Determining the QCD coupling from lattice vacuum polarization

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motivation

α_s is a fundamental parameter of QCD.

Its numerical value is crucial input for most practical calculations in particle physics.

Interpretation of experimental data requires the value of α_s to handle QCD backgrounds.

Several lattice QCD methods have been employed to determine α_s, including

- the short-distance QCD potential
- Wilson loops
- the Schrödinger functional
- the ghost-gluon vertex
- current two-point functions with heavy valence quarks
- vacuum polarization at short distances ⬅

For a review and references, see FLAG Working Group, Eur. Phys. J. C74, 2890 (2014).

vacuum polarization at short distances

This method uses vacuum polarization as a function of Euclidean Q^2.

The perturbative expression is a function of α_s.

At small Q^2, there are also important non-perturbative (NP) contributions.

At large Q^2, there are also important lattice artifacts.

This method was pioneered by Shintani et al, Phys Rev D79, 074510 (2009);
82, 074505 (2010);
89, 099903 (2014).
and PoS (LATTICE 2013) 487.

They worked at Q^2 as low as $\sim$1 GeV^2 and included NP contributions via OPE in the fit.

This is problematic if successive OPE terms have comparable sizes with alternating signs.

Perhaps surprisingly, this phenomenon really does occur:

Boito, Golterman, Maltman, Osborne, Peris, Phys Rev D91, 034003 (2015):

[For numerical values of $C_{6,V}$ and $C_{8,V}$, see Table III.]

$$
\begin{aligned}
\Pi^{(1+0)}_{\rm OPE}(Q^2) &= \sum_{k=0}^{\infty} \frac{C_{2k}}{Q^{2k}} \\
C_{4,V+A} &= +0.00268\ {\rm GeV}^4 \\
C_{6,V+A} &= -0.0125\ {\rm GeV}^6 \\
C_{8,V+A} &= +0.0349\ {\rm GeV}^8 \\
C_{10,V+A} &= -0.0832\ {\rm GeV}^{10} \\
C_{12,V+A} &= +0.161\ {\rm GeV}^{12} \\
C_{14,V+A} &= -0.191\ {\rm GeV}^{14} \\
C_{16,V+A} &= -0.233\ {\rm GeV}^{16}
\end{aligned}
$$

In the present project we work at larger Q^2 where all NP OPE terms are negligible.

method

The vector current two-point correlation function with $I{=}1$ and $m_u{=}m_d$ is

$$\begin{aligned}\langle V_\mu V_\nu\rangle &\equiv \Pi_{\mu\nu}(Q) \\ &= (Q^2\delta_{\mu\nu} - Q_\mu Q_\nu)\Pi(Q^2)\end{aligned}$$

The QCD coupling will be obtained from $\Pi(Q^2)$.

There is a close relation to tau decay α_s determination
(which uses experimental spectral data and finite-energy sum rule analysis of same $\Pi(Q^2)$)
but here we have certain systematic advantages.

The perturbative expression up to 6 loops (in $\overline{\mathrm{MS}}$ at scale μ) is

$$\Pi(Q^2) = C - \frac{1}{4\pi^2}\left(t + \sum_{k=1}^{5}\left(\frac{\alpha_s(\mu)}{\pi}\right)^k \sum_{m=0}^{k-1} c^A_{km}\frac{t^{m+1}}{m+1}\right)$$

where C is a constant and $t = \ln(Q^2/\mu^2)$.

All coefficients are known except c^A_{50} (which has been estimated).

Baikov, Chetyrkin, Kuhn, Phys Rev Lett 101, 012002 (2008)

managing lattice artifacts

Discretization errors grow as Q^2 increases, but they can be managed.
Choosing the momentum to be along a single lattice axis is particularly undesirable.

- We have generated $\Pi(Q^2)$ for all possible momenta Q_μ.

