The ϑ- term contribution to the EDM with the gradient flow

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Lattice 2015

12.05.2015

A.S., de Vries, Luu: 2014

Lattice 2014

A.S., de Vries, Luu: 2014

Lattice 2014

A.S., de Vries, Luu: 2014

Lattice 2014

BSM and nuclear physics

Nuclear observables induced by BSM operators

A.S., de Vries, Luu: 2014

Lattice 2014

BSM and nuclear physics

Nuclear observables induced by BSM operators

- EDM of nucleon and few body-system
- Understanding of heavy-quarks content of nucleons (Dark Matter)

A.S., de Vries, Luu: 2014

Lattice 2014

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Measurement of nucleon/nuclear EDM

?

ϑ term

New sources of CP-violation

Lattice 2014

Measurement of nucleon/nuclear EDM

?

ϑ term

New sources of CP-violation

EDM from theta-term

$$\langle N^\theta(\underline{p}', s')|J^\mu_{\rm em}|N^\theta(\underline{p}, s)\rangle = \bar{u}^\theta_N(\underline{p}', s')\Gamma^\mu(q^2)u^\theta_N(\underline{p}, s),$$

$$\begin{aligned}\Gamma^\mu(q^2) &= h(\theta^2)\left[F_1(q^2)\gamma^\mu + \frac{1}{2M_N}F_2(q^2)i\sigma^{\mu\nu}q_\nu\right] \\ &+ i\theta g(\theta^2)\frac{1}{2M_N}F_3(q^2)\sigma^{\mu\nu}\gamma_5 q_\nu\end{aligned}$$

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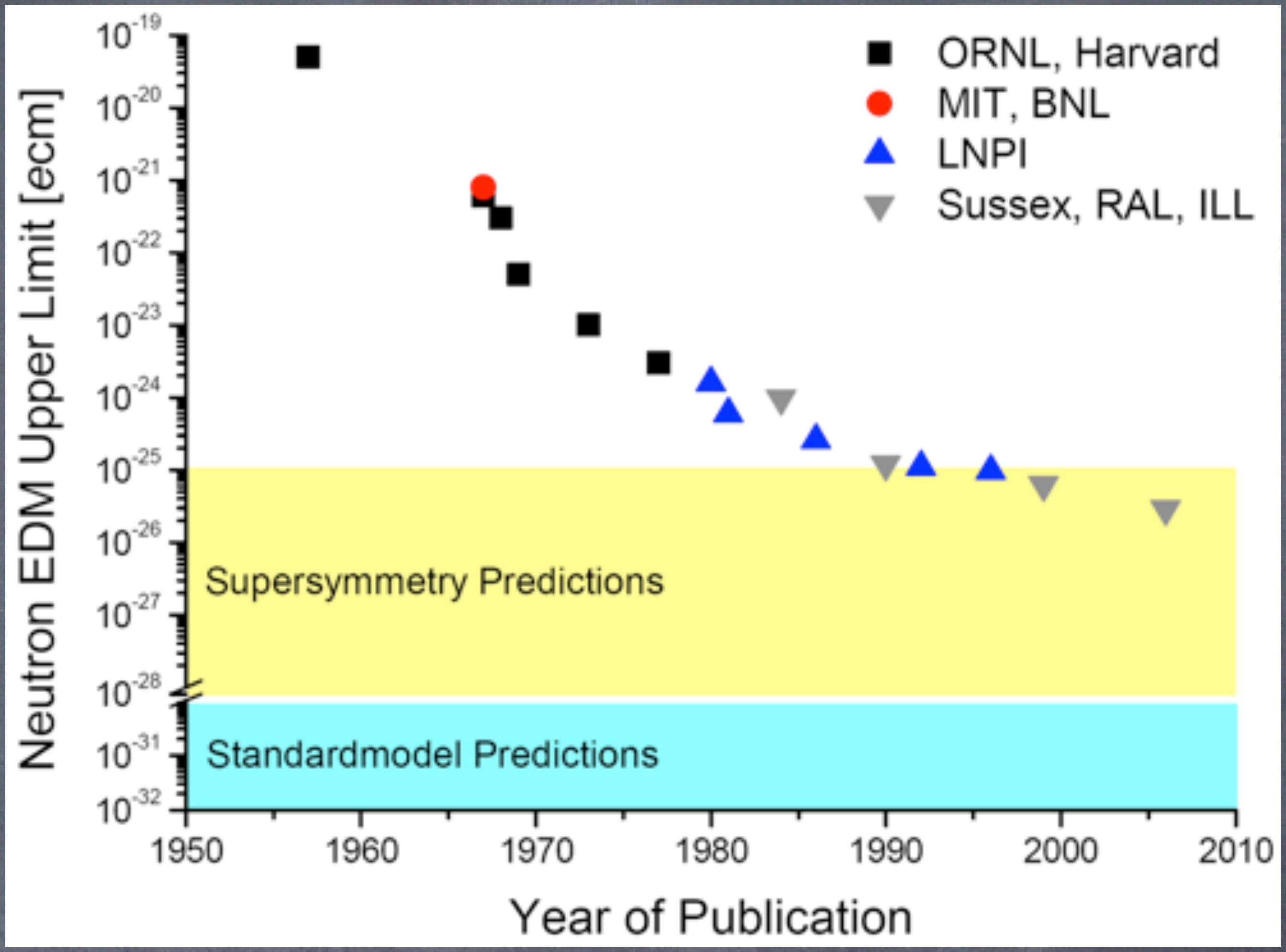
$$\boxed{|d_N| = F_3(0)/2M_N}$$

$$|d_N| = c_n\,\theta\,e\,\mathrm{fm}$$

CP violation within SM and nEDM

$$|d_N| < 2.9 \times 10^{-13}\, e\,\mathrm{fm}$$



Knecht

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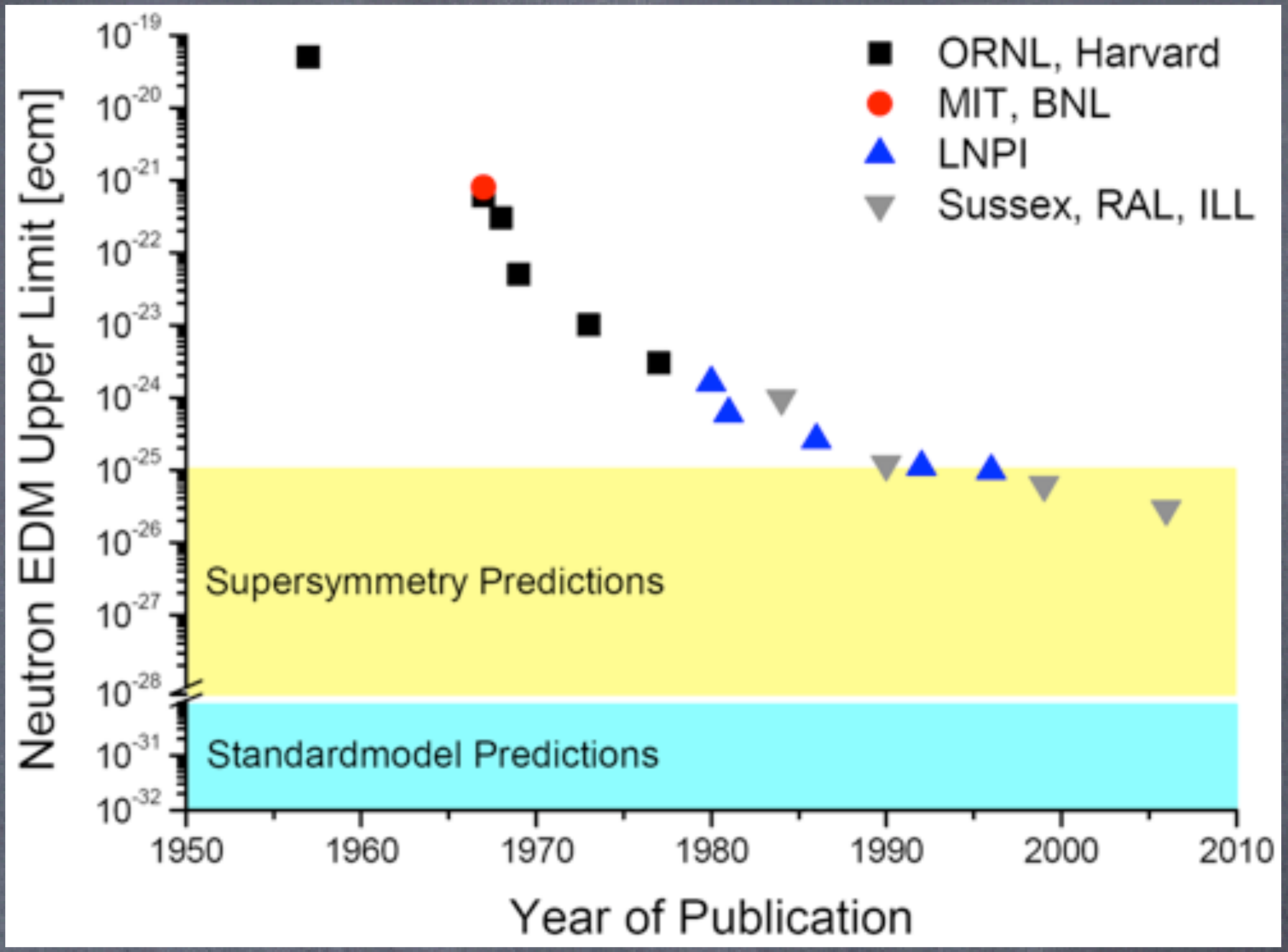

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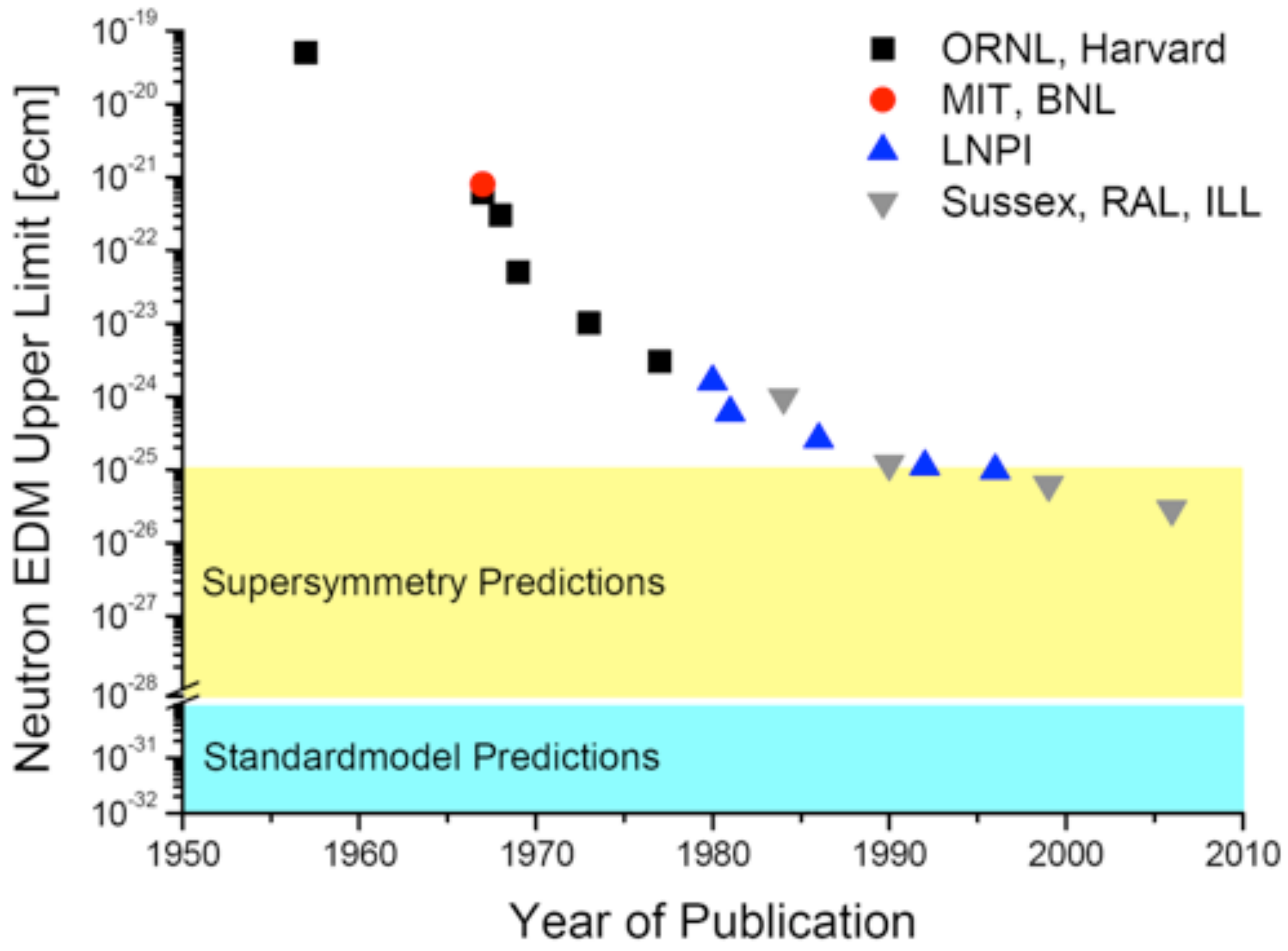

