G_2-QCD at finite temperature and density

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FSU Jena

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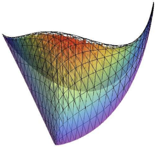
Kobe, Japan, 18.06.2015

Research Training Group
Quantum and Gravitational Fields



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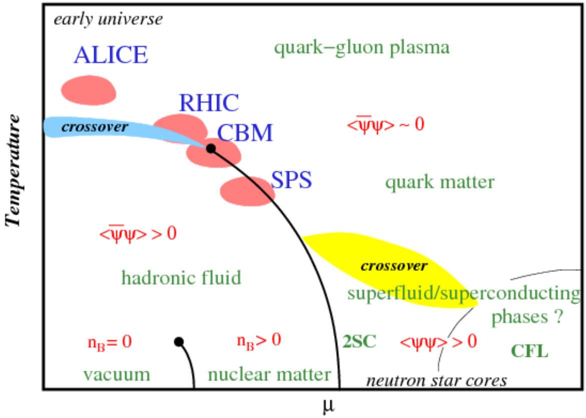


QCD sign problem at finite density:
Standard Monte-Carlo methods only for $\mu/T \leq 1$ applicable

- Monte-Carlo methods for complex weight functions (Complex Langevin, Lefschetz thimbles)
- Hopping parameter expansion, strong coupling expansion, fugacity expansion ...
- Isospin chemical potential
- ...

QCD-like theories where the gauge group / fermion representation is replaced by some other gauge group / representation such that fermion determinant $\det D[\mu, m] \geq 0$

- Adjoint QCD : already at $\mu = 0$ very different to QCD
- two-color QCD : no fermionic baryons
- G_2-QCD : in this talk

What is G_2-QCD ?

Replace $SU(3)$ by the exceptional Lie group G_2

$$\det D[\mathcal{U}, \mu, m] \geq 0$$

Investigate the full phase diagram of a gauge theory with
fermionic baryons and fundamental quarks with Monte-Carlo methods

Why G_2 ?

rank 2, $SU(3)$ is a subgroup of G_2,
first order deconfinement transition in Yang-Mills
fermionic baryons

What are the differences between G_2-QCD and QCD?

What is the contribution of fermionic baryons to the G_2-QCD phase diagram ?

Can we learn something about the QCD sign problem ?

G_2-QCD

	G_2	$SU(3)$
rank	2	2
dimension	14	8
fundamental reps	(7), (14)	(3),($\bar{3}$)
adjoint rep	(14)	(8)
center	$\mathbb{1}\,(k_{\mathcal{R}}=0)$	$\mathbb{Z}(3)\,(k_3=1)$
quark rep (N_c)	(7)	(3)
anti-quark rep	(7)	($\bar{3}$)
mesons	$(7)\otimes(7)=(1)\oplus\cdots$	$(3)\otimes(\bar{3})=(1)\oplus\cdots$
baryons	$(7)\otimes(7)\otimes(7)=(1)\oplus\cdots$	$(3)\otimes(3)\otimes(3)=(1)\oplus\cdots$

$$SU(3)\subset G_2\subset SO(7)$$

Lagrange density for N_f (Dirac) flavour G_2-QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}\, D[A, m, \mu]\, \Psi$$

$$D[A, m, \mu] = \gamma^\mu (\partial_\mu - g A_\mu) - m + \gamma_0 \mu$$

As in QCD:

$$\gamma_5 D[A, m, \mu] = D^\dagger[A, m, -\mu^*] \gamma_5$$

For G_2 every representation is real:

$$C \gamma_5 D[A, m, \mu] = D^*[A, m, \mu] C \gamma_5$$

$$\det D[A, m, \mu] \geq 0$$

Chiral symmetry for $N_f = 1$

$$SU(2) \to U(1)$$

Goldstone bosons:

$$d(0^{++}) \sim \bar{\psi}^{\mathrm{C}}\gamma_5\psi + \bar{\psi}\gamma_5\psi^{\mathrm{C}} \quad \text{and} \quad d(0^{+-}) \sim \bar{\psi}^{\mathrm{C}}\gamma_5\psi - \bar{\psi}\gamma_5\psi^{\mathrm{C}}$$

Massive state:

$$f(0^{++}) \sim \bar{\psi}\psi$$

$$\begin{tikzcd}
\boxed{SU(2)_{\mathrm{L=R^*}}} \arrow[r, "m"] \arrow[d, "\mu"] & \boxed{U(1)_{\mathrm{B}}} \arrow[d, "\mu"] \\
\boxed{U(1)_{\mathrm{B}}} \arrow[r, "m"] & \boxed{U(1)_{\mathrm{B}}}
\end{tikzcd}$$

Spectroscopy

- Wilson fermions
- Symanzik improved gauge action
- Rational Hybrid Monte-Carlo Algorithm
- $\beta = 0.96$, $\kappa = 0.159$ fixed

Spectroscopy ensemble

16×8^3 lattice
Proton mass $m_N = 938\,\mathrm{MeV}$

Diquark mass $m_{d(0^+)} = 247\,\mathrm{MeV}$
Lattice spacing $a = 0.343\,\mathrm{fm} \sim (575\mathrm{MeV})^{-1}$

Finite temperature and density

Lattice size: $N_T \times 8^3$ with $N_T = 2 \ldots 16$
$T = 36 \ldots 287$ MeV
$\mu = 0 \ldots 354$ MeV (below lattice saturation)

