D meson semileptonic decays from lattice QCD with chiral fermions

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for JLQCD Collaboration

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Plan of talk

- Introduction
- Our recipe of Form factors
- Form factors
- Form factors at $q^2=0$
- Summary

Introduction

CKM matrix

- Precise determination of the CKM matrix elements provides us a test of the SM.
- The elements Vcd and Vcs can be obtained from the D->Pi and D->K process, respectively.

$$\frac{d\Gamma(D \to \pi)}{dq^2} \propto |V_{cd}|^2 |f_+^{D\to\pi}(q^2)|^2$$

by experiments (→ $\frac{d\Gamma(D \to \pi)}{dq^2}$)

by lattice QCD (→ $f_+^{D\to\pi}(q^2)$)

We calculate $f_+(q^2)$ from lattice simulation with chiral fermions.

Introduction

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by experiments

by lattice QCD

We calculate $f_+(q^2)$ from lattice simulation with chiral fermions.

Today's goal

Lattice Set up

- 2+1 Möbius Domain Wall fermion
- Symanzik action

β	a	Volume	am	am
4.17	2.453(4)	32	0.030	0.070
				0.012
				0.019
			0.040	0.0035
				0.070
				0.012
				0.019
		48	0.040	0.0035
4.35	3.610(9)	48	0.018	0.0042
				0.0080
				0.0120
			0.025	0.0042
				0.0080
				0.0120
4.47	4.496(9)	64	0.015	0.0030

Introduction

Lattice Set up

- 2+1 Möbius Domain Wall fermion
- Symanzik action

Today's talk

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Plan of talk

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Our recipe of form factors

Form factors from Matrix elements

$$\langle \pi(p_\pi)|V_\mu|D(p_D)\rangle$$
$$= f_+^{D\to\pi}(q^2)\left[(p_D+p_\pi)_\mu - \frac{m_D^2-m_\pi^2}{q^2}q_\mu\right] + f_0^{D\to\pi}(q^2)\frac{m_D^2-m_\pi^2}{q^2}q_\mu$$

$$q = p_D - p_\pi$$

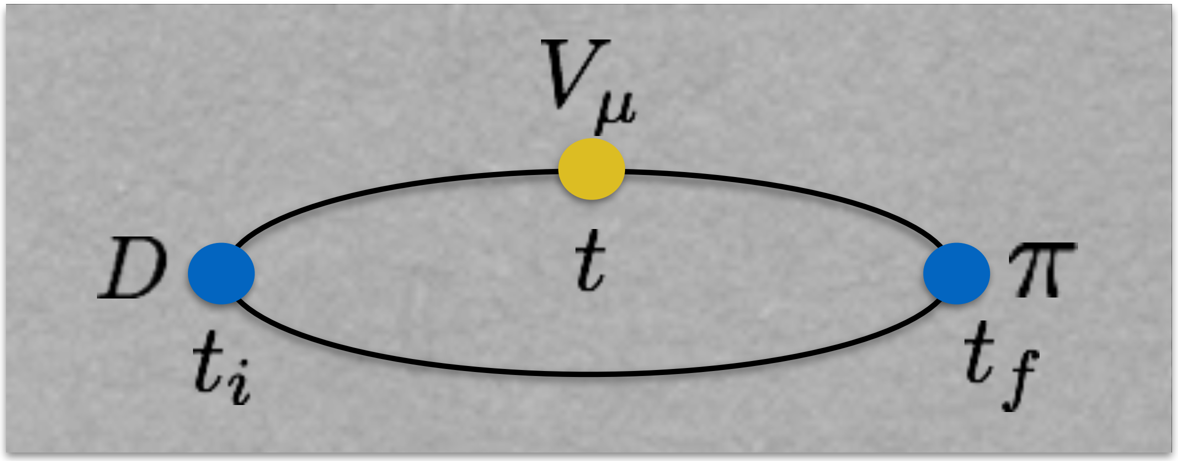
$$f_+(q^2) = \frac{(E_D - E_\pi)\langle \pi(p_\pi)|V_k|D(p_D)\rangle - (p_D - p_\pi)^k \langle \pi(p_\pi)|V_0|D(p_D)\rangle}{2E_D\, p_\pi^k - 2E_\pi\, p_D^k}$$

$$(k = 1, 2, 3)$$

Form factors can be extracted from the matrix elements.

Our recipe of form factors

Correlation functions



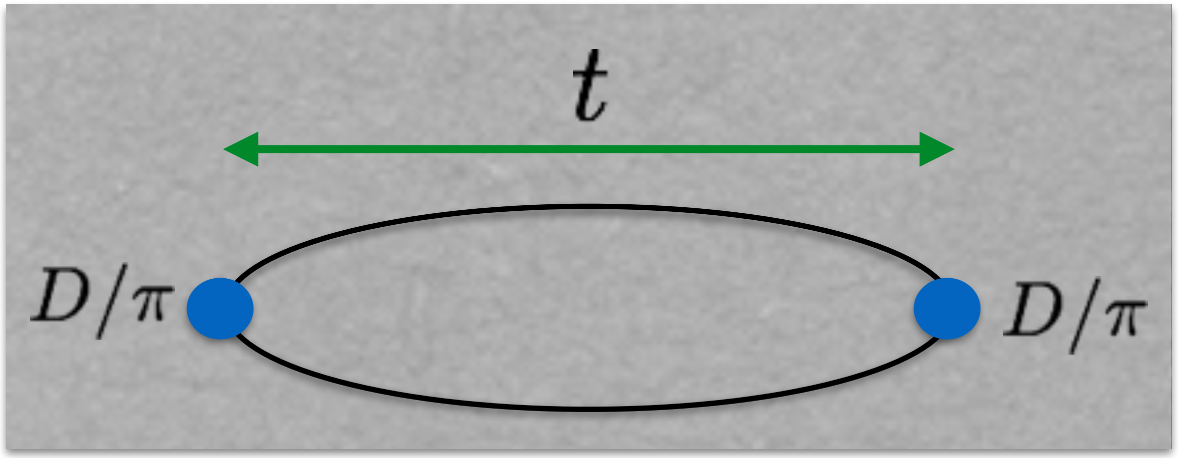
$$C_{3pt}^{DV_\mu\pi}(t_i, t, t_f : \mathbf{p}_D, \mathbf{p}_\pi)$$

$$= \frac{Z_D(p_D)^* Z_\pi(p_\pi)}{4E_D E_\pi} e^{-E_D(t-t_i)} e^{-E_\pi(t_f-t)} \langle \pi(p_\pi)|V_\mu|D(p_D)\rangle$$

What we want

$$C_{2pt}^{D/\pi}(t : \mathbf{p}) = \frac{|Z(p)_{D/\pi}|^2}{2E(\mathbf{p})} e^{-E(\mathbf{p})t}$$

$$Z_{D/\pi}(p) \equiv \langle P(0)|P(p)\rangle$$



Our recipe of form factors

Matrix elements from a ratio

$$A_{3\mathrm{pt}}^{D\pi\,\mu}(\mathbf{p}_i,\mathbf{p}_f)=\frac{C_{3\mathrm{pt}}^{D\pi}(t_i,t,t_f;\mathbf{p}_i,\mathbf{p}_f)}{\exp\{-E_D(t-t_i)-E_\pi(t_f-t)\}}\qquad B_{2\mathrm{pt}}^{D/\pi}(\mathbf{p})=\frac{C_{2\mathrm{pt}}^{D/\pi}(t;\mathbf{p})}{\exp\{-E_{D/\pi}t\}}$$

$$R_{D\,\pi}^{\mu}(\mathbf{p}_i,\mathbf{p}_f)\equiv\mathcal{N}\sqrt{\frac{\left[A_{3\mathrm{pt}}^{D\pi\,\mu}(\mathbf{p}_i,\mathbf{p}_f)\right]^2}{B_{2\mathrm{pt}}^{D}(\mathbf{p}_i)B_{2\mathrm{pt}}^{\pi}(\mathbf{p}_f)}}=\langle\pi(p_\pi)|V_\mu|D(p_D)\rangle$$

input parameter

Matrix elements can be extracted from the ratio of factors A and B.

Our recipe of form factors

Matrix elements from a ratio

$$A_{3\text{pt}}^{D\pi\,\mu}(\mathbf{p}_i, \mathbf{p}_f) = \frac{C_{3\text{pt}}^{D\pi}(t_i, t, t_f; \mathbf{p}_i, \mathbf{p}_f)}{\boxed{\exp\{-E_D(t-t_i) - E_\pi(t_f - t)\}}} \qquad B_{2\text{pt}}^{D/\pi}(\mathbf{p}) = \frac{C_{2\text{pt}}^{D/\pi}(t; \mathbf{p})}{\boxed{\exp\{-E_{D/\pi} t\}}}$$

$$R_{D\,\pi}^{\mu}(\mathbf{p}_i, \mathbf{p}_f) \equiv \boxed{\mathcal{N}} \sqrt{\frac{\left[A_{3\text{pt}}^{D\pi\,\mu}(\mathbf{p}_i, \mathbf{p}_f)\right]^2}{B_{2\text{pt}}^{D}(\mathbf{p}_i) B_{2\text{pt}}^{\pi}(\mathbf{p}_f)}} = \langle \pi(p_\pi) | V_\mu | D(p_D) \rangle$$

input parameter

Matrix elements can be extracted from the ratio of factors A and B.

Our recipe of form factors

Matrix elements from a ratio

$$\exp\{-E_D(t-t_i)-E_\pi(t_f-t)\}$$

$$\exp\{-E_{D/\pi}t\}$$

$$\mathcal{N}$$

Our recipe of form factors

Matrix elements from a ratio

$$\exp\{-E_D(t-t_i) - E_\pi(t_f - t)\}$$

$$\exp\{-E_{D/\pi} t\}$$

$$\mathcal{N} = 2Z_V \sqrt{E_D(\mathbf{p}_i) E_\pi(\mathbf{p}_f)}$$

Our recipe of form factors

Matrix elements from a ratio

$$\exp\{-E_D(t-t_i)-E_\pi(t_f-t)\}$$

$$\exp\{-E_{D/\pi}t\}$$

the dispersion relation is used

$$\mathcal{N} = 2Z_V\sqrt{E_D(\mathbf{p}_i)E_\pi(\mathbf{p}_f)}$$

Tomii's talk on Wed.
in Standard Model Parameters
and Renormalization session

Our recipe of form factors

①

$$A_{3\text{pt}}^{D\pi\,\mu}(\mathbf{p}_i, \mathbf{p}_f) = \frac{C_{3\text{pt}}^{D\pi}(t_i, t, t_f; \mathbf{p}_i, \mathbf{p}_f)}{\exp\{-E_D(t - t_i) - E_\pi(t_f - t)\}} \qquad B_{2\text{pt}}^{D/\pi}(\mathbf{p}) = \frac{C_{2\text{pt}}^{D/\pi}(t; \mathbf{p})}{\exp\{-E_{D/\pi} t\}}$$

②

$$R_{D\,\pi}^{\mu}(\mathbf{p}_i, \mathbf{p}_f) \equiv \mathcal{N} \sqrt{\frac{\left[A_{3\text{pt}}^{D\pi\,\mu}(\mathbf{p}_i, \mathbf{p}_f)\right]^2}{B_{2\text{pt}}^{D}(\mathbf{p}_i) B_{2\text{pt}}^{\pi}(\mathbf{p}_f)}}$$

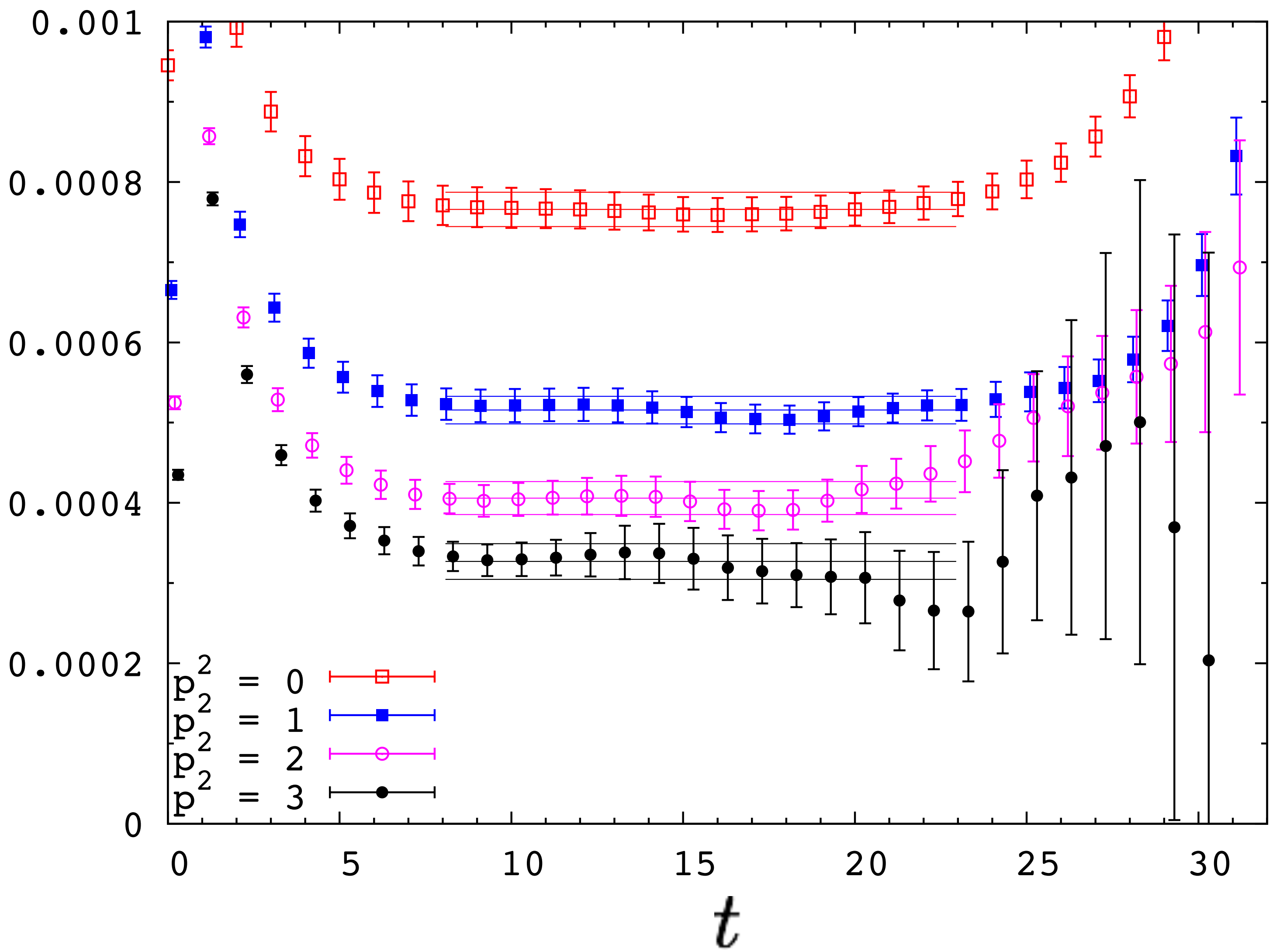
③

$$f_+(q^2) = \frac{(E_D - E_\pi)\, R_{D\pi}^{k}(\mathbf{p}_i, \mathbf{p}_f) - (p_D - p_\pi)^k\, R_{D\pi}^{0}(\mathbf{p}_i, \mathbf{p}_f)}{2E_D p_\pi^k - 2E_\pi p_D^k}$$

