The chirally rotated Schrödinger functional at work

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33rd International Symposium on Lattice Field Theory,
14th of July 2015, Kobe, Japan.

Outline

Introduction:

The chirally rotated Schrödinger functional in $N = 2$ slides.

Topics:

- Renormalization of the non-singlet local currents for $N_\mathrm{f} = 2 + 1$ non-perturbatively $\mathrm{O}(a)$ improved Wilson-fermions.
- Towards the non-perturbative computation of the RG-running of a complete basis of $\Delta F = 2$ four-quark operators.

The chirally rotated Schrödinger functional

Given the isospin doublets ψ and $\overline{\psi}$ satisfying standard SF b.c.'s, we consider the **chiral rotation**,

$$\psi \equiv R\chi \equiv e^{i\frac{\pi}{2}\gamma_5\frac{\tau^3}{2}}\chi, \quad \overline{\psi} \equiv \overline{\chi}R \equiv \overline{\chi}e^{i\frac{\pi}{2}\gamma_5\frac{\tau^3}{2}}.$$

The fields χ and $\overline{\chi}$ satisfy the **chirally rotated SF** ($\chi\mathrm{SF}$) b.c.'s,

$$\begin{aligned} \widetilde{Q}_{+}\chi(x)|_{x_0=0} &= 0, \\ \overline{\chi}(x)\widetilde{Q}_{+}|_{x_0=0} &= 0, \end{aligned} \qquad \widetilde{Q}_{\pm} \equiv \tfrac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3),$$

which are **invariant** under the (rotated-)parity transformation,

$$P_5 : \chi(x) \to i\gamma_0\gamma_5\tau^3\,\chi(\tilde{x}), \quad \overline{\chi}(x) \to -i\overline{\chi}(\tilde{x})\,\gamma_0\gamma_5\tau^3, \qquad \tilde{x} = (x_0, -\mathbf{x}).$$

Since R is a non-anomalous symmetry transformation of massless QCD, in the **continuum** we expect the **universality relations**,

$$\boxed{\langle O[\psi,\overline{\psi}]\rangle_{\mathrm{SF}} = \langle O[R\chi,\overline{\chi}R]\rangle_{\chi\mathrm{SF}}.}$$

On the **lattice** with Wilson-fermions these relations hold among properly **renormalized** correlation functions, **up to discretization effects**!

The chirally rotated Schrödinger functional

Renormalization and O(a) improvement

(Sint '05, '10)

- For Wilson-fermions, the $\chi\mathrm{SF}$ b.c.'s are realized by **fine-tuning** a finite dim. 3 boundary counterterm (e.g. at $x_0 = 0$)

$$\overline{\chi}\widetilde{Q}_{-}\chi \xrightarrow{R} -i\overline{\psi}\gamma_5\tau^3 P_{-}\psi,$$

$\Rightarrow$ **breaks** parity and flavour symmetry: its coefficient, $z_f(g_0)$, can be fixed by imposing parity/flavour symmetry restoration.

- **Automatic (bulk) O(a) improvement:**
  - $\Rightarrow$ **NO** bulk O(a) effects for P_5-even obs. $(O \xrightarrow{P_5} +O)$,
  - $\Rightarrow$ bulk O(a) effects are located in P_5-odd obs. $(O \xrightarrow{P_5} -O)$.
- Full O(a) improvement needs in practice the tuning of a **couple** of O(a) boundary counterterms. PT seems to be good! (Sint, Vilaseca '12, '14)
- The set-up has been recently studied to 1-loop order in perturbation theory, and for $N_\mathrm{f} = 2$ dynamical fermions: (Sint, Vilaseca '14; Sint, MDB '14)
  - ✓ Automatic O(a) improvement and universality.
  - ✓ Competitive determinations of several finite renormalization constants.

Renormalization in the χSF

The correlation functions we need ...