Mesons $n_q = 0$

Name	Operator	Pos.	Spin	Colour	Flavour	J	P
η	$\bar{u}\gamma_5 u$	S	A	S	S	0	-
f	$\bar{u}u$	S	A	S	S	0	$+$
ω	$\bar{u}\gamma_\mu u$	S	S	S	A	1	-
h	$\bar{u}\gamma_5\gamma_\mu u$	S	S	S	A	1	$+$
π	$\bar{u}\gamma_5 d$	S	A	S	S	0	-
a	$\bar{u}d$	S	A	S	S	0	$+$
ρ	$\bar{u}\gamma_\mu d$	S	S	S	A	1	-
b	$\bar{u}\gamma_5\gamma_\mu d$	S	S	S	A	1	$+$

$$(7) \otimes (7) = (1) \oplus \dots$$

Baryons $n_q = 1$

Name	Operator	Pos.	Spin	Col.	Flav.	J	P
Hybrid	$\epsilon_{abcdefg} u^a F^p_{\mu\nu} F^q_{\mu\nu} F^r_{\mu\nu} T^{bc}_p T^{de}_q T^{fg}_r$	S	S	A	S	1/2	$\pm$
$\tilde{\Delta}$	$T^{abc}(\bar{u}_a \gamma_\mu u_b) u_c$	S	S	A	S	3/2	$\pm$
$\tilde{N}$	$T^{abc}(\bar{u}_a \gamma_5 d_b) u_c$	S	A	A	A	1/2	$\pm$

$$(7) \otimes (7) \otimes (7) = (1) \oplus \dots$$
$$(7) \otimes (14) \otimes (14) \otimes (14) = (1) \oplus \dots$$

Baryons $n_q = 2$

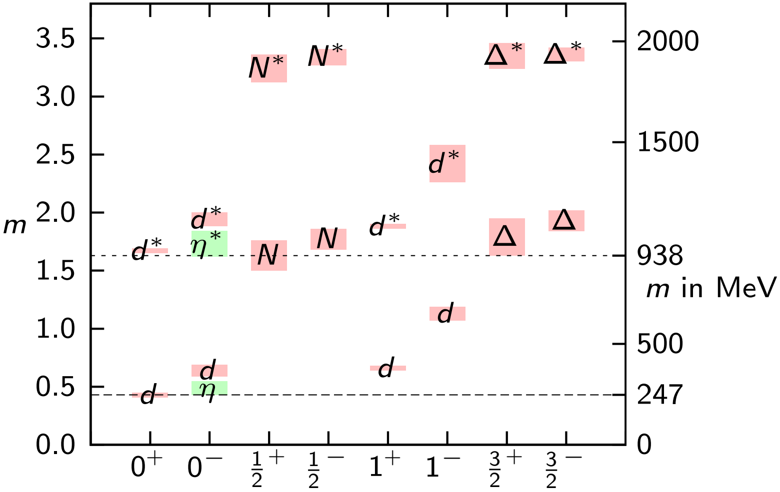
Name	Operator	Pos.	Spin	Colour	Flavour	J	P	C
$d(0^{++})$	$\bar{u}^C\gamma_5 u + \bar{u}\gamma_5 u^C$	S	A	S	S	0	+	+
$d(0^{+-})$	$\bar{u}^C\gamma_5 u - \bar{u}\gamma_5 u^C$	S	A	S	S	0	+	-
$d(0^{-+})$	$\bar{u}^C u + \bar{u} u^C$	S	A	S	S	0	-	+
$d(0^{--})$	$\bar{u}^C u - \bar{u} u^C$	S	A	S	S	0	-	-
$d(1^{++})$	$\bar{u}^C\gamma_\mu d + \bar{u}\gamma_\mu d^C$	S	S	S	A	1	+	+
$d(1^{+-})$	$\bar{u}^C\gamma_\mu d - \bar{u}\gamma_\mu d^C$	S	S	S	A	1	+	-
$d(1^{-+})$	$\bar{u}^C\gamma_5\gamma_\mu d + \bar{u}\gamma_5\gamma_\mu d^C$	S	S	S	A	1	-	+
$d(1^{--})$	$\bar{u}^C\gamma_5\gamma_\mu d - \bar{u}\gamma_5\gamma_\mu d^C$	S	S	S	A	1	-	-

$$(7) \otimes (7) = (1) \oplus \dots$$

Baryons $n_q = 3$

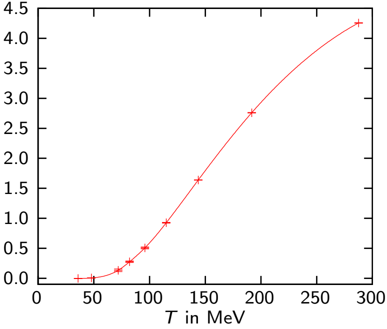
Name	Operator	Pos.	Spin	Colour	Flavour	J	P
Δ	$T^{abc}(\bar{u}_a^C \gamma_\mu u_b) u_c$	S	S	A	S	3/2	$\pm$
N	$T^{abc}(\bar{u}_a^C \gamma_5 d_b) u_c$	S	A	A	A	1/2	$\pm$

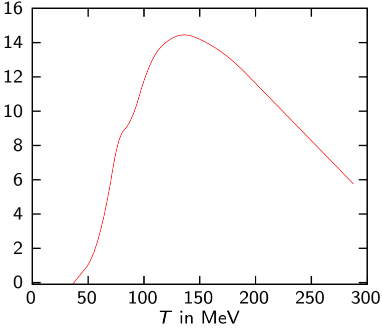
$$(7) \otimes (7) \otimes (7) = (1) \oplus \ldots$$


3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
m
2000
1500
938
m in MeV
500
247
0
d^* d^* η^* N^* N^* d^* Δ^* Δ^*
N N d^* Δ Δ
d d η d d
0^+ 0^- $\frac{1}{2}^+$ $\frac{1}{2}^-$ 1^+ 1^- $\frac{3}{2}^+$ $\frac{3}{2}^-$

Phase diagram

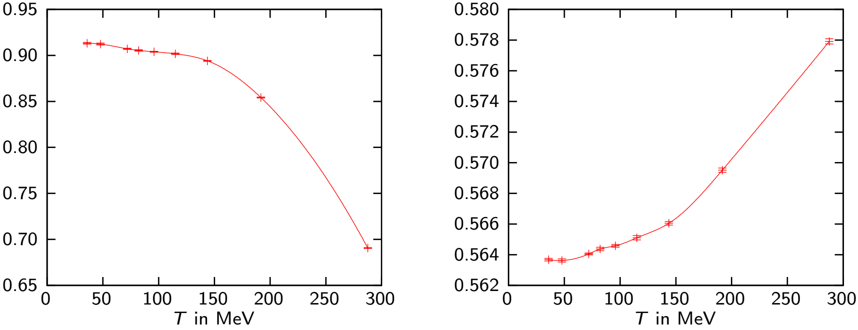
Polyakov loop $\langle P \rangle$ and susceptibility $\frac{\partial \langle P \rangle}{\partial T}$ at $\mu = 0$





