Nucleon axial and tensor charges with the overlap fermions

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Kobe

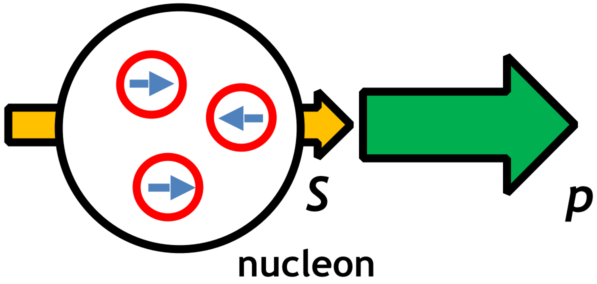
Quark axial and tensor charges

Probe the spin structure of nucleon, non/relativistic polarized component

Axial charge:

Nucleon axial charge probes the quark longitudinal polarization (helicity)

$$\langle N(p,S)|\bar{q}\gamma^{\mu}\gamma_5 q|N(p,S)\rangle = 2p^{\mu}\Delta q$$



Important problem:

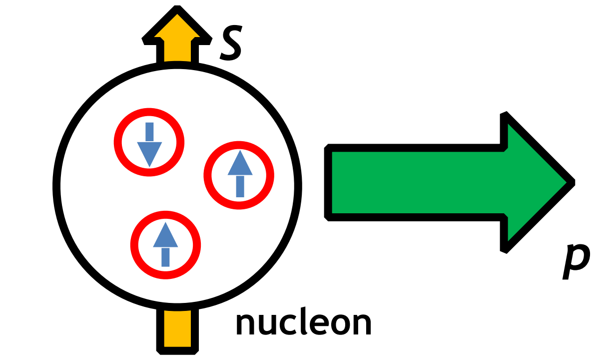
Proton spin crisis $\sum_q \Delta q \sim 0.3 \neq 1$

⇒ Why quark spin fraction so small ?

Tensor charge:

Nucleon tensor charge probes the quark transverse polarization (transversity)

$$\langle N(p,S)|\bar{q}i\sigma^{\mu\nu}\gamma_5 q|N(p,S)\rangle = 2(S^{\mu}p^{\nu} - S^{\nu}p^{\mu})\delta q$$



Why important:
Related to the quark EDM contribution to the nucleon EDM
(EDM is a powerful probe of new physics beyond standard model)

Object of study

The nucleon axial and tensor charges have extensively been worked in Lattice QCD.

Analysis including disconnected diagram contributions is required to control the systematics of the nucleon charges,
But, disconnected diagrams are very noisy…

New techniques to improve the statistical accuracy are available (use of all-to-all propagator, low mode averaging, all-mode-averaging).

Object of study:

Evaluate the nucleon axial and tensor charges including disconnected diagrams in lattice QCD with overlap fermions.

Setup

Setup:

N_f = 2+1 QCD using overlap quarks + Iwasaki gauge action

Simulation parameters:

Lattice spacing : a = 0.112(1) fm

Fixed topology Q = 0

16^3 x 48 lattice, m_π = 450, 540 MeV (calculation for m_π = 380, 290 MeV is on-going, please wait …)

All-to-all propagators:

Low and high mode contributions:

$$D^{-1} = \sum_{k}^{160} \frac{1}{\lambda_k} u_k u_k^\dagger + \text{“high modes”}$$

- 160 low Dirac eigenmodes
- High mode contribution with noise method

Nucleon charges on lattice

Nucleon charge matrix element:

$$\langle N | O | N \rangle = \lim_{|t_{snk}-t_{src}|,\ |t_{vtx}-t_{src}| \to \infty} 2m_N \frac{\langle 0 | N(y,t_{snk})\, O(x,t_{vtx})\, N^\dagger(0,t_{src}) | 0 \rangle}{\langle 0 | N(y,t_{snk})\, N^\dagger(0,t_{src}) | 0 \rangle}$$

(Δq and δq are dimensionless, factor out 2 m_N)

⇒ **Ratio of 3pt and 2pt functions**

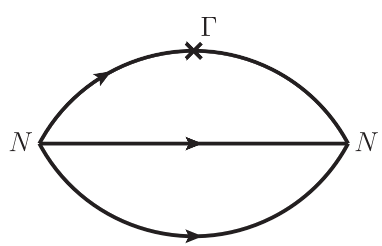
O(x) : quark charge operator

- $O(x) = \bar{q}\, \gamma^3 \gamma_5\, q\, (x)$ (Axial charge)
- $O(x) = \bar{q}\, i\sigma^{03} \gamma_5\, q\, (x)$ (Tensor charge)

Polarize the nucleon of the 3-point function in the z-axis to measure the axial, tensor charges

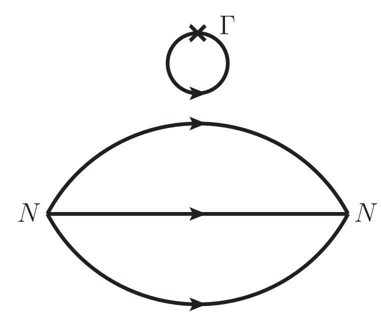
Renormalize to μ = 2 GeV J. Noaki et al., Phys. Rev. D 81, 034502 (2010).

3-point function:



Connected diagrams

For isovector, isoscalar charges



Disconnected diagram

For isoscalar, strange contributions

Improvement of the statistics

Decompose the nucleon correlator into low and high modes:

$$< C_{3pt} > = \underbrace{< C_{3pt} >_{llll}}_{\text{low modes : LMA}} + \underbrace{< C_{3pt} >_{hlll} + < C_{3pt} >_{lhll} + \ldots}_{\text{high modes : AMA}}$$

Low mode averaging:

$$< C_{3pt} >_{llll} \rightarrow < C_{3pt} >_{LMA} = \frac{1}{N_t} \sum_{t_{src}} < C_{3pt} (t_{src}) >_{llll}$$

$\Rightarrow$ Average over source points (N_t = 48 time slices)