(Leder, Sint '10)

SF:

$$f_X(x_0) = -\frac{1}{2}\langle\overline{\psi}_{f_1}(x)\Gamma_X\psi_{f_2}(x)\,\mathcal{O}_5^{f_2 f_1}\rangle,$$

$$k_Y(x_0) = -\frac{1}{6}\sum_k\langle\overline{\psi}_{f_1}(x)\Gamma_{Y_k}\psi_{f_2}(x)\,\mathcal{O}_k^{f_2 f_1}\rangle,$$

χSF:

$$g_X^{f_1 f_2}(x_0) = -\frac{1}{2}\langle\overline{\chi}_{f_1}(x)\Gamma_X\chi_{f_2}(x)\,\mathcal{Q}_5^{f_2 f_1}\rangle,$$

$$l_Y^{f_1 f_2}(x_0) = -\frac{1}{6}\sum_k\langle\overline{\chi}_{f_1}(x)\Gamma_{Y_k}\chi_{f_2}(x)\,\mathcal{Q}_k^{f_2 f_1}\rangle,$$

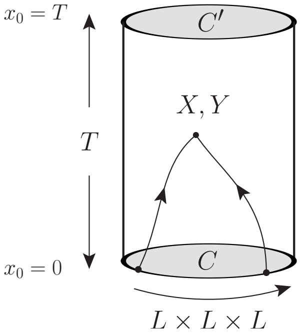
where,

$X = A_0, V_0, P, S,$

$Y_k = A_k, V_k, T_{0k}, \widetilde{T}_{0k}.$

and,

$f_1, f_2 = u, d, u'd',\ f_1 \neq f_2.$



Bilinears of boundary quark-fields

$$\mathcal{O}_5^{f_1 f_2}, \mathcal{O}_k^{f_1 f_2} \xrightarrow{R} \mathcal{Q}_5^{f_1 f_2}, \mathcal{Q}_k^{f_1 f_2}$$

Renormalization in the χSF

Renormalization conditions from universality relations

Universality relations:

We consider P_5-even correlation functions: $(\widetilde{V} \equiv$ conserved current$)$

$$(f_A)_R = (g_A^{uu'})_R = (-ig_V^{ud})_R \quad \Rightarrow \quad Z_A\, g_A^{uu'} = -ig_{\widetilde{V}}^{ud} + \mathrm{O}(a^2),$$

$$(k_V)_R = (l_V^{uu'})_R = (-il_A^{ud})_R \quad \Rightarrow \quad Z_A\, l_A^{ud} = il_{\widetilde{V}}^{uu'} + \mathrm{O}(a^2).$$

Renormalization conditions: (Leder, Sint '10; Sint, MDB '14)

The universality relations suggest to us simple renormalization conditions for the definition of Z_A, e.g.,

$$Z_A^g \equiv \left.\frac{-ig_{\widetilde{V}}^{ud}(x_0)}{g_A^{uu'}(x_0)}\right|_{x_0=\frac{T}{2}}, \qquad Z_A^l \equiv \left.\frac{il_{\widetilde{V}}^{uu'}(x_0)}{l_A^{ud}(x_0)}\right|_{x_0=\frac{T}{2}}.$$

N.B.: The Z_A's so obtained are fully **O(a) improved**:

- **NO** need for $\mathrm{O}(a)$ operator improvement i.e. $c_A(g_0)$ or $c_{\widetilde{V}}(g_0)$.
- $\mathrm{O}(a)$ boundary effects **cancel** out in the ratios.

Renormalization in the χSF

Universality relations:

We consider P_5-even correlation functions: $(\widetilde{V} \equiv$ conserved current$)$

$$(f_A)_R = (g_A^{uu'})_R = (-ig_V^{ud})_R \quad \Rightarrow \quad Z_V\, g_V^{ud} = g_{\widetilde{V}}^{ud} + \mathrm{O}(a^2),$$
$$(k_V)_R = (l_V^{uu'})_R = (-il_A^{ud})_R \quad \Rightarrow \quad Z_V\, l_V^{uu'} = l_{\widetilde{V}}^{uu'} + \mathrm{O}(a^2).$$

Renormalization conditions: (Leder, Sint '10; Sint, MDB '14)

The universality relations suggest to us simple renormalization conditions for the definition of Z_V, e.g.,

$$Z_V^g \equiv \left.\frac{g_{\widetilde{V}}^{ud}(x_0)}{g_V^{ud}(x_0)}\right|_{x_0=\frac{T}{2}}, \qquad Z_V^l \equiv \left.\frac{l_{\widetilde{V}}^{uu'}(x_0)}{l_V^{uu'}(x_0)}\right|_{x_0=\frac{T}{2}}.$$

N.B.: The Z_V's so obtained are fully **O(a) improved**:

- **NO** need for O(a) operator improvement i.e. $c_V(g_0)$ or $c_{\widetilde{V}}(g_0)$.
- O(a) boundary effects **cancel** out in the ratios.