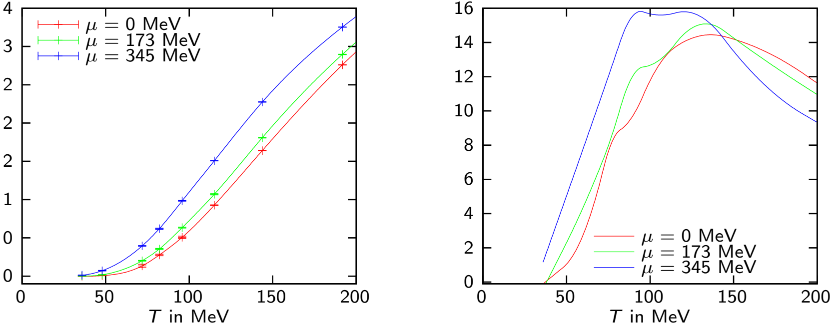
Deconfinement transition at $T_c(\mu = 0) \sim 137\,\mathrm{MeV}$

Chiral Condensate $\langle \Sigma \rangle - \Sigma_{\mathrm{SB,Latt}}$ and Plaquette $\langle \mathrm{Plaq} \rangle$ at $\mu = 0$



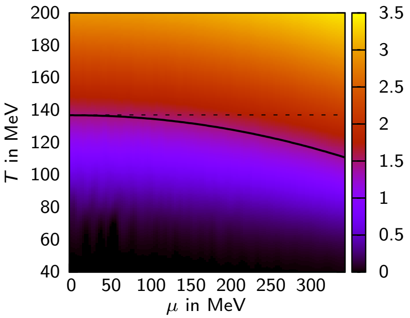
Still not clear whether deconfinement and chiral transition temperatures agree

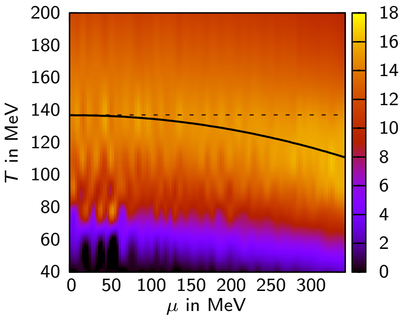
Polyakov loop $\langle P \rangle$ and susceptibility $\frac{\partial \langle P \rangle}{\partial T}$



Deconfinement transition shifts to smaller T for larger μ and peak of the susceptibility increases

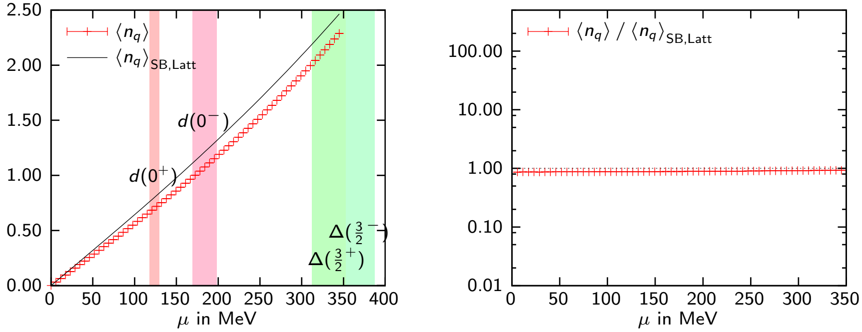
Polyakov loop $\langle P \rangle$ and susceptibility $\frac{\partial \langle P \rangle}{\partial T}$





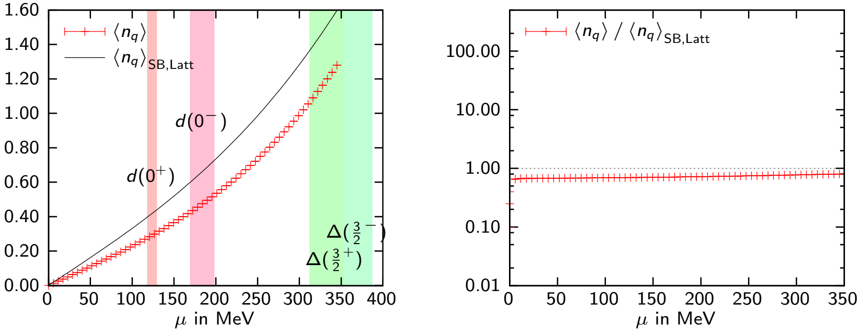
$$\frac{T_c(\mu)}{T_c(0)} \sim 1 + 0.001(8)\frac{\mu}{T_c(0)} - 0.031(6)\left(\frac{\mu}{T_c(0)}\right)^2$$