All-mode-averaging for high modes:

$$<O>_{AMA} = <O>_{Str} - <O>_{Rel} + \frac{1}{N_G} \Sigma_G <O_G>_{Rel}$$

$<O>_{Str}$: calculated with strict stopping condition in the inversion of D

$<O>_{Rel}$: calculated with relaxed stopping condition in the inversion of D

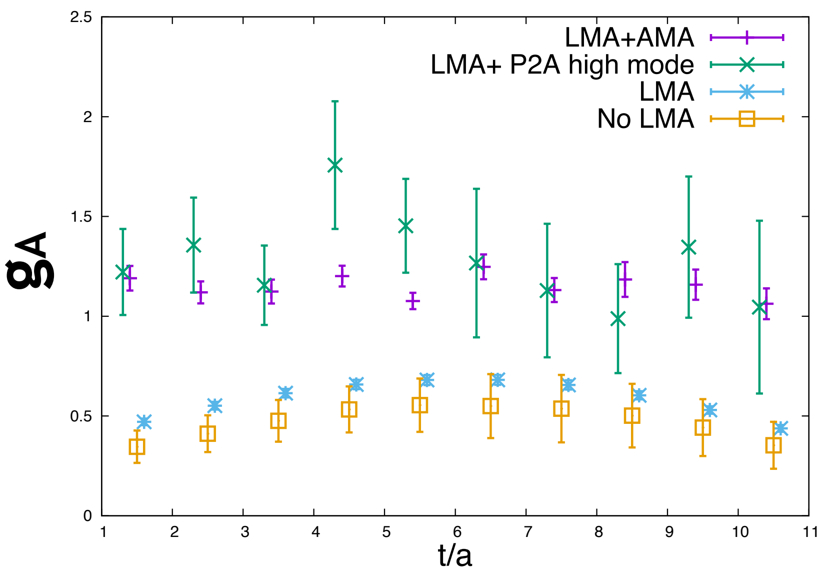
O_G : Shift source points (24 points in our work)

G. S. Bali, S. Collins, A. Schafer, Comp. Phys. Com. **181**, 1570 (2010).

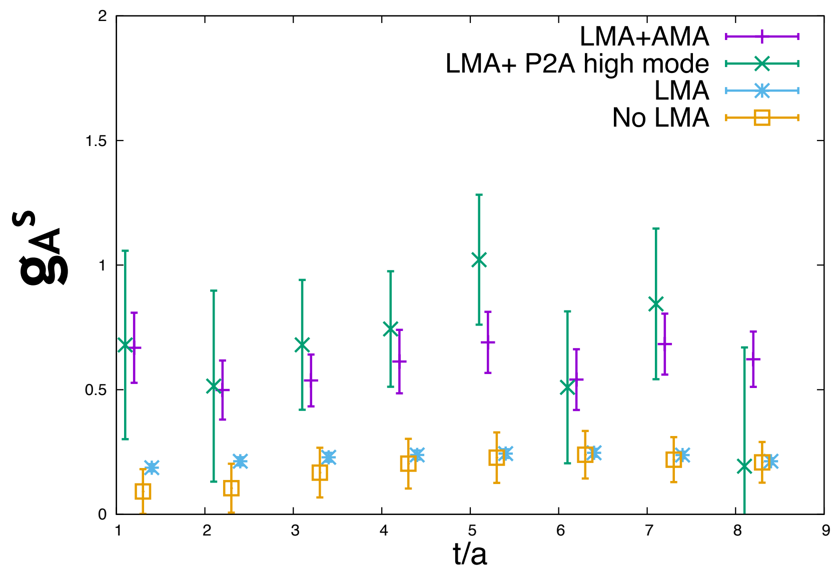
T. Blum, T. Izubuchi, E. Shintani, Phys. Rev. D **88**, 094503 (2013).

Improvement of the signal with LMA and AMA

Isovector axial charge:



Isoscalar axial charge:



Improvement of LMA:

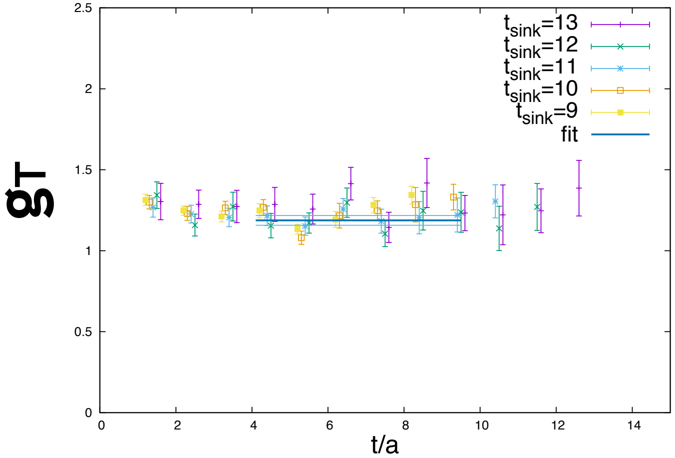
The reduction of the error bars up to
1/5 for isovector charges,
1/5 for isoscalar charges.

Improvement of AMA:

The reduction of the error bars up to
1/4 for isovector charges,
1/2 for isoscalar charges.

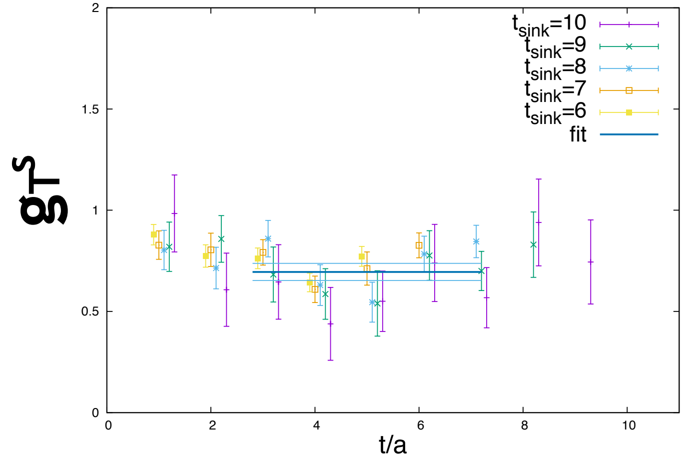
Result of isoscalar and isovector charges (m_π=540 MeV)

Isovector tensor charge:



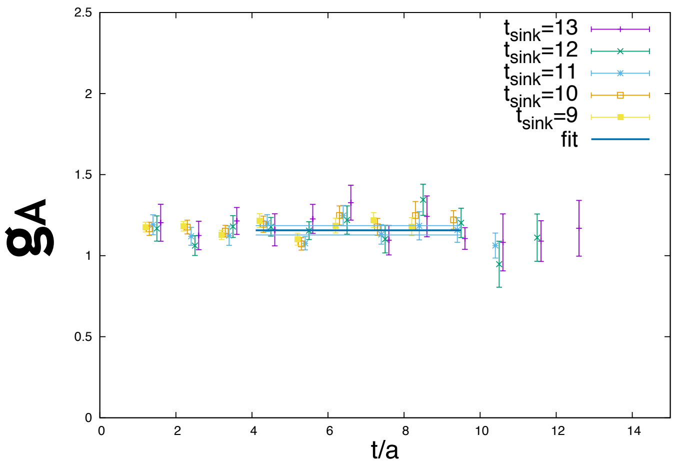
g_T = 1.187 ± 0.030

Isoscalar tensor charge:



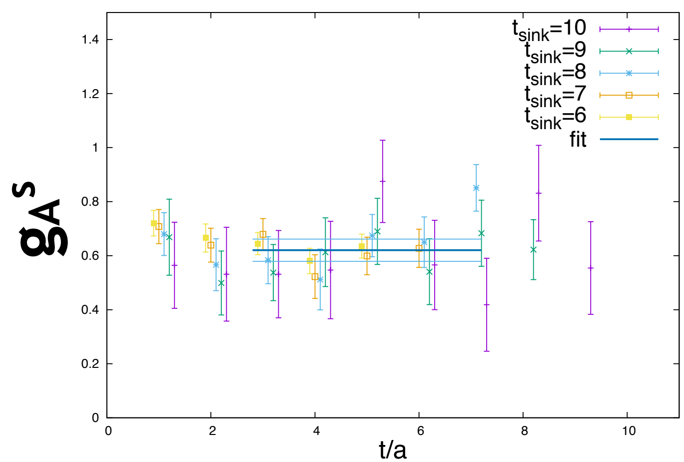
g_T^S = 0.695 ± 0.042

Isovector axial charge:



g_A = 1.157 ± 0.029

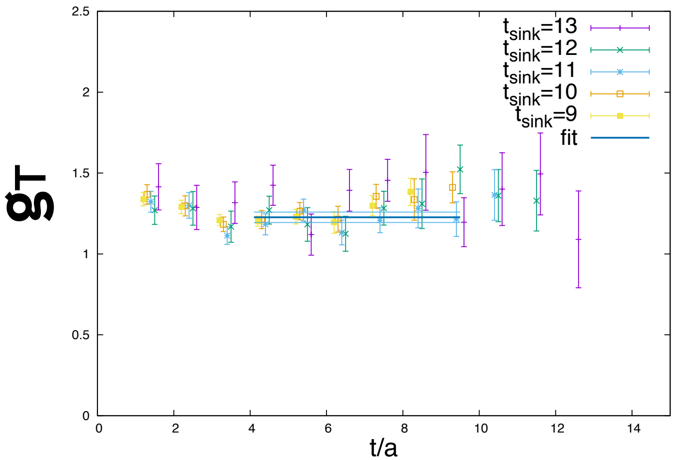
Isoscalar axial charge:



g_A^S = 0.620 ± 0.041

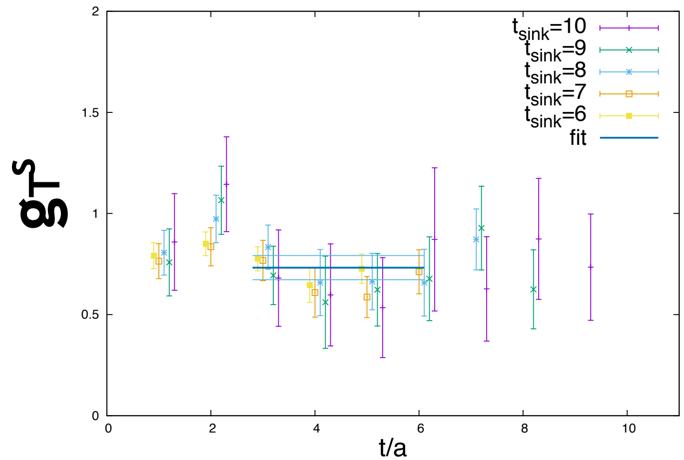
Result of isoscalar and isovector charges (m_π=450 MeV)

Isovector tensor charge:



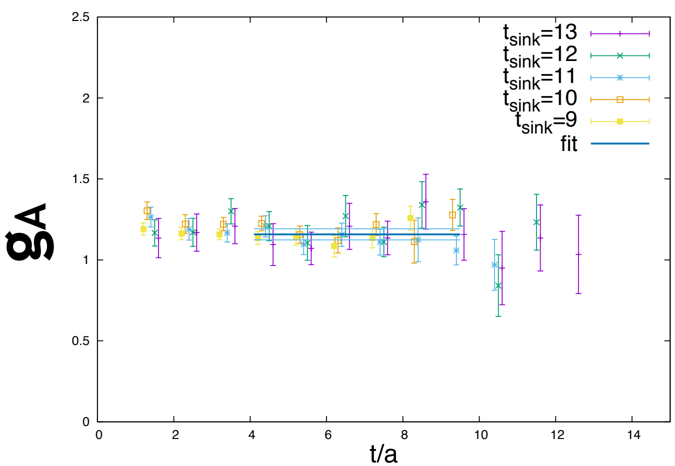
g_T = 1.227 ± 0.032

Isoscalar tensor charge:



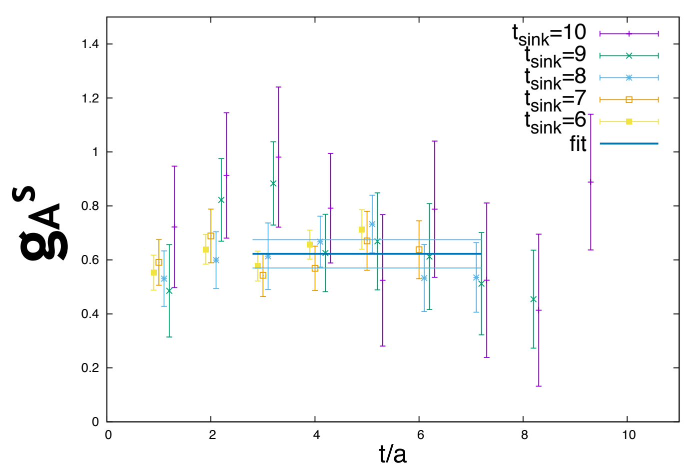
g_T^S = 0.732 ± 0.060

Isovector axial charge:



g_A = 1.158 ± 0.034

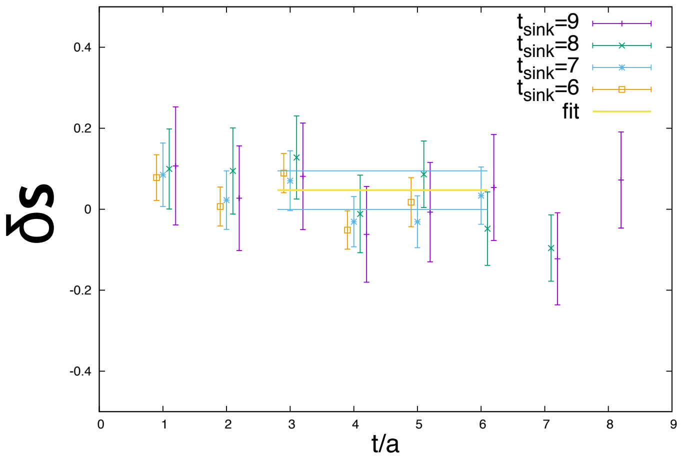
Isoscalar axial charge:



g_A^S = 0.622 ± 0.052

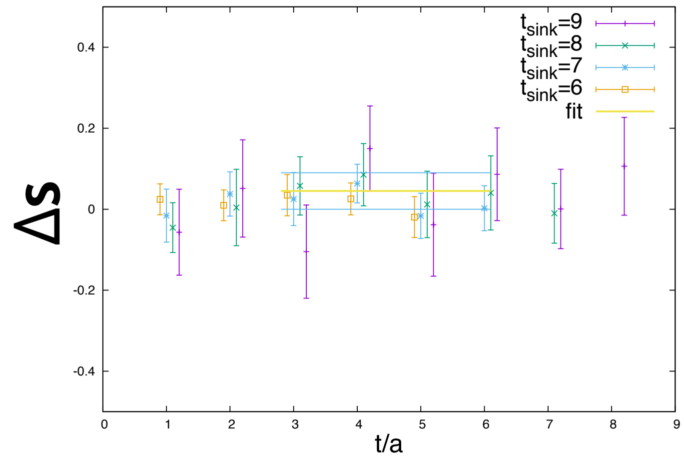
Strange quark contributions

Strange tensor charge: (m_π=540MeV)



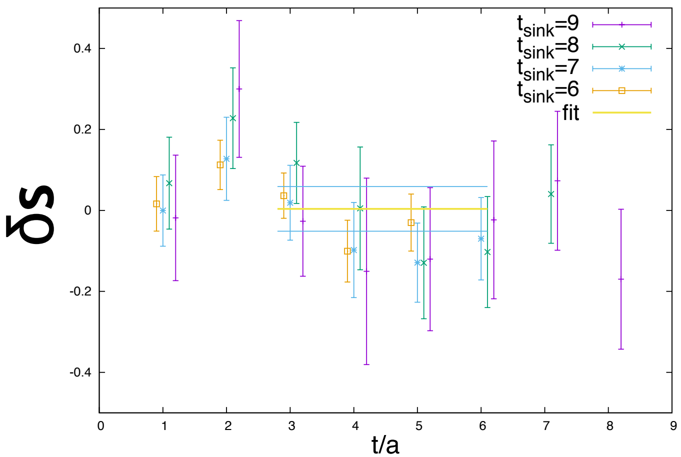
$\delta s = 0.047 \pm 0.048$

Strange axial charge: (m_π=540MeV)



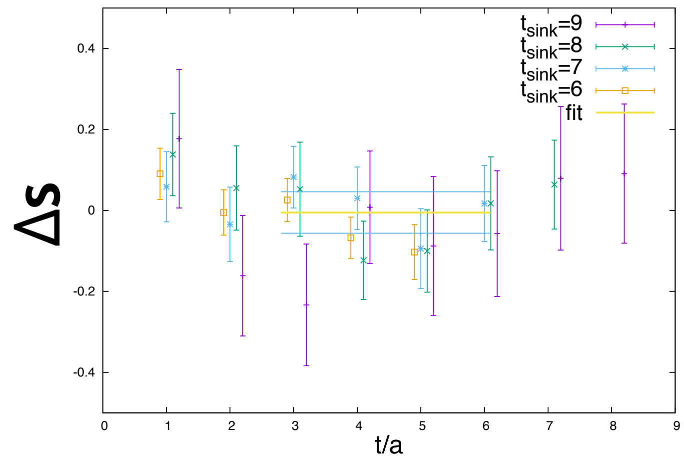
$\Delta s = 0.045 \pm 0.045$

Strange tensor charge: (m_π=450MeV)