Renormalization in the χSF

Renormalization conditions from universality relations

Universality relations:
We consider P_5-even correlation functions:

$$(f_P)_R = (ig_S^{uu'})_R = (g_P^{ud})_R \quad \Rightarrow \quad Z_S\, ig_S^{uu'} = Z_P g_P^{ud} + \mathrm{O}(a^2),$$
$$(k_T)_R = (il_{\widetilde{T}}^{uu'})_R = (l_T^{ud})_R \quad \Rightarrow \quad Z_{\widetilde{T}}\, il_{\widetilde{T}}^{uu'} = Z_T l_T^{ud} + \mathrm{O}(a^2).$$

Renormalization conditions: (Leder, Sint '10; Sint, MDB '14)

The universality relations suggest to us simple renormalization conditions for the definition of Z_P/Z_S, or $Z_T/Z_{\widetilde{T}}$, e.g.,

$$\frac{Z_P}{Z_S} \equiv \left.\frac{ig_S^{uu'}(x_0)}{g_P^{ud}(x_0)}\right|_{x_0=\frac{T}{2}}, \qquad \frac{Z_T}{Z_{\widetilde{T}}} \equiv \left.\frac{il_{\widetilde{T}}^{uu'}(x_0)}{l_T^{ud}(x_0)}\right|_{x_0=\frac{T}{2}}.$$

N.B.: The finite ratios so obtained are fully **O(a) improved**:

- **NO** need for O(a) operator improvement i.e. $c_T(g_0)$ or $c_{\widetilde{T}}(g_0)$.
- O(a) boundary effects **cancel** out in the ratios.

Adding strangeness . . .

Lattice action and other details

Question: How do we get $N_{\rm f} = 2 + 1$?

Answer: We consider 2 χSF + 1 SF Wilson-fermions. (Sint '10)

What's new?

- O(a) improved determinations now require an improved bulk action.
  $\Rightarrow$ This has to be considered in any case (s. below)!
- Twice as many O(a) boundary counterterm coefficients to be tuned.
  $\Rightarrow$ They all do not contribute to $Z_A, Z_V, \ldots$, at O(a), we are grand!
- If renormalization conditions are defined only in terms of χSF fields, **NO** need for O(a) operator improvement!

Lattice action (cf. CLS) (S. Schaefer's talk; Bruno, *et. al.* '14)

Fermionic action: $N_{\rm f}$=2+1 NPT O(a) improved Wilson-fermions.

Gauge action: Tree-level O(a^2) improved Lüscher-Weisz action.

χSF-specific: $T = L$; $C = C' = 0$; c_t @ 1-loop; d_s, $\tilde{c}_t$ @ tree-level.

Line of constant physics

Renormalization conditions in a fixed topological sector

LCP:

β	L^{trg}/a	L/a
3.40	8	6, 8, 10, 12
3.46	9.04	6, 8, 10, 12
3.55	10.76	8, 10, 12, 16
3.70	13.89	8, 10, 12, 16

β	t_0^{sym}/a^2	a (fm)
3.40	2.8468(61)	≈ 0.08
3.46	3.635(31)	≈ 0.07
3.55	5.150(23)	≈ 0.06
3.70	8.555(23)	≈ 0.05