With those data in hand, we define $\hat{v} = (1,1,1,1)/2$ and calculate

$$(Q_\perp)_\mu = Q_\mu - (Q\cdot\hat{v})\hat{v}_\mu$$

If Q_μ points along the lattice diagonal then $Q_\perp = 0$.
For fixed $|Q^2|$, those Q_μ options closest to $(Q_\perp)_\mu$ have the smallest lattice artifacts.
Therefore we choose a maximum radius $|Q_\perp|$.

- We handle O(4)-breaking lattice artifacts via reflection averaging:

$$\Pi_{\rm lat}(Q^2) = \frac{1}{12}\sum_{\mu=x,y,z,t}\sum_{\nu\neq\mu}\left(\frac{\Pi_{\mu\nu}(Q) - \Pi_{\mu\nu}(R_\mu Q)}{2Q_\mu Q_\nu}\right)$$

where R_μ is a reflection operator in the μ direction.
Note: $Q=0$ is no problem because it will be outside the maximum radius.

- O(4)-preserving artifacts remain to be fitted:

$$\Pi_{\rm lat}(Q^2) = \Pi(Q^2) + c_1 a^2 Q^2 + c_2 a^4 Q^4 + \ldots$$

lattice data

We use ensembles from RBC/UKQCD, Phys. Rev. D87, 094514 (2013)

lattice size	β	m_s	m_ℓ	a^{-1} [GeV]	Z_V	# configurations
$24^3\times 64$	2.13	0.04	0.005,0.01,0.02	1.78	0.714	900
$32^3\times 64$	2.25	0.03	0.004,0.006,0.008	2.38	0.745	940

We calculate $\langle V_\mu^L V_\nu^C \rangle$. The local current removes a contact term.
The conserved current preserves the Ward-Takahashi identity.

$$\sum_\nu \hat{Q}_\nu e^{iQ_\nu/2}\Pi_{\mu\nu} = 0$$

$$\hat{Q}_\nu = 2\sin(Q_\nu/2)$$

$\Pi(Q^2)$ only depends on α_s at subleading orders.

We use a renormalization-independent function where α_s appears at leading order:

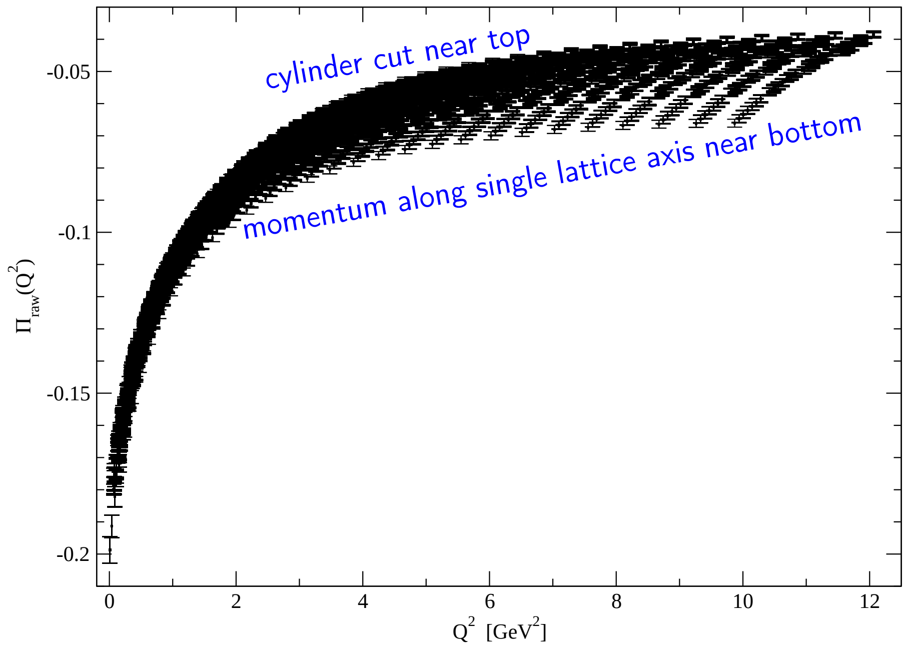
$$\Delta(Q_1^2, Q_2^2) \equiv -4\pi^2 \left(\frac{\Pi_{\rm lat}(Q_1^2) - \Pi_{\rm lat}(Q_2^2)}{\ln(\hat{Q}_1^2/\hat{Q}_2^2)} \right) - 1$$

$$= \frac{\alpha_s(\mu)}{\pi} + \text{ higher orders}$$

removing O(4)-breaking lattice artifacts

The raw lattice data show a fishbone pattern due to lattice artifacts.