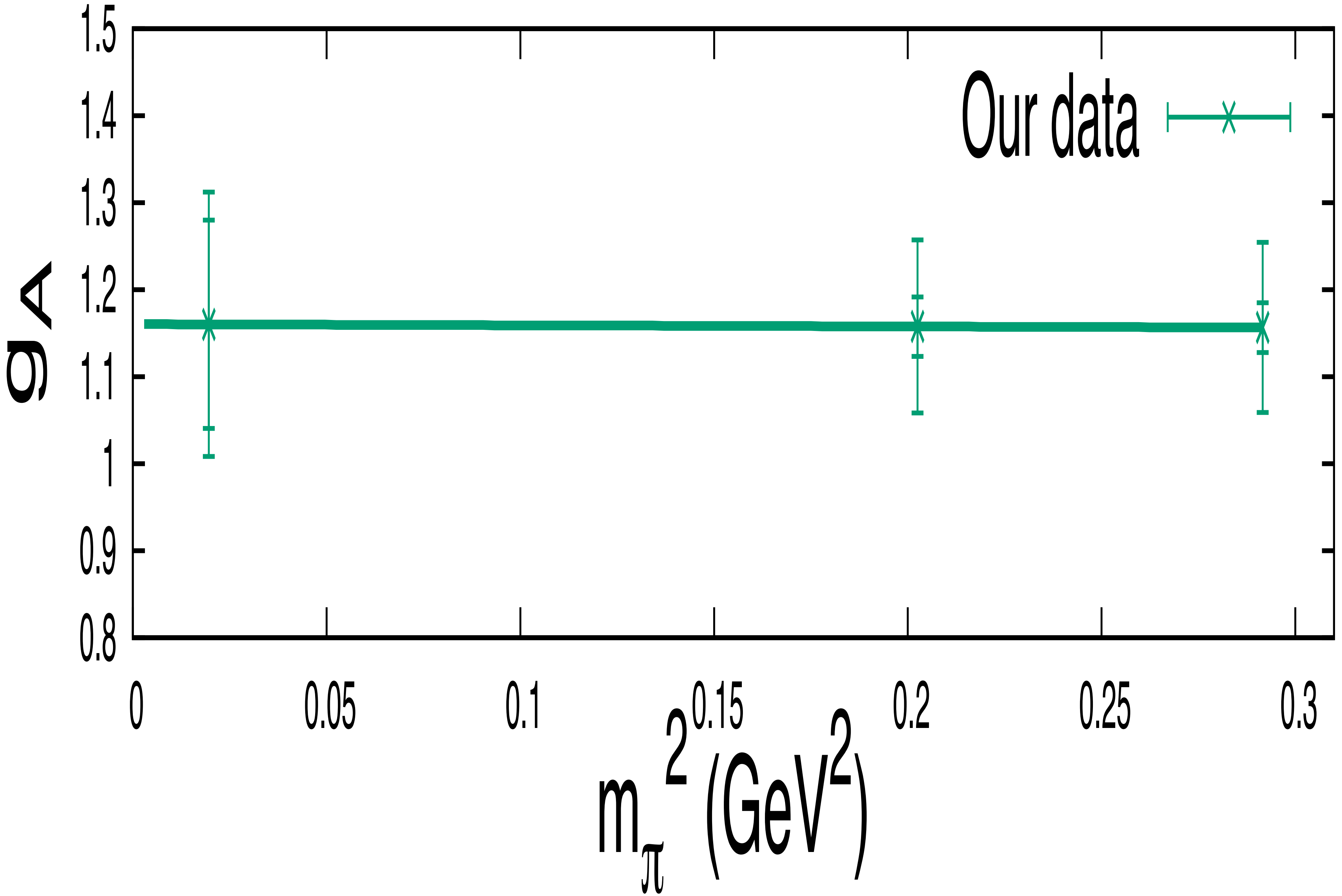
$\delta s = 0.004 \pm 0.055$

Strange axial charge: (m_π=450MeV)

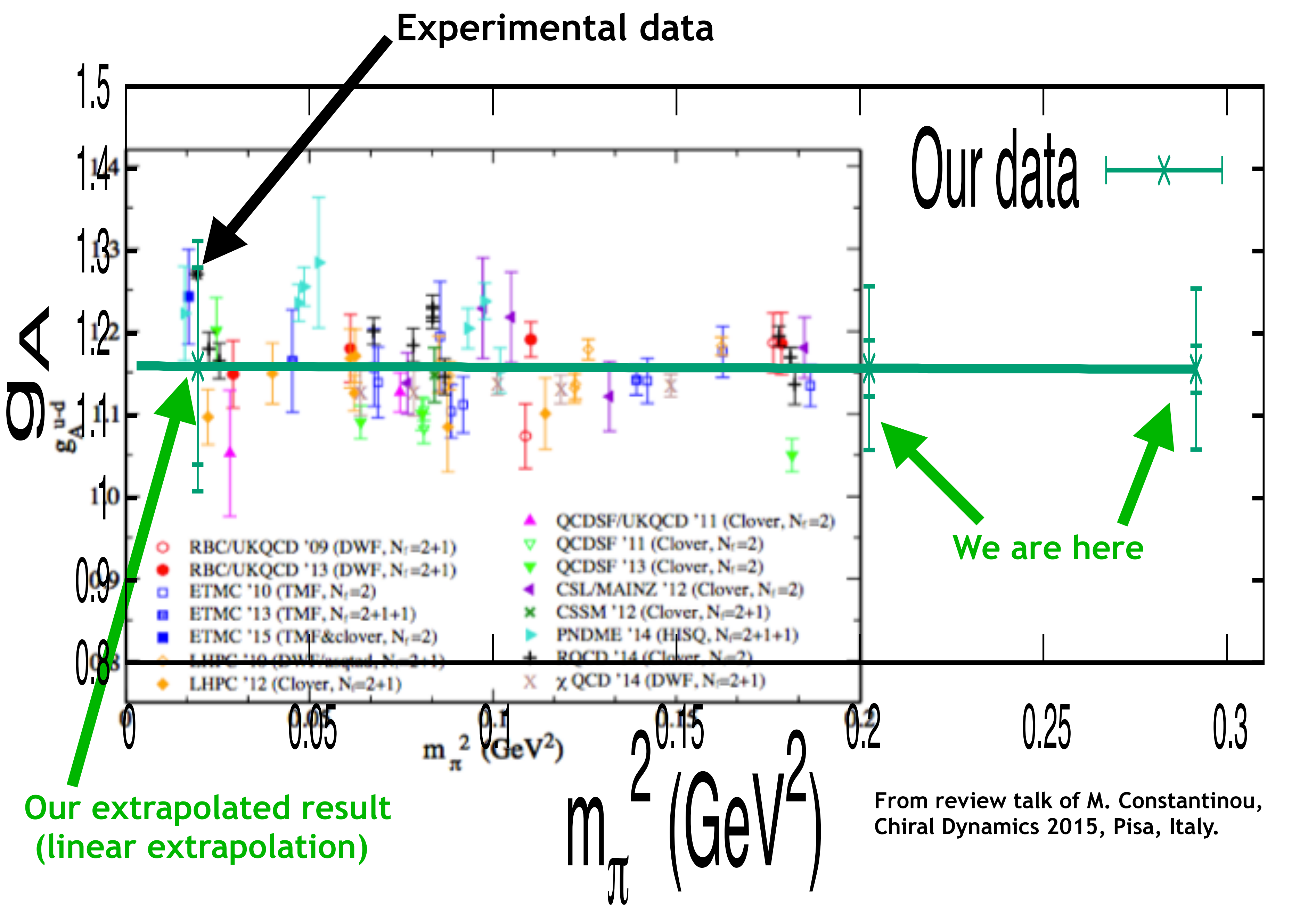


$\Delta s = -0.008 \pm 0.033$

Comparison with previous works (isovector axial charge)



Comparison with previous works (isovector axial charge)



From review talk of M. Constantinou, Chiral Dynamics 2015, Pisa, Italy.

Analysis : isovector charges

Error bars and extrapolation:

Statistical error ~ 3% at each points

Discretization error by order counting: $O(a^2\Lambda_{QCD}^2)$ ~ 8%

Quark mass dependence within error bar:
Here we employ simple linear extrapolation.

We currently simulate rather large quark masses,
Extrapolation to physical point enlarges the error.

