Large-scale computation of the exponentially expanding universe in a simplified Lorentzian IIB matrix model

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Introduction

- Superstring theory
  - describes quantum gravity and requires 10d spacetime.
  - too many vacua leading to 4d effective field theories
- Perturbative approach

  → No guiding principle to pick up one vacuum.
- Non-perturbative approach needed

  → matrix model

Theory	QCD	String theory
Non-perturbative formulation	Lattice gauge theory	Matrix model

Type IIB matrix model [N.Ishibashi, H.Kawai, Y.Kitazawa, A.Tsuchiya (1996)]

- 0-dimensional model $\longrightarrow$ Eigenvalues of matrices describe 10d spacetime dynamically.
- Partition function

$$\mathcal{Z} = \int \mathcal{D}A\mathcal{D}\Psi e^{-S[A,\Psi]}$$ (Euclidean version)

$$\begin{cases} S_b = -\frac{1}{4g^2}\mathrm{tr}\,[A_\mu, A_\nu]^2 \\ S_f = -\frac{1}{2g^2}\mathrm{tr}\,\Psi\mathcal{C}\Gamma^\mu\,[A_\mu, \Psi] \end{cases}$$

$A_\mu, \Psi : N \times N$ Hermitian matrices

$\mu = 0, \cdots, 9$

Lorentzian version [S.-W.Kim, J.Nishimura, A.Tsuchiya, (2011)]

- metric : Lorentzian signature $\eta_{\mu\nu} = \mathrm{diag}\,(-1, 1, \cdots, 1)$
- Partition function

$$\mathcal{Z} = \int \mathcal{D}A\mathcal{D}\Psi e^{iS[A,\Psi]}$$

✓ SO(9,1) symmetry

✓ suitable for real-time dynamics

✓ unbounded action

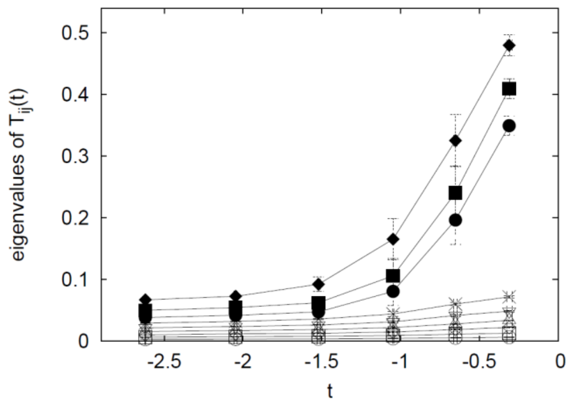
$$S_b \simeq 2\underbrace{\mathrm{tr}\left([A_0, A_i]^2\right)}_{\leq 0} - \underbrace{\mathrm{tr}\left([A_i, A_j]^2\right)}_{\leq 0}$$

$\Rightarrow$ regularized by introducing IR cutoffs

- The matrices describe dynamically the spacetime coordinates.

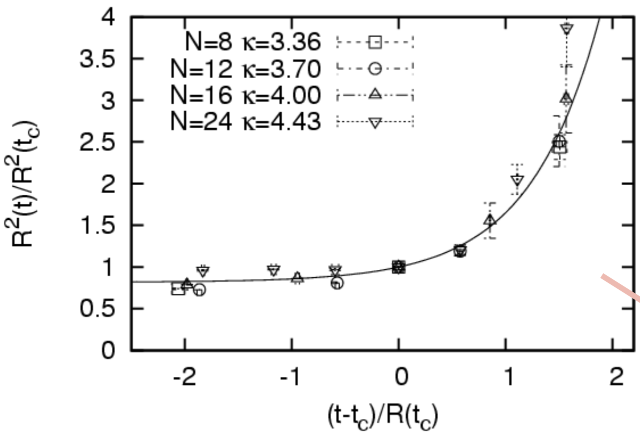
Previous works

- extent in each direction



[S.-W.Kim, J.Nishimura, A.Tsuchiya, (2011)]