Quark Number Density



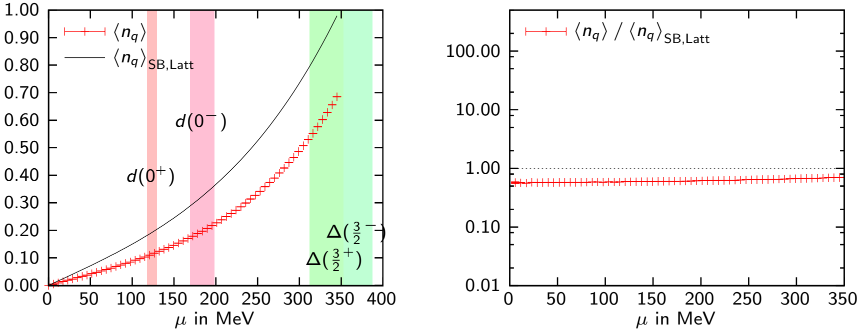
$T = 287$ MeV

Quark Number Density



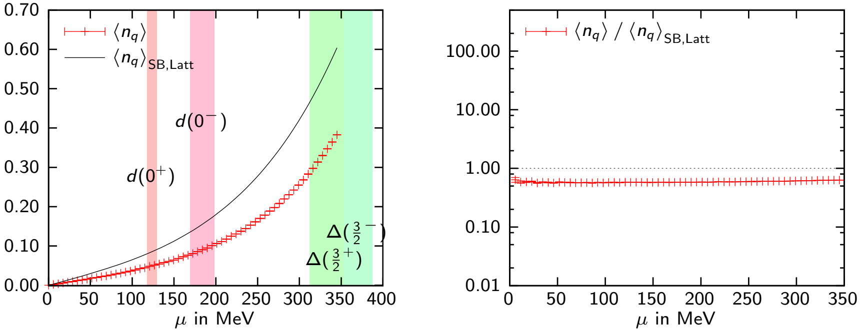
$T = 192$ MeV

Quark Number Density



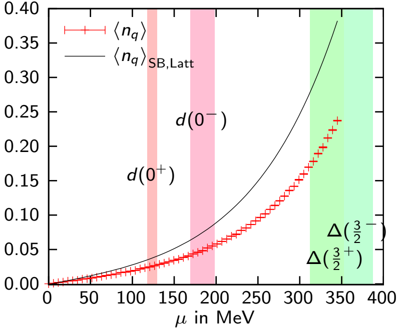
$T = 144$ MeV

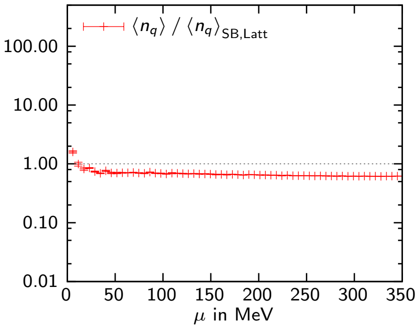
Quark Number Density



$T = 115$ MeV, $\mu_{\mathrm{deconf}} = 317$ Mev

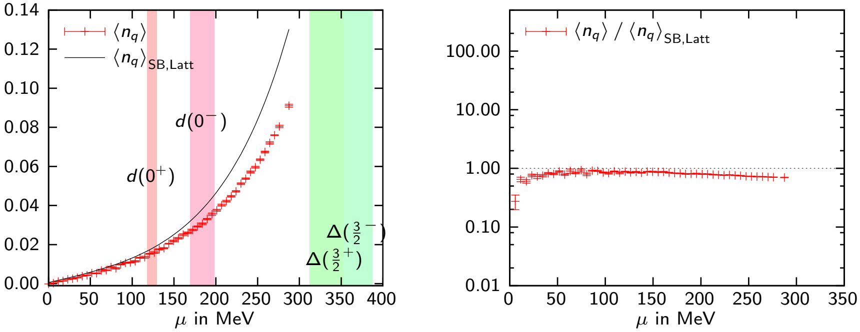
Quark Number Density





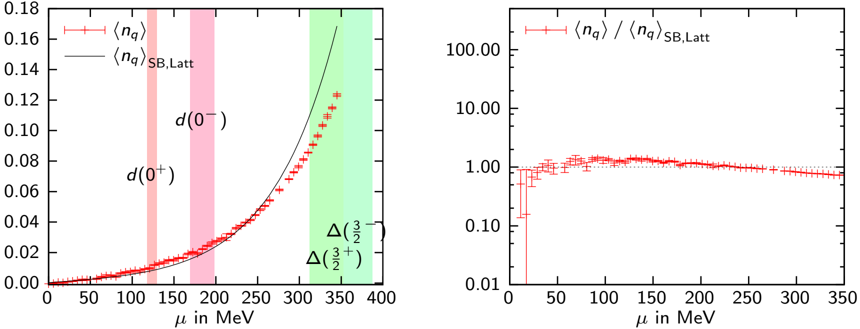
$T = 96$ MeV

Quark Number Density



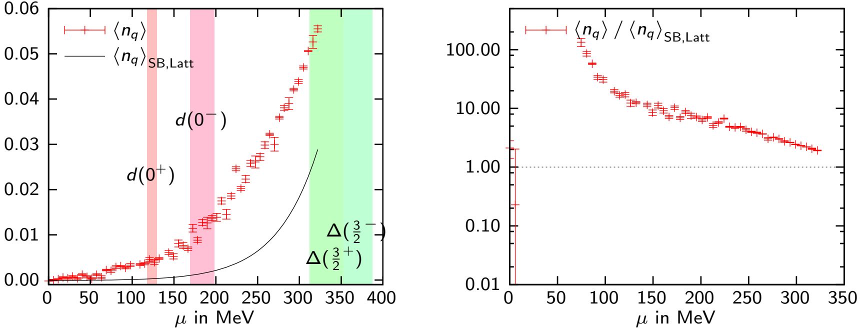
$T = 82$ MeV

Quark Number Density



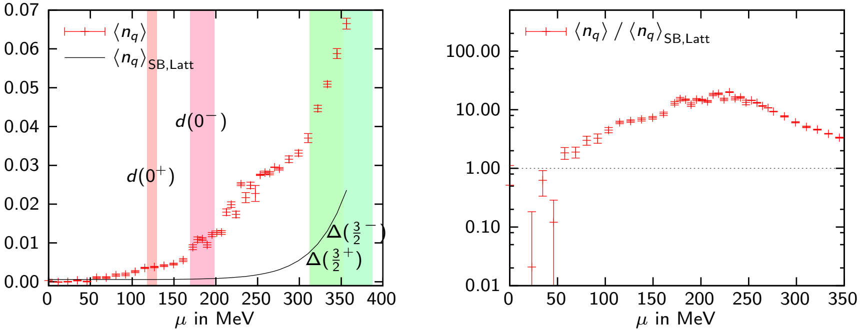
$T = 72$ MeV

Quark Number Density



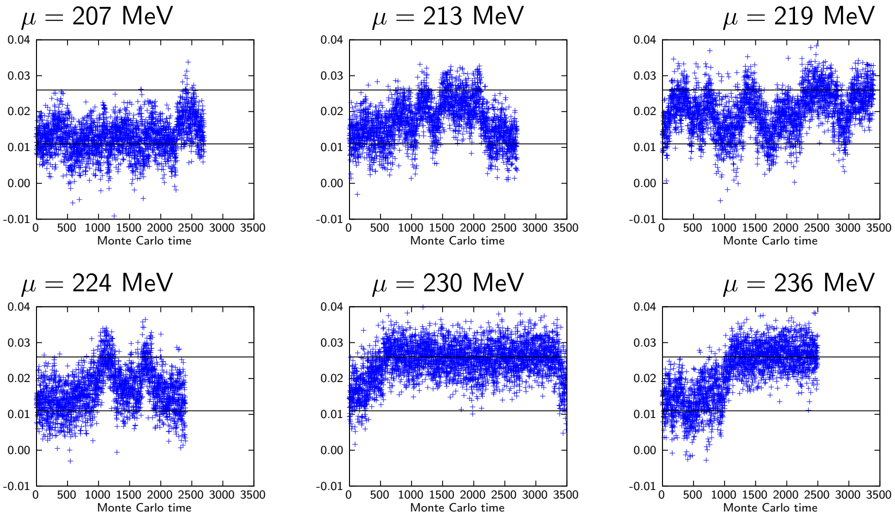
$T = 48$ MeV

Quark Number Density



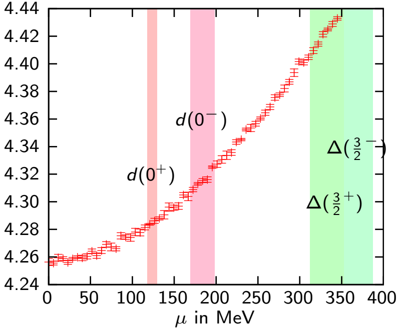
$T = 36$ MeV

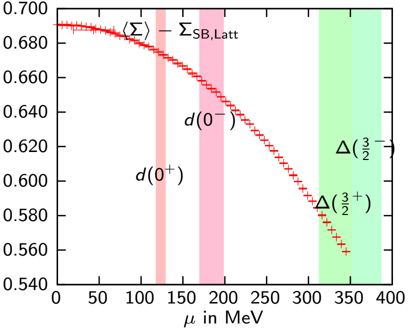
Quark number density at $T = 36$ MeV



First order nuclear matter transition?