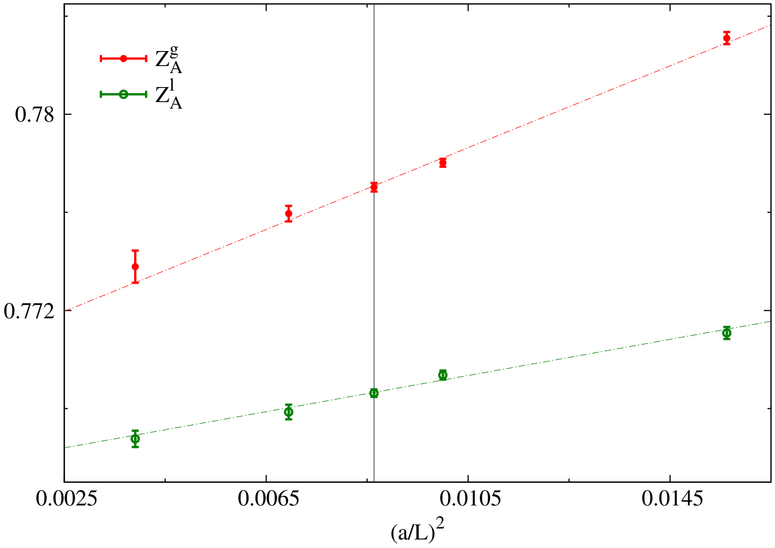
Renormalization conditions: (Fritzsch, Ramos, Stollenwerk '14)

$$\boldsymbol{m_{\mathrm{crit}}}\ :\ m_{\mathrm{PCAC}} = \left.\frac{\partial_0 g_{A,0}^{ud}(x_0)}{2 g_{P,0}^{ud}(x_0)}\right|_{x_0=\frac{T}{2}} \stackrel{!}{=} 0; \qquad \boldsymbol{z_f}\ :\ g_{A,0}^{ud}(x_0)\Big|_{x_0=\frac{T}{2}} \stackrel{!}{=} 0;$$

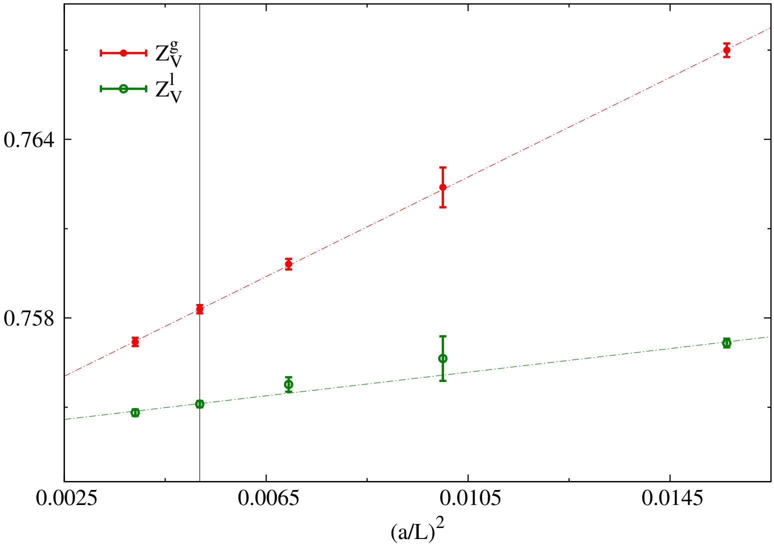
$$Z_A^g \equiv \left.\frac{-i g_{\widetilde{V},0}^{ud}(x_0)}{g_{A,0}^{uu'}(x_0)}\right|_{x_0=\frac{T}{2}}, \qquad \langle \mathcal{O}_0 \rangle \equiv \langle \mathcal{O}\cdot \delta_{Q_c,0}\rangle .$$

NOTE: Q_c is the topological charge defined through the gradient flow, evaluated at $c = \sqrt{8t}/L = 0.6$.