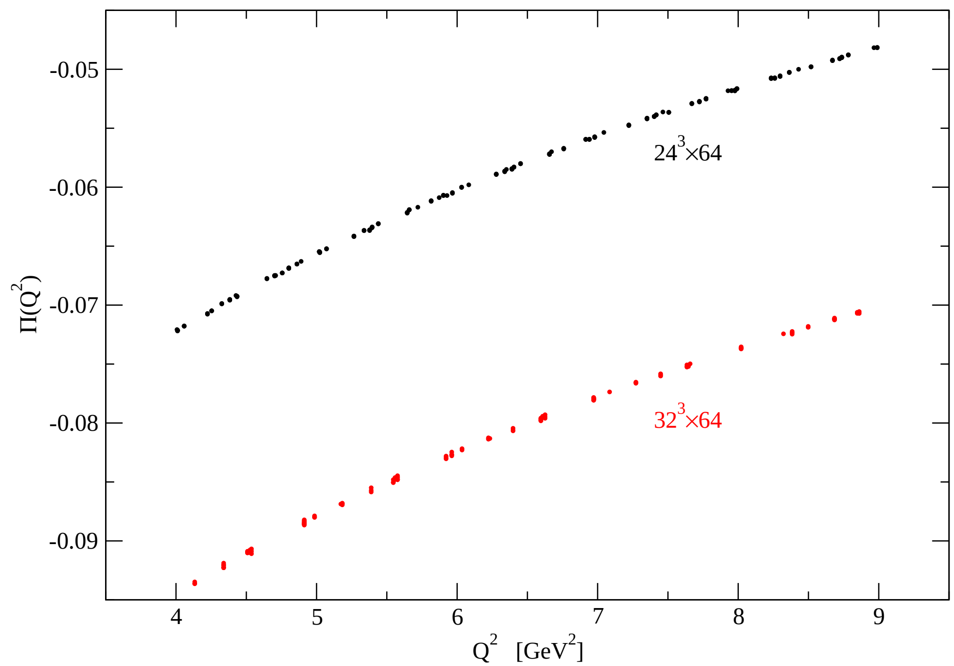
$$\Pi_{\rm raw}(Q^2) = \frac{-1}{3\hat{Q}^2}\left(\delta_{\mu\nu} - \frac{4\hat{Q}_\mu\hat{Q}_\nu}{\hat{Q}^2}\right)\Pi_{\mu\nu}(Q^2)$$



[5 configurations for $s\bar{s}$ valence quarks]

Reflection averaging produces a smooth curve.

$$\Pi_{\rm lat}(Q^2) = \frac{1}{12}\sum_{\mu=x,y,z,t}\sum_{\nu\neq\mu}\left(\frac{\Pi_{\mu\nu}(Q)-\Pi_{\mu\nu}(R_\mu Q)}{2\hat{Q}_\mu\hat{Q}_\nu}\right)$$