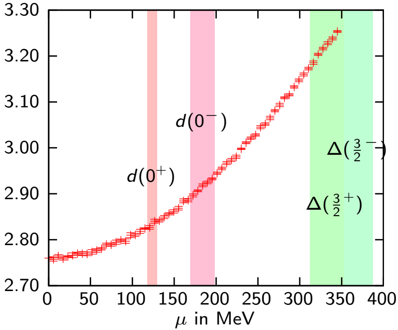
Polyakov Loop and Chiral Condensate

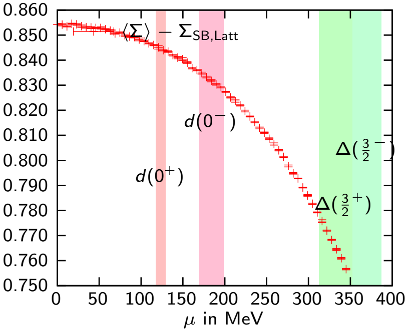




$T = 287$ MeV

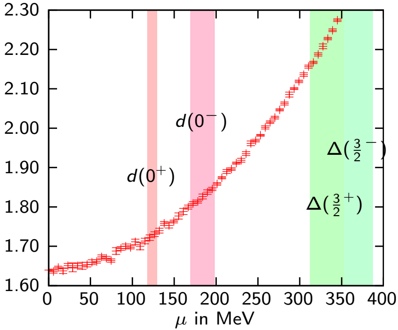
Polyakov Loop and Chiral Condensate

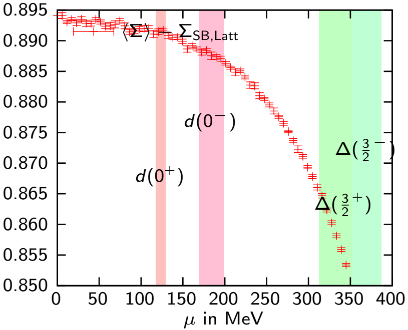




$T = 192$ MeV

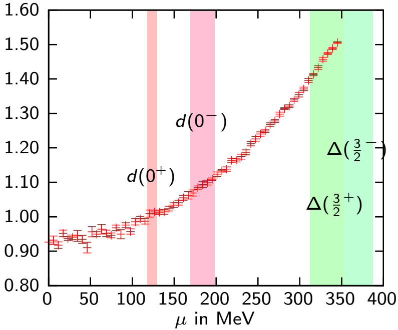
Polyakov Loop and Chiral Condensate

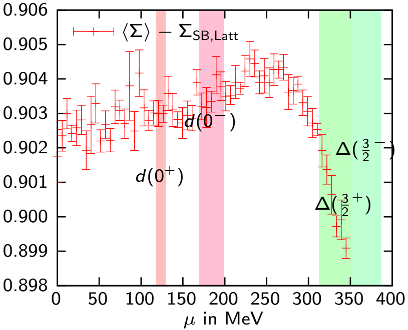




$T = 144$ MeV

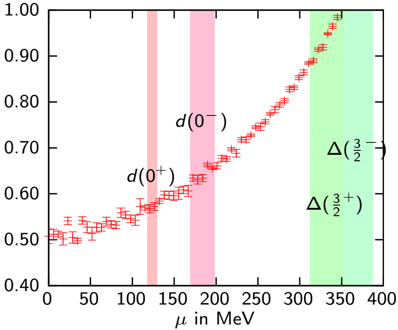
Polyakov Loop and Chiral Condensate

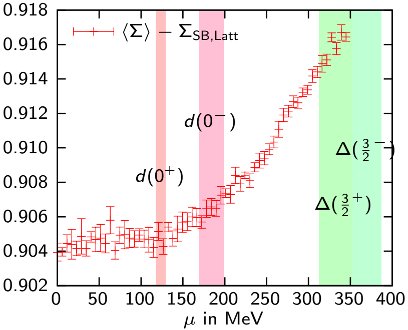




$T = 115$ MeV, $\mu_{\mathrm{deconf}} = 317$ Mev

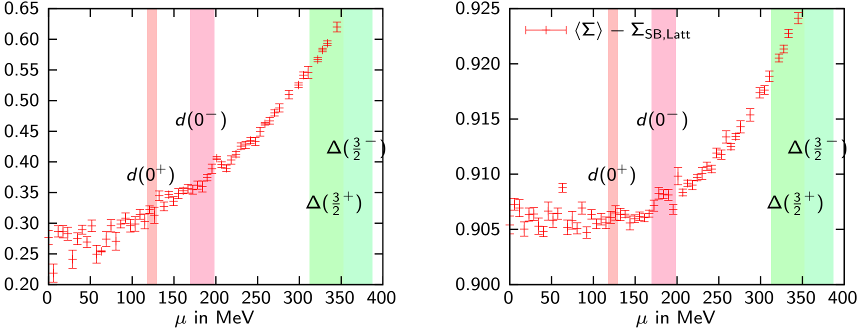
Polyakov Loop and Chiral Condensate





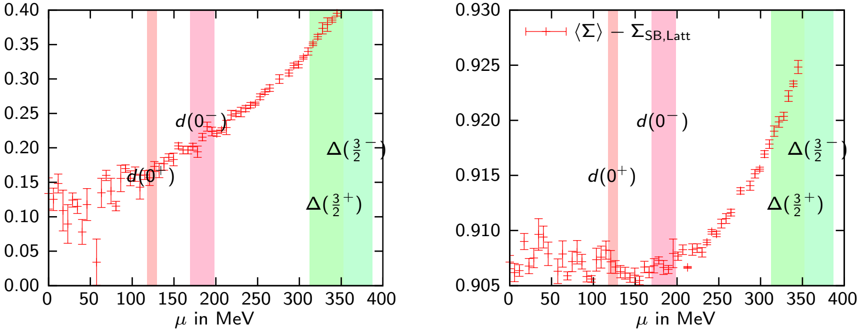