Determination of Z_A at $\beta = 3.55$

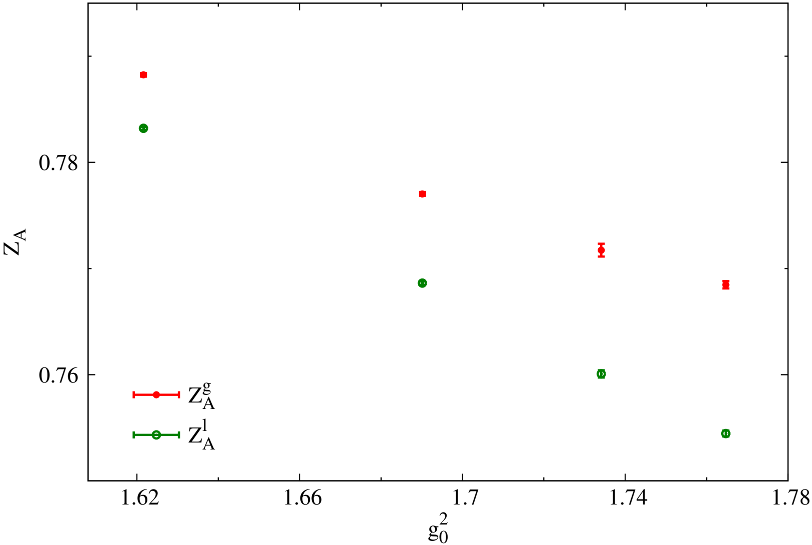


Determination of Z_V at $\beta = 3.70$



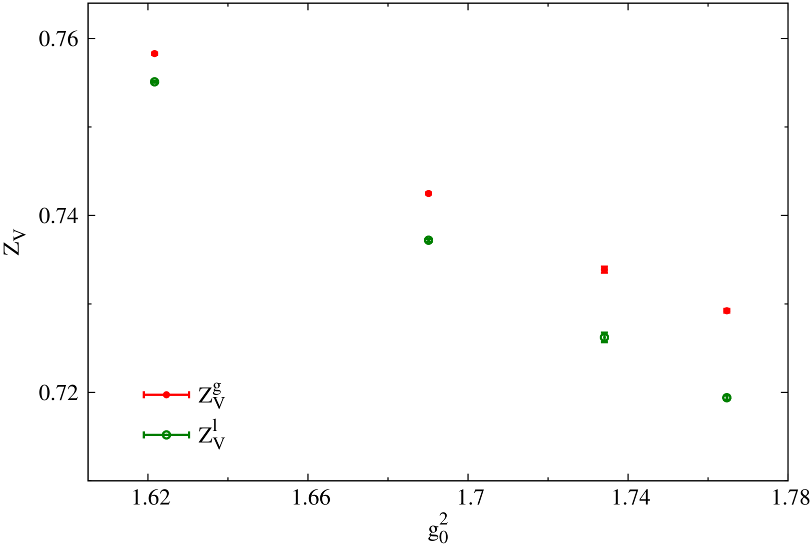
Renormalization constants

Comparison between two different definitions of Z_A as a function of g_0^2



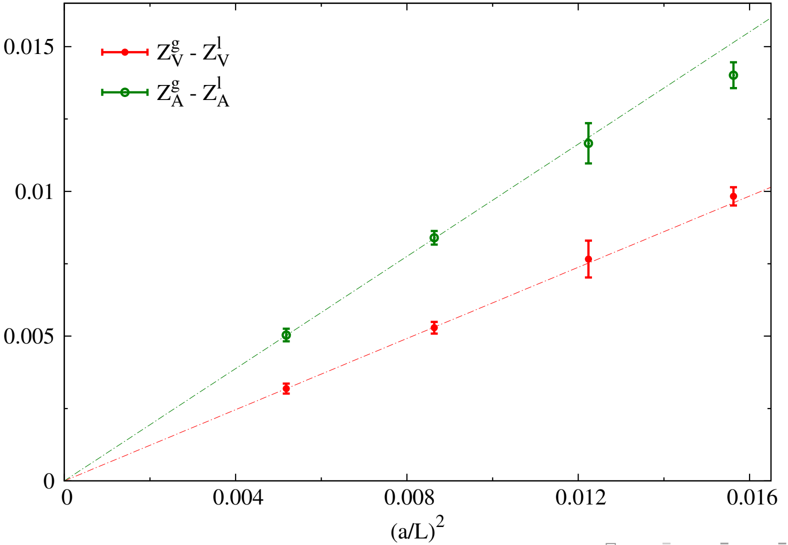
Renormalization constants

Comparison between two different definitions of Z_V as a function of g_0^2

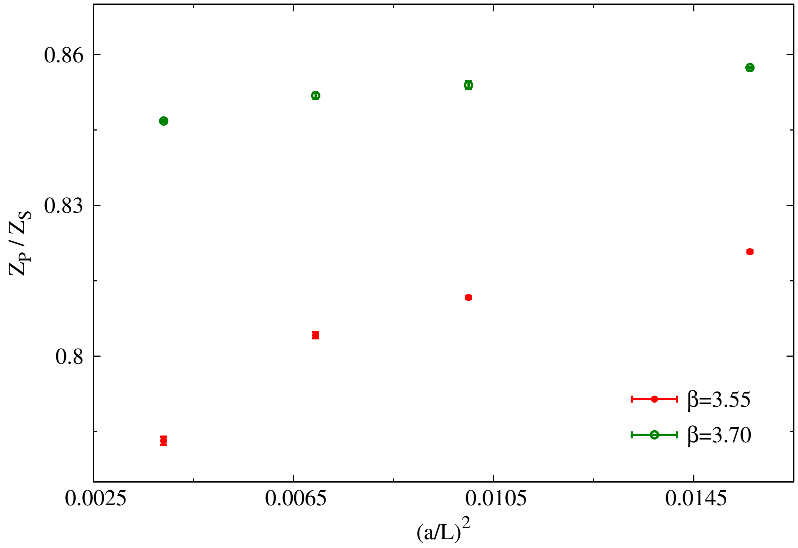


Test of universality

Continuum limit extrapolations of $Z_{V,A}$ differences



Results for Z_P/Z_S



Renormalization of $\Delta F = 2$ four quark-operators

First steps towards the non-perturbative running in the $N_\mathrm{f} = 2+1$ theory?

Start: The $\chi\mathrm{SF}$ allows for finite-volume, mass-independent renormalization schemes, compatible with auto. $\mathrm{O}(a)$ improvement.