- extent of the space



$N \leq 24$

[YI, S.-W.Kim, J.Nishimura, A.Tsuchiya, (2013)]

- IR cutoffs

$$\frac{1}{N}\mathrm{tr}\left[\left(A_0^2\right)^p\right] \leq \kappa^p$$: temporal

$$\frac{1}{N}\mathrm{tr}\left[\left(A_i^2\right)^p\right] \leq 1$$: spatial

Our main finding in this work

- The exponential expansion is confirmed with larger N for $p = 1$.
- The results actually depend on p, **but**
  - $p = 1$ (above result)
  - $p = 1.3,\ 1.4,\ 1.5$ $\Rightarrow$ universal !

Plan of the talk

1. Introduction
2. Extracting the time evolution
3. SSB of SO(9) and exponential expansion
4. Dependence on p in the IR cutoffs
5. Summary

Definition of the model

- Lorentzian IIB MM

$$\mathcal{Z} = \int \mathcal{D}A\, \mathrm{Pf}\mathcal{M}(A)\, e^{iS_{\mathrm{b}}[A]} \xrightarrow[\text{the scale factor}]{\text{integrating out}} \int \mathcal{D}A\, \mathrm{Pf}\mathcal{M}(A)\, \delta(S_b)$$

- simplified model
  - At early times, the space is not expanded yet.

⇩ neglect this term

$$S_{\mathrm{f}} = \mathrm{tr}\, \Psi_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \Psi_\beta]$$

$$= \mathrm{tr}\, \Psi_\alpha \left(\Gamma^0\right)_{\alpha\beta} [A_0, \Psi_\beta] + \cancel{\mathrm{tr}\, \Psi_\alpha \left(\Gamma^i\right)_{\alpha\beta} [A_i, \Psi_\beta]}$$

Then,

$$\mathrm{Pf}\, \mathcal{M}(A) \Longrightarrow \Delta(\alpha)^{2(d-1)} = \prod_{I<J} (\alpha_I - \alpha_J)^{2(d-1)}$$

