Transverse and longitudinal spectral functions of charmonia at finite temperature with maximum entropy method

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Motivation

Relativistic Heavy Ion Collisions

- Dynamical property of QGP medium
- Charmonium
  - J/ψ suppression [Matsui & Satz 1986]
  - Color Debye screening
    Bound state melts at upper T_c

Dynamical property of chrmonium at finite temperature with lattice QCD

- Charmonium at rest frame
  - dissosiation
  - transport coefficient
- Charmonium at moving frame
  - dispersion relation
  - decomposition into transverse and longitudinal components with vector channel
  - dissosiation

Spectral function and Lattice

Correlator and Spectral function

$$D(\tau, \vec{k}) = \int d^3x e^{i\vec{k}\cdot\vec{x}} \left\langle J_i(\tau, \vec{x}) J_i^\dagger(0, \vec{0}) \right\rangle$$

$$= \int_0^\infty K(\tau, \omega) A(\omega, \vec{k}) d\omega$$

$$K(\tau, \omega) = \frac{e^{-\tau\omega} + e^{-(\beta-\tau)\omega}}{1 - e^{-\beta\omega}}$$

$D(\tau, \vec{p})$: Imaginary time Correlator
Lattice QCD

$J_i(\tau, \vec{x})$: $\bar{c} i \gamma_i c \ (i = 1, 2, 3)$
Vector current

$A(\omega, \vec{k})$: Spectral function

ill-posed problem

- Imaginary time correlator → $O(10)$ data points
- Spectral function → continuous

Inverse Laplace transform is difficult

Maximum Entropy Method

Reconstructed Image A_{out}

$$A_{\text{out}} = \int d\alpha \int [dA]\, A(\omega) P(A,\alpha)$$

$$P(A,\alpha) = [\text{Likelihood function}](A) \times \alpha[\text{Prior probability}](A)/Z$$

from Bayes Theorem

1. Likelihood function $\Longleftarrow \chi^2$
   - $\exp(-L) = \exp\left[-\frac{1}{2}\sum_{i,j}\left(D(\tau_i) - D_A(\tau_i)\right) C_{ij}^{-1} \left(D(\tau_j) - D_A(\tau_j)\right)\right]$
2. Prior probability $\Longleftarrow$ Shannon-Jaynes entropy
   - $\exp(S) = \exp\left(\alpha \int_0^\infty \left[A(\omega) - m(\omega) - A(\omega)\log\left(\frac{A(\omega)}{m(\omega)}\right)\right] d\omega\right)$
   - default model $m(\omega)$ dependence → error analysis

Error estimate

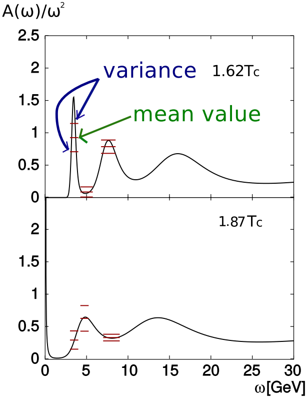
- Exprresion of MEM error

$$\langle A_{\text{out}}\rangle_I = \int d\alpha \int [dA] \int_I d\omega A(\omega) P(A,\alpha) \Big/ \int_I d\omega$$

$$\langle (\delta A_{\text{out}})^2\rangle_I = \int d\alpha \int [dA] \int_{I\times I} d\omega d\omega' \, \delta A(\omega)\delta A(\omega') P(A,\alpha) d\omega d\omega' \Big/ \int_{I\times I} d\omega d\omega'$$

$$\delta A(\omega) = A(\omega) - A_\alpha(\omega)$$

$$I = [\omega_1, \omega_2]$$



M.Asakawa and T.Hatsuda, PRL. 92, (2004).