$\Rightarrow$ **Step 1.** Identify valuable schemes within PT; both in terms of anomalous dimensions, as well as cutoff effects in the SSFs. (P. Vilaseca's talk)

$\Rightarrow$ **Step 2.** Feasibility study for a NPT determination of the running using $N_\mathrm{f} = 2$ NPT $\mathrm{O}(a)$ improved Wilson-fermions. (Della Morte *et. al.* '05)

Renormalization conditions:

$$\mathcal{G}_{i;\alpha} = \mathcal{N}_\alpha^{-1} \langle \mathcal{O}'_5 \, \mathcal{Q}_i \mathcal{O}_5 \rangle$$

$$\mathcal{L}_{i;\alpha} = \mathcal{N}_\alpha^{-1} \langle \mathcal{O}'_k \, \mathcal{Q}_i \mathcal{O}_k \rangle \qquad \begin{pmatrix} \mathcal{Z}_{22} & \mathcal{Z}_{23} \\ \mathcal{Z}_{32} & \mathcal{Z}_{33} \end{pmatrix} \begin{pmatrix} \mathcal{G}_{2;\alpha} & \mathcal{L}_{2;\beta} \\ \mathcal{G}_{3;\alpha} & \mathcal{L}_{3;\beta} \end{pmatrix} = \begin{pmatrix} \mathcal{G}_{2;\alpha} & \mathcal{L}_{2;\beta} \\ \mathcal{G}_{3;\alpha} & \mathcal{L}_{3;\beta} \end{pmatrix}_{g_0=0} .$$