Extrapolation to physical point:

$g_A = 1.160 \pm 0.152$

$g_T = 1.307 \pm 0.157$

(Consistent with previous works)

Prospects:

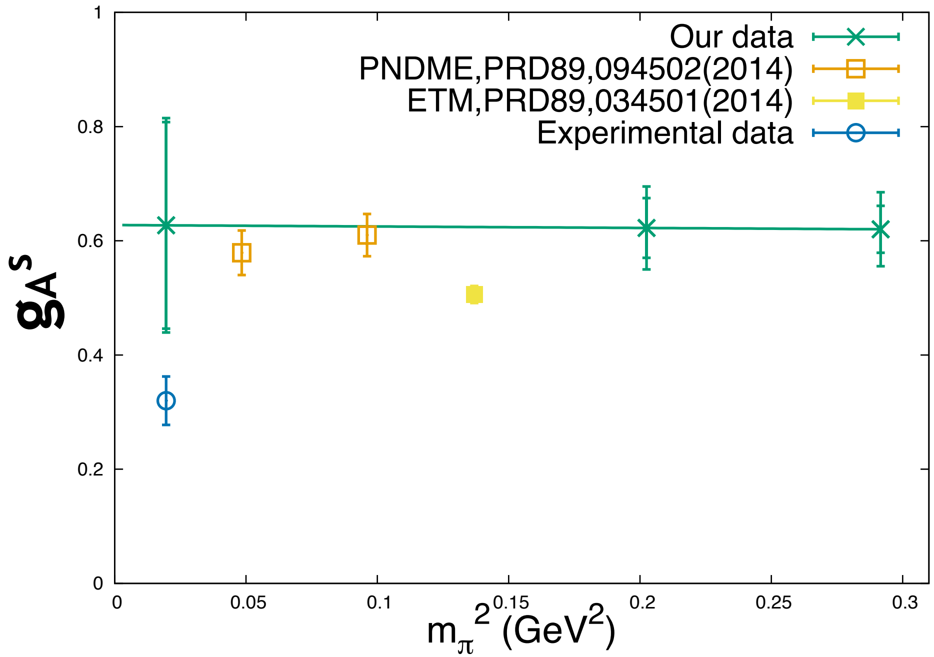
Calculation at lighter quark masses

⇒ Reduce error of extrapolation.

⇒ Study the quark mass dependence with chiral perturbation theory.

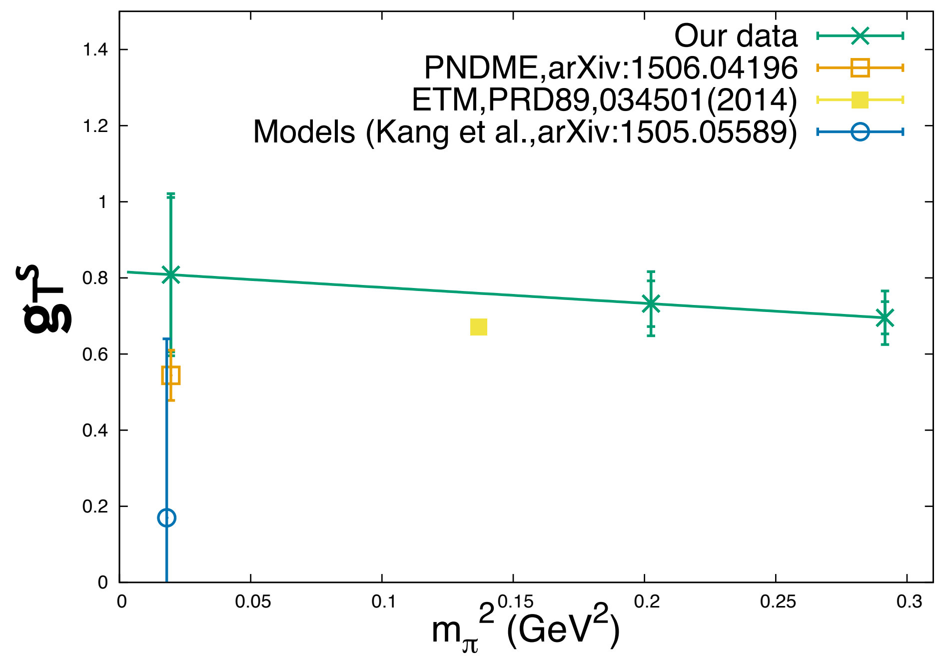
Analysis : isoscalar charges

Axial charge:



g_A^s = 0.627±0.188

Tensor charge:



g_T^s = 0.808±0.213

Analysis : isoscalar charges

- **Error bars and extrapolation:**

  Statistical error ~ 10% at each points

  Larger statistical error due to disconnected graph

  Discretization error by order counting: $O(a^2\Lambda_{QCD}^2)$ ~ 8%

  Quark mass dependence within error bar:
  Here we employ simple linear extrapolation.

  We currently simulate rather large quark masses,
  Extrapolation to physical point enlarges the error.

- **Extrapolation to physical point:**

  $$g_A^s = 0.627 \pm 0.188$$

  $$g_T^s = 0.808 \pm 0.213$$

  (Consistent with previous works)

- **Prospects:**

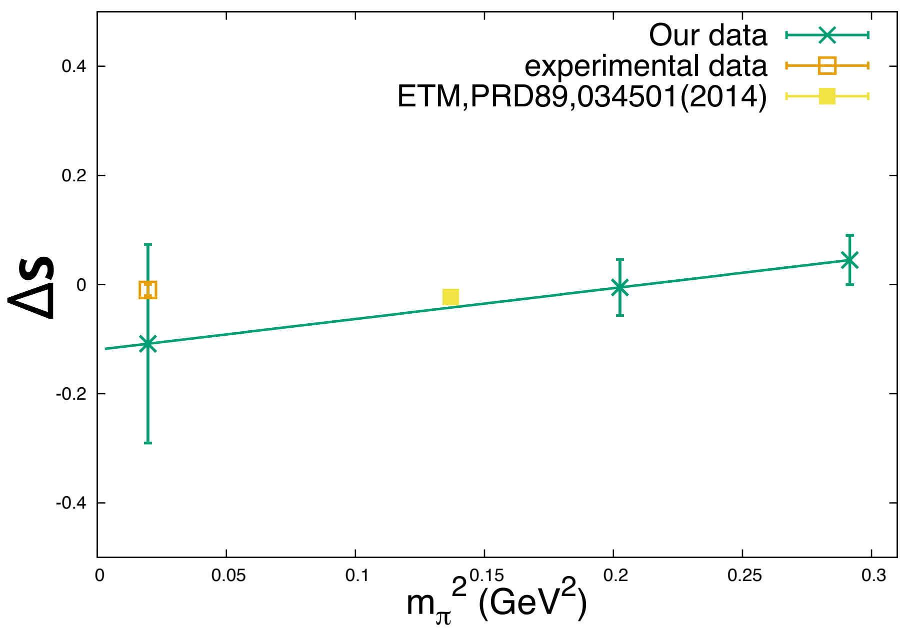
  Reduction of the error bar at physical point:

  ⇒ Reduction of the statistical error with more source points in AMA.