Knecht

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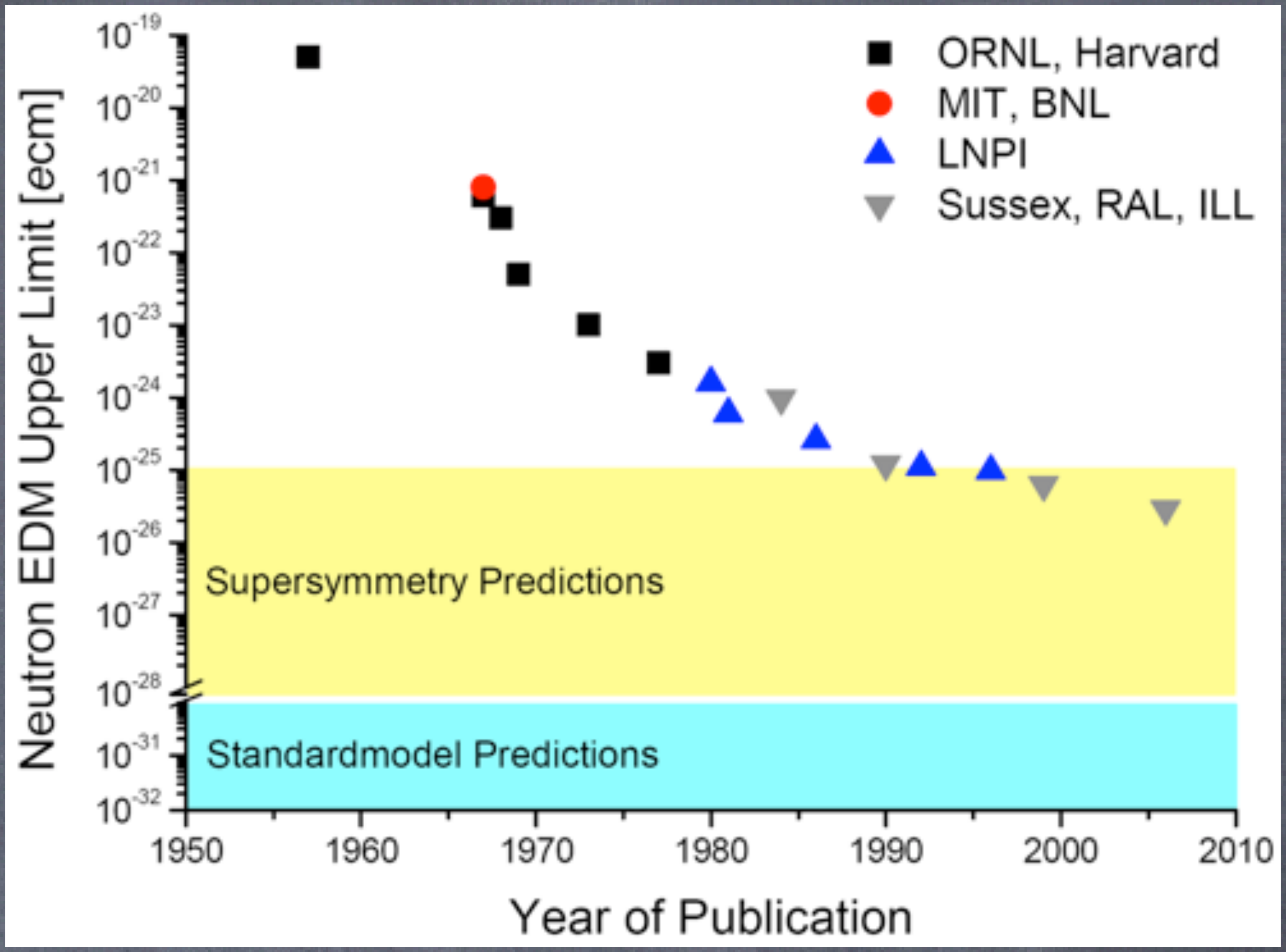
$$|c_n| \gtrsim 0.001$$



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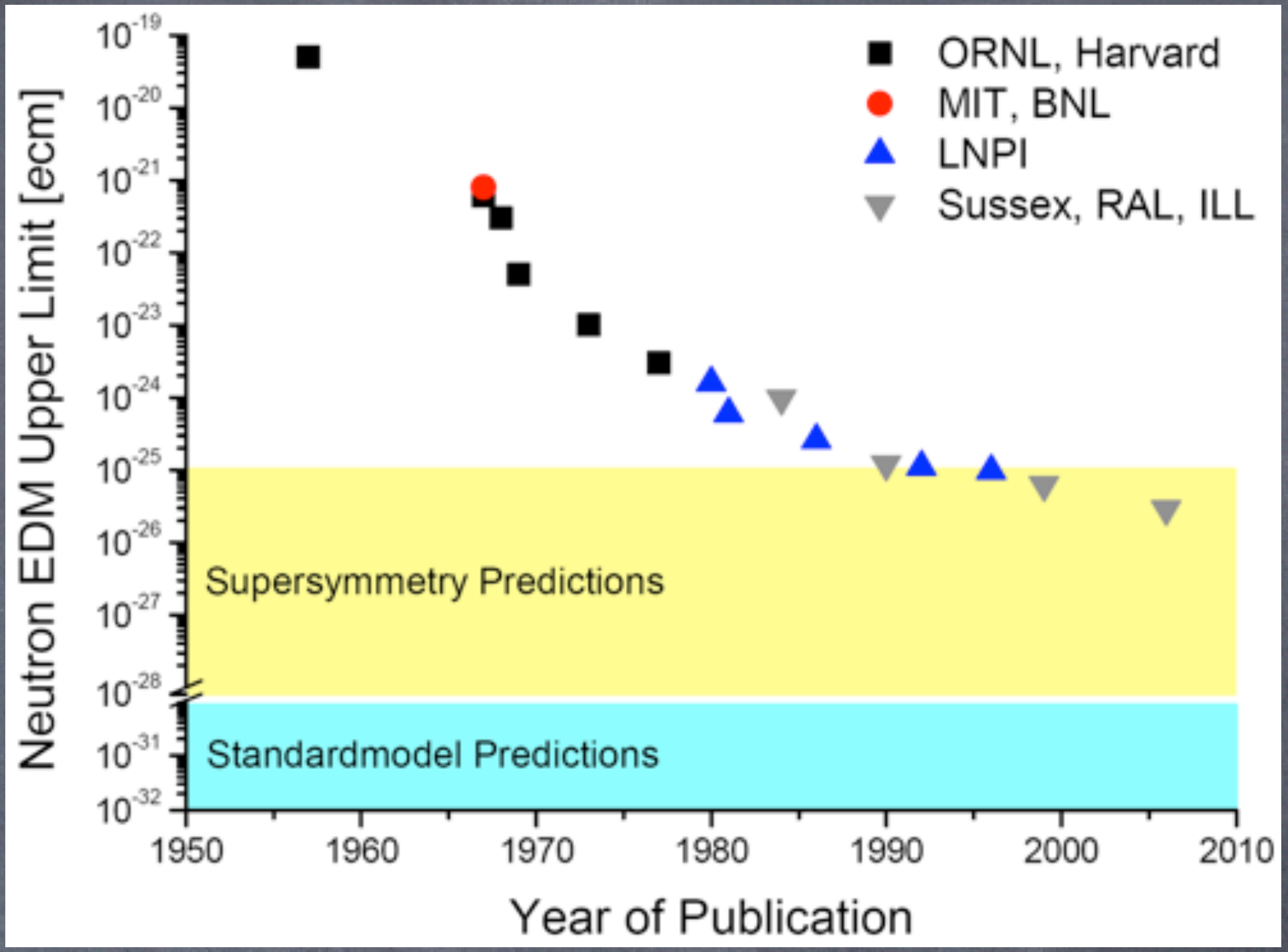
$$|c_n| \gtrsim 0.001 \quad \Rightarrow \quad |\theta| \lesssim \mathrm{O}(10^{-10})$$



Knecht

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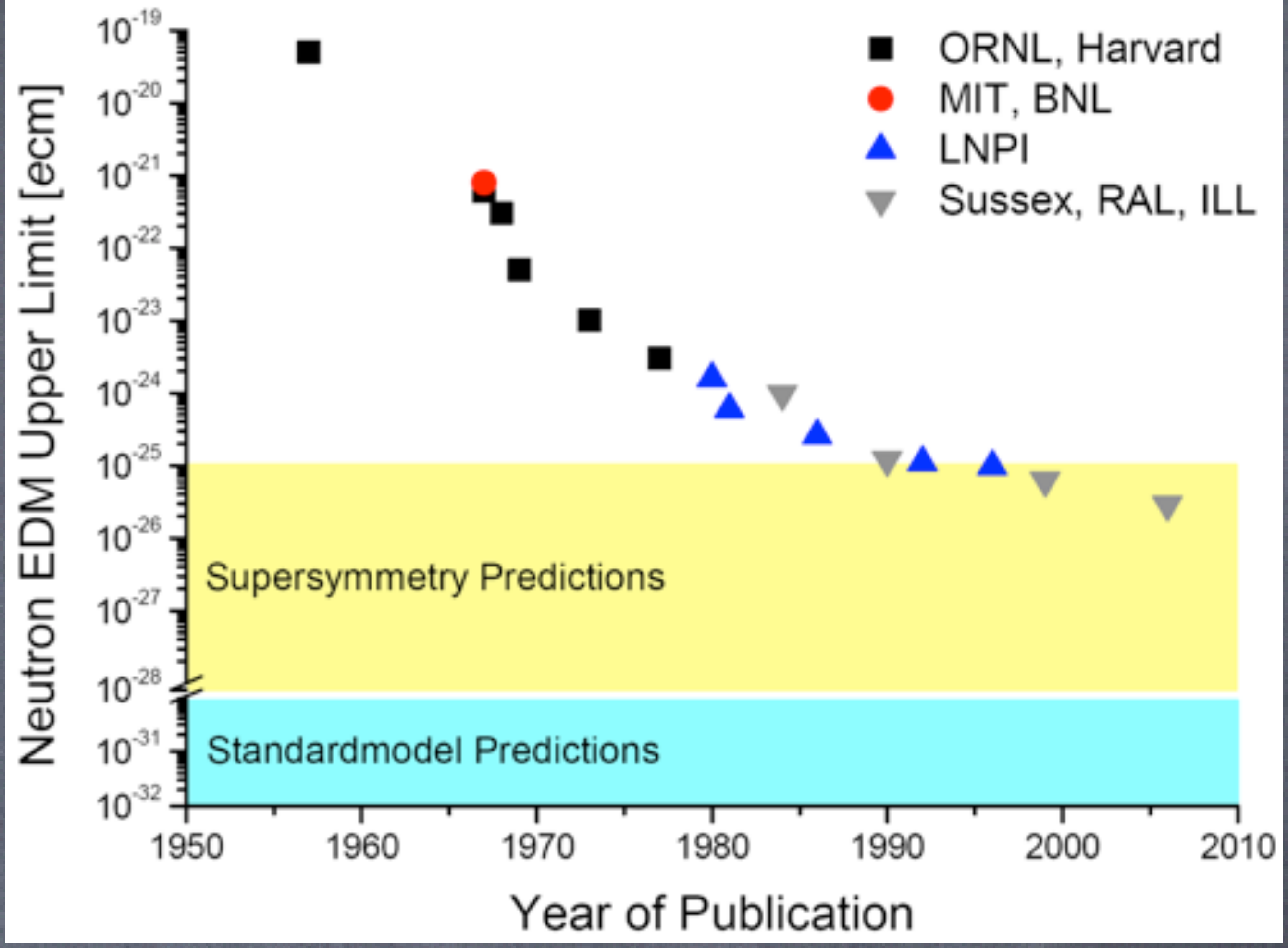

Knecht

$$|c_n| \gtrsim 0.001 \;\Rightarrow\; |\theta| \lesssim \mathrm{O}(10^{-10})$$

Very difficult theoretical calculation and experimental measurement

Izubuchi: plenary

EDM lattice calculation

$$\langle N^\theta(\underline{p}', s')|J^\mu_{\rm em}|N^\theta(\underline{p}, s)\rangle = \bar{u}^\theta_N(\underline{p}', s')\Gamma^\mu(q^2)u^\theta_N(\underline{p}, s),$$

EDM lattice calculation

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$$G^\theta_{NJ_\mu N} = \langle N(y_0, \underline{p}_2)J^\mu_{\rm em}(x_0, \underline{q})N^\dagger(0, \underline{p}_1)\rangle_\theta$$

EDM lattice calculation

$$\langle N^\theta(\underline{p}', s')|J^\mu_{\rm em}|N^\theta(\underline{p}, s)\rangle = \bar{u}^\theta_N(\underline{p}', s')\Gamma^\mu(q^2)u^\theta_N(\underline{p}, s),$$

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$$S = \int d^4x \ \left[\mathcal{L}_{\rm QCD} - i\theta q(x)\right]$$

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$$S = \int d^4x \; [\mathcal{L}_{\rm QCD} - i\theta q(x)]$$

$$\mathrm{e}^{-S} \simeq \mathrm{e}^{-S_{\rm QCD}}[1 + i\theta Q] \qquad Q = \int d^4x \; q(x)$$

Shintani et al.: 2005

Berruto, Blum, Orginos, Soni: 2005

Observables in θ vacuum

$$\langle \mathcal{O} \rangle_\theta \simeq \langle \mathcal{O} \rangle_{\theta=0} + i\theta \langle \mathcal{O} Q \rangle_{\theta=0} + \mathrm{O}(\theta^2)$$

$$q(x) = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{tr} \left\{ F_{\mu\nu}(x) F_{\rho\sigma}(x) \right\} \qquad Q = \int d^4x \; q(x)$$

How does Q renormalize?

Can we safely remove the regulator?