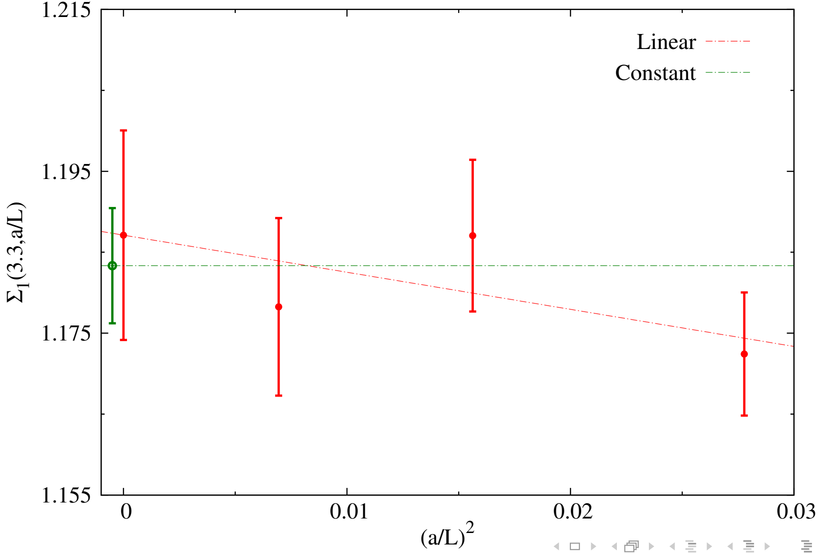
$$\mathcal{N}_\alpha = g_1^{\alpha_1} \, l_1^{\alpha_2} \, g_{\widetilde{V}}^{\alpha_2} \, l_{\widetilde{V}}^{\alpha_3}$$

Step-scaling function (SSF):

$$\sigma(u) = \lim_{a\to 0} \Sigma(u, a/L), \quad \Sigma(u, a/L) = \mathcal{Z}(g_0, 2L/a)\mathcal{Z}^{-1}(g_0, L/a)\big|_{u=\bar{g}^2(L)} .$$

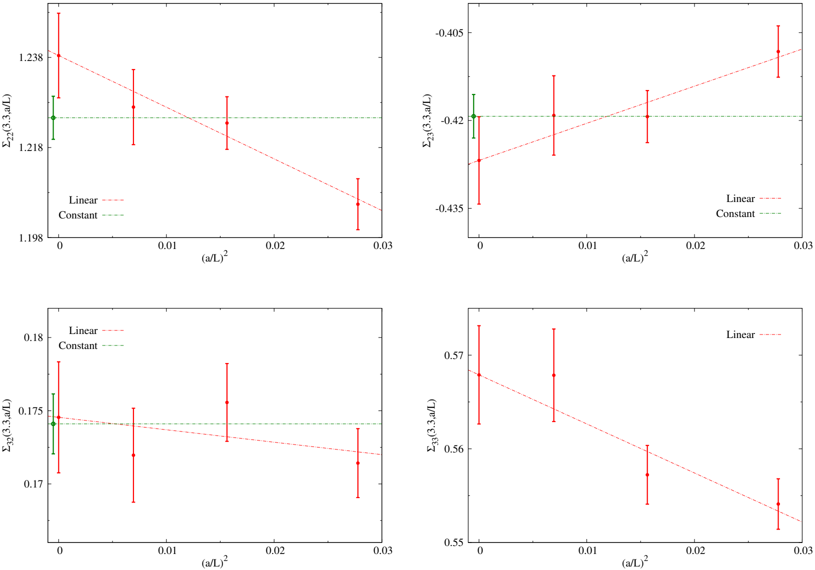
Renormalization of $\mathcal{Q}_1^+$

Continuum limit extrapolation for the lattice step-scaling function for $\bar{g}^2(L) = 3.3$



Renormalization of $\mathcal{Q}_2^-$, and $\mathcal{Q}_3^-$

Continuum limit extrapolations for the lattice step-scaling functions for $\bar{g}^2(L) = 3.3$



Outlook

Outlook:

- Little extra work is needed to finalize the results for Z_V, and Z_A.
- The ensembles generated can be used for several other purposes: Z_P/Z_S, low-energy matchings, ...
- Still a lot to do and to understand for the non-perturbative running of four-quark operators:

  $\Rightarrow$ crucial to understand whether perturbation theory at NLO provides a good picture of the high-energy regime we can reach.

  $\Rightarrow$ make use of several different schemes to improve the determination of the RGI-operators.

  ... while we are thinking computers are running!
- Investigate other applications of the $\chi\mathrm{SF}$, as for example the determination of some improvement coefficients.


Thanks