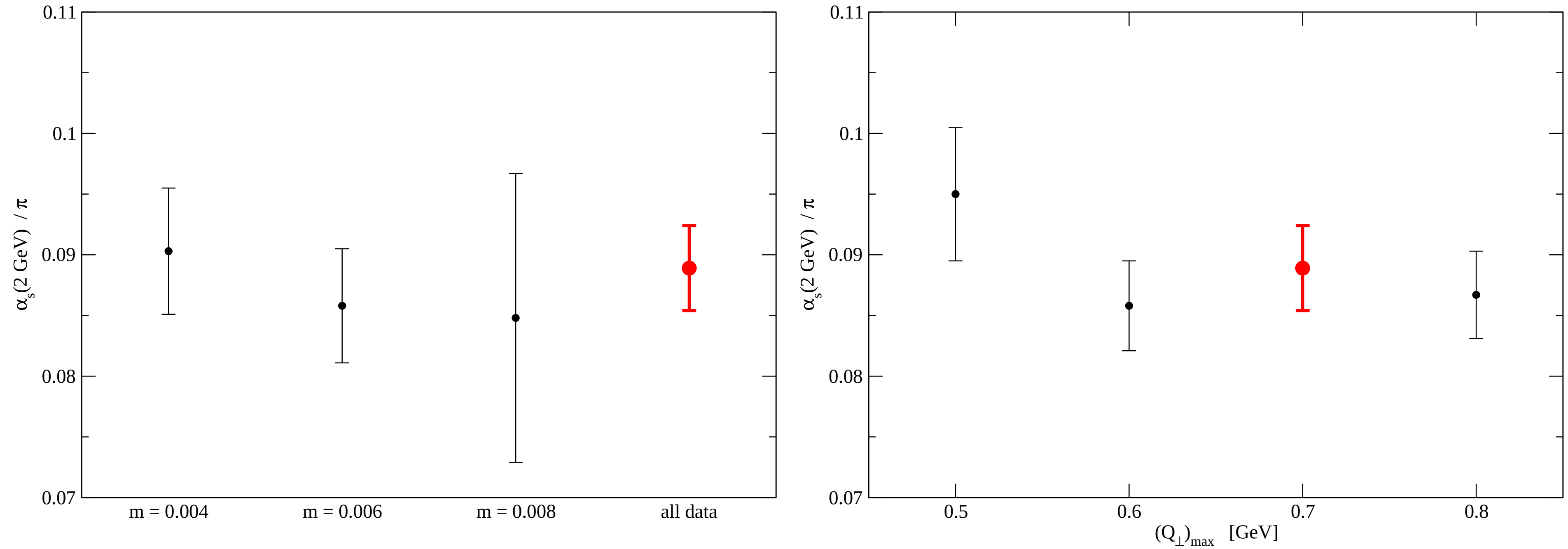


[cylinder cut: $|Q_\perp| < (Q_\perp)_{\rm max}$]

stability of fitted α_s

2-parameter fits to $\Pi_{\rm lat}(Q^2) = \Pi(Q^2) + c_1 a^2 \hat{Q}^2$ on fine lattice

varying the light quark mass varying the cylinder cut



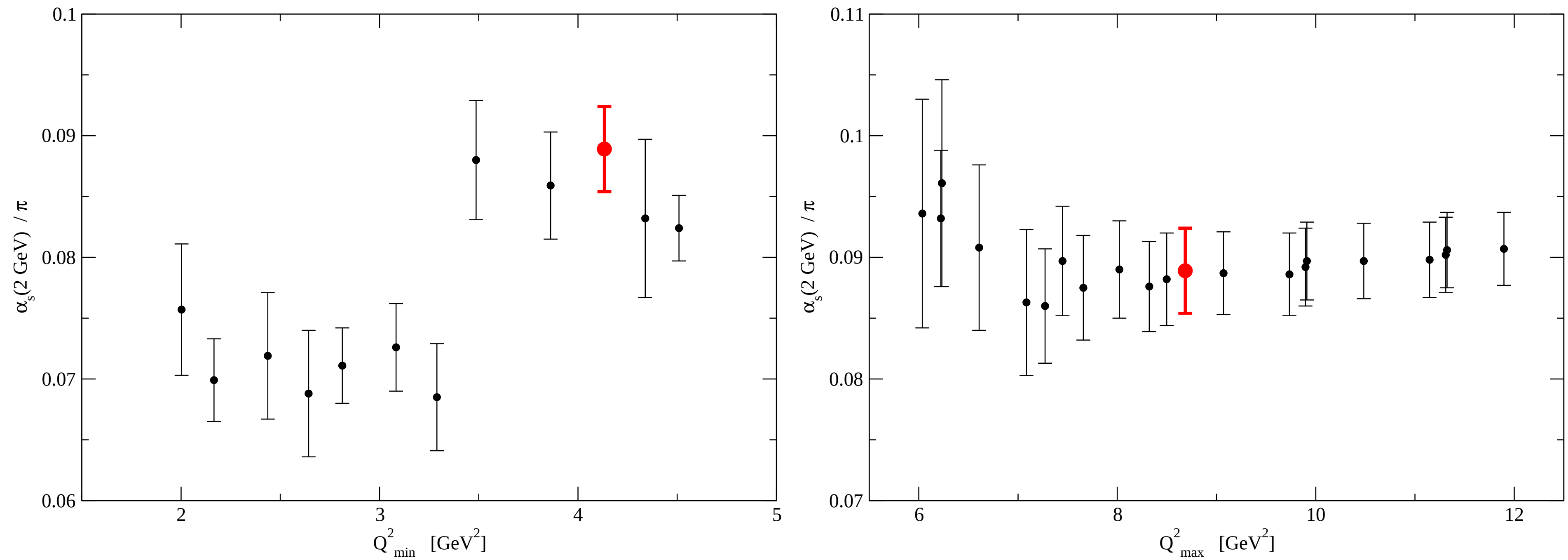
(The thick red data point is our default value that reappears in several graphs.)