$T = 96$ MeV

Polyakov Loop and Chiral Condensate



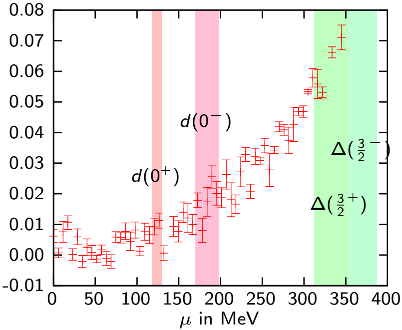
$T = 82$ MeV

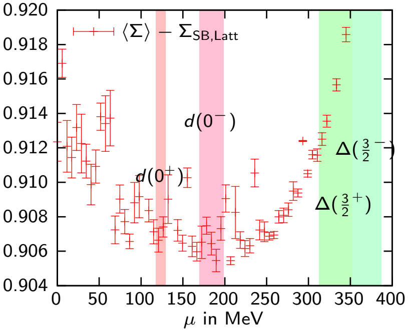
Polyakov Loop and Chiral Condensate



$T = 72$ MeV

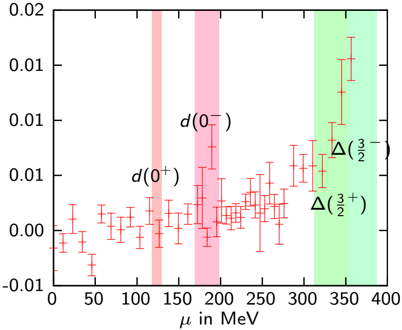
Polyakov Loop and Chiral Condensate

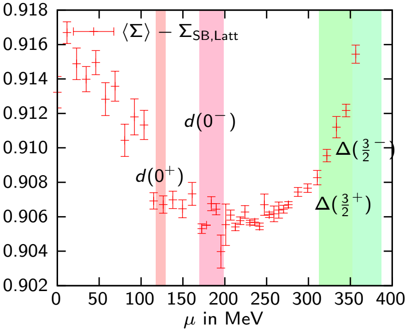




$T = 48$ MeV

Polyakov Loop and Chiral Condensate





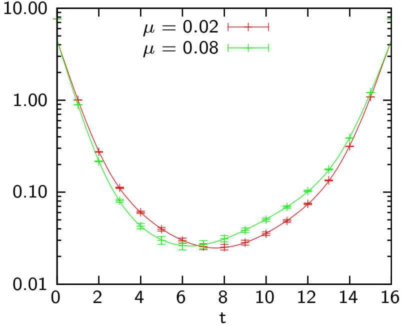
$T = 36$ MeV

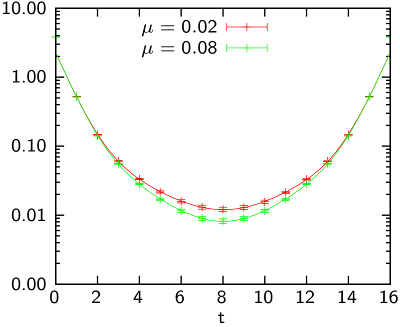
Correlation functions at finite density

Lattice correlation function for operator with quark number n_q

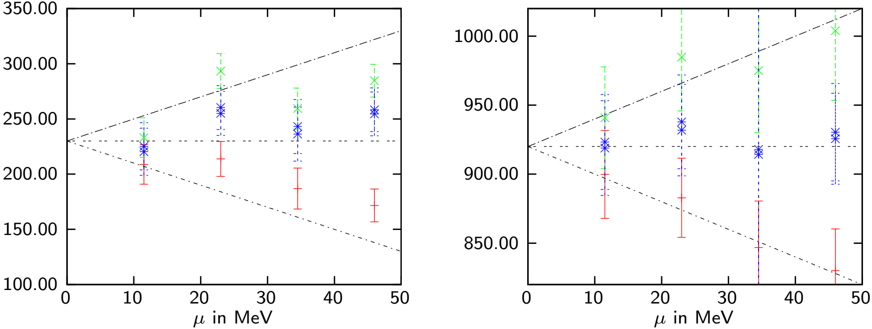
$$C(\mu, n_q) \sim a\exp^{-(m(\mu)+n_q\mu)t} + b\exp^{(m(\mu)-n_q\mu)t}$$