Our recipe of form factors

Factor B of Pion

$$B^{\pi}_{2\mathrm{pt}}(\mathbf{p}) = \frac{C^{\pi}_{2\mathrm{pt}}(t;\mathbf{p})}{\exp\{-E_\pi t\}} \qquad \beta = 4.17,\ M_\pi = 500[\mathrm{MeV}]$$



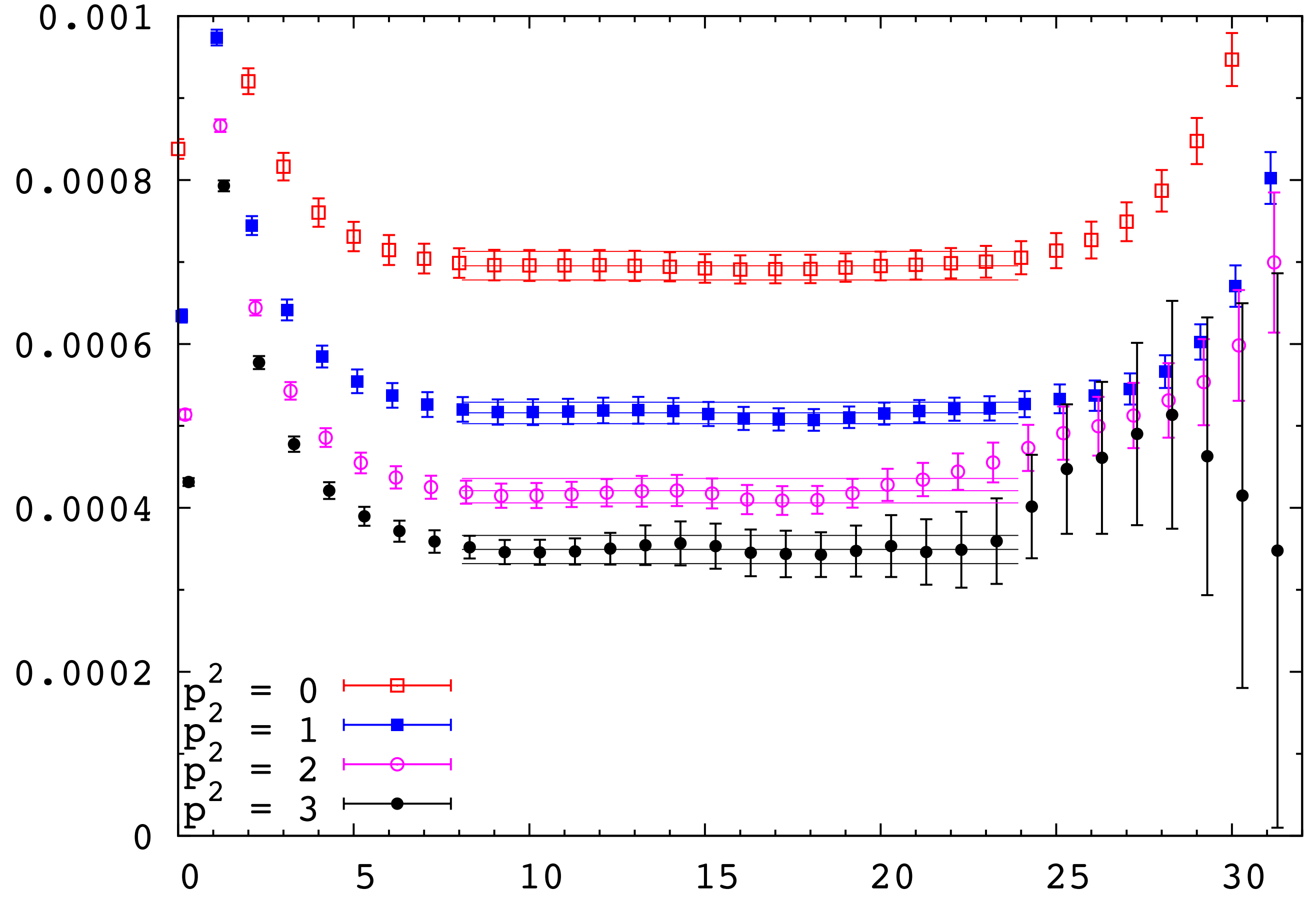
plateau gives

$$\frac{|Z_\pi(p)|^2}{2E(\mathbf{p})}$$

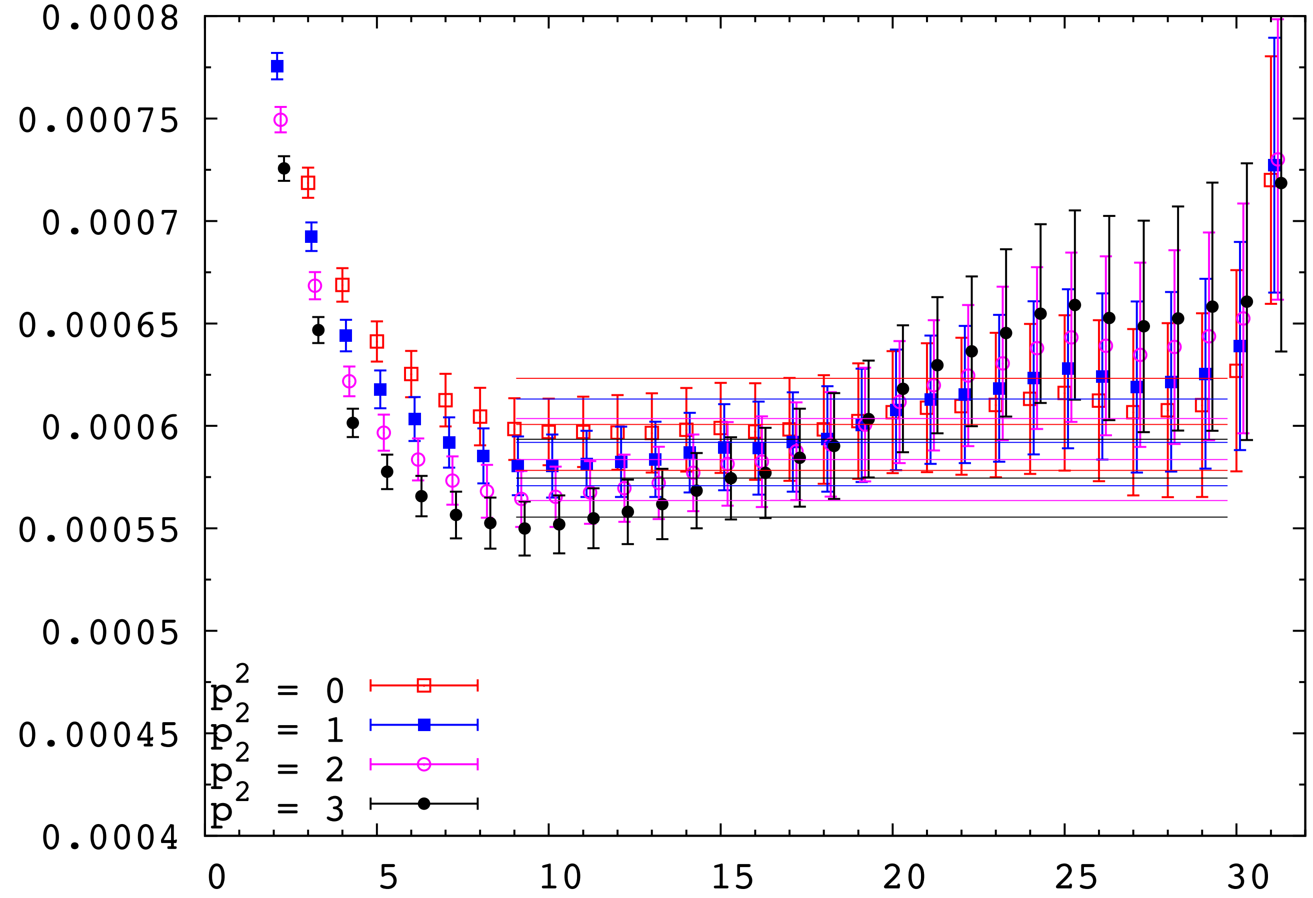
Our recipe of form factors

Factor B of Kaon and D meson

Kaon



D meson

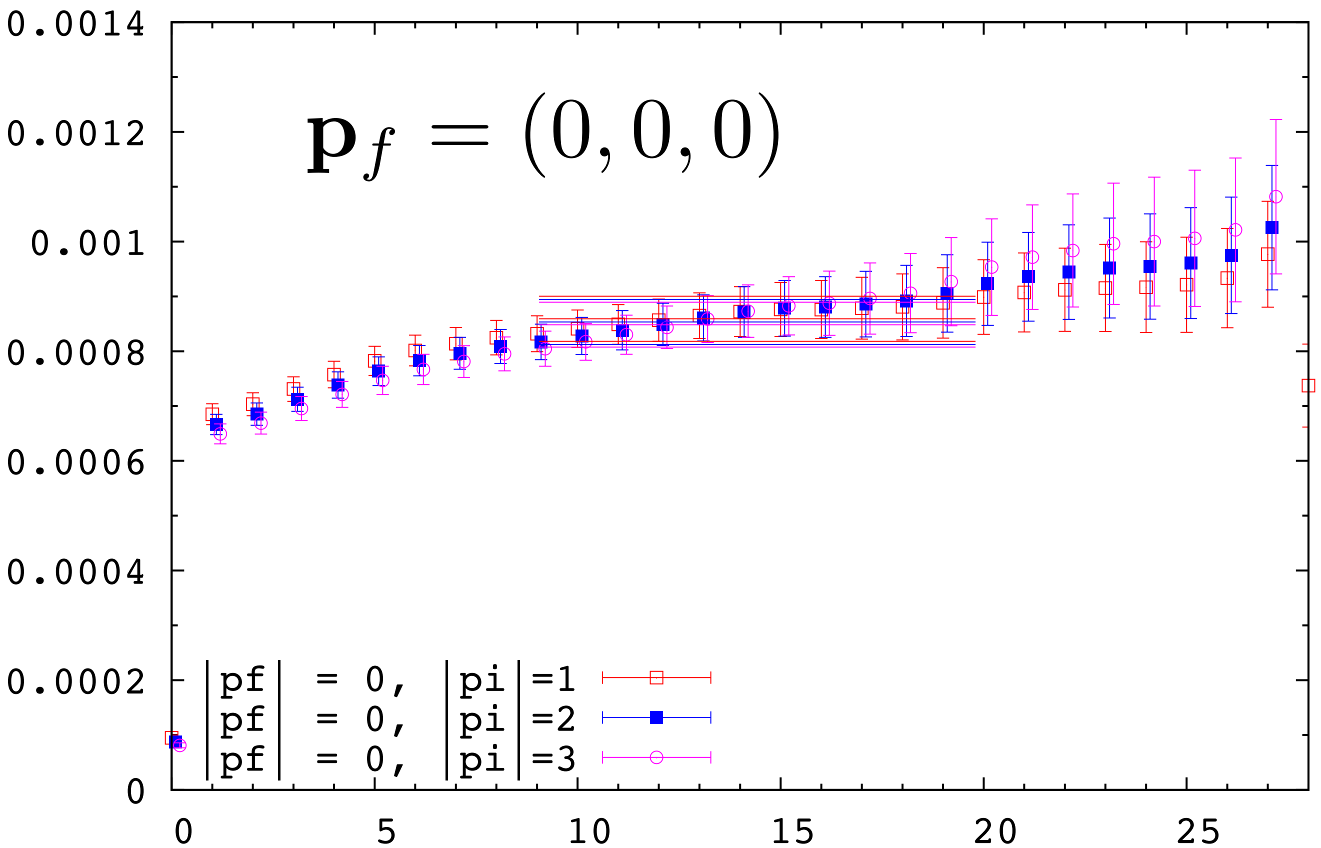


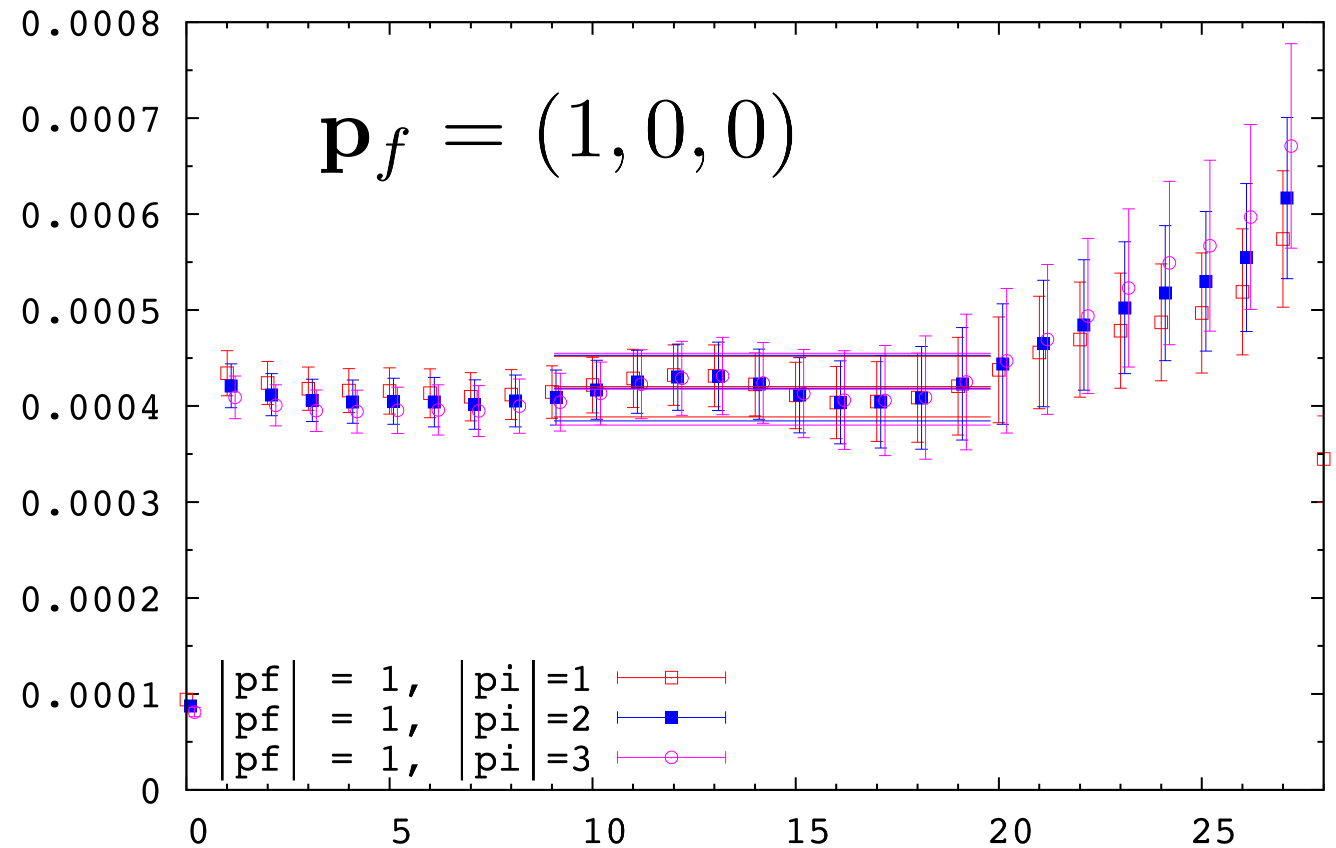
Our recipe of form factors

Factor A of D->Pi

$$A_{3\mathrm{pt}}^{D\pi\,\mu}(\mathbf{p}_i,\mathbf{p}_f)=\frac{C_{3\mathrm{pt}}^{D\pi}(t_i,t,t_f;\mathbf{p}_i,\mathbf{p}_f)}{\exp\{-E_D(t-t_i)-E_\pi(t_f-t)\}}$$

$\beta = 4.17,\ M_\pi = 500[\mathrm{MeV}]$





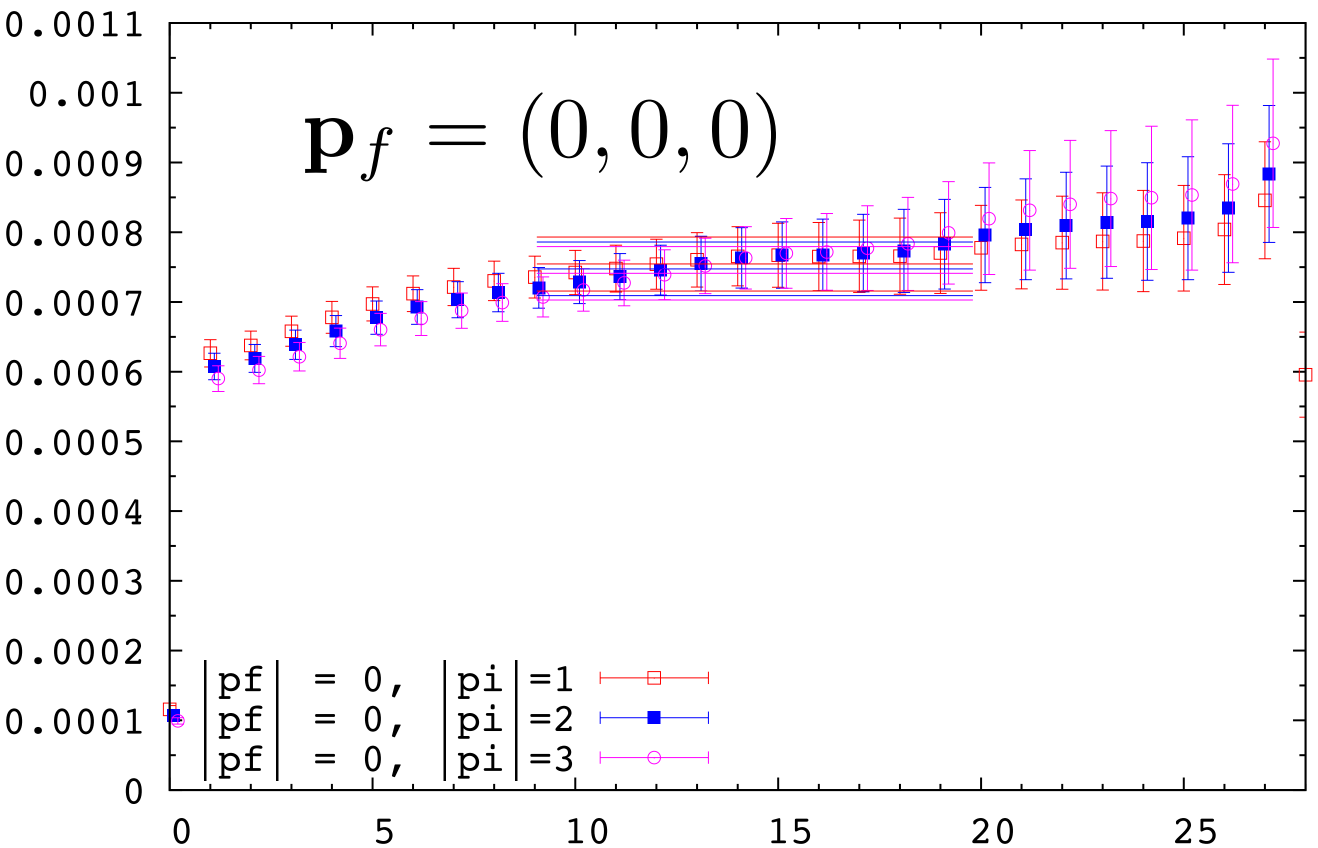
plateau → gives

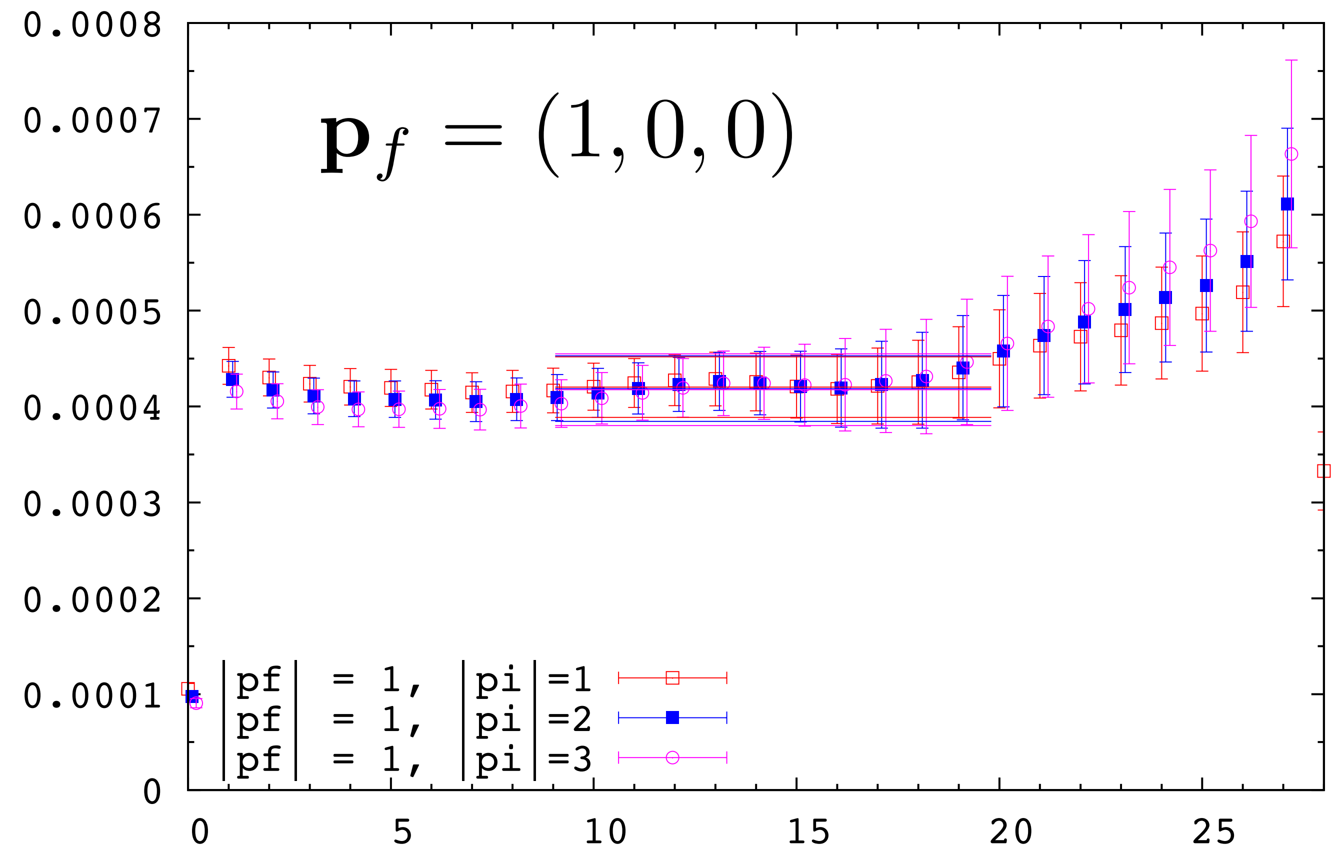
$$\frac{Z_D(p_D)^* Z_\pi(p_\pi)}{4E_D E_\pi}\langle\pi(p_\pi)|V_\mu|D(p_D)\rangle$$

Our recipe of form factors

Factor A of D->K