Can we perform the continuum limit a -> 0

Continuum limit VERY important to constrain the chiral interpolation

Gradient flow

Lüscher 2010-2013

$$\partial_t B_{t,\mu} = D_{\nu,t} G_{t,\nu\mu}$$

$$B_{t,\mu}(x)|_{t=0} = A_\mu(x)$$

$$D_{\nu,t} = \partial_\nu + [B_\nu, \ \cdot \ ]$$

$x_\mu = (x_0, \mathbf{x})$ $\quad t = \text{flow} - \text{time}$ $\quad [t] = -2$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

Symanzik improvement

Nogradi et al: 2014

Ramos, Sint: 2014

Continuous form of stout-smearing

Morningstar, Peardon: 2004

EDM

A.S., de Vries, Luu: 2014

$$\langle \mathcal{O} \rangle_\theta \approx \langle \mathcal{O} \rangle_{\theta=0} + i\theta \langle \mathcal{O} Q \rangle_{\theta=0} + \mathrm{O}(\theta^2)$$

EDM

A.S., de Vries, Luu: 2014

$$\langle \mathcal{O} \rangle_\theta \approx \langle \mathcal{O} \rangle_{\theta=0} + i\theta \langle \mathcal{O} Q \rangle_{\theta=0} + \mathrm{O}(\theta^2)$$

$$q(x,t) = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{tr} \left\{ G_{\mu\nu}(x,t) G_{\rho\sigma}(x,t) \right\}$$

$$Q(t) = \int d^4x \; q(x,t) \qquad Q = \int d^4x \; q(x,t)$$

Lüscher: 2010

Giusti: plenary

Lattice 2015

Numerical details

Proof-of-principle calculation in Yang-Mills

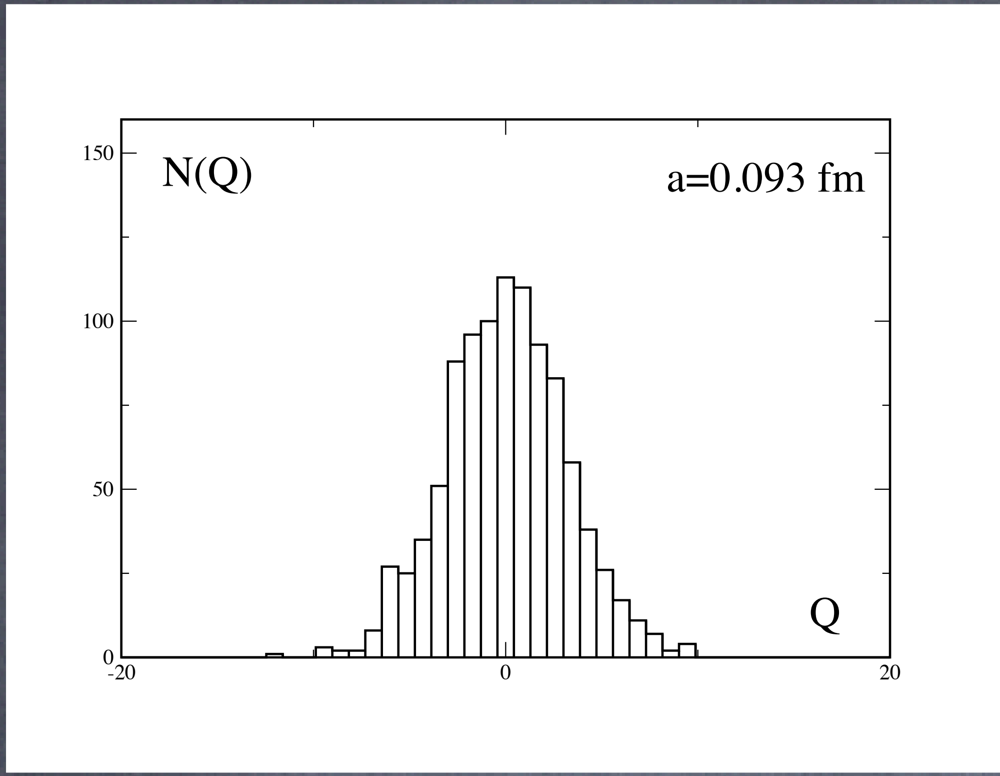
NP improved Wilson +Wilson gauge

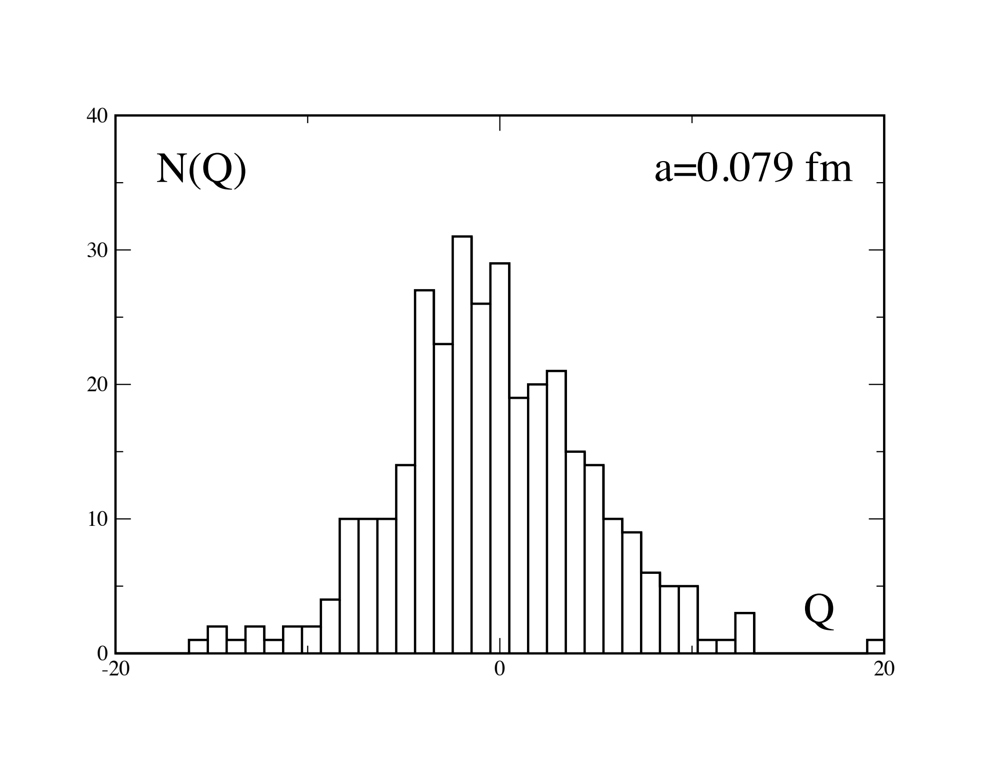
a=0.1-0.05 fm L/a=16,24,24,32 T/L=2
@ Mpi ≃ 800 MeV

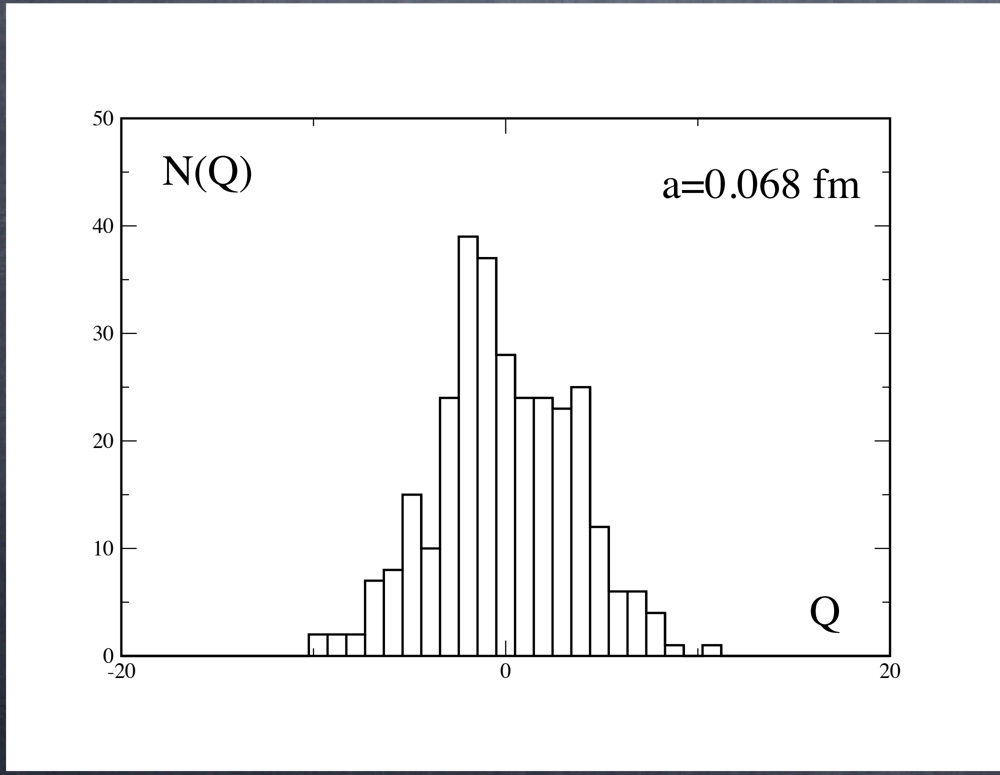
O(L/2a) Stochastic source locations
3 Gaussian smearings

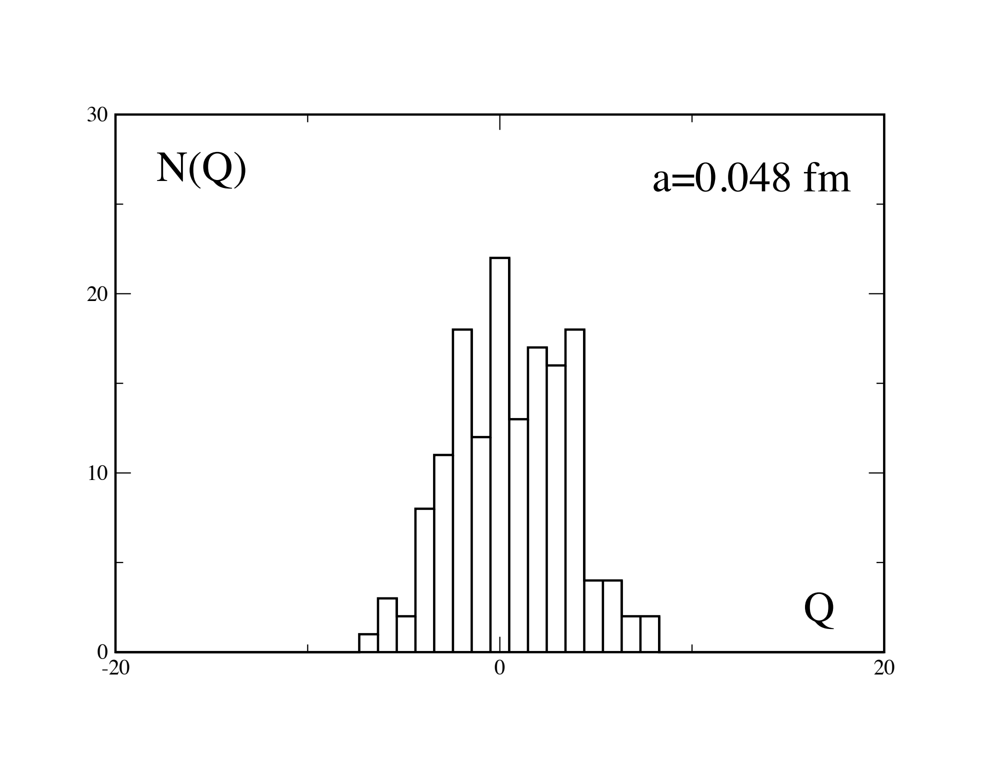
β	$N_{\rm th}$	$N_{\rm up}$	N_g	$N_{\rm meas}$
6.0	2000	200000	1000	1000
6.1	2000	65000	325	325
6.2	2000	60000	300	300
6.45	2000	122400	612	153