$d(0^+)$ and π correlation functions fitted with 4 exponentials for ground and excited states





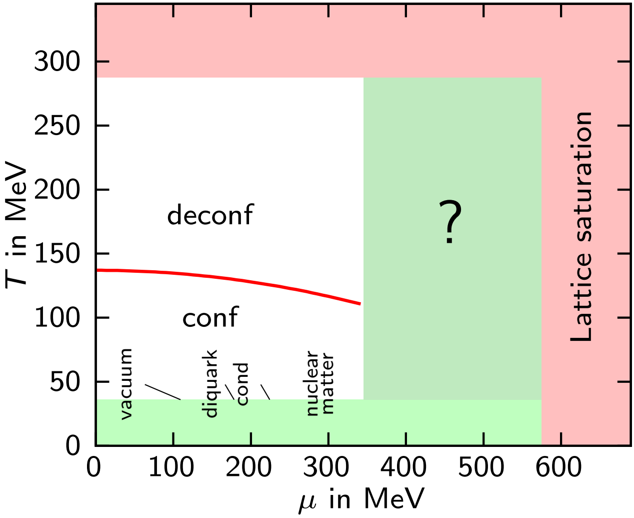
Ground state and first excited state masses of $d(0^+)$ and π



Below the silver blaze transition the masses do not depend on chemical potential

Conclusions and Outlook

G_2-QCD Phase diagram with $V = (2.7\text{fm})^3$ and $m_{d(0^+)} = 247$ MeV



- Finite size effects?
- First order nuclear matter transition?
- Chiral symmetry restoration?
- Deconfinement transition at lower temperatures / critical endpoint?

- Hybrid spectroscopy
- Spectroscopy at finite density
- Diquark condensation
- $N_f = 2$ Phase diagram
- Equations of state
- Comparison with different expansion methods
- and many more ...

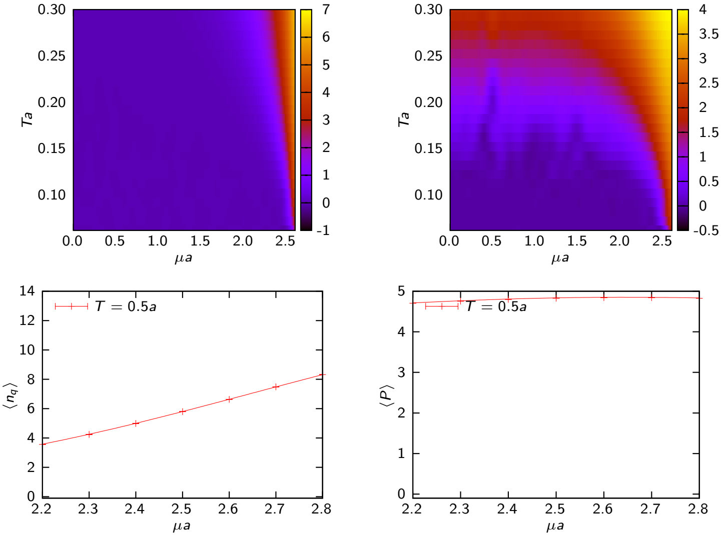
A. Maas, L. von Smekal, B. H. Wellegehausen and A. Wipf, *The phase diagram of a gauge theory with fermionic baryons*, arXiv:1203.5653 [hep-lat], 2012.

A. Maas, L. von Smekal, B. H. Wellegehausen and A. Wipf, *Hadron masses and baryonic scales in G2-QCD at finite density*, arXiv:1312.5579 [hep-lat], 2014.

Heavy quarks

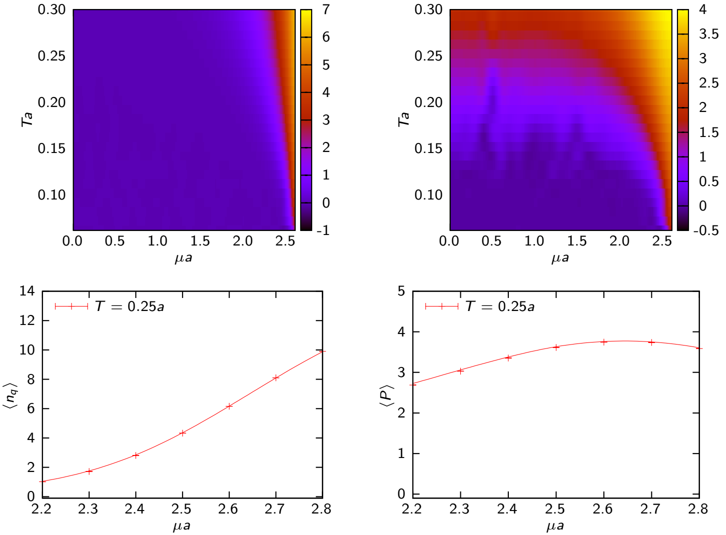
with Philipp Scior

Quark number density and Polyakov loop in Heavy G_2-QCD



G_2-QCD provides an important test for the strong coupling / hopping parameter expansion at finite temperature - quite good agreement in first simulations

Quark number density and Polyakov loop in Heavy G_2-QCD



G_2-QCD provides an important test for the strong coupling / hopping parameter expansion at finite temperature - quite good agreement in first simulations