$$A_{3\mathrm{pt}}^{DK\,\mu}(\mathbf{p}_i,\mathbf{p}_f) = \frac{C_{3\mathrm{pt}}^{DK}(t_i,t,t_f;\mathbf{p}_i,\mathbf{p}_f)}{\exp\{-E_D(t-t_i)-E_K(t_f-t)\}} \quad \beta = 4.17,\ M_\pi = 500[\mathrm{MeV}]$$





plateau → gives

$$\frac{Z_D(p_D)^* Z_K(p_K)}{4E_D E_K}\langle K(p_K)|V_\mu|D(p_D)\rangle$$

Our recipe of form factors

①

$$A^{D\pi\,\mu}_{3\mathrm{pt}}(\mathbf{p}_i,\mathbf{p}_f) = \frac{C^{D\pi}_{3\mathrm{pt}}(t_i,t,t_f;\mathbf{p}_i,\mathbf{p}_f)}{\exp\{-E_D(t-t_i)-E_\pi(t_f-t)\}} \qquad B^{D/\pi}_{2\mathrm{pt}}(\mathbf{p}) = \frac{C^{D/\pi}_{2\mathrm{pt}}(t;\mathbf{p})}{\exp\{-E_{D/\pi}t\}}$$

②

$$R^{\mu}_{D\,\pi}(\mathbf{p}_i,\mathbf{p}_f) \equiv \mathcal{N}\sqrt{\frac{\left[A^{D\pi\,\mu}_{3\mathrm{pt}}(\mathbf{p}_i,\mathbf{p}_f)\right]^2}{B^{D}_{2\mathrm{pt}}(\mathbf{p}_i)B^{\pi}_{2\mathrm{pt}}(\mathbf{p}_f)}}$$

③

$$f_+(q^2) = \frac{(E_D-E_\pi)\,R^k_{D\pi}(\mathbf{p}_i,\mathbf{p}_f) - (p_D-p_\pi)^k\,R^0_{D\pi}(\mathbf{p}_i,\mathbf{p}_f)}{2E_D p^k_\pi - 2E_\pi p^k_D}$$

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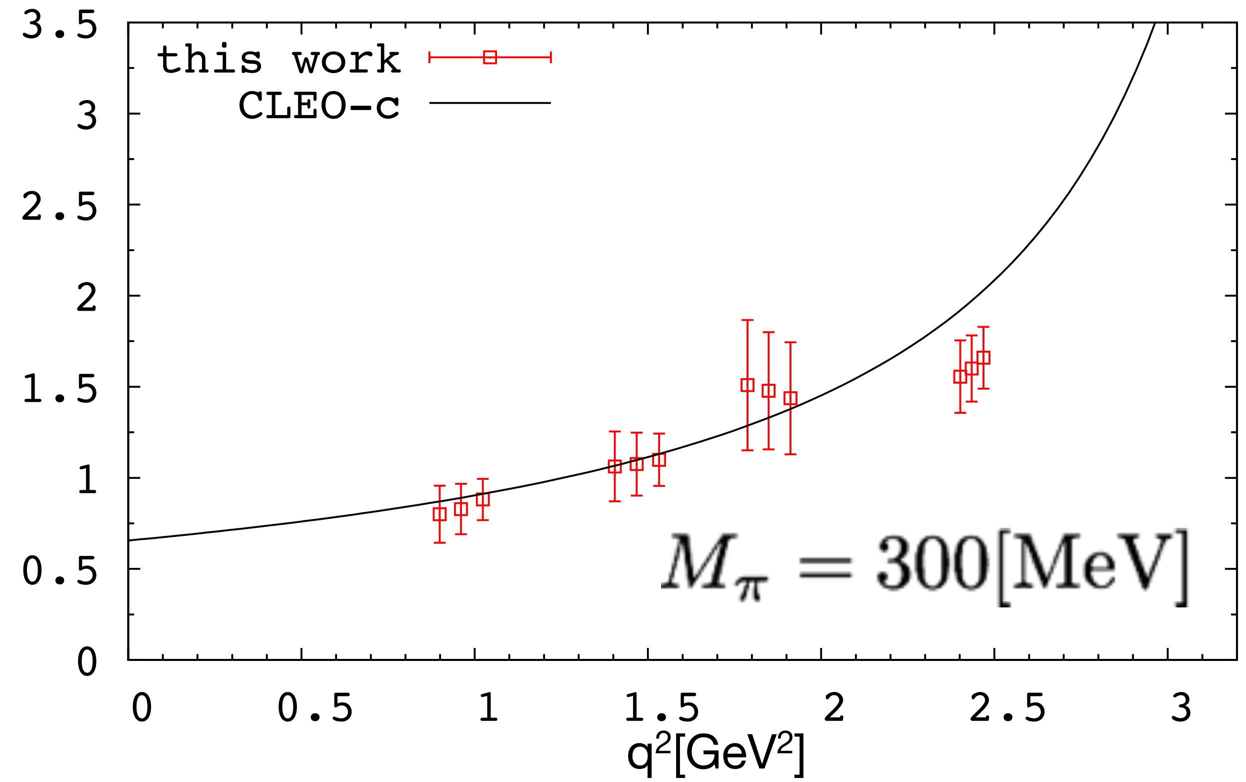
Form factors