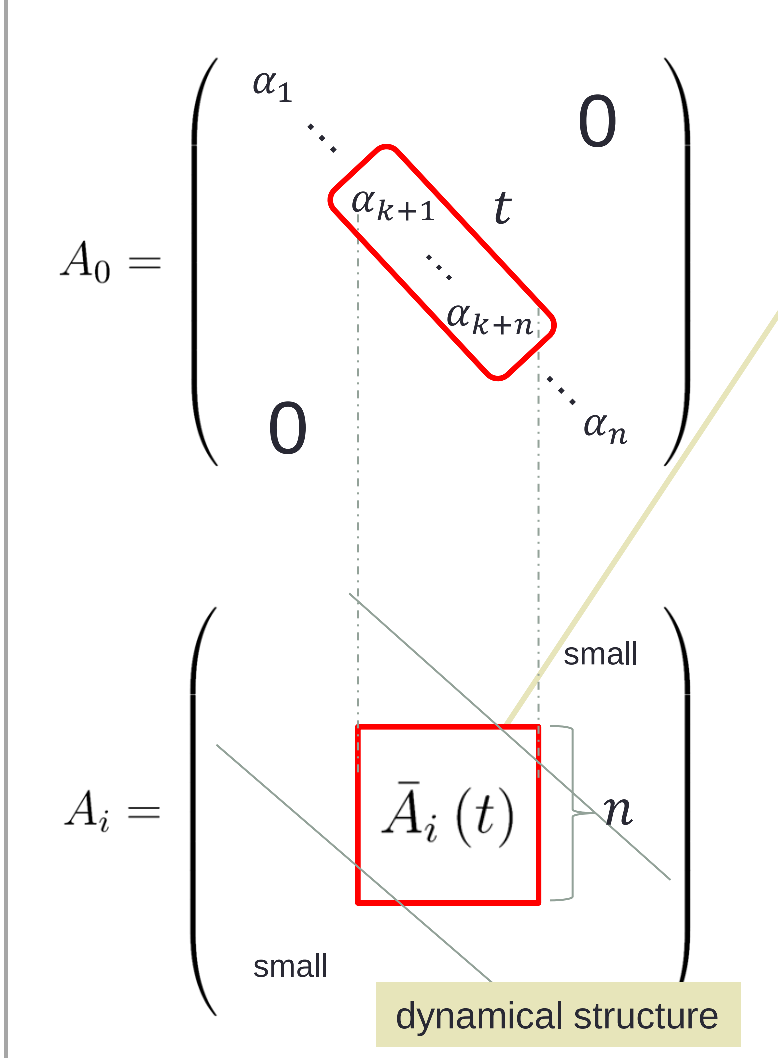
Gauge fixing

$$A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N)$$

$$\boxed{\mathcal{Z} \sim \int dA_i d\alpha\, \Delta(\alpha)^{-2d}\, \delta(S_b) \quad + \quad \text{IR cutoffs} \begin{cases} \frac{1}{N}\mathrm{tr}\left[(A_0^2)^p\right] \le \kappa^p \\ \frac{1}{N}\mathrm{tr}\left[(A_i^2)^p\right] \le 1 \end{cases}}$$

Extracting the time evolution

Structure of matrices



$$t = \frac{1}{n}\sum_{i=1}^{n} \alpha_{k+i}$$: averaged time

$\bar{A}_i(t)$:state of the universe at time t

- the spatial extent at time t

$$R^2(t) = \frac{1}{n}\sum_i \mathrm{tr}\,\bar{A}_i^2(t)$$

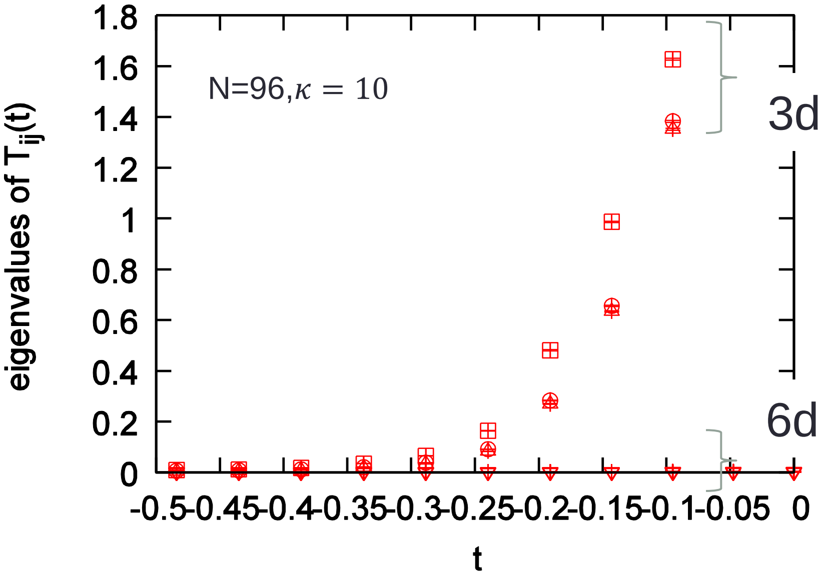
- the SSB occurs at $t = t_c$
- physical scale $R(t_c)$