Charmonium melt between $1.62T_c$ and $1.87T_c$

Vector channel: decomposition into transverse and longitudinal

Decomposition

$$A^{\mu\nu}(\omega, \vec{k}) = P_L^{\mu\nu} A_L(\omega, \vec{k}) + P_T^{\mu\nu} A_T(\omega, \vec{k})$$

- In vacuum
  - No rest frame
  - $A \propto g^{\mu\nu} - k^\mu k^\nu / k^2$
    $\rightarrow A_L = A_T$
- In medium
  - Rest frame ($u^\mu = (1, 0, 0, 0)$) exists.
    $\rightarrow A_L, A_T$ can depend on $k^0 = u \cdot k, |\vec{k}|^2 = \sqrt{(u \cdot k)^2 - k^2}$
  - Decomposition is work.
  - for $k = (\omega, p, 0, 0)$
    1. Transverse: $A_T(\omega, p) = \frac{A_2(\omega,p)+A_3(\omega,p)}{2}$
    2. Longitudinal: $A_L(\omega, p) = \frac{\omega^2-p^2}{\omega^2} A_1(\omega, p)$

Lattice setup

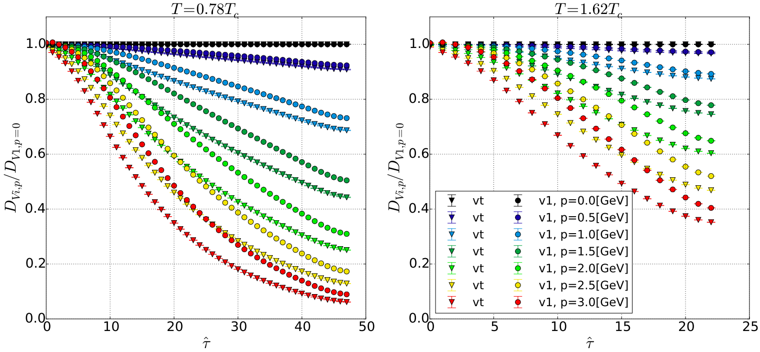
1. Gauge configuration
   - quenched QCD
   - Anisotropic lattice:
     $a_\tau = 9.75 \times 10^{-3}$[fm],
     $a_\sigma / a_\tau = 4.0$
   - $\beta = 7.0$
   - (Over relaxation$\times 4$
     + pseudo heatbath)$\times 500$
2. Correlator measurement
   - Wilson Fermion (iroiro++)
   - $\kappa_\sigma = 8.285 \times 10^{-2}$,
     $\gamma_F = 3.476$ [Asakawa, Hatsuda 2004]
   - $p = 0 \sim 3.0$[GeV]
   - Average of 8 source positions

setup

N_τ	T/T_c	N_σ	N_{conf}
40	1.87	64	500
46	1.62	64	500
50	1.49	64	500
96	0.78	64	500