Topological distribution





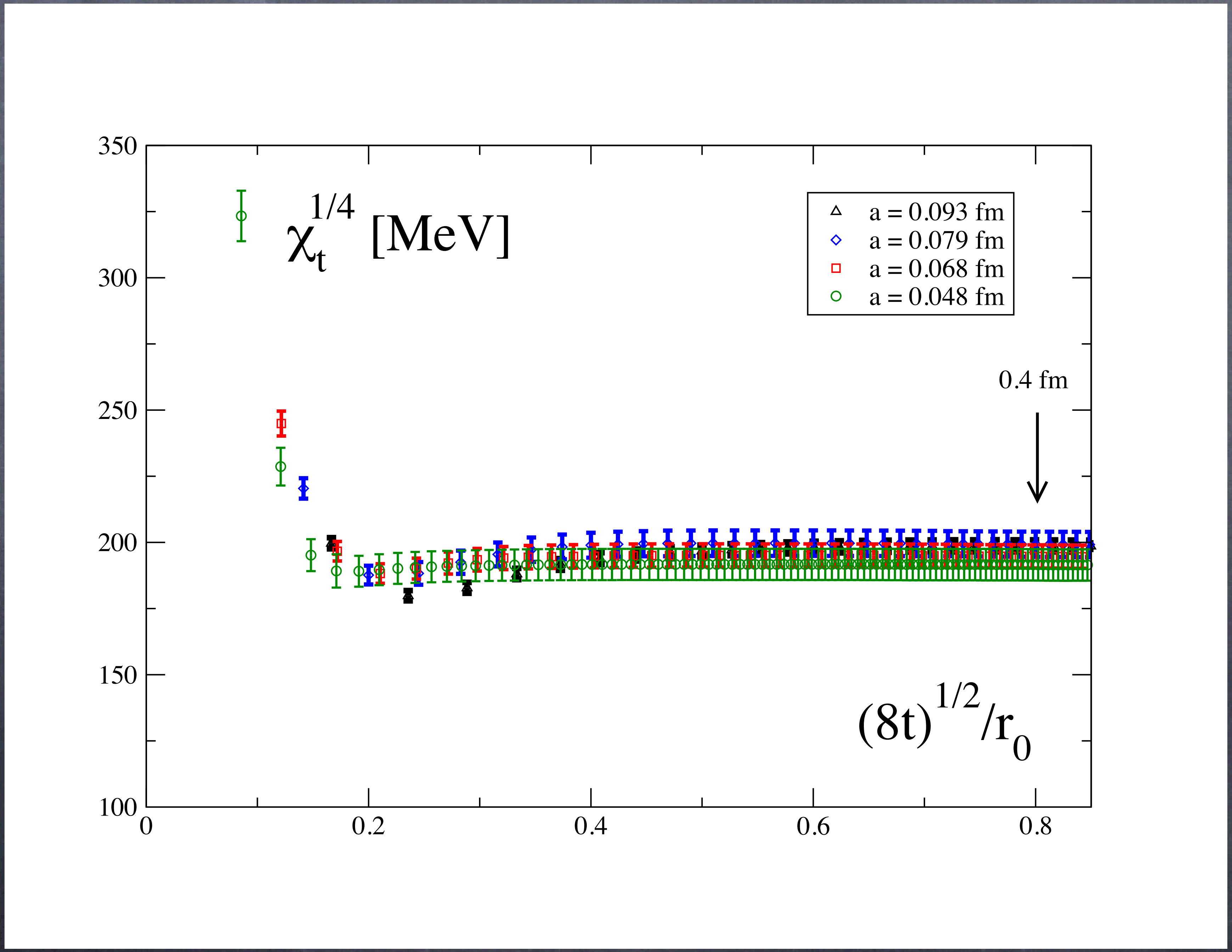




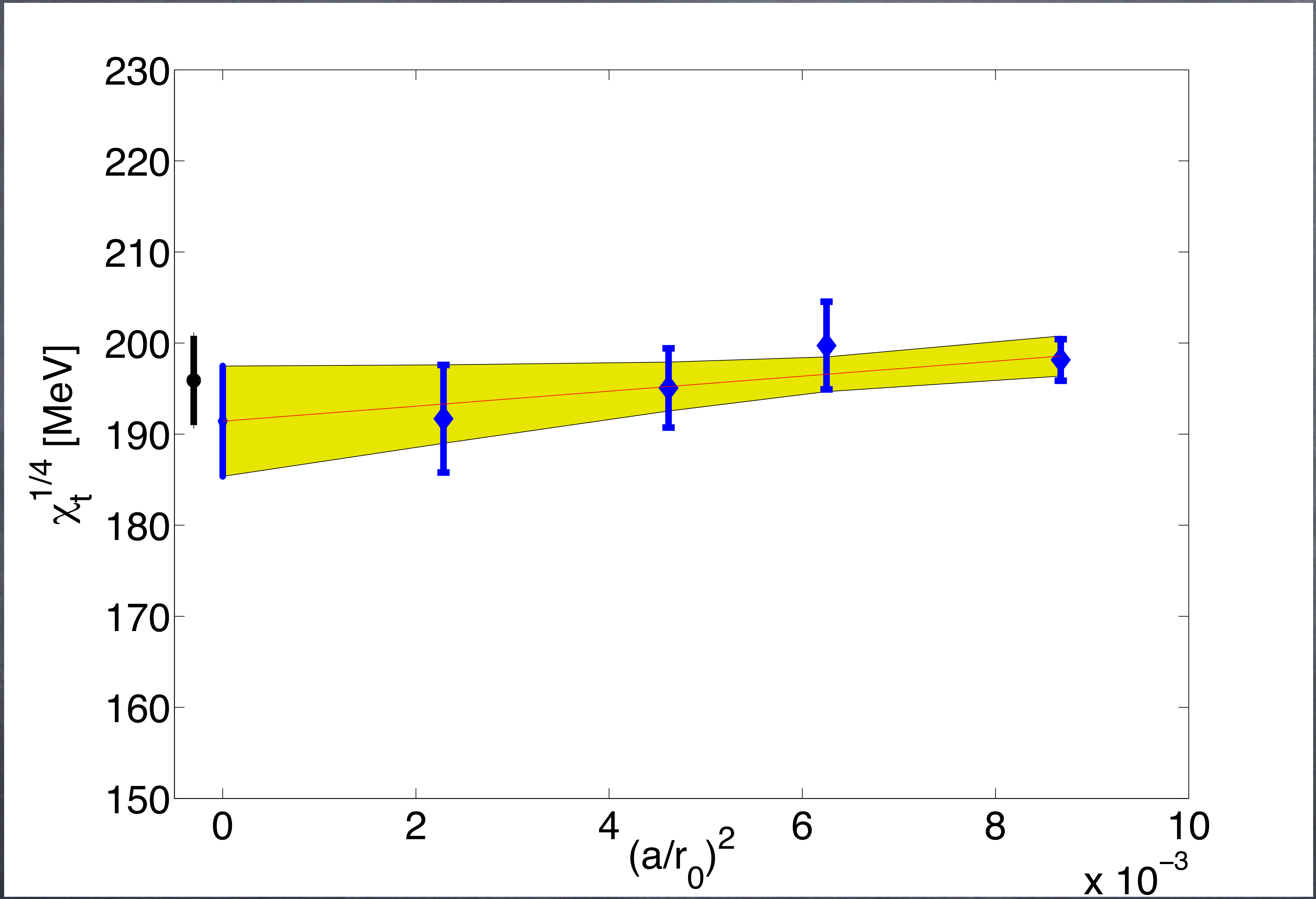
Bilson-Thompson, Leinweber, Williams : 2002

Topological susceptibility

$$\chi_{\rm top}(t) = \frac{1}{V}\int d^4x\; d^4y\, \langle q(t,x)q(t,y)\rangle$$



Continuum limit



$$\chi_t^{1/4} = 191(7)\ \mathrm{MeV}$$

Parity violating mixing

$$\langle\theta|\mathcal{N}|N^\theta(\underline{p},s)\rangle = \mathcal{Z}_N(\theta;\underline{p})u_N^\theta(\underline{p},s) = \mathcal{Z}_N(\theta;\underline{p})\,\mathrm{e}^{i\alpha_N(\theta)\gamma_5}u_N(\underline{p},s)$$

Pospelov

Mixing of the ϑ=0 eigenstates in the CP-broken vacuum

Unphysical mixing of electric and magnetic dipole moment form factors

Determine the phase of the nucleon mass

$$\sum_s u_N^\theta(\underline{p},s)\overline{u}_N^\theta(\underline{p},s) = E_\theta(\underline{p})\gamma_0 - i\gamma_k p_k + M_N \mathrm{e}^{2i\alpha_N(\theta)\gamma_5}$$

Parity violating mixing

$$G^{\theta}_{NN}(x_0) = a^3 \sum_{\underline{x}} \left\langle \mathcal{N}(\underline{x}, x_0) \mathcal{N}^{\dagger}(0) \right\rangle_{\theta} = G_{NN} + i\theta G^{Q}_{NN} + \mathrm{O}(\theta^2)$$

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Parity violating mixing

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$$\mathrm{tr}\,[P_+ G_{NN}] = 2|Z_N|^2 \mathrm{e}^{-M_N x_0} + \cdots$$

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$$\mathrm{tr}\left[P_+ G_{NN}\right] = 2|Z_N|^2 \mathrm{e}^{-M_N x_0} + \cdots$$

$$\mathrm{tr}\left[P_+ \gamma_5 G^{Q}_{NN}\right] = 2|Z_N|^2 \alpha_N \mathrm{e}^{-M_N x_0} + \cdots$$

$$\mathrm{tr}\left[P_+ G_{NN}\right] = |Z_N|^2 \mathrm{e}^{-M_N x_0} + \cdots$$

A.S., de Vries, Luu: 2014

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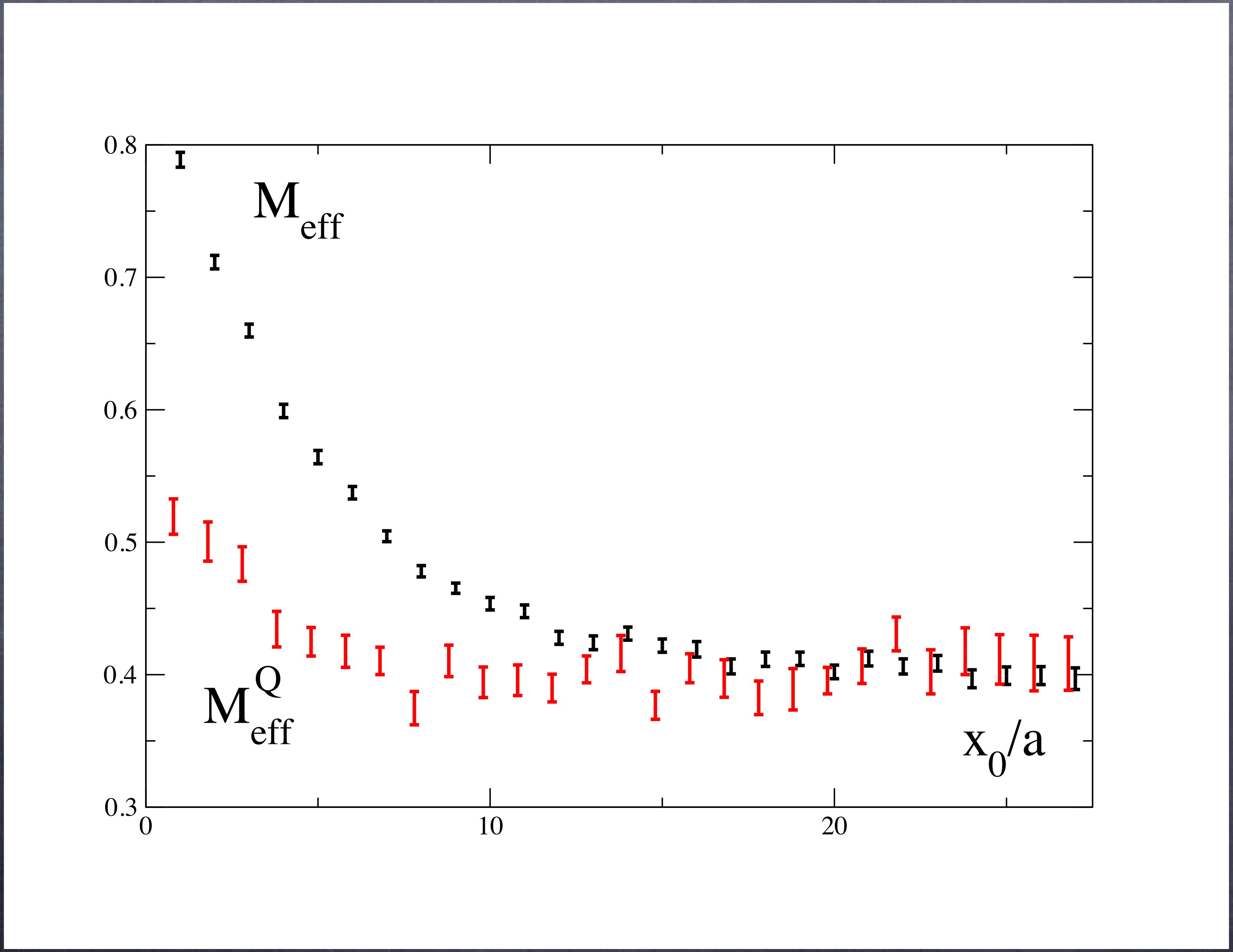
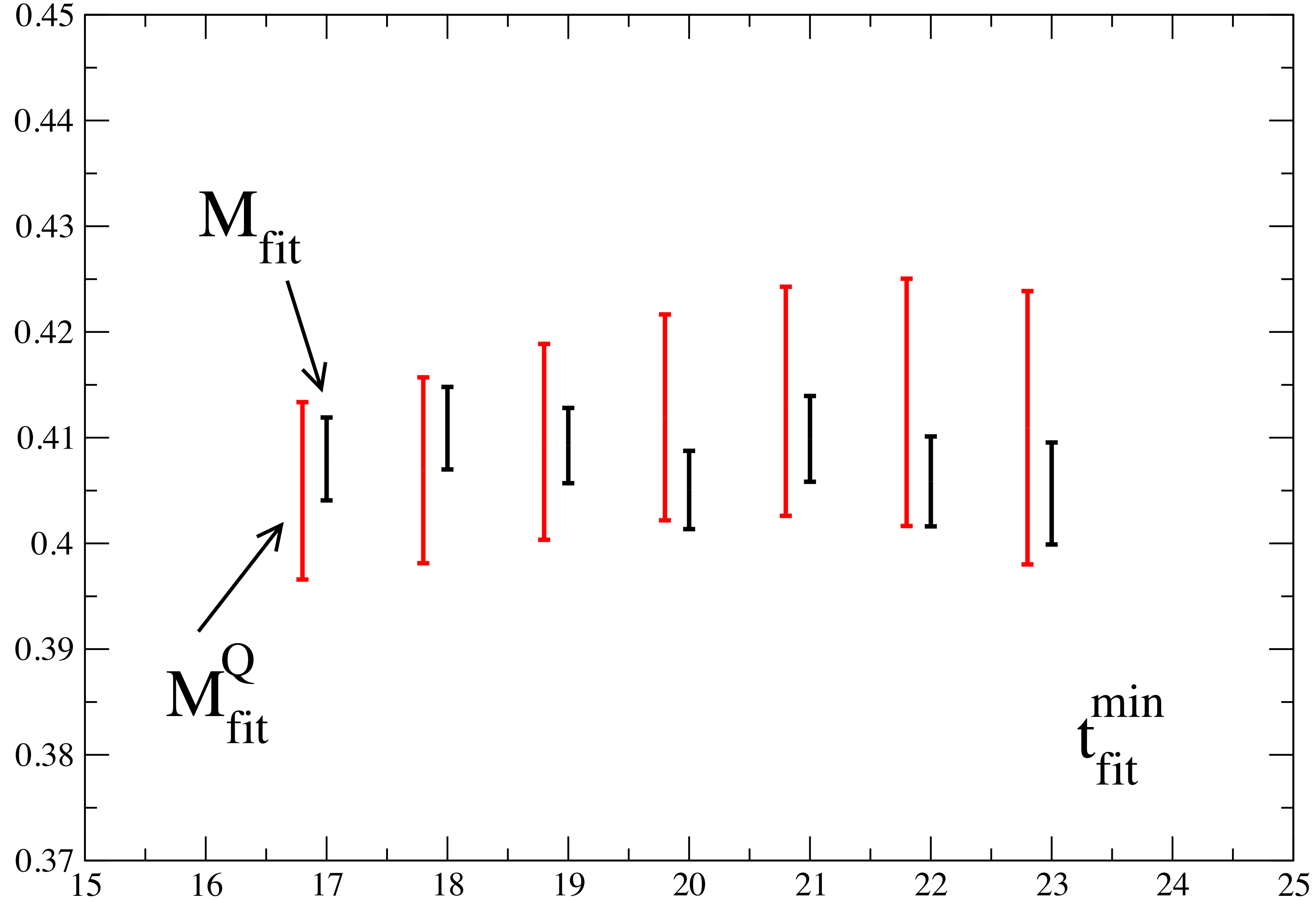
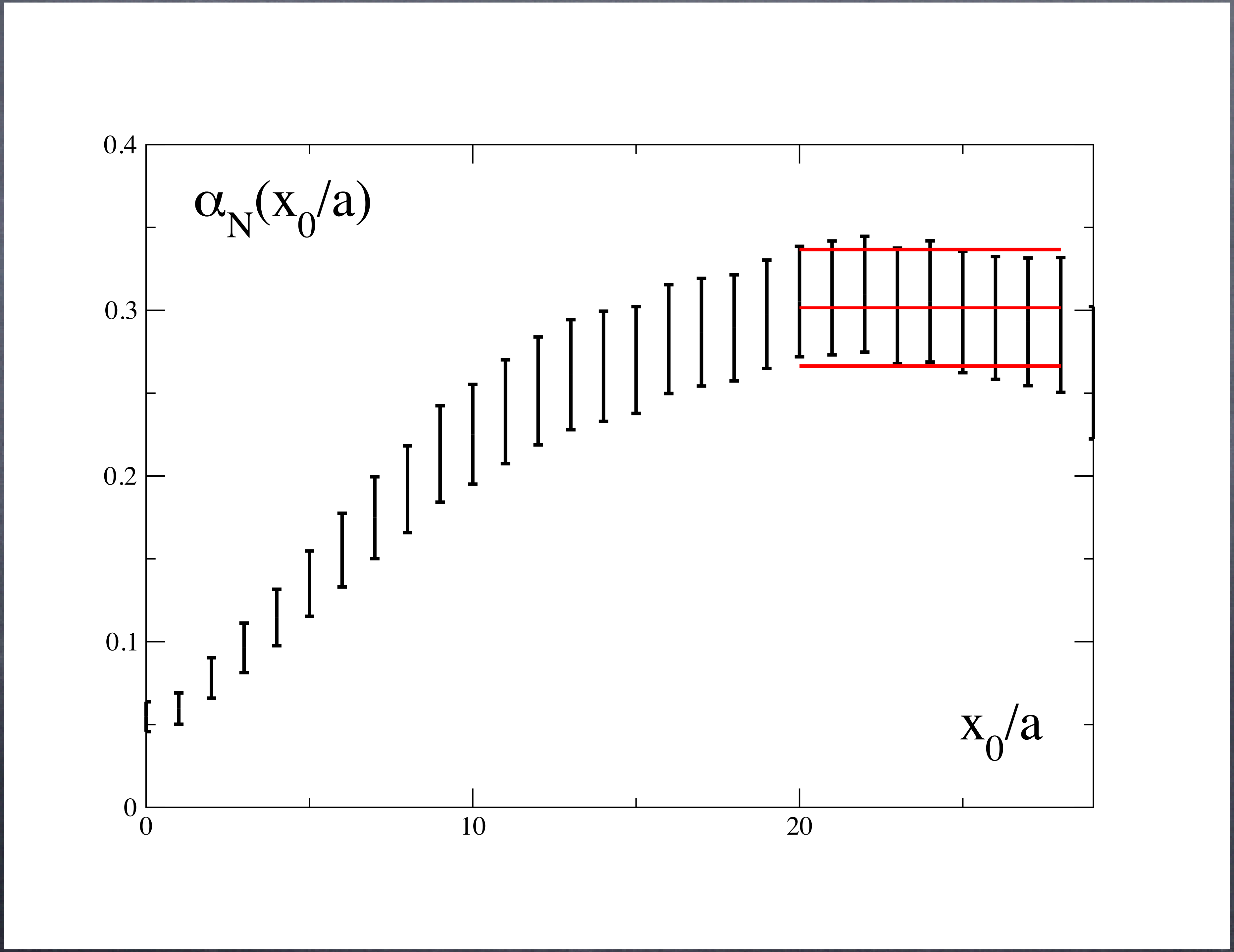


