Investigating some technical improvements to glueball calculations.

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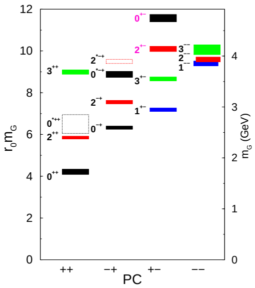
July 14, 2015

Introduction

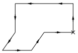
In pure gauge theory there is rich spectrum of glueball states. From Morningstar and Peardon (hep-lat/9901004). Can we find evidence for glueball degrees of freedom in nature?



Unquenched lattice QCD calculations which include glueball degrees of freedom are hard because

- Glueball correlators have a poor signal to noise ratio, hence high statistics are required or better techniques.

Introduction to glueballs



- Glueballs are bound states of glue.
- The holy grail of hadron spectroscopy is to find experimental evidence for glueballs.
- Unfortunately in the real world, glueballs will probably mix with quark-antiquark states with the same quantum numbers.
- Also in unquenched QCD the glueball will decay with the strong force, hence resonance effects need to be considered.

Note that pure Yang-Mills theory (QCD with no quarks) is a consistent quantum field theory, but because it doesn't describe reality, it is not easy to compute the lattice spacing.

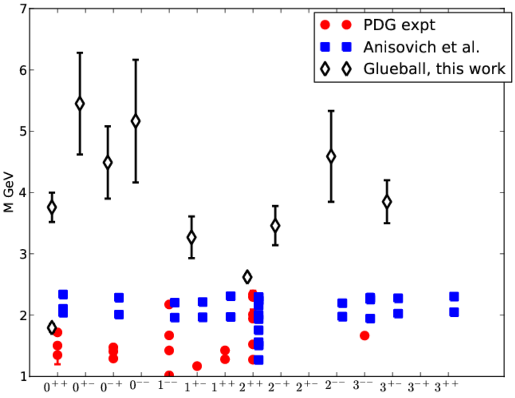
Unquenched glueball calculation

- The ASQTAD improved staggered fermion action with the one loop tadpole improved gauge action was used. (Same as MILC collaboration)
- The strange quark mass was not very well tuned. The parameters were chosen to be part of a larger (but with lower statistics) set of calculations performed by the MILC collaboration (arXiv:0903.3598 for an overview).
- First paper (arXiv:1005.2473), the high statistics allowed us to us to see a signal for the pseudo-scalar 0^{-+} glueball.
- Second paper (1208.1858) used a bigger basis of glue based operators including bi-torelon operator and two body glueball states (using code developed in 1007.3879).

a fm	m_π MeV	L fm	Number of configs
0.12	280	2.9	5000
0.09	360	2.9	3000

Summary plot (1208.1858)

A comparison of lattice results with PDG and results from Crystal Barrel collaboration.



η η' Glueball mixing

- The lightest glueballs in quenched QCD are the 0^{++} (1710(80) MeV), 2^{++} (2390(120) MeV), and 0^{+-} (2560(120) MeV).
- The dynamics of 0^{++} states is complicated, so might be better to look at η-η'-Glueball mixing.

For example the KLOE experiment (hep-ex/0612029) wrote the physical η' meson state in terms of light and strange quark states and glueball degrees of freedom.

$$| \eta' \rangle = X_{\eta'} \mid \overline{q}q \rangle + Y_{\eta'} \mid \overline{s}s \rangle + Z_{\eta'} \mid glueball \rangle$$

- Fit $X_{\eta'}$, $Y_{\eta'}$, and $Z_{\eta'}$ to experimental branching fractions.
- There are different parameterizations, but KLOE used one with $X_{\eta'}^2 + Y_{\eta'}^2 = 1$
- KLOE claimed $Z_{\eta'}^2 = 0.14 \pm 0.04$

η η' Glueball mixing

- It is not clear how to write down a mixing angle between fermionic operator and glueball operator with 0^{-+}.
- Many different mixing schemes proposed. For example: Kentucky-Taiwan, arXiv:0811.2577.
- It is possible to include quark and glueball degrees of freedom in the same variational calculation.
- Some ($\eta - \eta'$ - G) mixing schemes suggest large unquenching effects in the mass of the 0^{-+} glueball degrees of freedom.