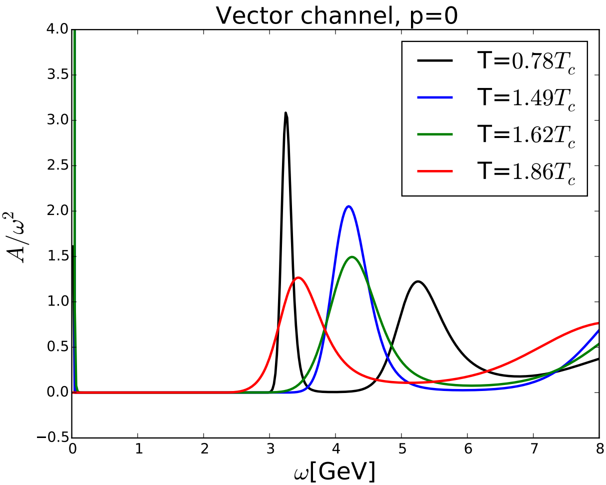
performed on Blue Gene/Q @ KEK

Correlator: Vector channel

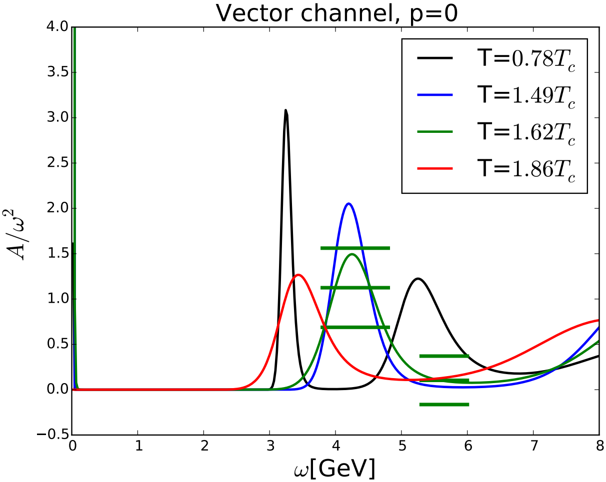


- Normalized by $D_{p=0}(\hat{\tau})$
- Correlators decompose at finite momentum
- $D_{Vt} = \frac{D_{V2}+D_{V3}}{2}$

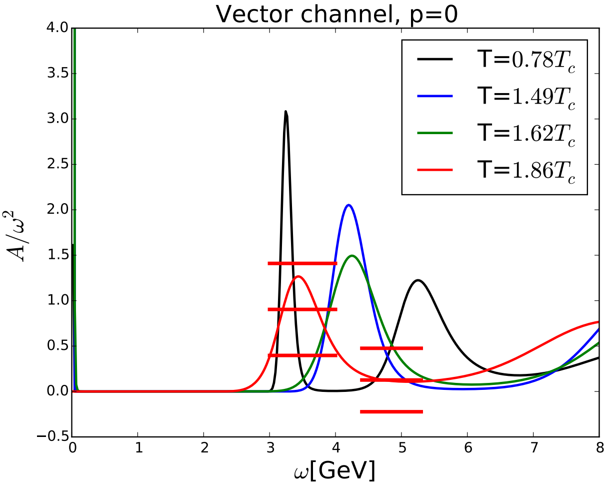
Spectral function with 0 momentum



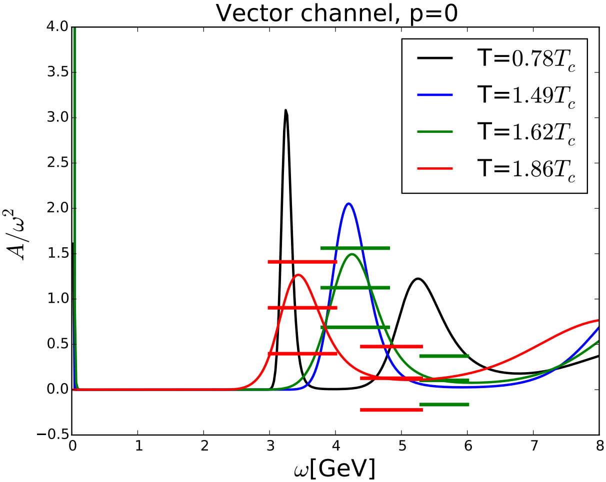
Spectral function with 0 momentum



Spectral function with 0 momentum

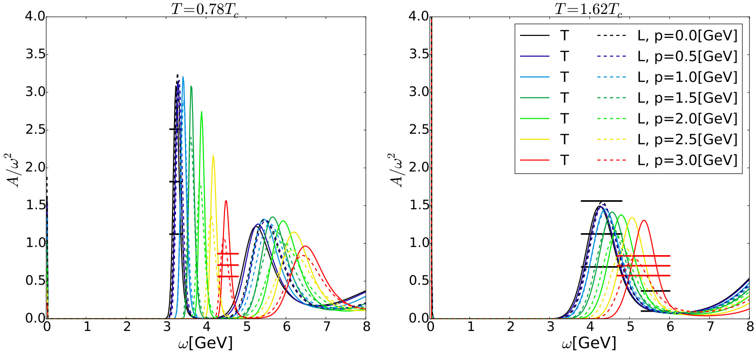


Spectral function with 0 momentum



- J/ψ suvives up to $T = 1.62T_c$

Spectral function with momentum

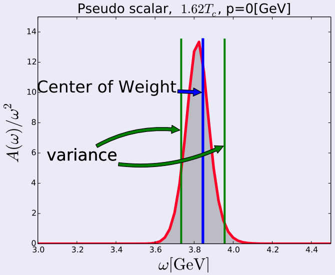


- If a peak is exists at $\boldsymbol{p} = \mathbf{0}$, the peak remains at nonzero momenta.