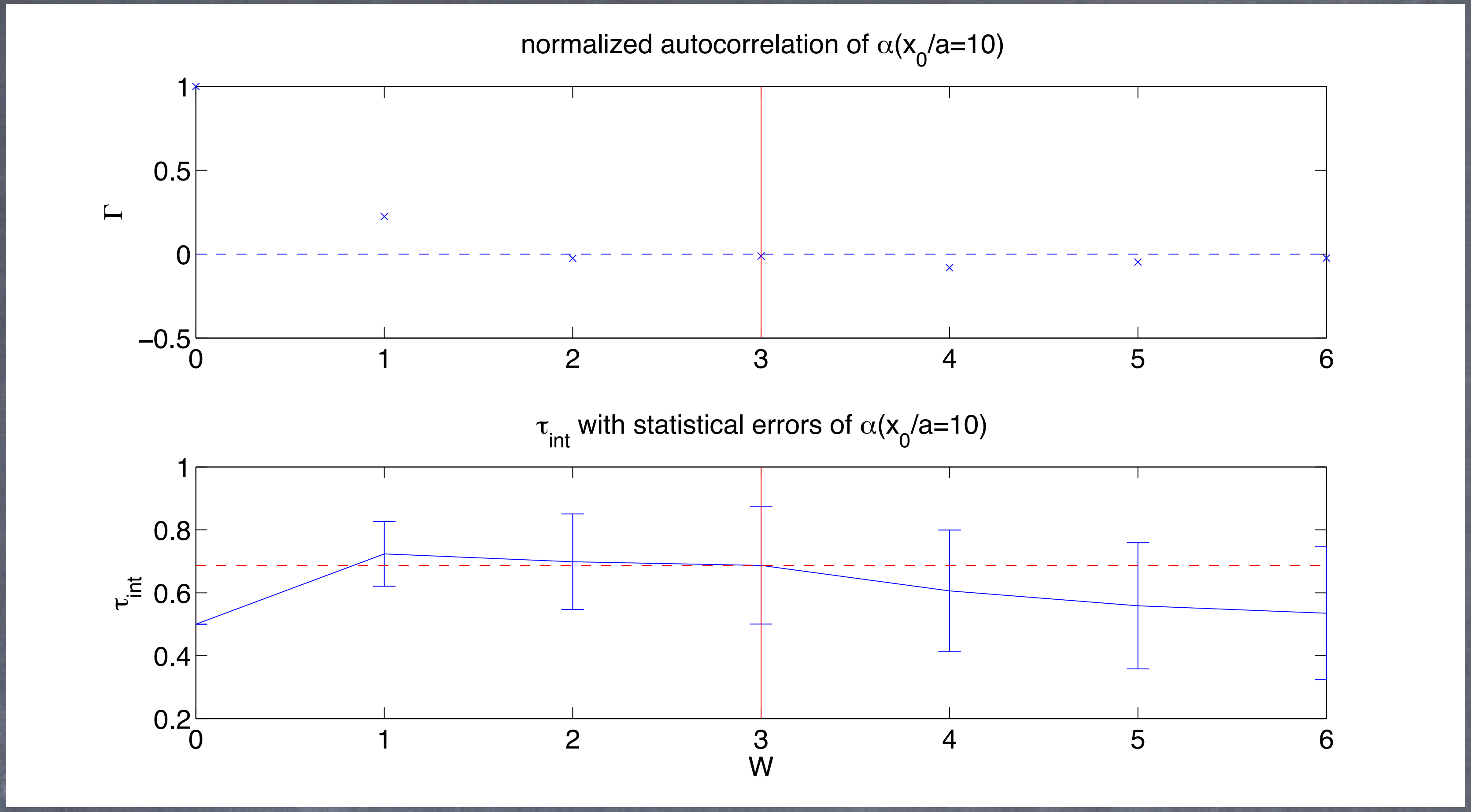
Fit window

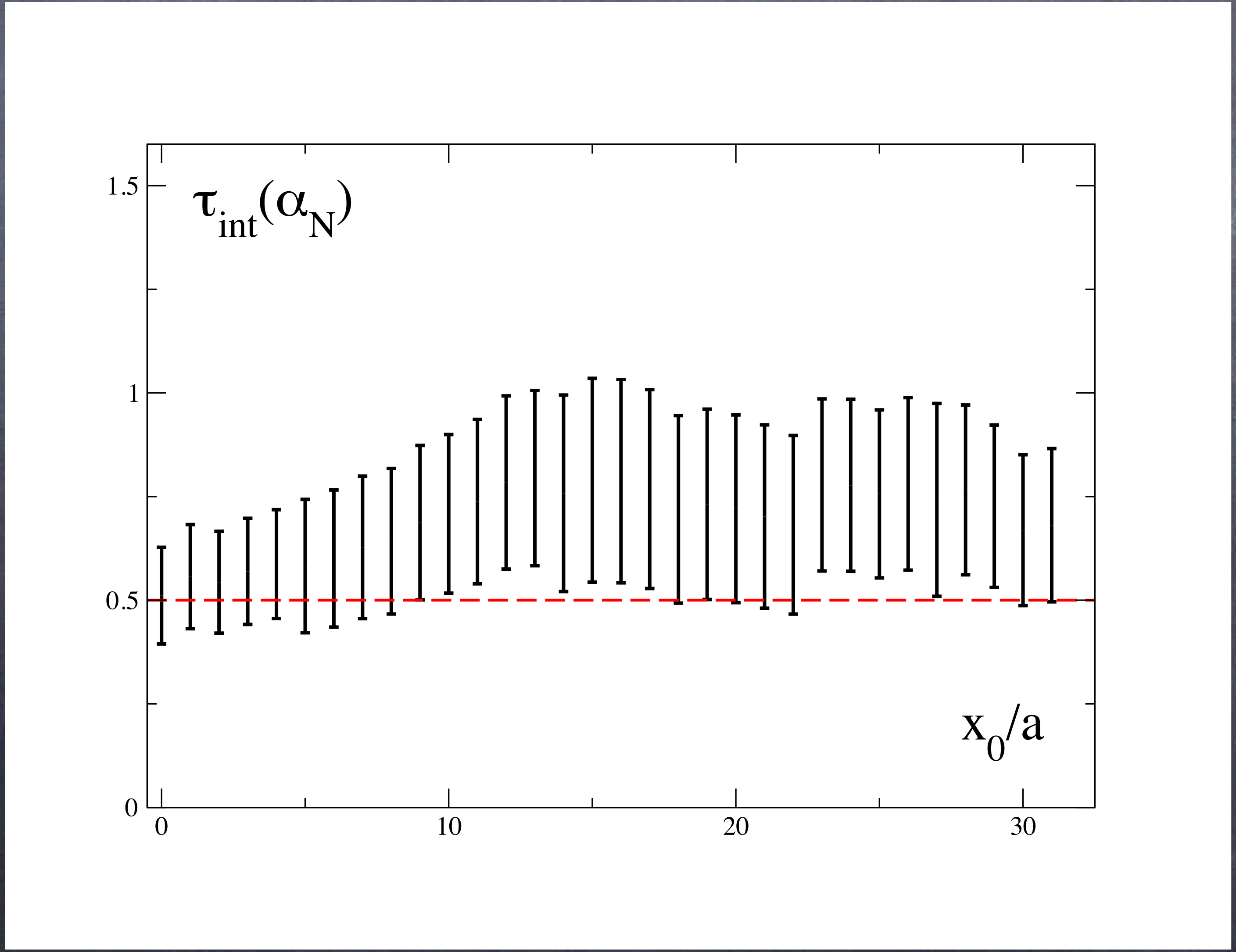


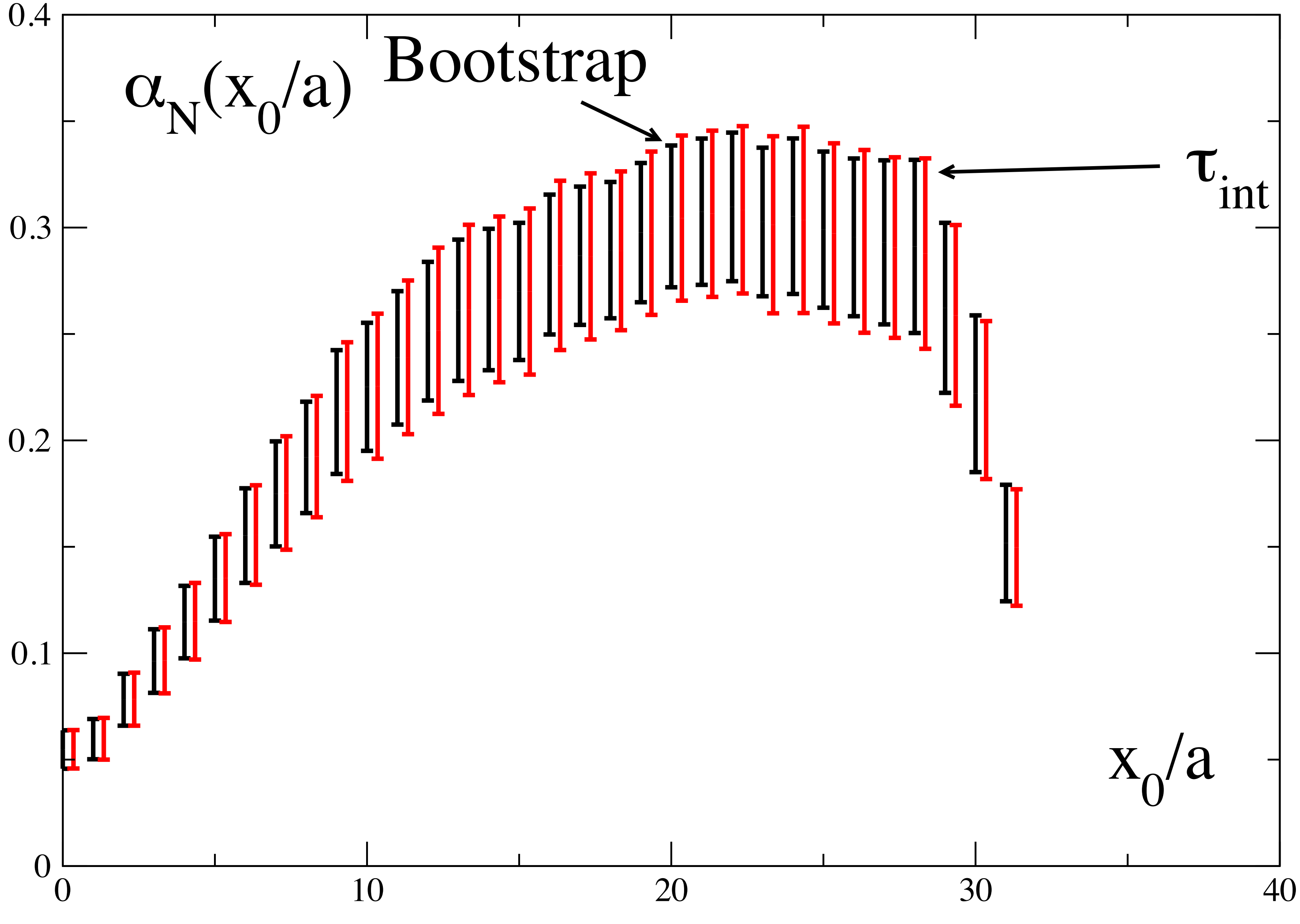
0.45
0.44
0.43
0.42
0.41
0.4
0.39
0.38
0.37
15
16
17
18
19
20
21
22
23
24
25
M_{fit}
M^Q_{fit}
t^{min}_{fit}

$$\alpha_N^{(1)} = \frac{\mathrm{tr}\left[P_+ \gamma_5 G^Q_{NN}\right]}{\mathrm{tr}\left[P_+ G_{NN}\right]} + \cdots$$