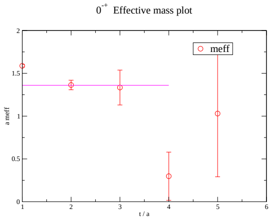
Look at the glueball operators on the same configurations from ETMC ($n_f = 2 + 1 + 1$ sea quarks) (arXiv:1310.1207) used to compute the η and η' mass.

- ETMC found $m_\eta = 551(8)(6)$ MeV, $m_{\eta'} = 1006(54)(38)(+61)$ MeV and mixing angle 46(1)(3).

Very preliminary results using 0^{-+} glueball

Two $n_f = 2 + 1 + 1$ twisted mass ensembles, from ETMC, run through glueball code:

1. iwasaki b2.10 L32T64 k0.156315 mu0.0045 musigma0.0937 mudelta0.1077, 1100 configs. $1/a \sim 3.2$ GeV. (analysis in progress).
2. iwasaki b1.95 L32T64 k0.161236 mu0.0055 musigma0.135 mudelta0.17, 2200 configs, from ETMC, $1/a \sim 2.5$ GeV.



Open boundary conditions

- The basis of our numerical lattice QCD calculations is the Monte Carlo process.
- It is important that the sample size is large enough.
- Different observables require a different number of Monte Carlo samples (long autocorrelation times).
- People worry the most about the topology of the gauge fields.
- It is not clear that the gauge field topology is important for the masses, but is important to check.

The topological charge Q is defined by

$$Q = \frac{1}{32\pi^2} \int F^a_{\mu\nu} \tilde{F}^a_{\mu\nu} d^4x.$$

in terms of the Euclidean color gauge field $F^a_{\mu\nu}$ and its dual $\tilde{F}^a_{\mu\nu}$,

Open boundary conditions

- Consider a lattice with dimensions $T \times L \times L \times L$
- Normally we use periodic boundary conditions in space and anti-periodic conditions in time for the quark fields.

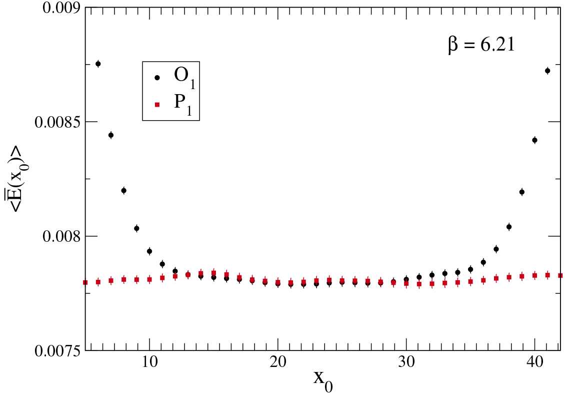
Lüscher and Schaefer (CERN) proposed the use of open boundary conditions to "solve" the problem with topology (JHEP 1107 (2011) 036).

$$F_{0k}(x) \mid_{x0=0} = F_{0k}(x) \mid_{x0=T} = 0$$

for all k=1,2,3
There are more complicated boundary conditions for the quark fields.
First use of open boundary conditions for glueballs, arXiv:1402.7138, Chowdhury et al. Pure SU(3) simulations with Wilson gauge action.

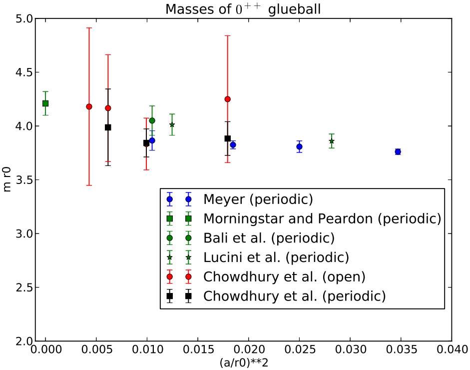
Continuum limit (1402.7138, Chowdhury et al.)