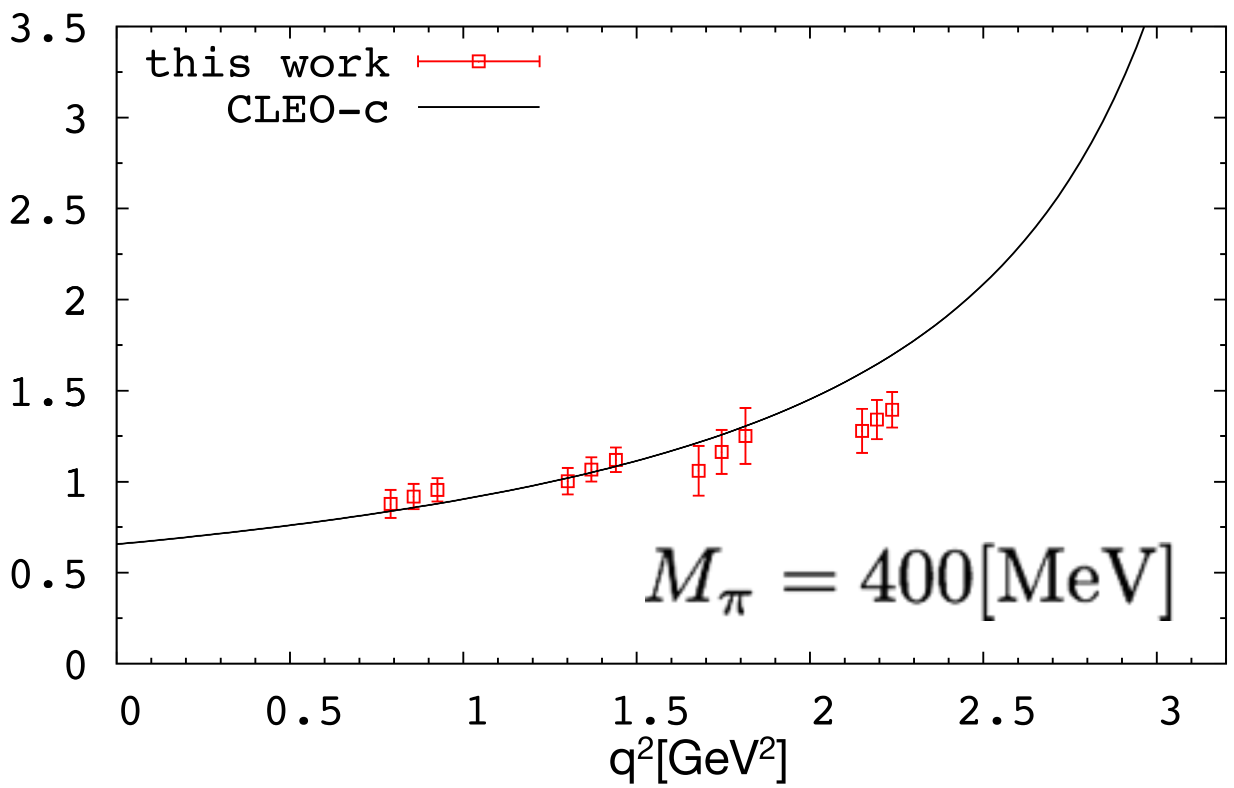
Compare with CLEO-c

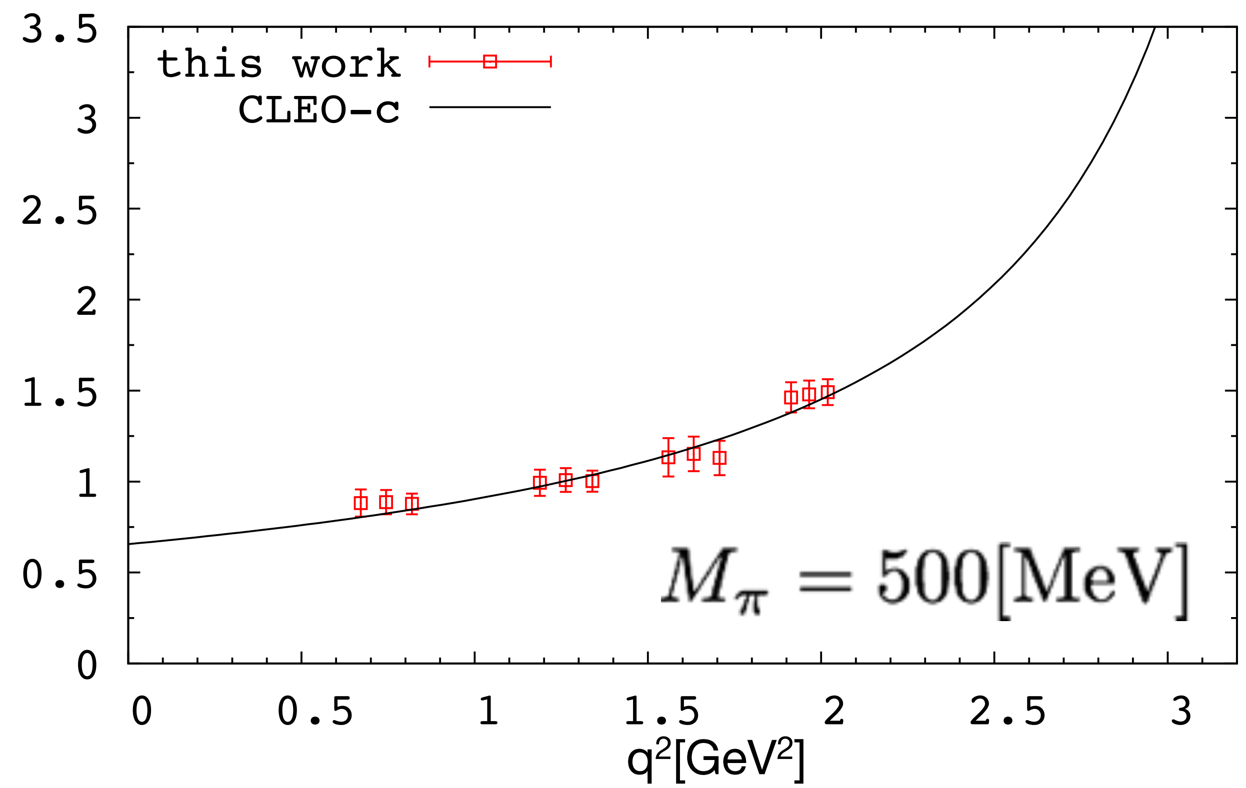
[arXiv:0906.2983]

- D -> Pi
- $f_+(q^2)$ vs q^2[GeV^2]

solid curve
= single pole fit by CLEO-c







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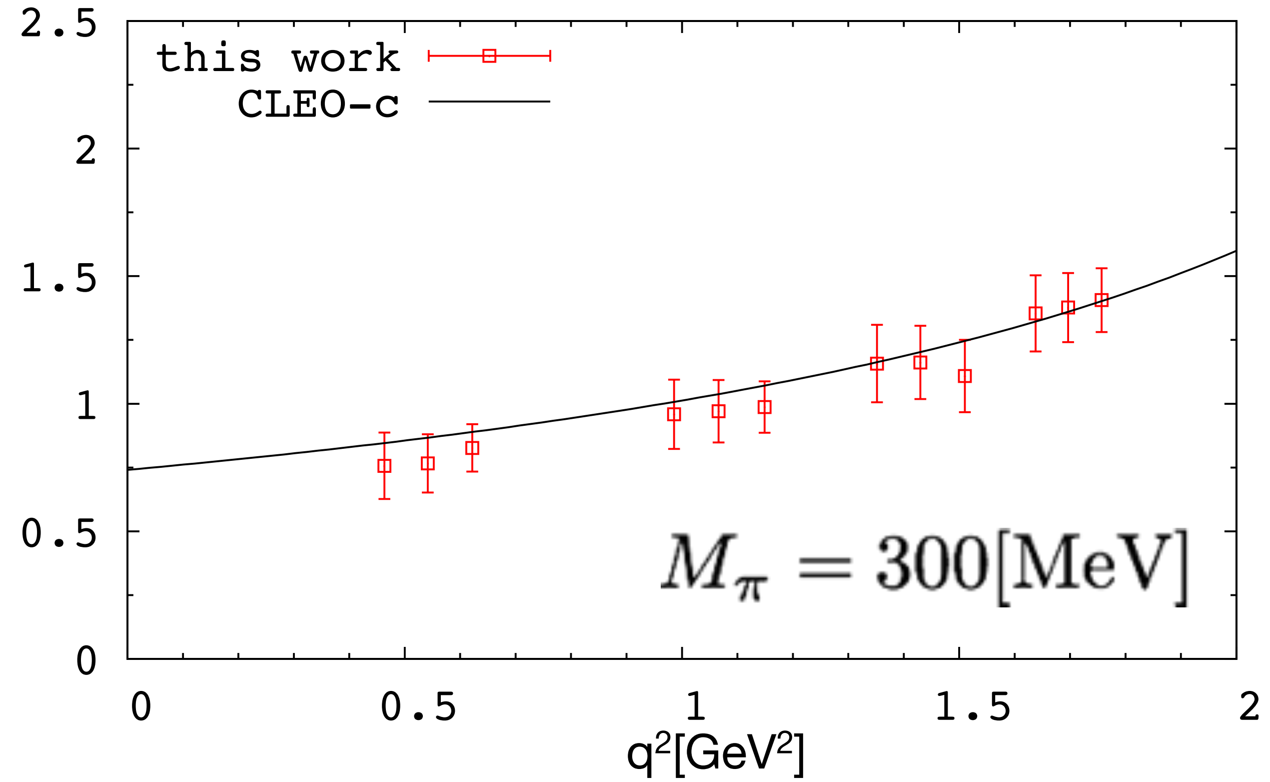
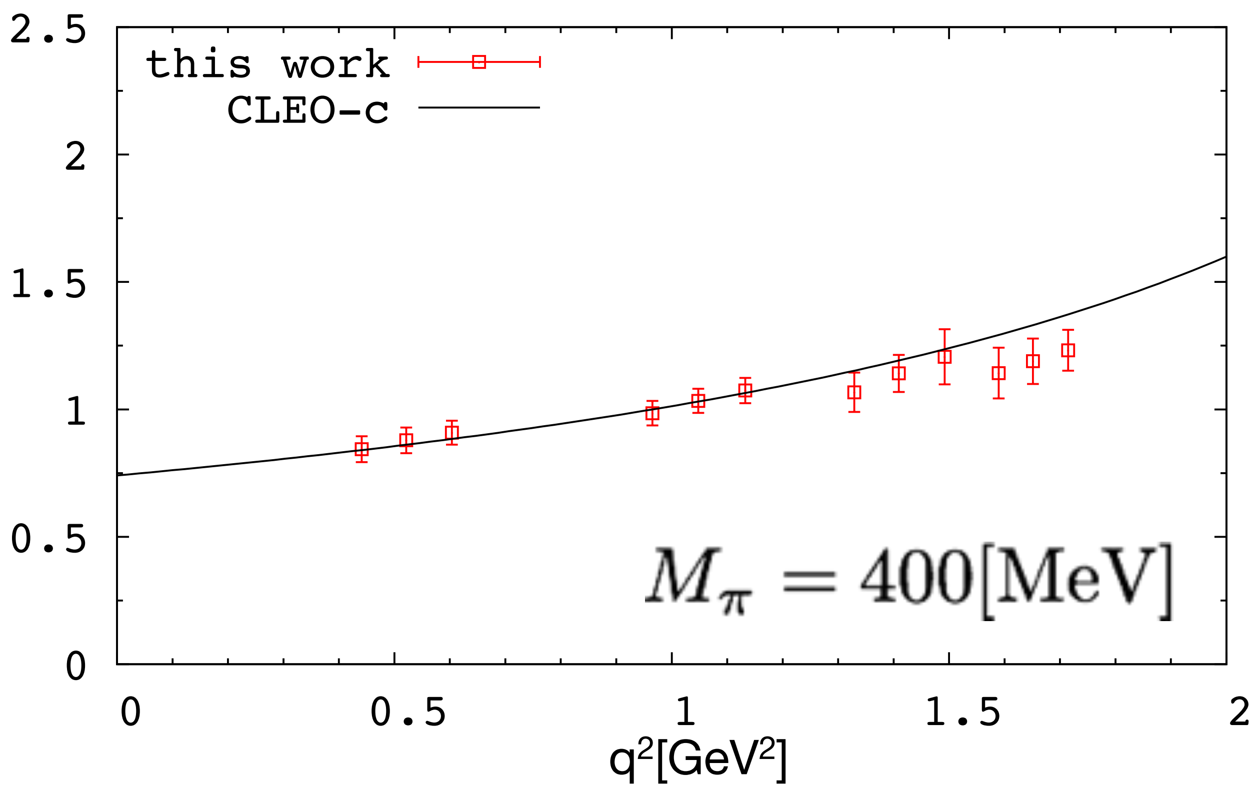
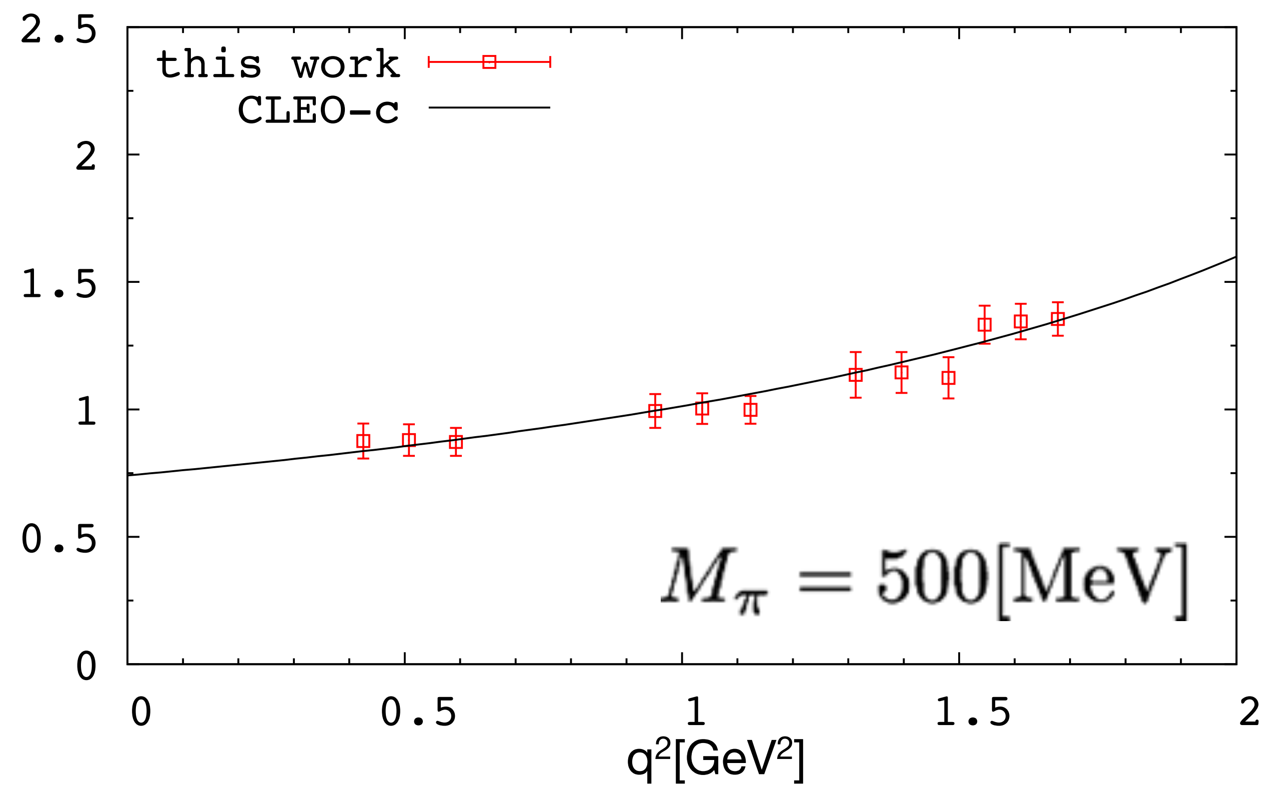






Form factors at q^2=0

Fit functions

We perform fit with below functions,
and we use “VMD + linear.