(c.f.) pion mass in QCD

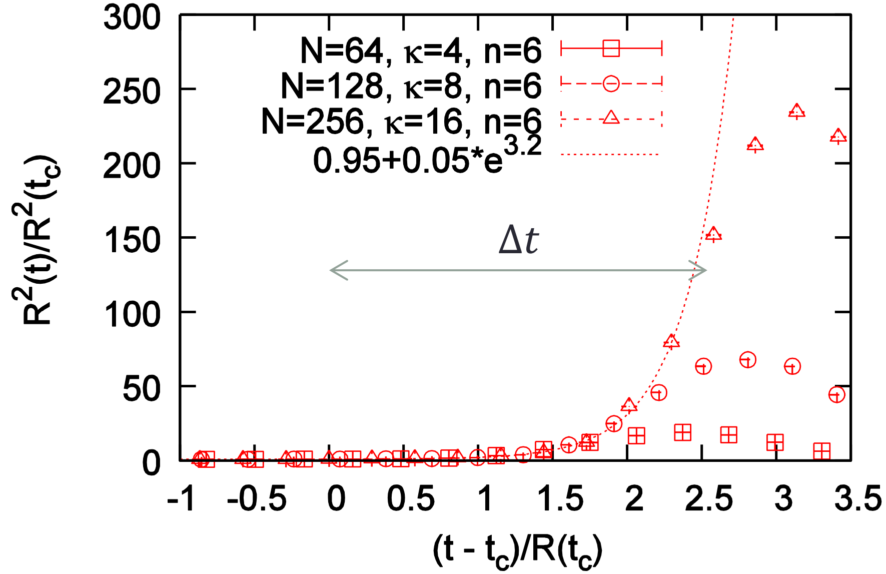
SSB of SO(9) and exponential expansion

- For $p = 1$ case

extent in each direction



the extent of space



- ✓ SSB from SO(9) to SO(3)
- ✓ Exponential expansion

Similar to the original Lorentzian IIB MM

- These behaviors were confirmed with much larger N. (Using the K computer)