A problem with open boundary conditions is that you can't use correlators close to time boundaries. (O_1 open BC and P_1 periodic BC).



Continuum limit

From arXiv:1402.7138, compare results from Chowdhury et al. with results from other groups. Use $r_0 \sim 0.48$ fm for quenched QCD. Higher statistics required for a definitive test.



Why think about statistics and noise?

- Calculations which include glueball degrees of freedom require large statistics
- Gregory et al. (arXiv:0709.4224, arXiv:1112.4384) found tails of distributions were important for signal and error of η and η'.
- Progress in molecules, nuclear type lattice calculations is limited by noise in the correlators. (Endres, 1112.4023).

But note however,

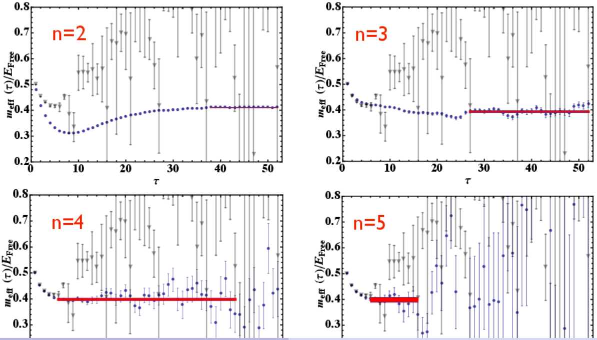
- In lattice QCD spectroscopy calculations we measure some correlators and then average to get the central value.
- The law of large numbers protects us from needing to know about the detailed form of the probability distribution.

Histogram the correlators at specific timeslices and fit a Guassian and use Kolmogorov-Smirnov test for normality.

In an ideal world

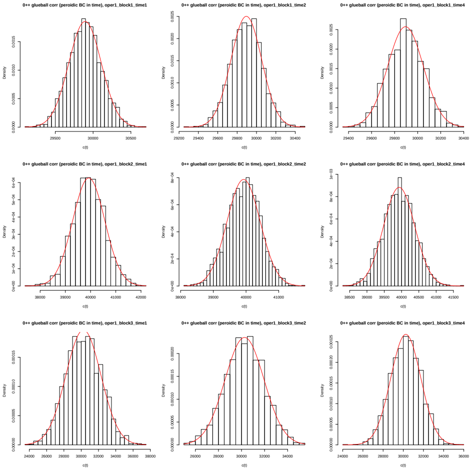
- From paper by Kaplan et al. 1112.4023
- Some lattice data for unitary fermions (not QCD).
- Moments based analysis technique (estimator has reduced errors, but biased)

DeGrand (rXiv:1204.4664) looked at his BSM data and found much evidence for lognormal behavior, but moments method of Kaplan et al. didn't work.



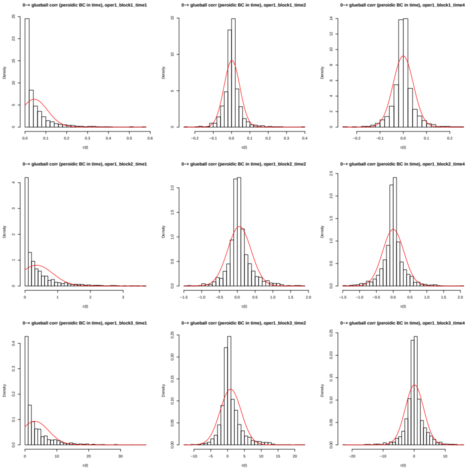
16^4 quenched $a \approx 0.1fm$ Wilson 0^{++}

- 3 basis states and t = 1,2,4 (glueball correlators)