D->Pi : D^*
D->K : D_s^*

- Vector Meson Dominance (VMD) : $f_+(q^2) = \frac{f_+(0)}{1 - q^2/m_V^2}$

- VMD + linear : $f_+(q^2) = \frac{f_+(0)}{1 - q^2/m_V^2}(1 + aq^2)$

- VMD + linear + quadratic : $f_+(q^2) = \frac{f_+(0)}{1 - q^2/m_V^2}(1 + aq^2 + b(q^2)^2)$

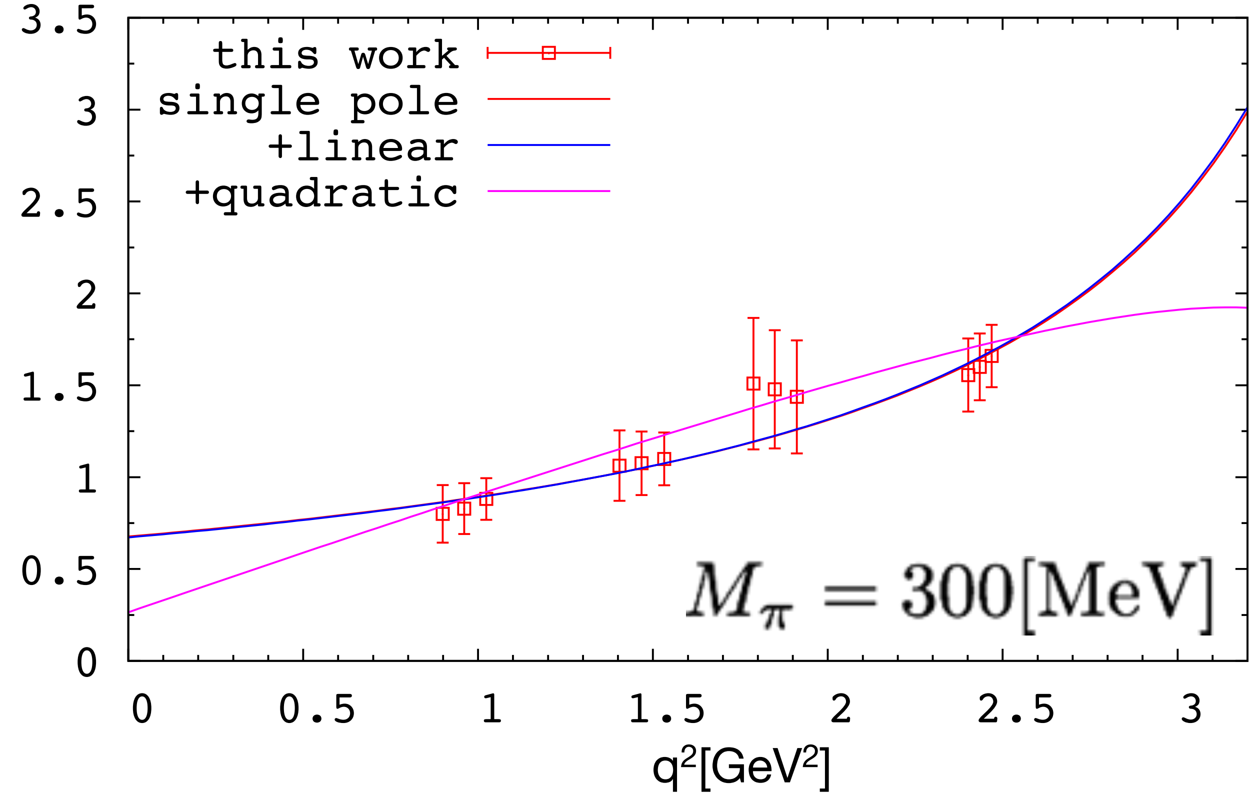
Form factors at q^2=0

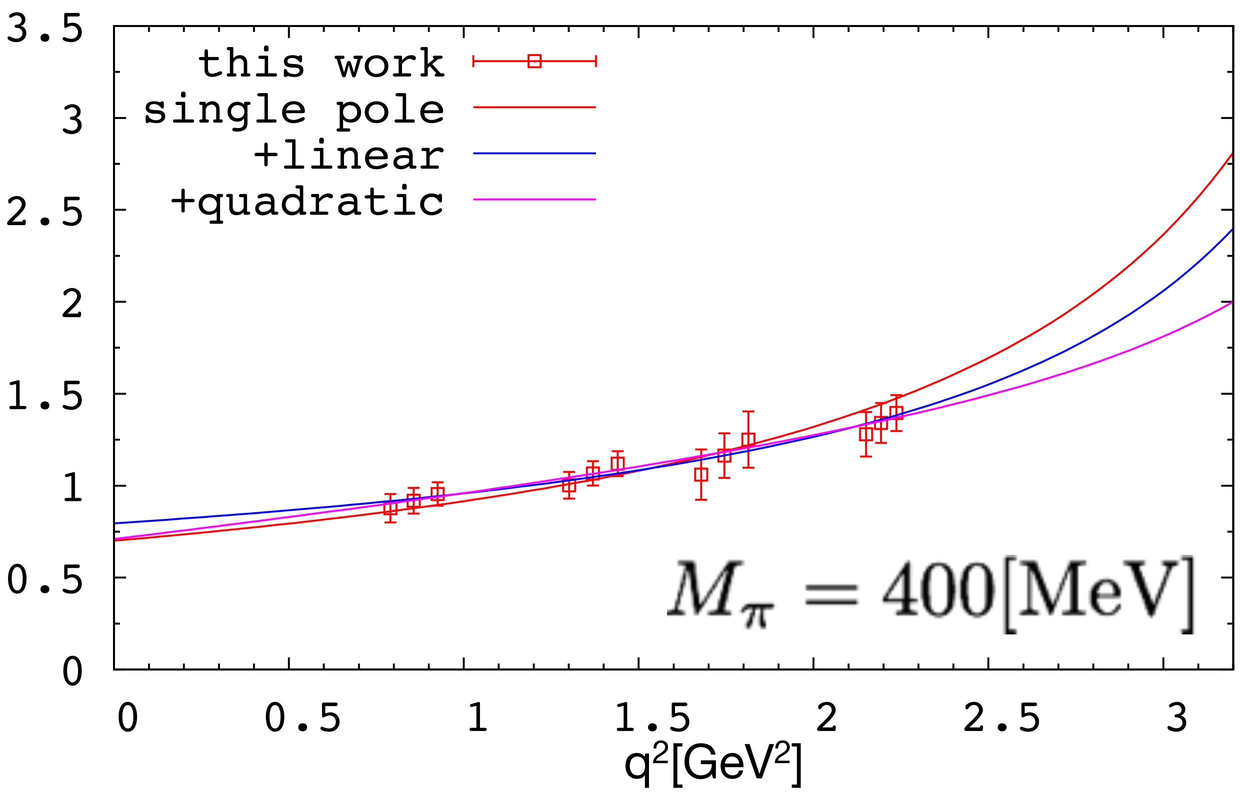
Fit results

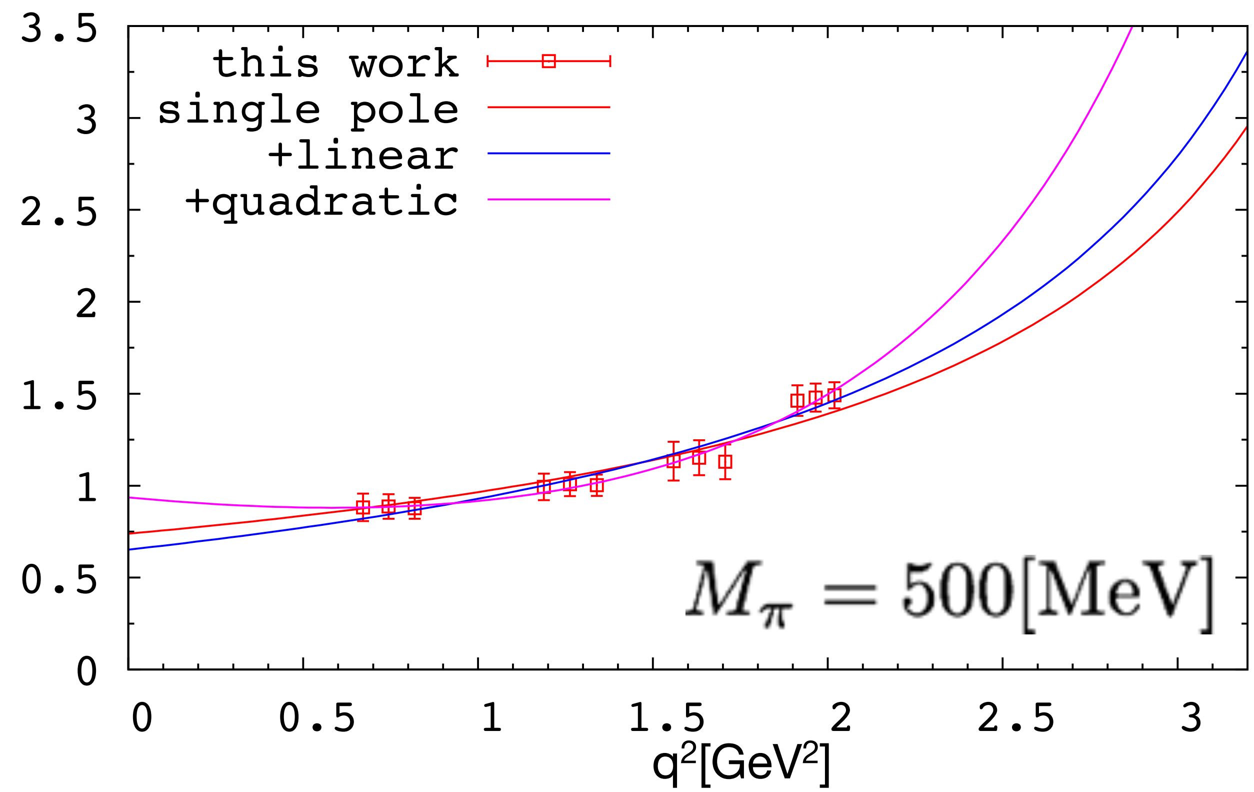
- D -> Pi
- $f_+(q^2)$ vs q^2[GeV^2]

$\chi^2/\text{d.o.f.}$

polynomial	$M_\pi = 300$[MeV]	$M_\pi = 400$[MeV]	$M_\pi = 500$[MeV]
1	0.23	0.54	0.92
$1 + aq^2$	0.24	0.21	0.64
$1 + aq^2 + b(q^2)^2$	0.11	0.19	0.25