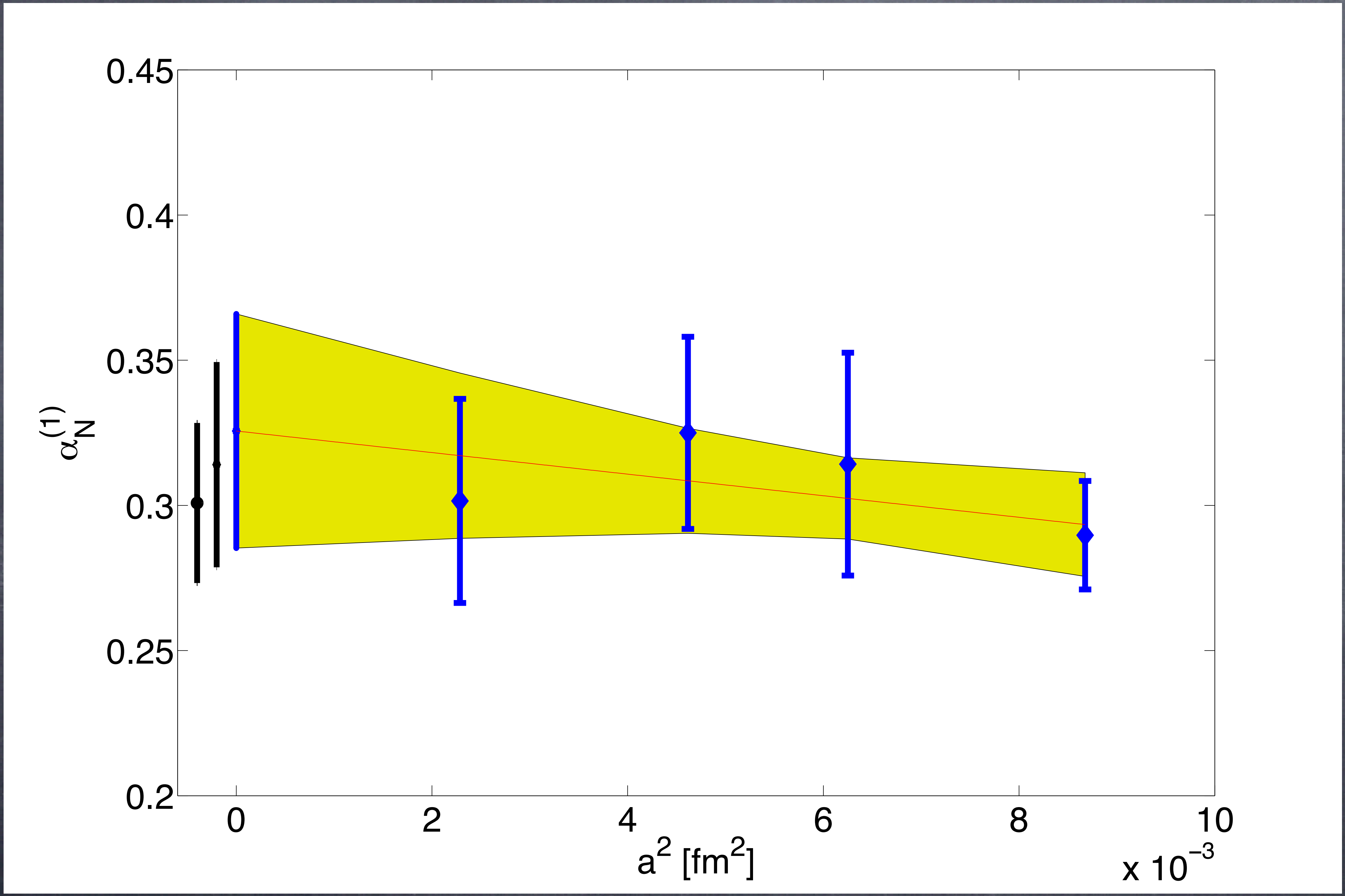

normalized autocorrelation of α(x0/a=10)
Γ
τint with statistical errors of α(x0/a=10)
τint
W


τint(αN)
x0/a

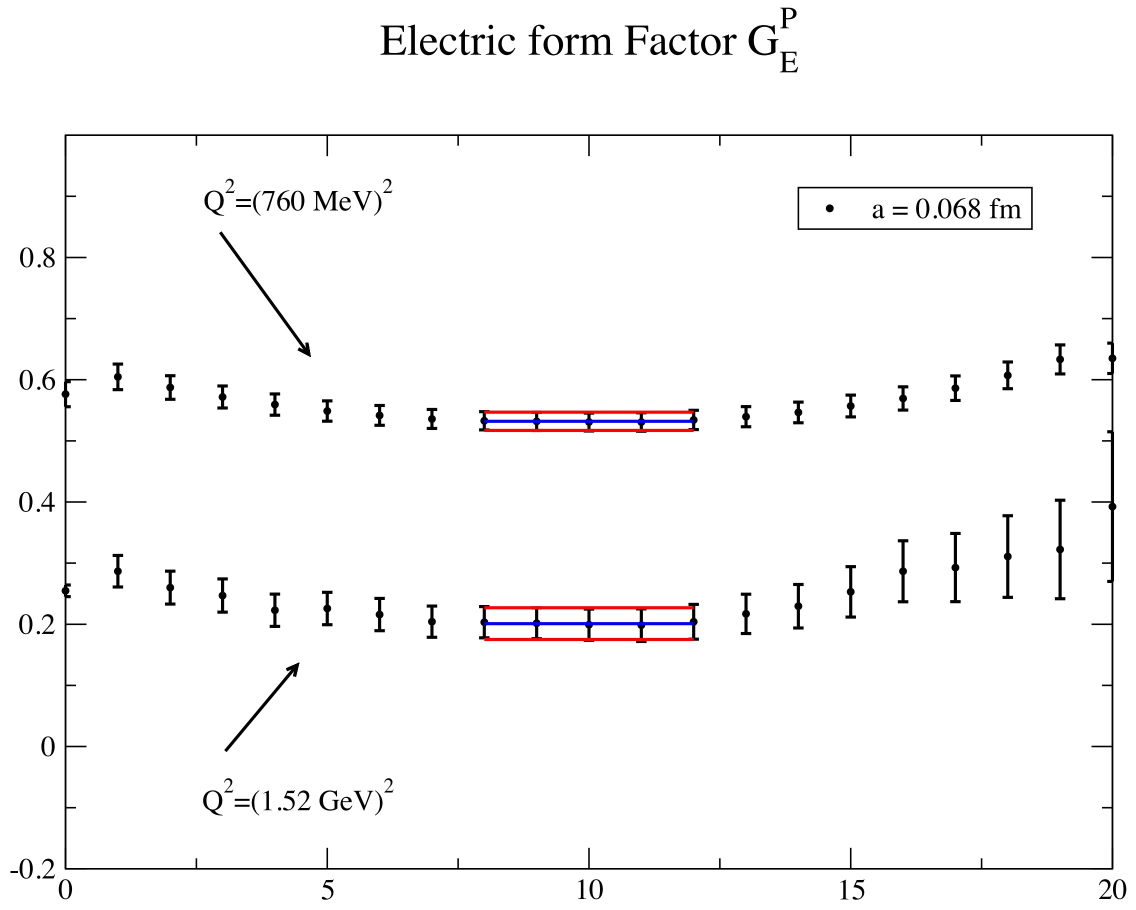


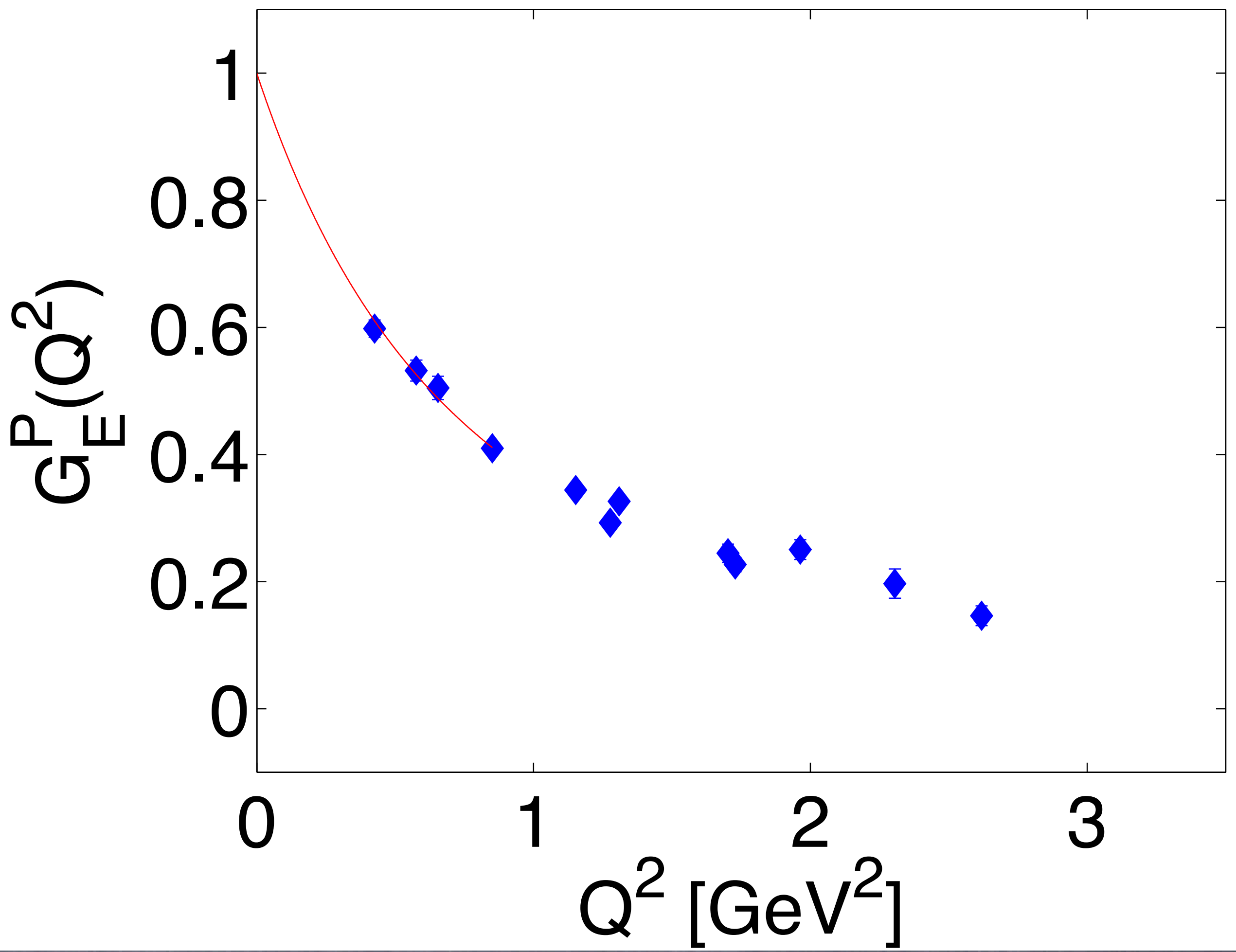
$\alpha_N(x_0/a)$
Bootstrap
τ_{int}
x_0/a
0.4
0.3
0.2
0.1
0
0
10
20
30
40