Estimation of a peak position and its error

- Define the peak position by a center of weight of the peak
- Error can be estimated in MEM.
- Check the dependence on the range $I = [\omega_1, \omega_2]$

Error estimate

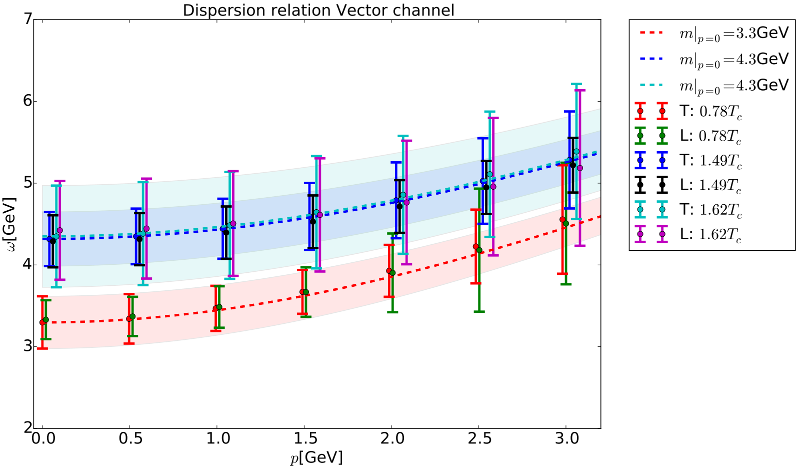


$$\text{(Center of weight)} = \frac{\langle \omega \frac{A(\omega)}{\omega^2} \rangle_I}{\langle \frac{A(\omega)}{\omega^2} \rangle_I}$$

$$\text{(variance)} = \frac{\sqrt{\langle \{\omega \delta (\frac{A(\omega)}{\omega^2})\}^2 \rangle_I}}{\langle \frac{A(\omega)}{\omega^2} \rangle_I}$$

- $\langle A(\omega)/\omega \rangle_I$ relates to a residue of the peak

Dispersion relation



- Consistent with the Lorentz-invariant dispersion relation

$$\omega = \sqrt{m|_{p=0}^2 + p^2}$$

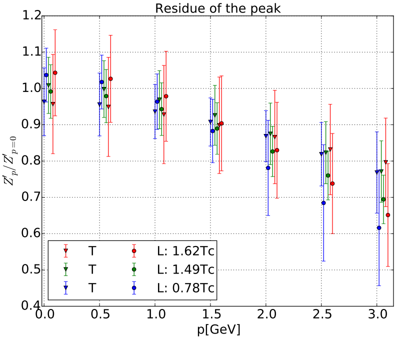
Residue of the bound state peaks

In vaccum,

$$A(\omega) = Z\delta(\omega^2 - m^2) = \frac{Z}{2\omega}\delta(\omega - m),$$

$Z' \equiv Z/2\omega$ at zero temperature should be constant .
We determined Z' in MEM by

$$Z' \sim \langle A(\omega)/2\omega \rangle_I$$



Summary

- We measure the current-current correlators with finite momenta at finite temperature which corresponds to J/ψ, and reconstruct the spectral functions.
- The bound state suvives up to $T = 1.62T_c$.
- When the bound state exists, the bound state remains with high momentum.
- The residue of the bound states with finite momenta have same behavior above and below T_c, althogh the correlators decompose.
- The form of dispersion relations at finite temperature is same with vaccum.
- The medium effect on the momentum dependence is not observed in the present statistics.