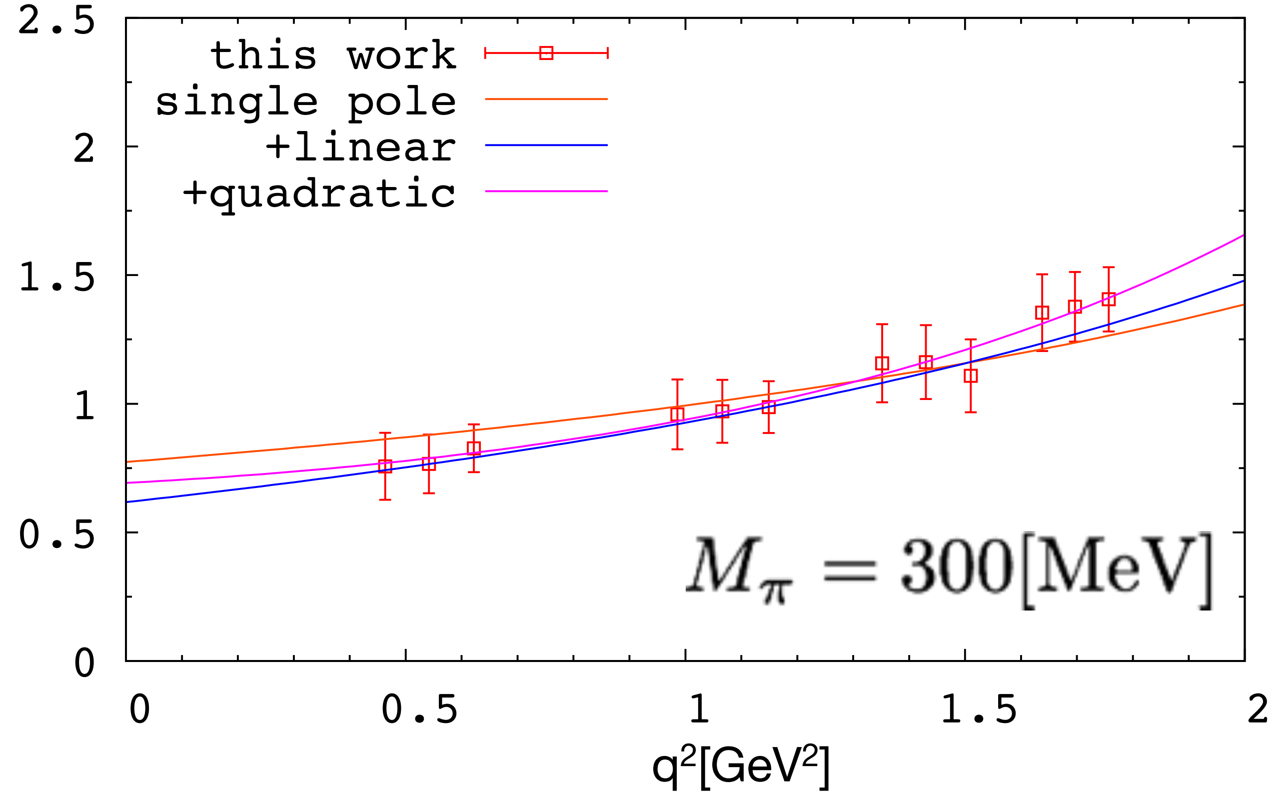
Form factors at q^2=0

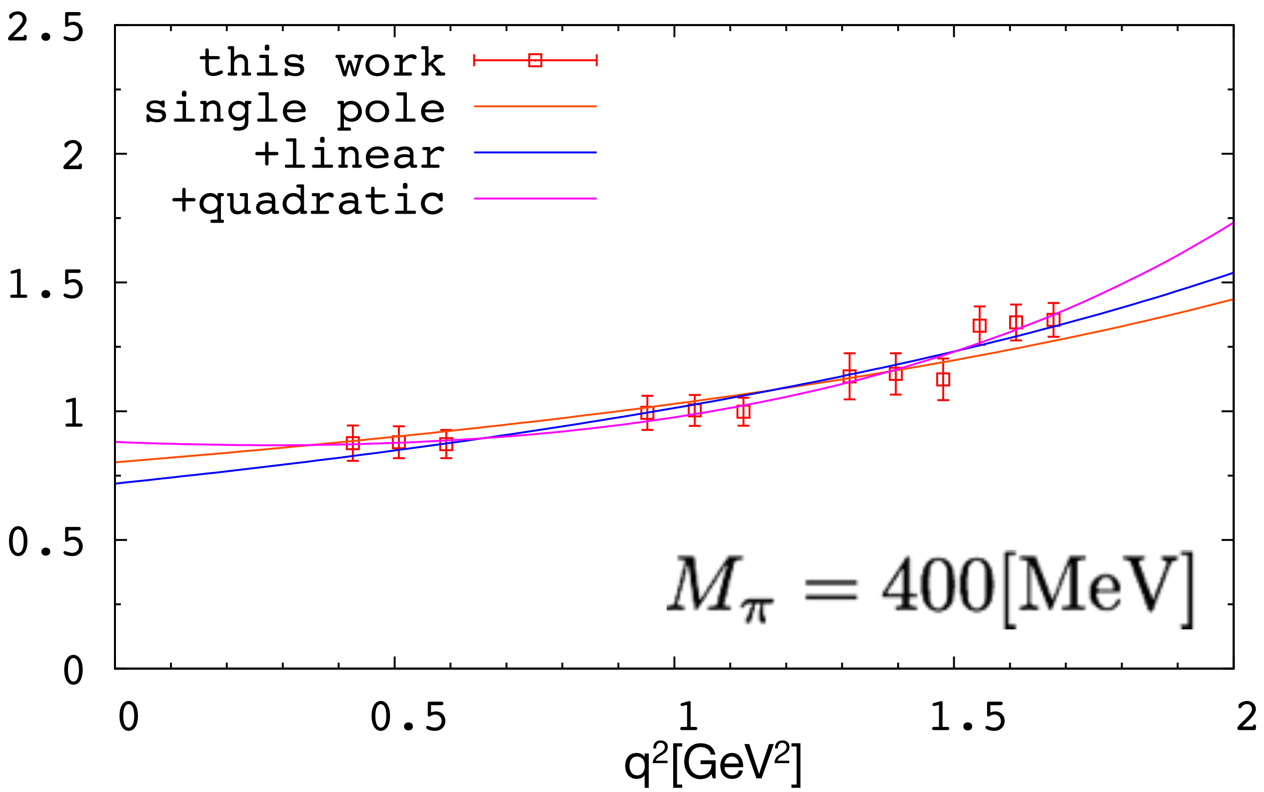
Fit results

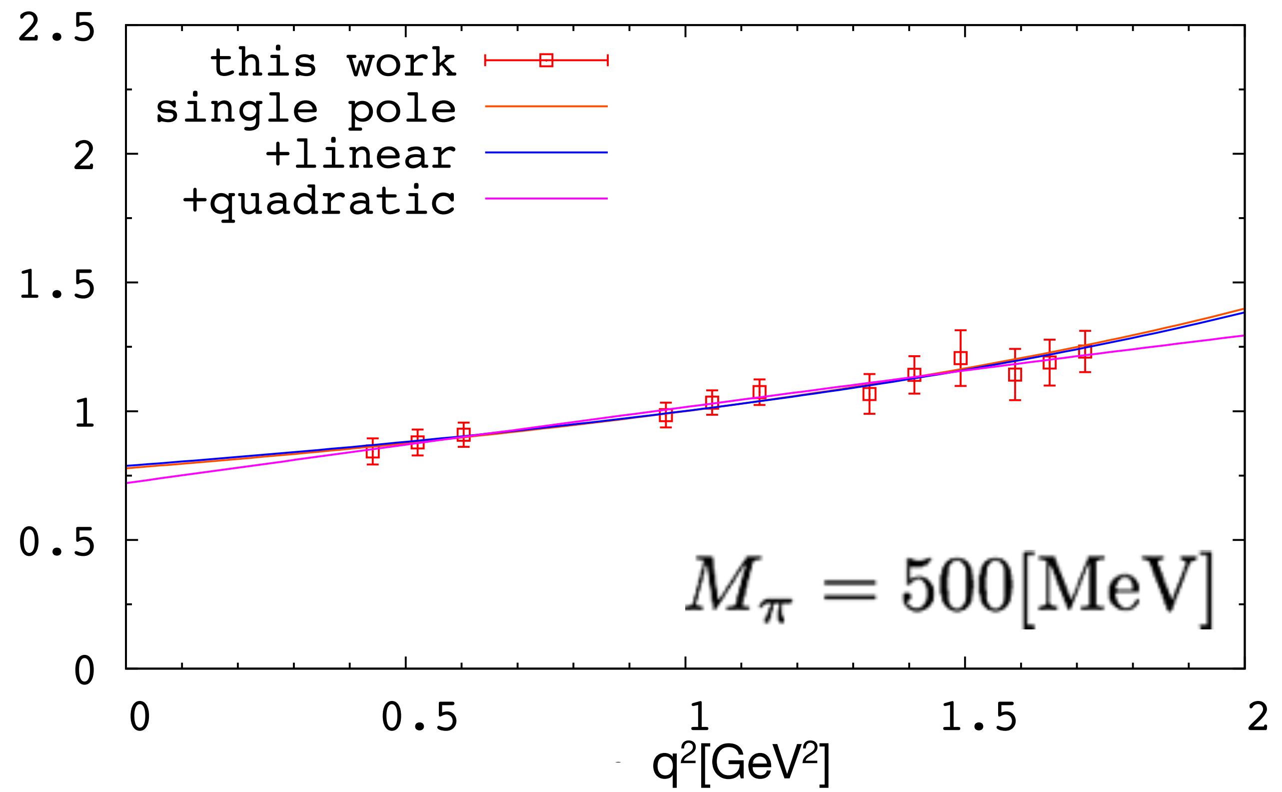
- D -> K
- $f_+(q^2)$ vs q^2[GeV^2]

χ^2/d.o.f.

polynomial	$M_\pi = 300$[MeV]	$M_\pi = 400$[MeV]	$M_\pi = 500$[MeV]
1	0.57	0.19	0.89
$1 + aq^2$	0.12	0.20	0.58
$1 + aq^2 + b(q^2)^2$	0.11	0.14	0.34







Form factors at $q^2=0$

$f_+(0)$ results

- D -> Pi

polynomial	$M_\pi = 500[\mathrm{MeV}]$	$M_\pi = 400[\mathrm{MeV}]$	$M_\pi = 300[\mathrm{MeV}]$
1	0.7386 ± 0.0414	0.7007 ± 0.0373	0.6766 ± 0.0813
$1 + aq^2$	0.6521 ± 0.0728	0.7951 ± 0.0918	0.6720 ± 0.1651
$1 + aq^2 + b(q^2)^2$	0.9363 ± 0.1307	0.7096 ± 0.2316	0.2647 ± 0.2981

- D ->K

polynomial	$M_\pi = 500[\mathrm{MeV}]$	$M_\pi = 400[\mathrm{MeV}]$	$M_\pi = 300[\mathrm{MeV}]$
1	0.7386 ± 0.0414	0.7007 ± 0.0373	0.6766 ± 0.0813
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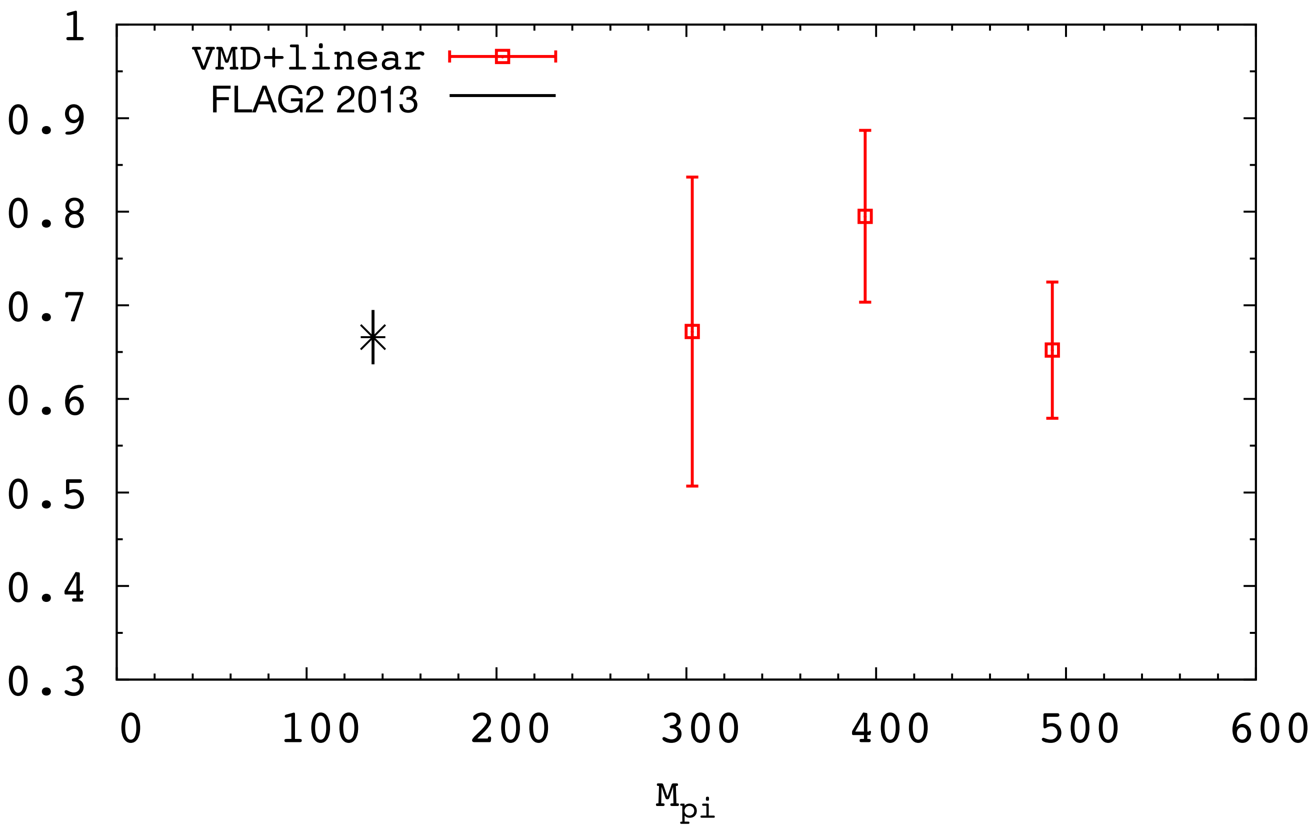
$1 + aq^2 + b(q^2)^2$	0.9363 ± 0.1307	0.7096 ± 0.2316	0.2647 ± 0.2981

Form factors at q^2=0

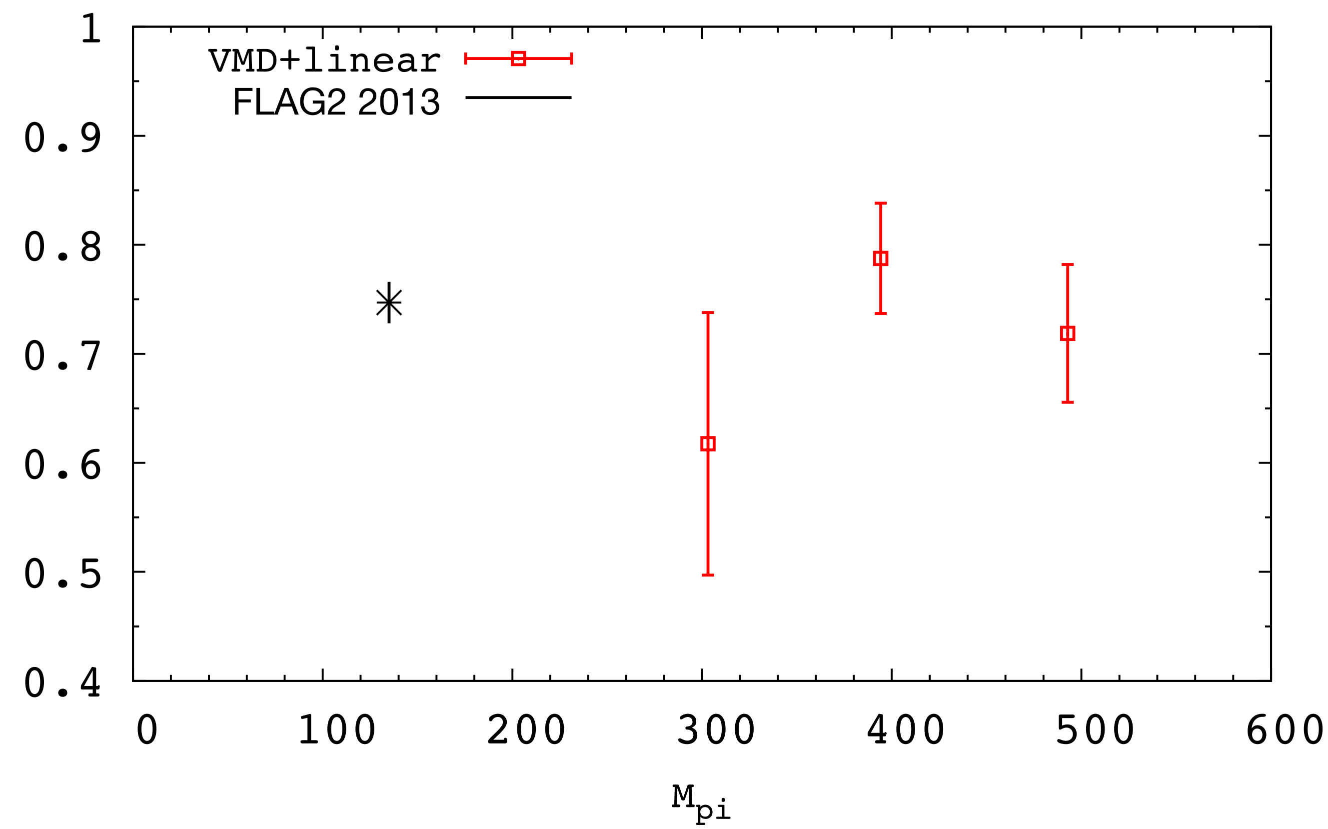
Pion mass dependence of $f_+(0)$

FLAG [arXiv:1310.8555]

D -> Pi



D -> K



Our results look to be consistent with previous lattice results within our errors.