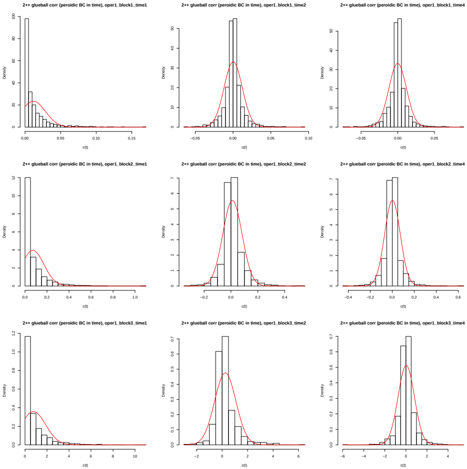
16^4 quenched $a \approx 0.1fm$ Wilson 0^{-+}

- 3 basis states and t = 1,2,4 (Preliminary results with open boundary conditions were less normal.) (glueball correlators)



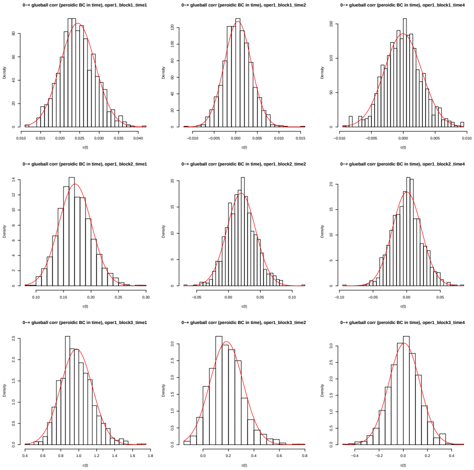
16^4 quenched $a \approx 0.1 fm$ Wilson 2^{++}

- 3 basis states and t = 1,2,4 (glueball correlators)



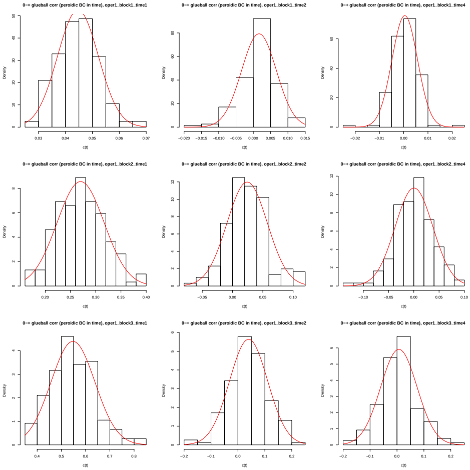
Unquenched iwasaki b1.95 L32T64 k0.161236 mu0.0055 musigma0.135 mudelta0.17 from ETMC 0^{-+}

- 3 basis states and t = 1,2,4 (glueball correlators)



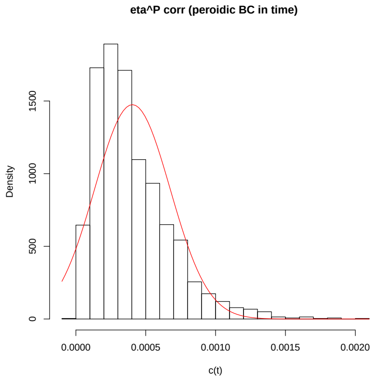
Unquenched ASQTAD fermions $32^3 \times 64$, 0^{-+}

- 3 basis states and $t = 1,2,4$ (glueball correlators)



Unquenched ASQTAD fermionic loops $32^3 \times 64$, 0^{-+}

- 1 basis states and t = 4 ($\overline{q}q$ correlators)
- Eric Gregory worked on distributions in arXiv:0709.4224.



Conclusions and Summary

- Some insight into η' - glueball mixing may be possible, if we can beat the signal to noise problem.
- A clear picture has not emerged from looking at the noise in glueball and disconnected correlators.
- It is not clear that the statistical distribution of a glueball correlator is physical.
- More work is required to determine the effect of open boundary conditions on the masses of glueballs.
- Perhaps run on other available high statistics ensembles.

Thanks

Glueball correlators generated on clusters at Plymouth and compute grids run by GridPP.