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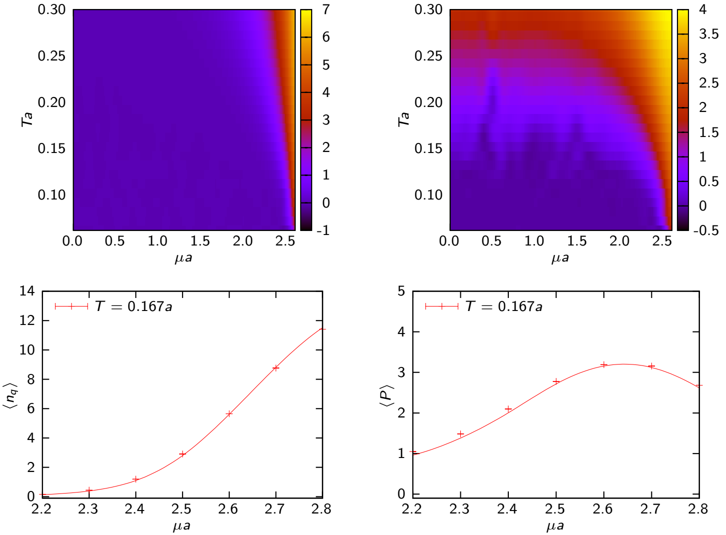
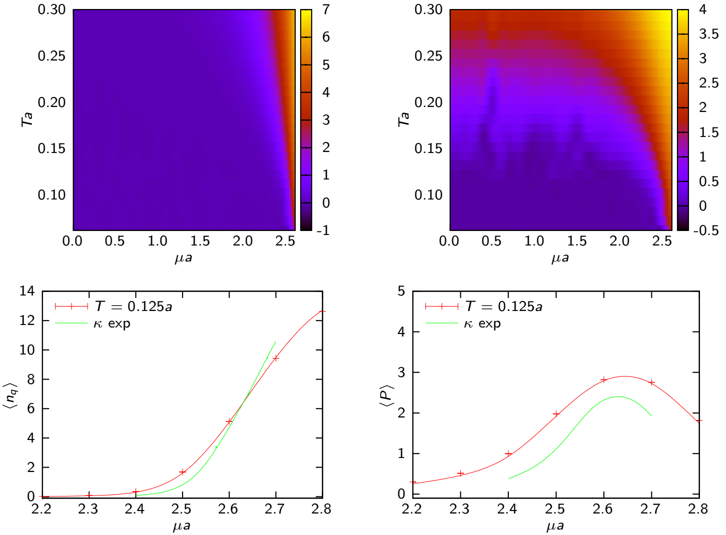


G_2-QCD provides an important test for the strong coupling / hopping parameter expansion at finite temperature - quite good agreement in first simulations

Quark number density and Polyakov loop in Heavy G_2-QCD



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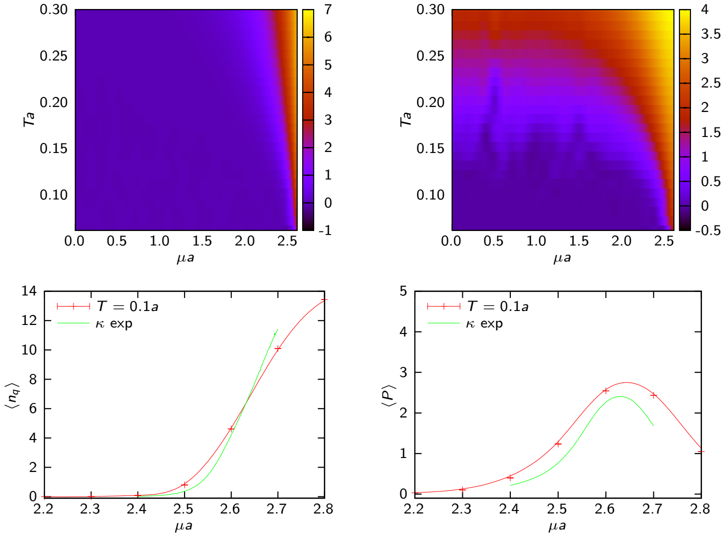

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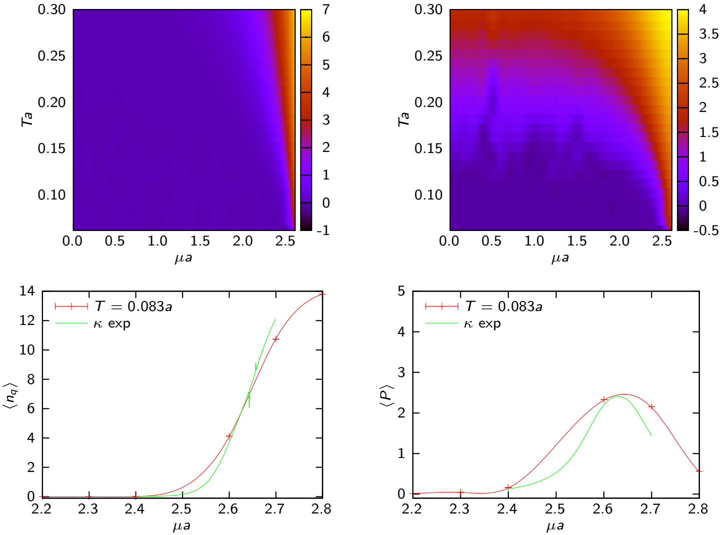

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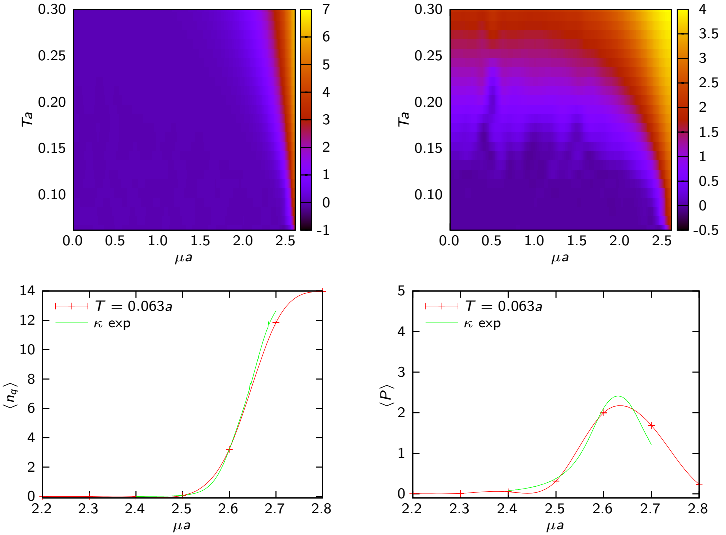

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