We will reduce errors by using data of smaller quark mass or other source points.

Summary

We compute form factors of D-decays.

- 2+1 Möbius Domain Wall fermion
- Symanzik action
- a = 0.08 [fm], V = 2.57^3x5.15 [fm^4]
- Mpi = 300, 400, 500 [MeV]

Current data

VMD with/without polynomial fit -> f+(0) consistent with previous lattice results within our errors.

To do

We have data of lighter pion mass and finer lattice, we increase statistics with more source point.

- reduce statistical errors
- take chiral or continuum limit

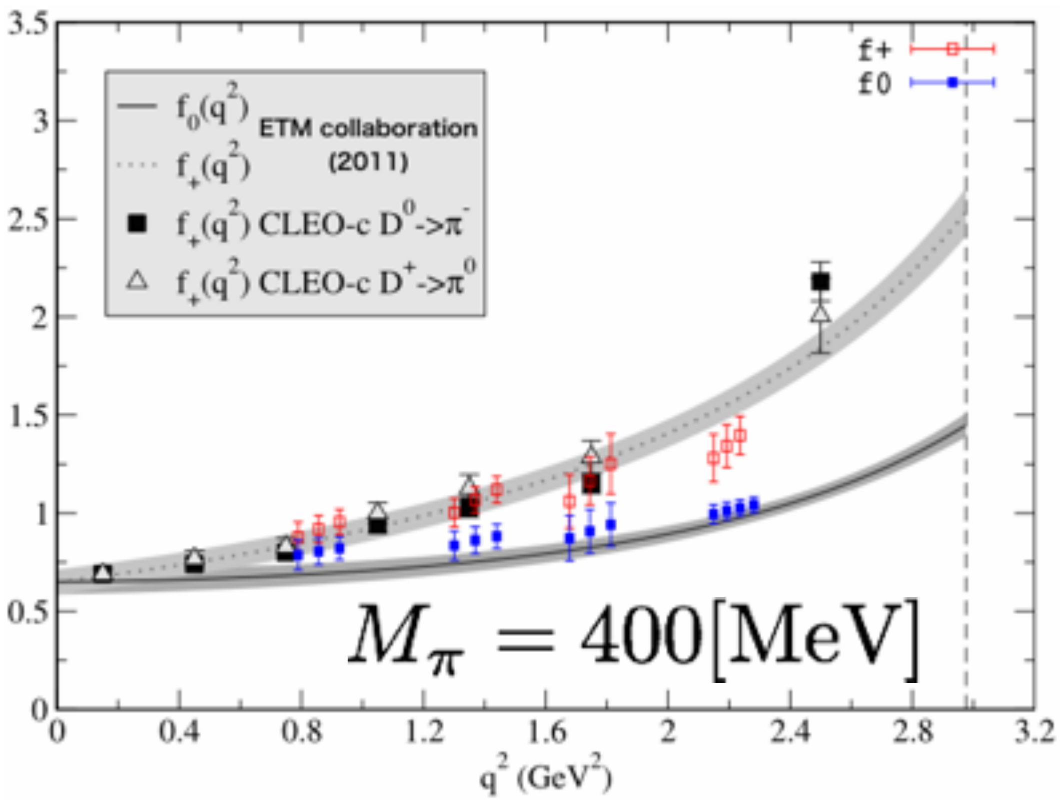
Thank you very much !

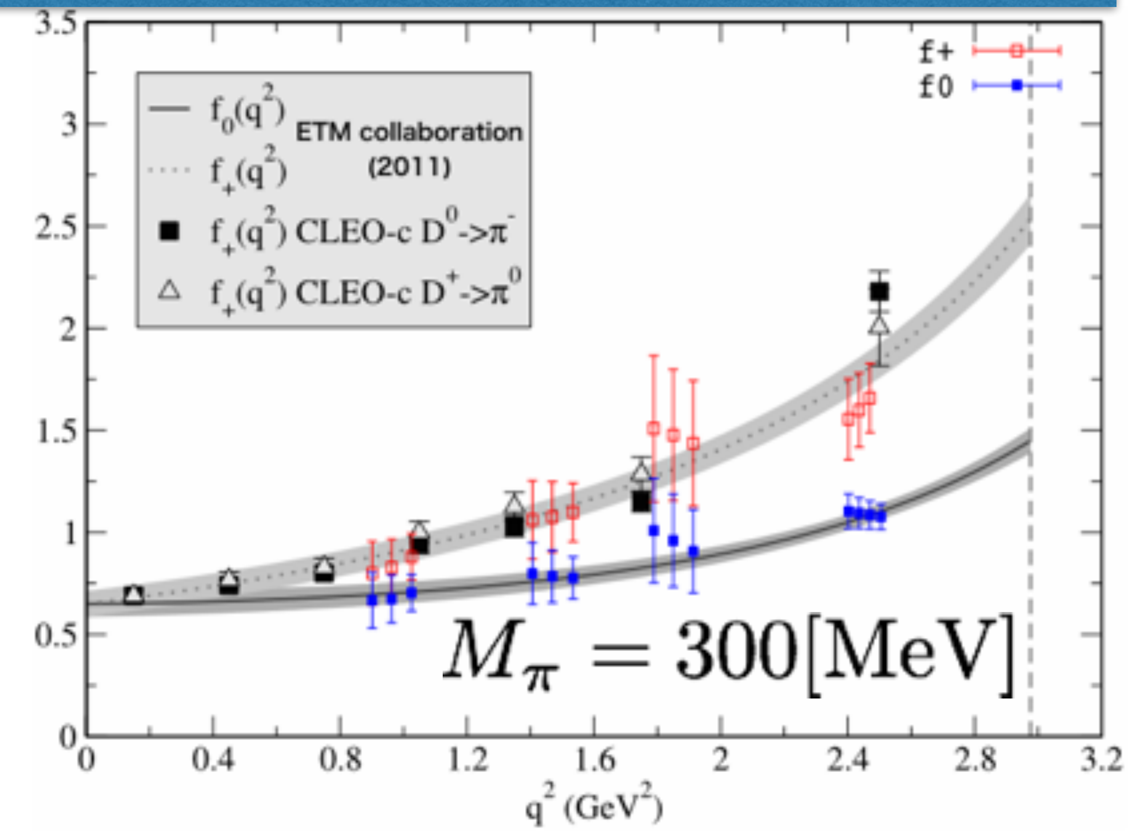
Back Up

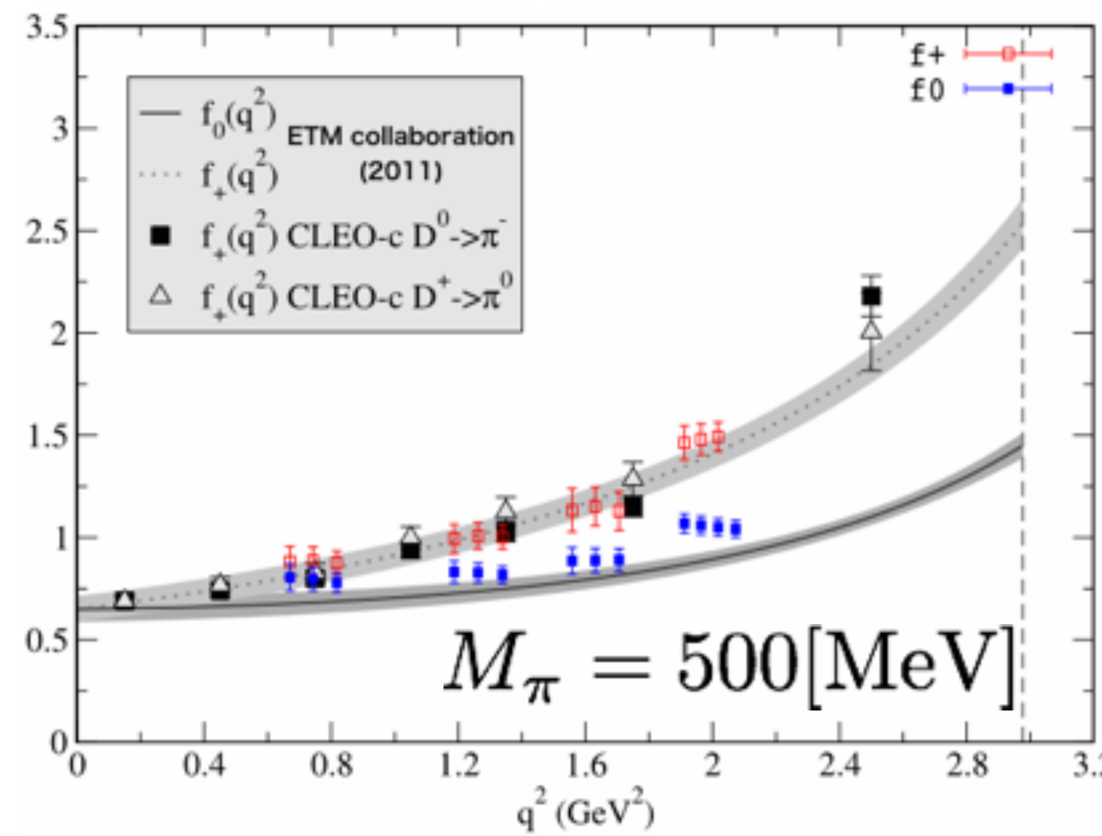
Compare with ETMC

[arXiv:1104.0869]