Continuum limit



$$\alpha_N^{(1)} = 0.326(40)$$

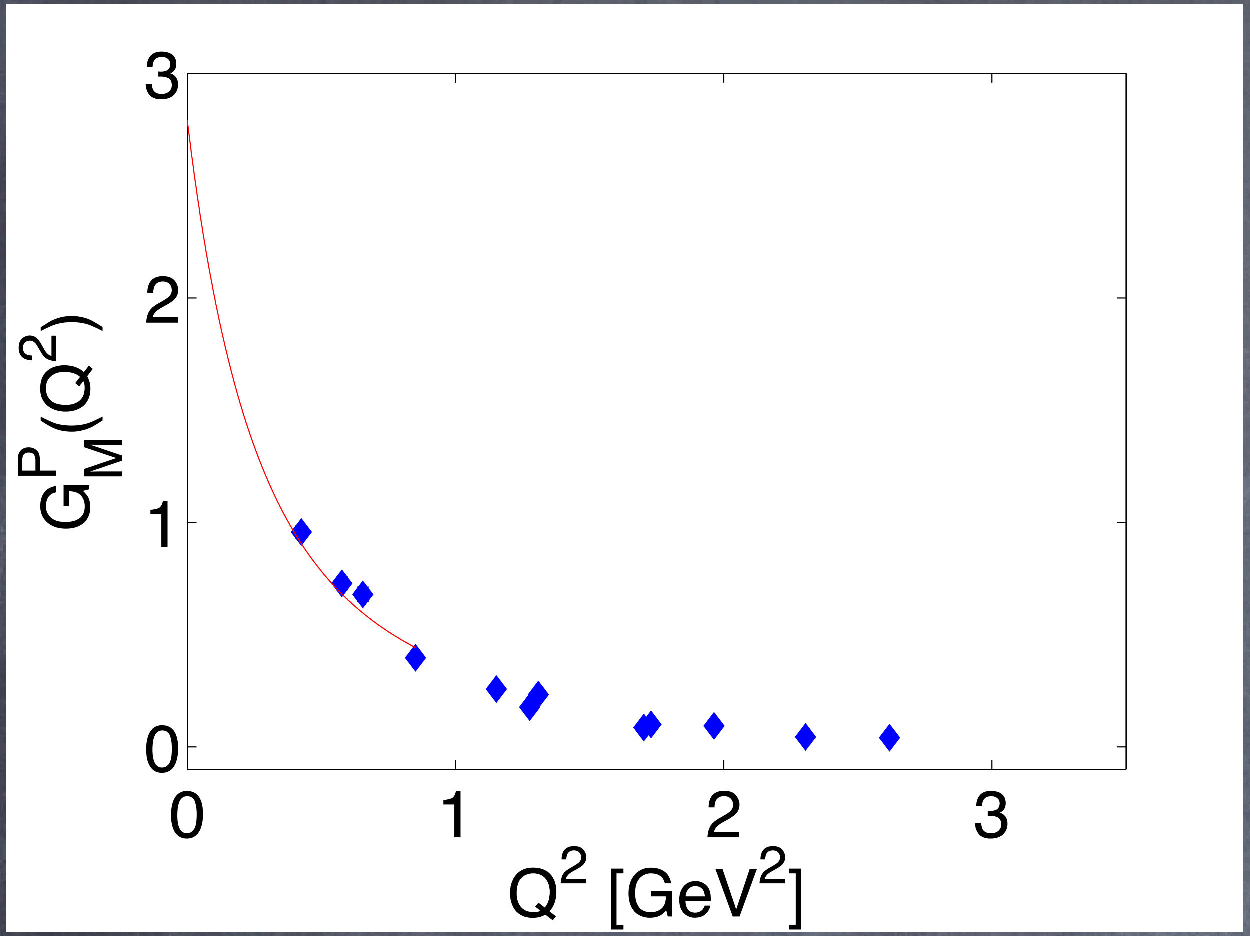


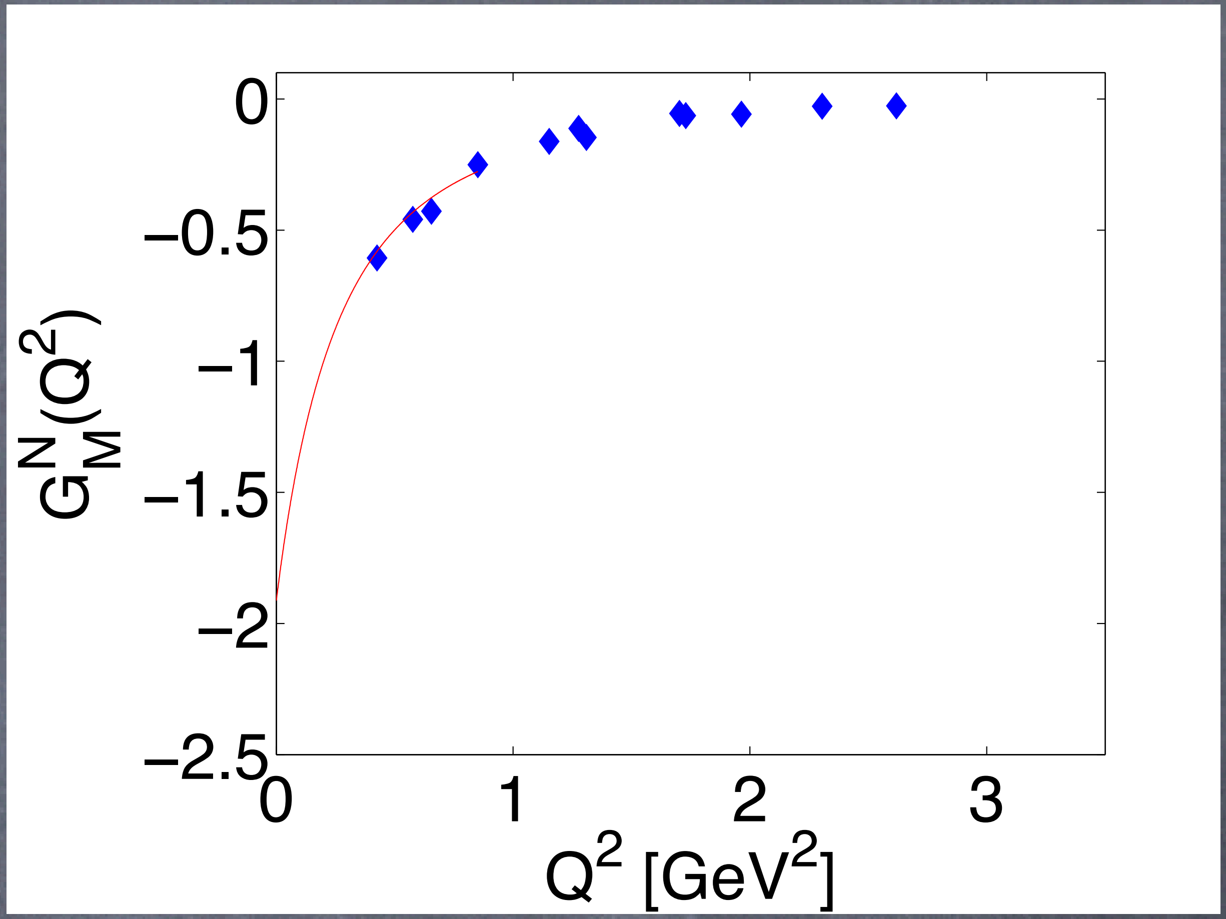


$$\Gamma_\mu(Q^2) = \gamma_\mu F_1(Q^2) + \sigma_{\mu\nu}\frac{Q_\nu}{2M}F_2(Q^2) + $$
$$+ \quad i\theta\sigma_{\mu\nu}\gamma_5\frac{Q_\nu}{2M}F_3(Q^2)$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)$$

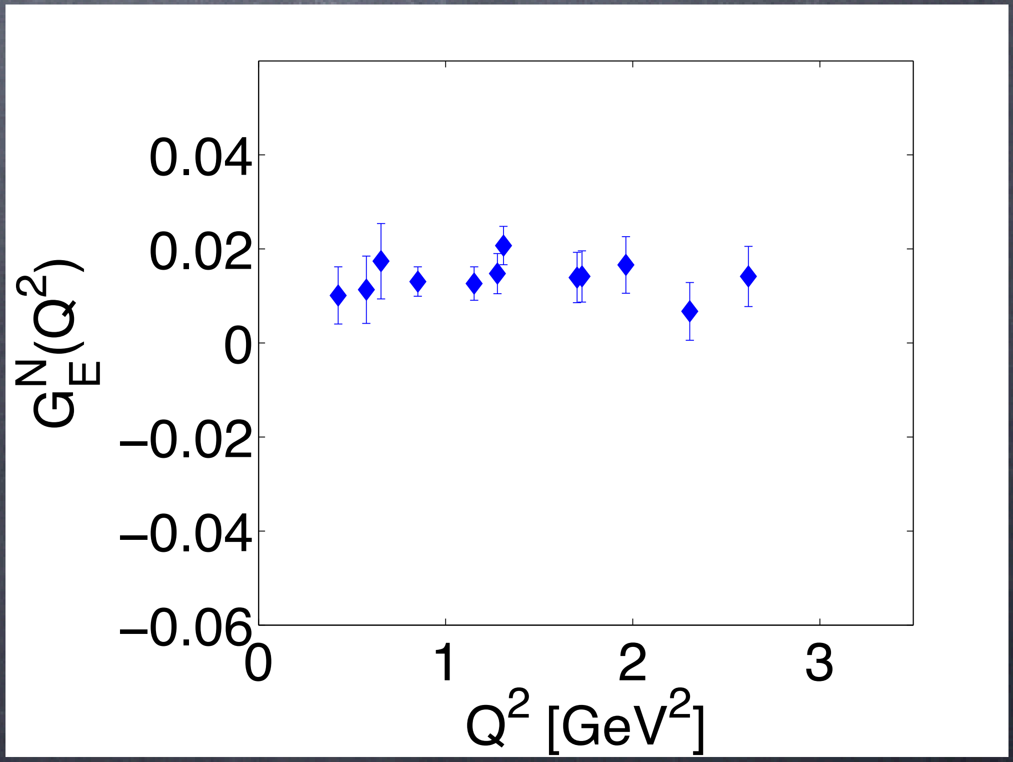
CP-even form factors



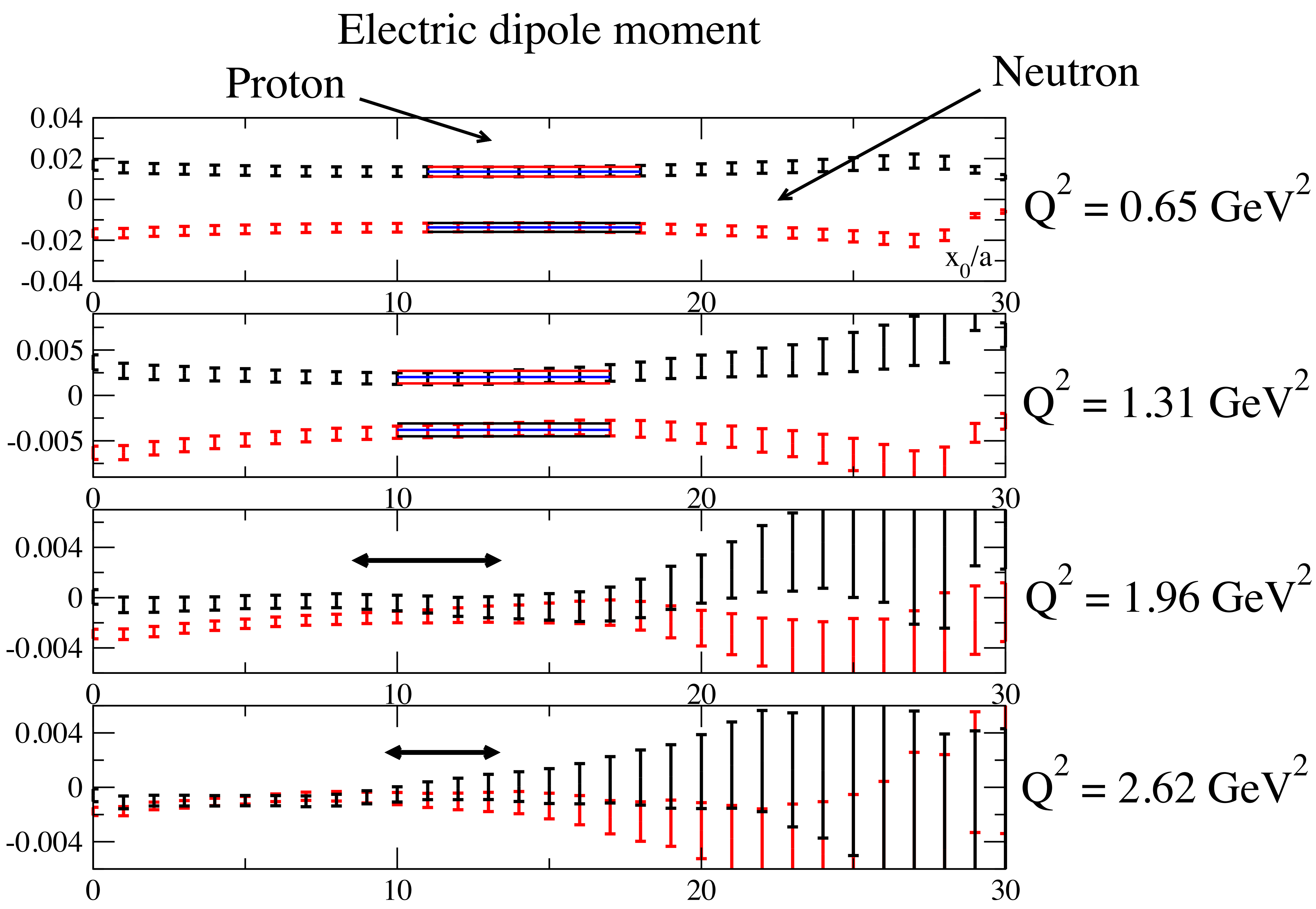


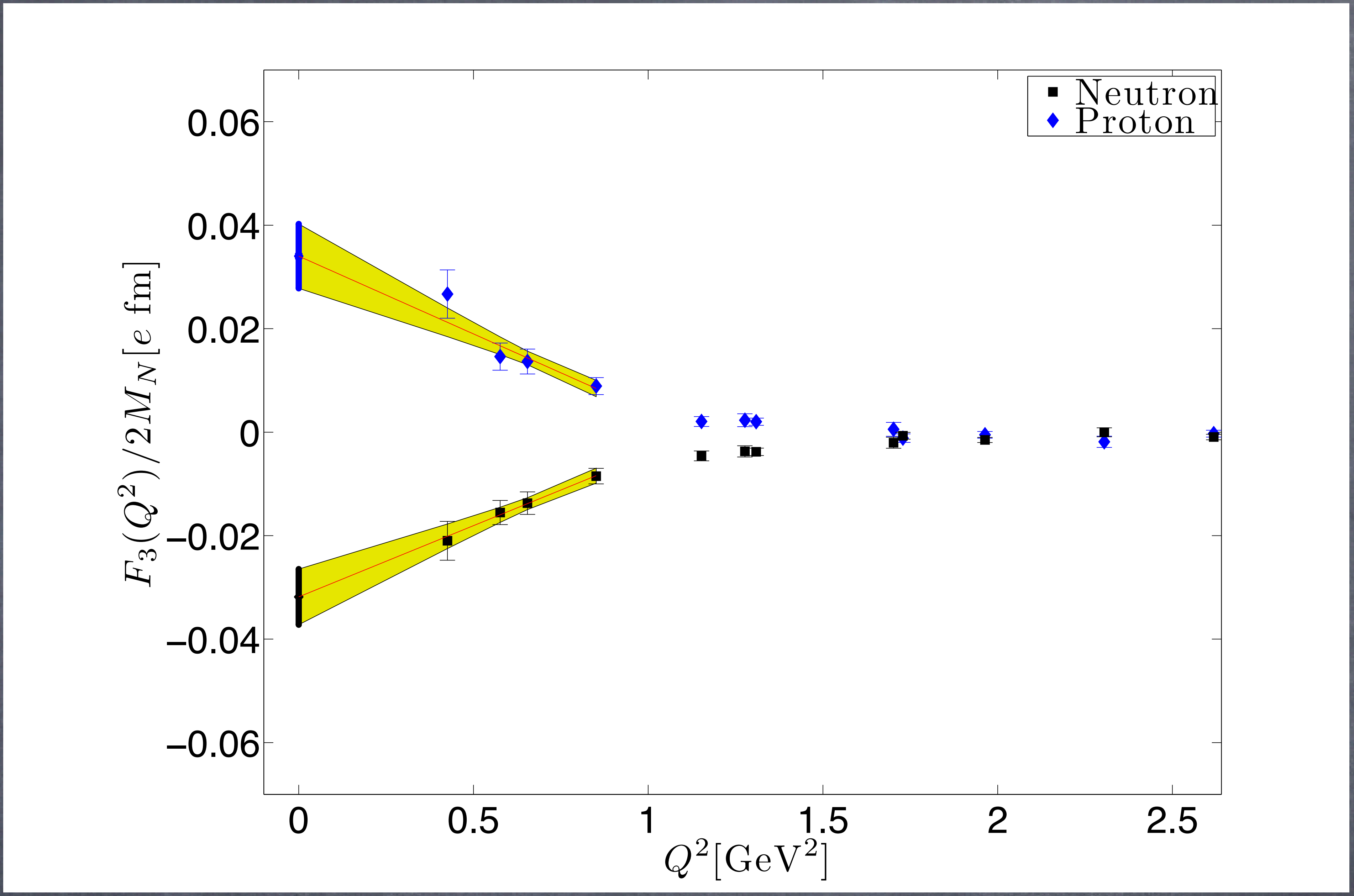
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$