previous works

- $N \leq 24$ for the original model
- $N \leq 64$ for the simplified model

$N \leq 64$
$\Delta t \sim 1.2$

⇨

$N \leq 256$
$\Delta t \sim 2.3$

Dependence on p in the IR cutoffs

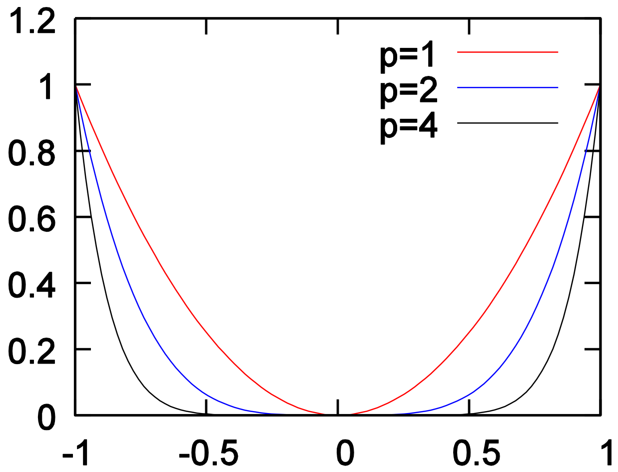
- IR cutoffs

$$\frac{1}{N}\mathrm{tr}\left[\left(A_0^2\right)^p\right] \le \kappa^p$$

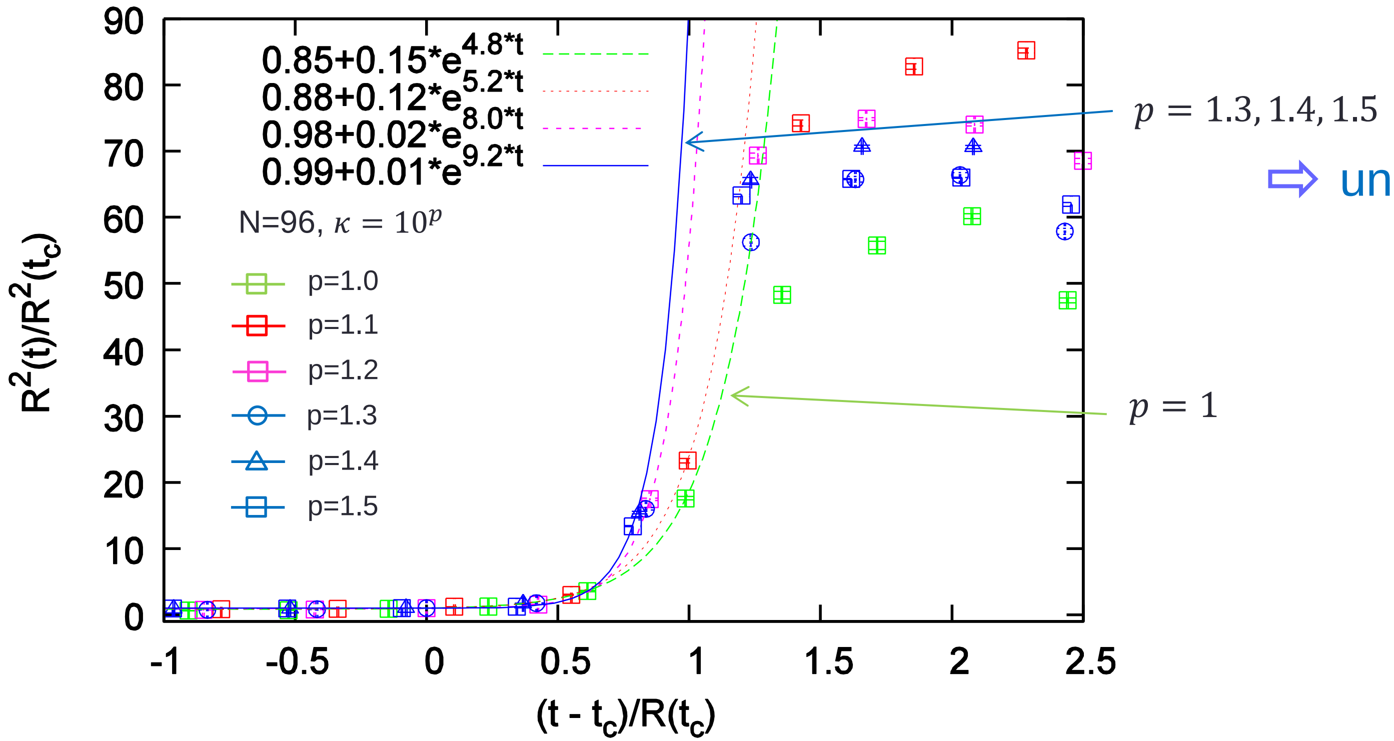
$$\frac{1}{N}\mathrm{tr}\left[\left(A_i^2\right)^p\right] \le 1$$

- larger p $\Rightarrow$ the IR cutoff affects only the boundary.

c.f.) $V(x) = x^{2p}$



Dependence on p in the IR cutoffs (cont'd)



⇨ universal !

This result suggests that
the IR cutoff effects disappear for sufficiently large p

(But not for $p = 1$!!)

Summary

- IIB matrix model
  - Non-perturbative formulation of superstring theory

- Lorentzian version
  - SSB from SO(9) to SO(3)
  - Exponential expansion

➢ However, IR cutoffs are needed to regularize the Lorentzian model.

$$\begin{cases} \frac{1}{N}\mathrm{tr}\left[\left(A_0^2\right)^p\right] \leq \kappa^p \\ \frac{1}{N}\mathrm{tr}\left[\left(A_i^2\right)^p\right] \leq 1 \end{cases}$$

- For $p = 1$, the exponential expansion was confirmed with much larger N.

- The results of $R(t)$ become universal for sufficiently large p, but not for $p = 1$.
  (qualitative behaviors for $p > 1$ are the same as $p = 1$.)

- Analogous studies for the original model, in progress. (Azuma, YI, Nishimura, Tsuchiya, work in progress)

➢ A more direct approach to see the effects of IR cutoffs: Schwinger-Dyson eq.

- The term arising from the IR cutoffs becomes smaller as N is increased for $p \gtrsim 1$