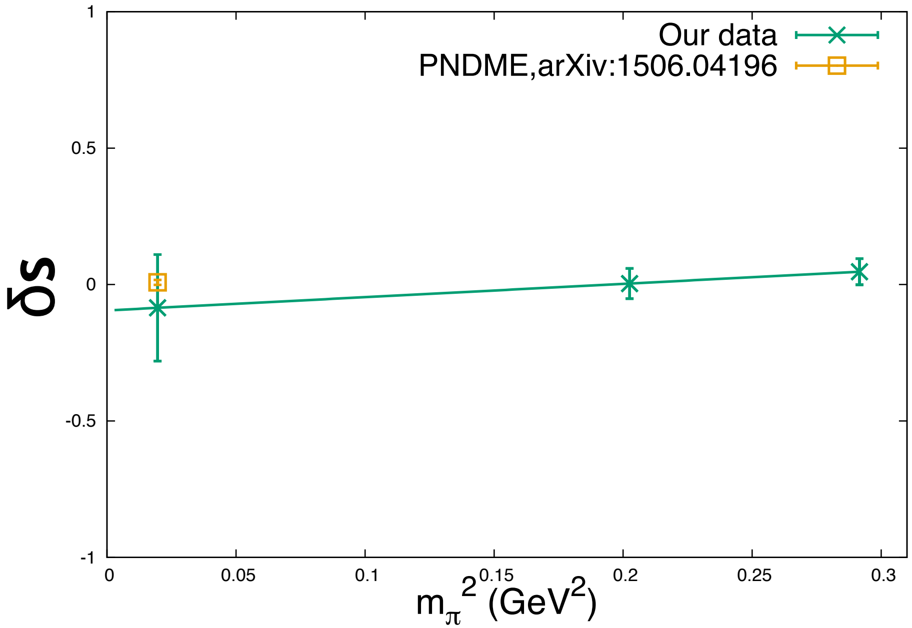
  ⇒ Calculation at lighter quark masses will reduce error of extrapolation

Analysis : strange quark contributions

Axial charge:



Tensor charge:



$\Delta s = -0.108 \pm 0.182$

$\Rightarrow$ Consistent with zero

$\delta s = -0.085 \pm 0.195$

Prospects:

Reduction of the error bar at physical point:

$\Rightarrow$ Reduction of the statistical error with more source points in AMA.

$\Rightarrow$ Calculation at lighter quark masses will reduce error of extrapolation.

Summary and future prospects

Summary:

- We have calculated the quark axial and tensor charges in lattice QCD with overlap fermions, using All-to-all propagator, LMA, and AMA.
- Our preliminary results at physical pion mass:

  g_A^s = 0.627±0.188
  g_A = 1.160±0.152
  g_T = 1.307±0.157
  g_T^s = 0.808±0.213

  Δs = -0.108±0.182
  δs = -0.085±0.195

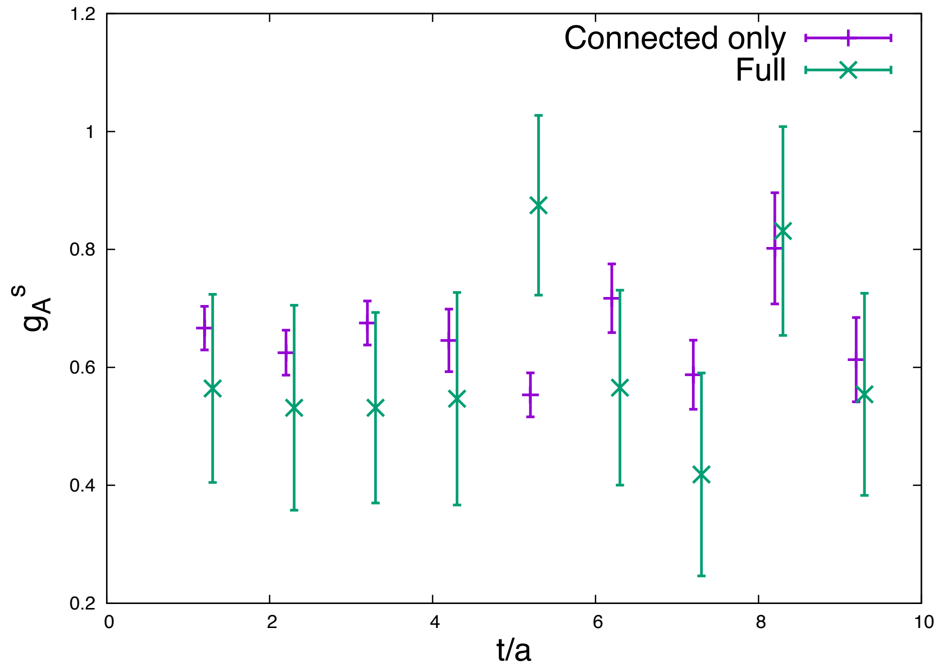
Future subjects:

- Improve statistics with more source points in AMA (?)
- Reduce the error of extrapolation by on-going calculations with lighter quarks.
- Study of volume effect : systematic due to fixed topology.

