotropic lattices
  - With quark mass for both charm and bottom
  - By using a stochastic method
  - At temperatures in a range between 0.73 and 2.2Tc
- There is large regularization parameter α dependence
- More studies are needed
  - To eliminate α
  - To investigate default model dependence
  - To compare to MEM

End