CP-odd form factor





$$\frac{F_3^{P/N}(Q^2)}{2M_N} = d_{P/N} + S_{P/N}Q^2 + H_{P/N}(Q^2)$$

$$\frac{d_P}{d_N} < 0 \qquad \frac{S_P}{S_N} < 0$$

Mereghetti et al.: 2011

Nucleon EDM and Schiff moments

$$|d_{\rm N}| < 2.9 \cdot 10^{-26} \; e \cdot {\rm cm}$$

$$d_{\rm P} = 0.0340(62) \; \theta \; e \cdot {\rm fm}$$

$$d_{\rm N} = -0.0318(54) \; \theta \; e \cdot {\rm fm}$$

$$d_N = \frac{e g_A \bar{g}_0^\theta}{16\pi^2 F_\pi^2} \left(\ln \frac{M_\pi^2}{\Lambda_{N,{\rm EDM}}^2} - \frac{\pi M_\pi}{2 M_N} \right)$$

$$d_P = -\frac{e g_A \bar{g}_0^\theta}{16\pi^2 F_\pi^2} \left(\ln \frac{M_\pi^2}{\Lambda_{P,{\rm EDM}}^2} - \frac{2\pi M_\pi}{M_N} \right)$$

$$d_{\rm P}^{\rm phys} = 0.96(18) \cdot 10^{-3} \; \theta \; e \cdot {\rm fm}$$

$$d_{\rm N}^{\rm phys} = -0.90(15) \cdot 10^{-3} \; \theta \; e \cdot {\rm fm}$$

$$\Rightarrow \quad \theta \lesssim 3.2 \cdot 10^{-10}$$

$$S_{\rm P} = -S_{\rm N} = -\frac{e g_A \bar{g}_0^\theta}{48\pi^2 F_\pi^2 M_\pi^2}$$

Ab-initio determination of $\bar{g}_0^\theta$

Some remarks

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- New method for the calculation of the EDM from theta-term

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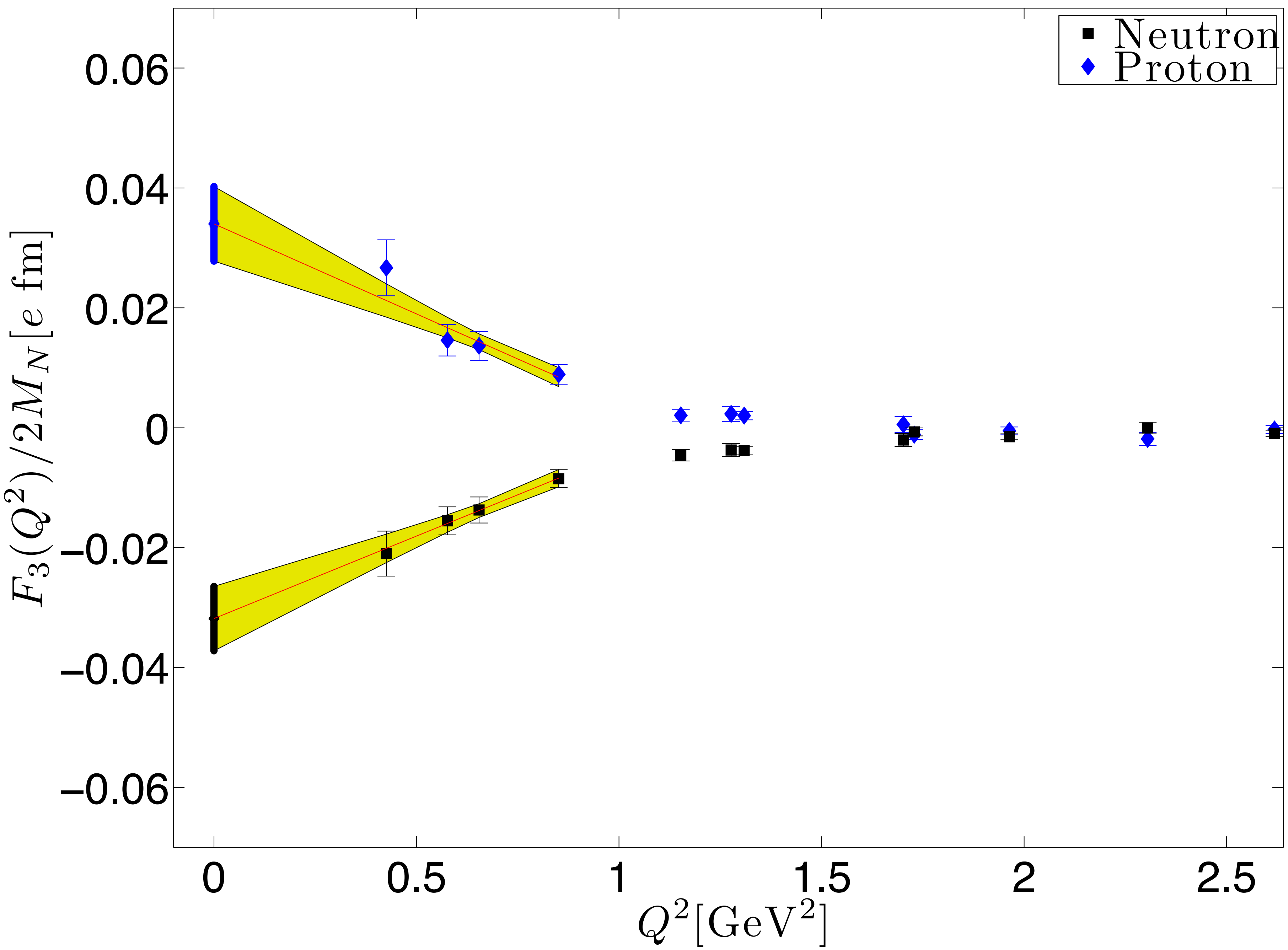
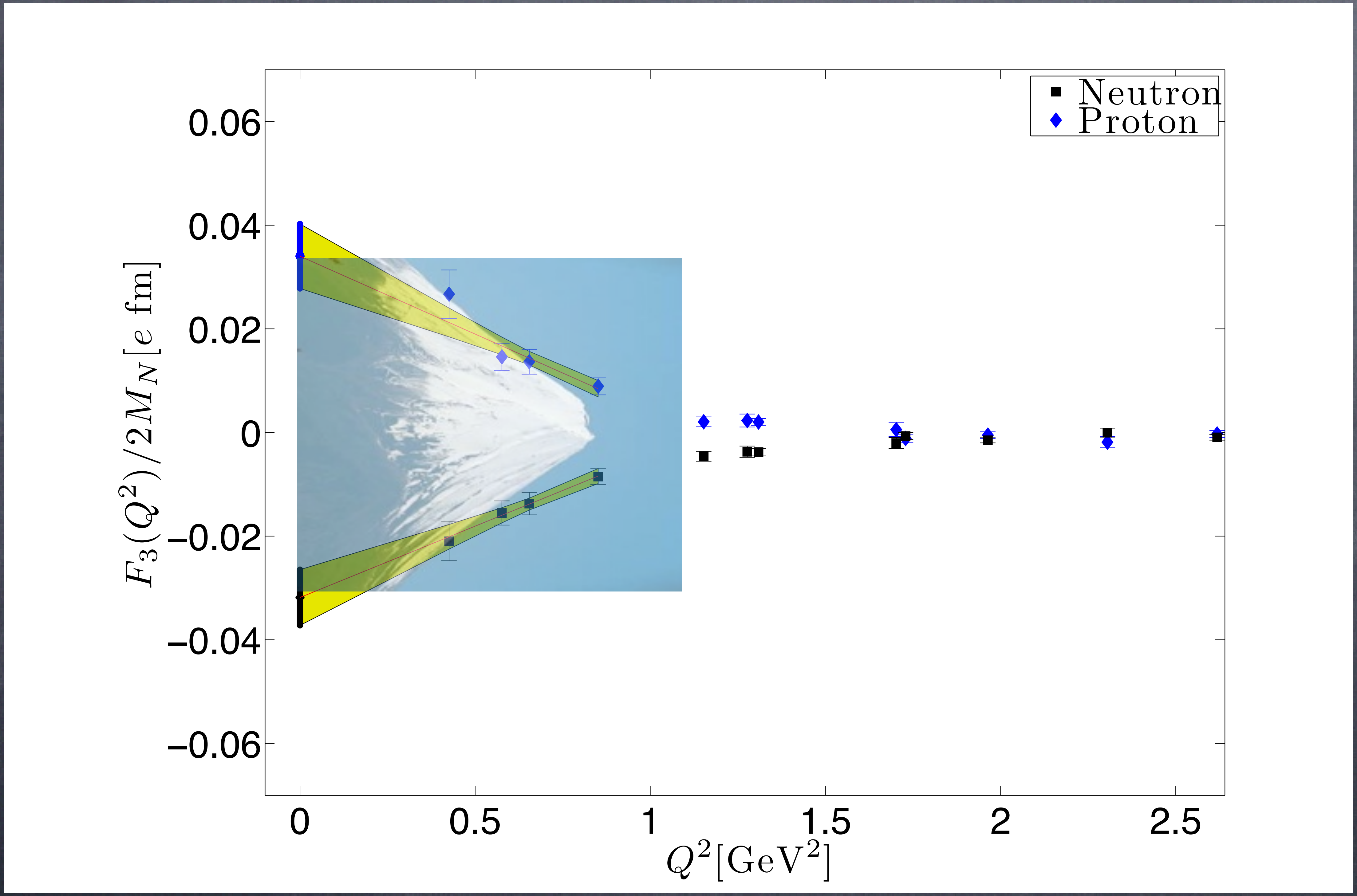
- Proof-of-principle calculation has been successful
- Extension to dynamical quarks started
- Stay tuned! The fun starts now!!



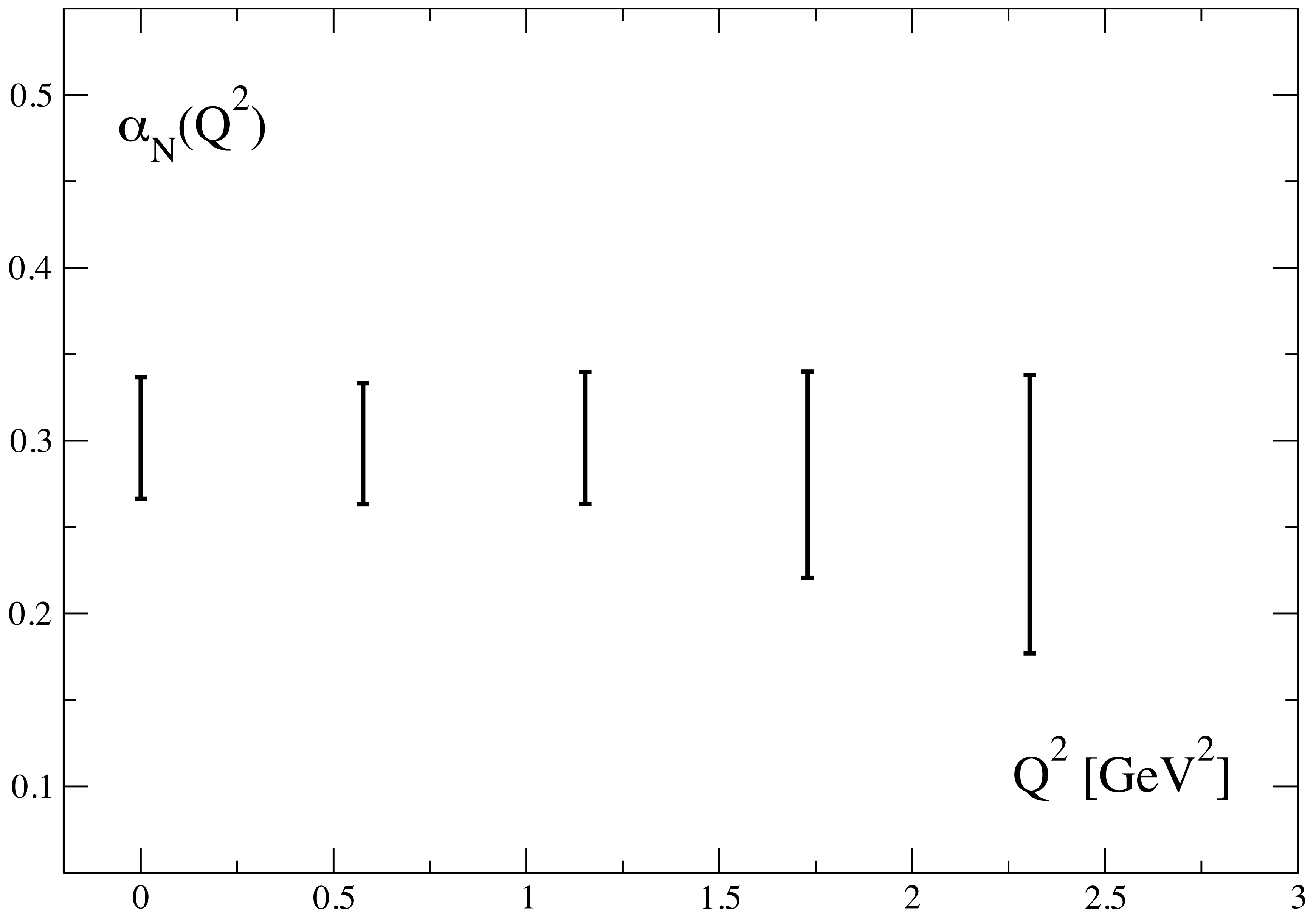
Neutron
Proton
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
$F_3(Q^2)/2M_N[e\ \mathrm{fm}]$
0
0.5
1
1.5
2
2.5
$Q^2[\mathrm{GeV}^2]$



Neutron
Proton
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
$F_3(Q^2)/2M_N[e\ \mathrm{fm}]$
0
0.5
1
1.5
2
2.5
$Q^2[\mathrm{GeV}^2]$

Backup slides

Momentum independence of alpha



Dispersion relation