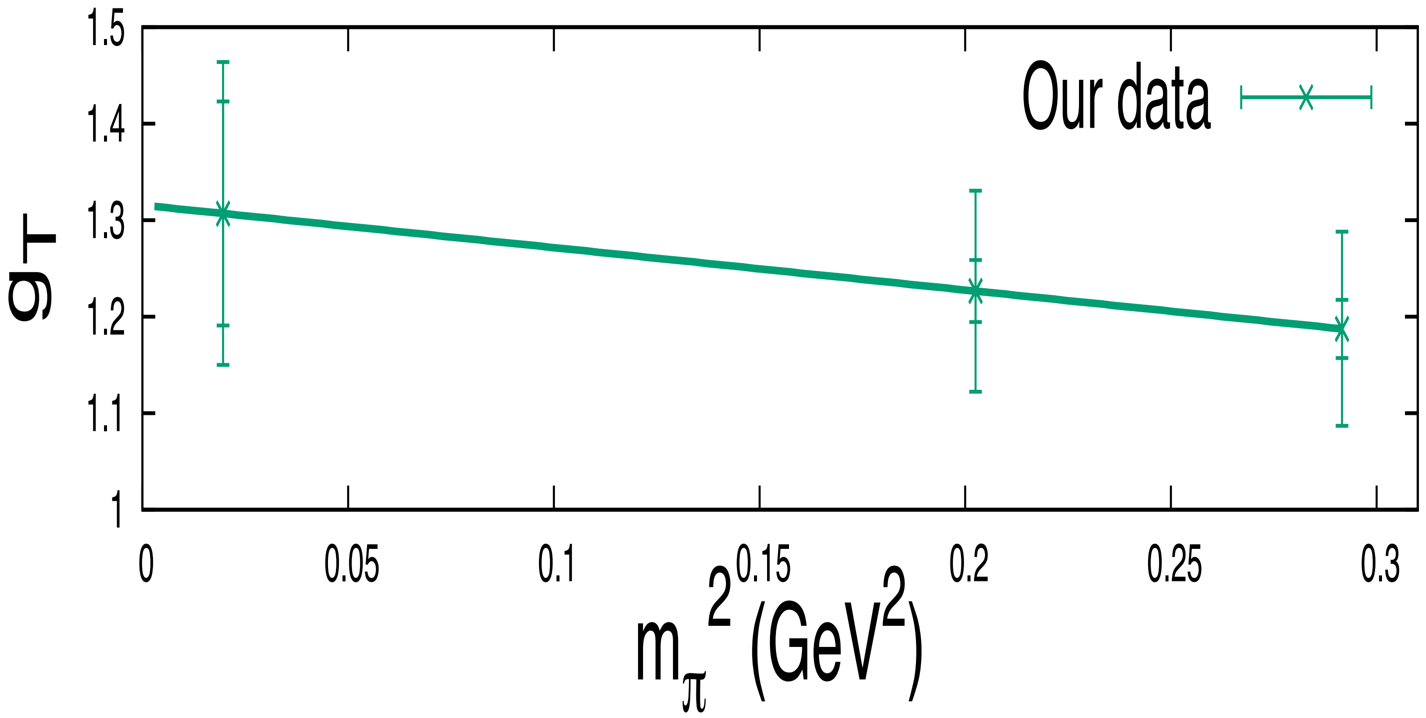
Backup slides

Disconnected and connected contributions

Example of isoscalar axial charge:



Comparison with previous works (isovector tensor charge)



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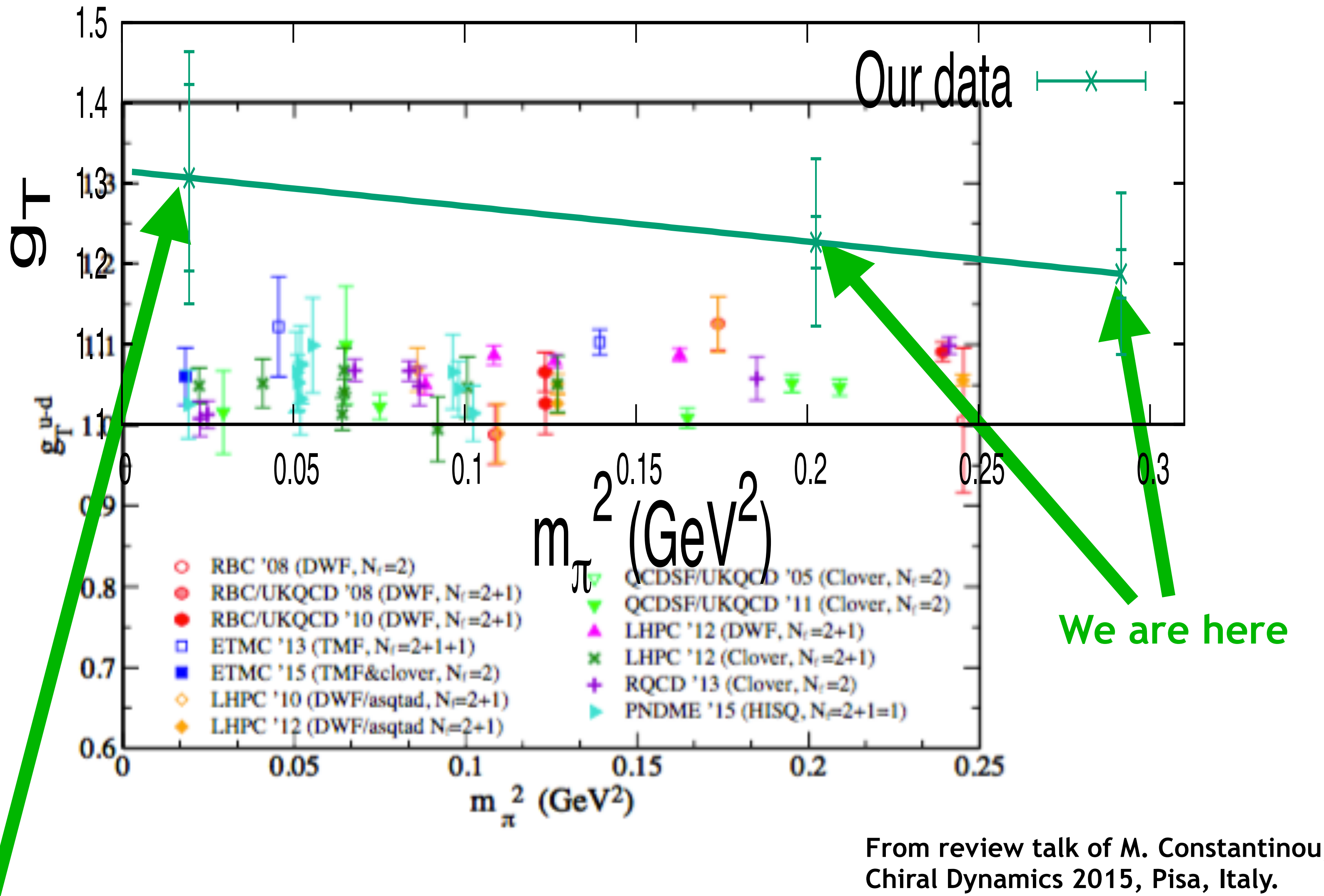

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Chiral Dynamics 2015, Pisa, Italy.

Our extrapolated result (linear extrapolation)