- D -> Pi





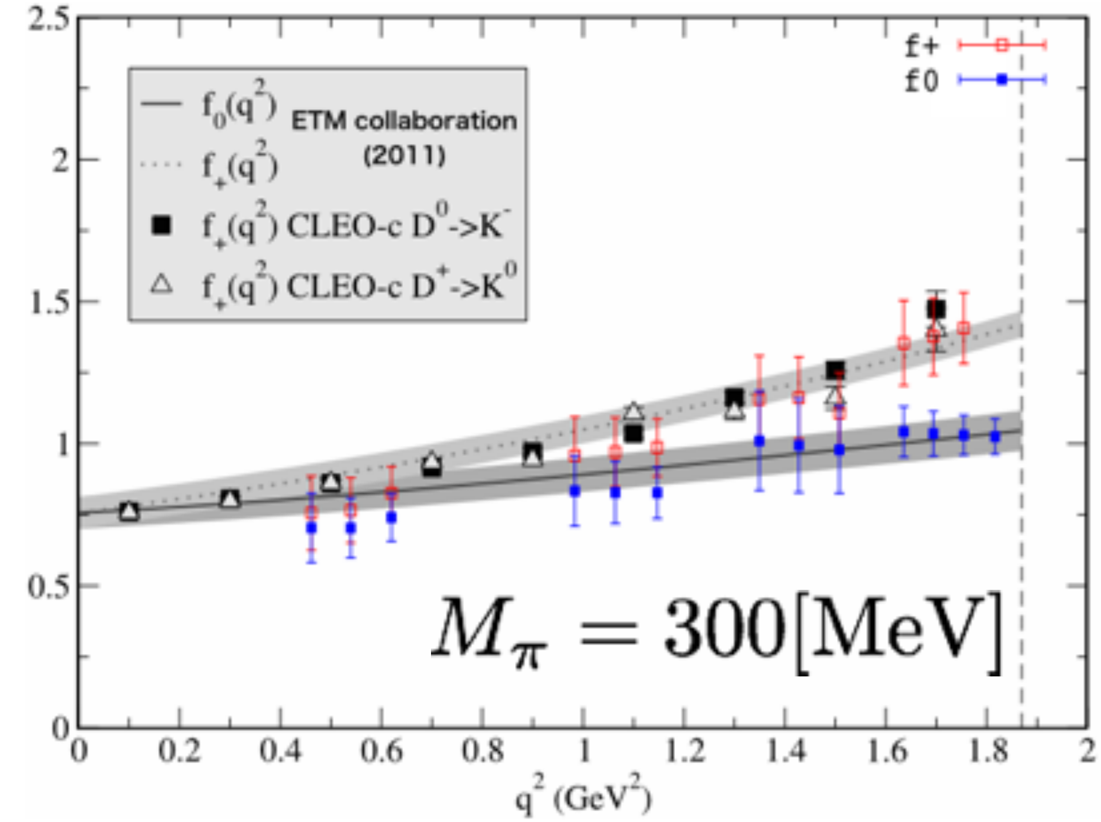


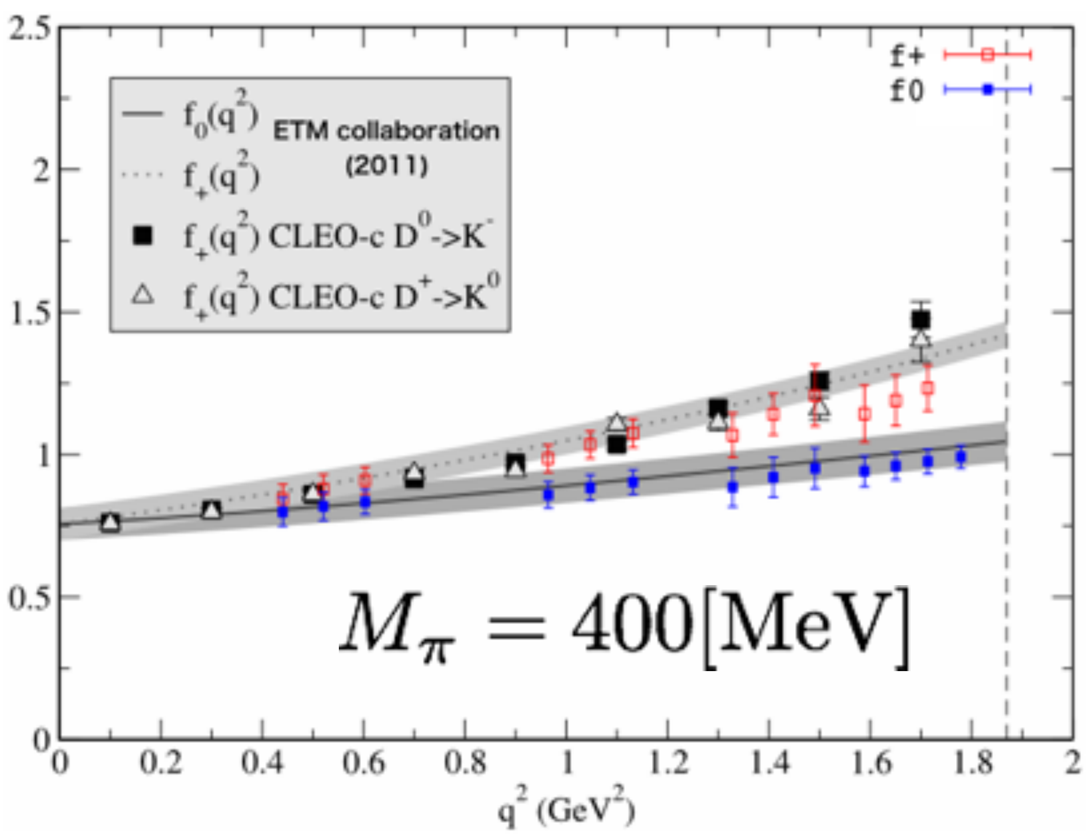
Back Up

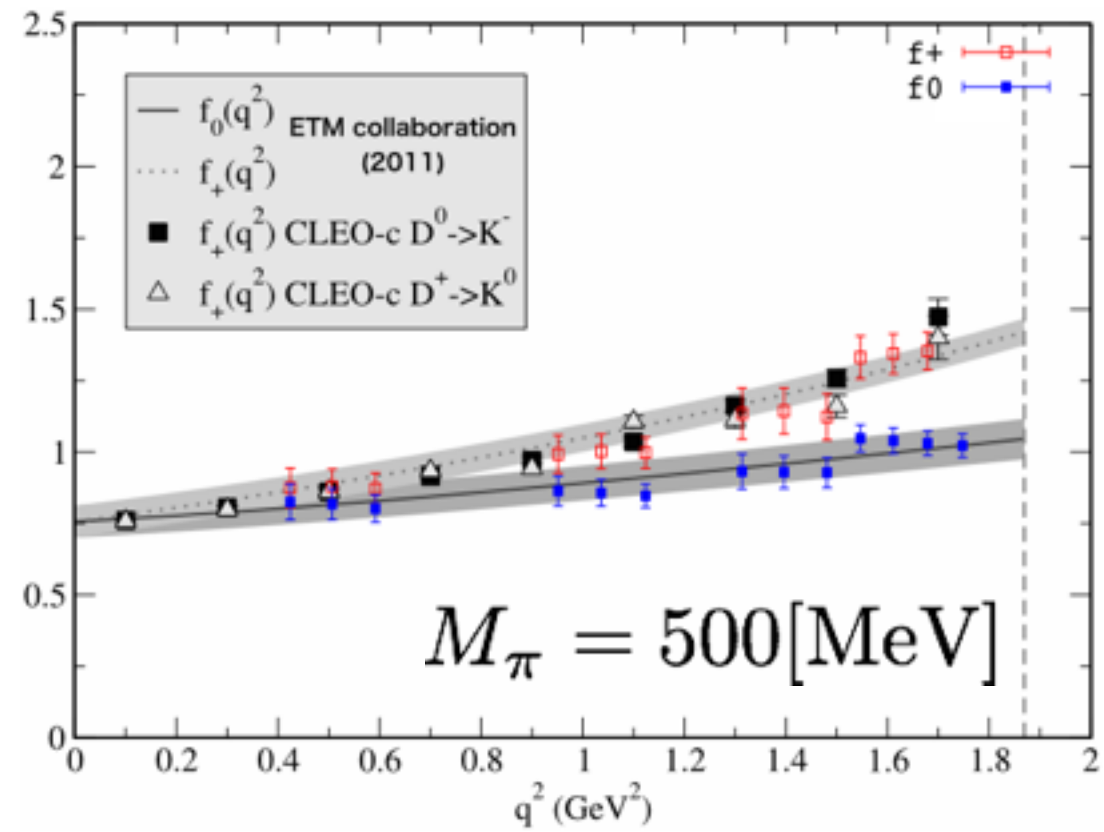
Compare with ETMC

[arXiv:1104.0869]

- D -> K



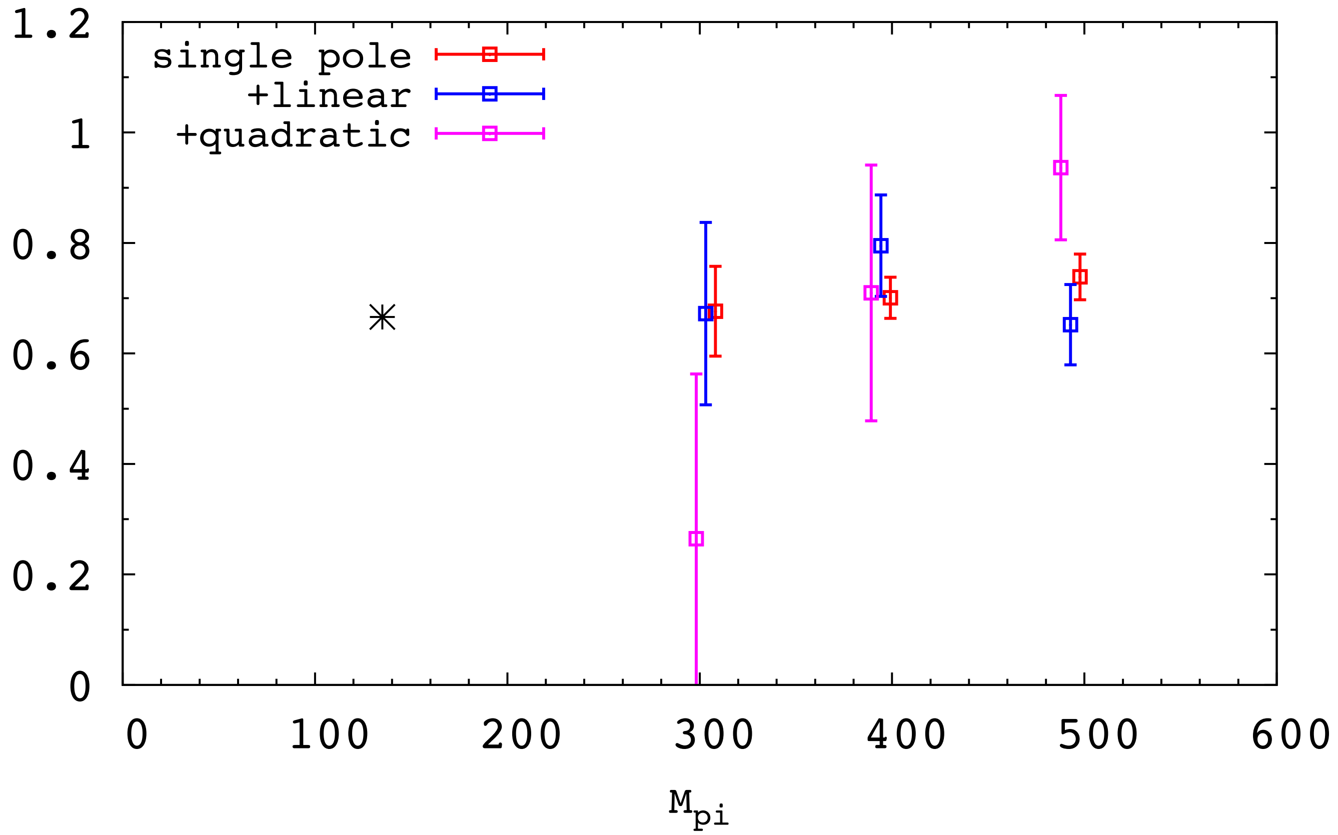




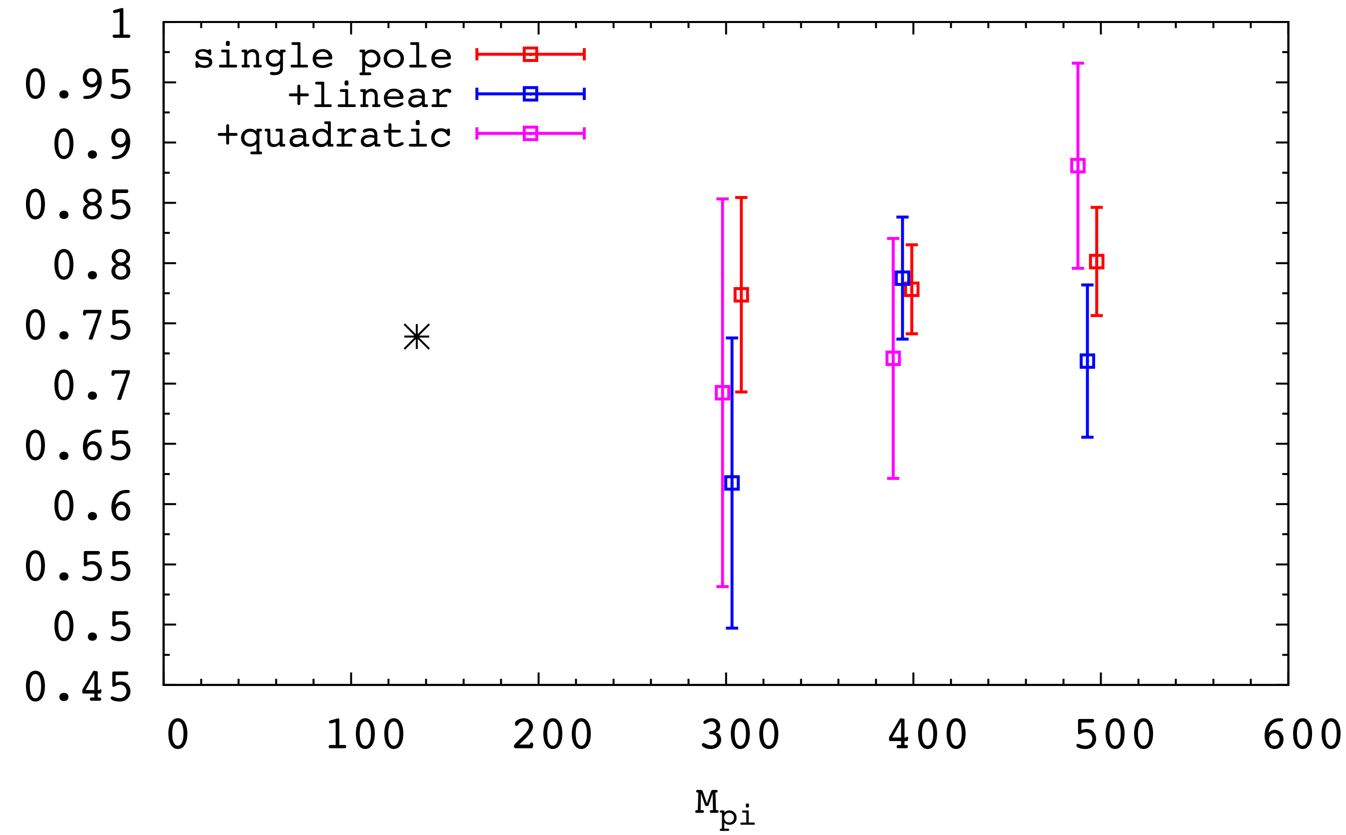
Back Up

Results of other parametrization

D -> Pi



D -> K



Back Up

fit parameters

$1 + aq^2$	$m_{ud} = 0.019$	$m_{ud} = 0.012$	$m_{ud} = 0.007$
a	0.0904583 ± 0.0602044	-0.0776441 ± 0.0482898	0.00468288 ± 0.1013706

TABLE VI: fit result of a with a function VMD$\times(1 + aq^2)$ for $D \to \pi$

$1 + aq^2$	$m_{ud} = 0.019$	$m_{ud} = 0.012$	$m_{ud} = 0.007$
a	0.0975518 ± 0.0484413	-0.0109076 ± 0.0387716	0.227427 ± 0.136424

TABLE VII: fit result of a with a function VMD$\times(1 + aq^2)$ for $D \to K$

$1 + aq^2 + b(q^2)^2$	$m_{ud} = 0.019$	$m_{ud} = 0.012$	$m_{ud} = 0.007$
a	-0.425850 ± 0.161230	0.092781 ± 0.525394	2.22395 ± 4.80083
b	0.175269 ± 0.058508	-0.058043 ± 0.185866	-0.632139 ± 1.39418

TABLE VIII: fit results of a and b with a function VMD$\times(1 + aq^2 + b(q^2)^2)$ for $D \to \pi$

$1 + aq^2 + b(q^2)^2$	$m_{ud} = 0.019$	$m_{ud} = 0.012$	$m_{ud} = 0.007$
a	-0.321912 ± 0.129044	0.193971 ± 0.293239	-0.0573307 ± 0.407134
b	0.185619 ± 0.057827	-0.0971861 ± 0.141083	0.112683 ± 0.178942

TABLE IX: fit results of a and b with a function VMD$\times(1 + aq^2 + b(q^2)^2)$ for $D \to K$