varying the fit range

2-parameter fits to $\Pi_{\rm lat}(Q^2) = \Pi(Q^2) + c_1 a^2 \hat{Q}^2$ on fine lattice

Due to experience from τ decay phenomenology, we need $Q^2_{\rm min} \gtrsim 4$ GeV^2.
Here we explore beyond that range for interest.

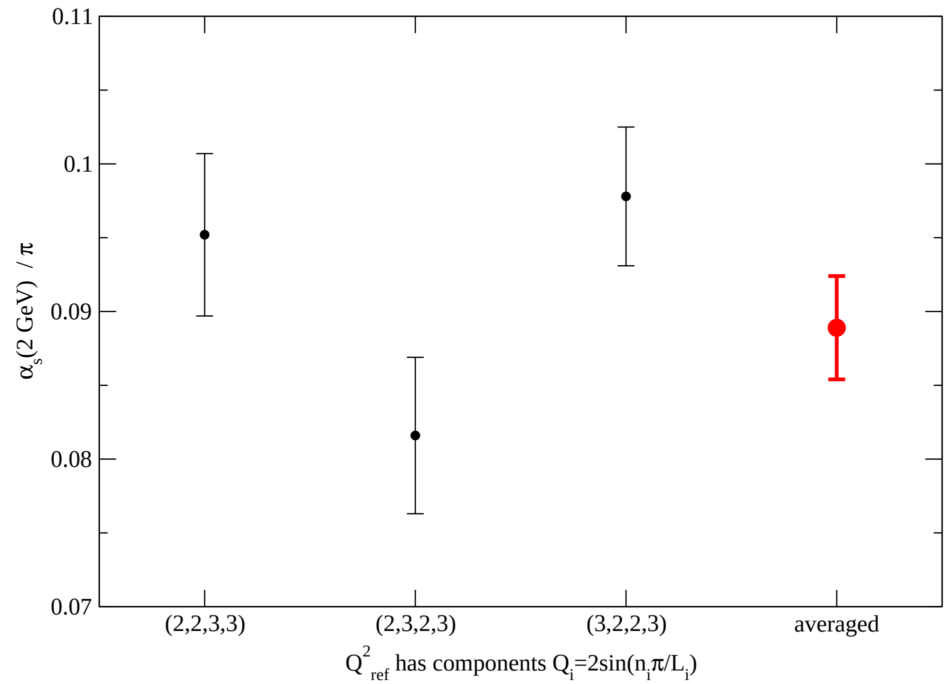
Results should be insensitive to $Q^2_{\rm max}$ if lattice artifacts are under control.

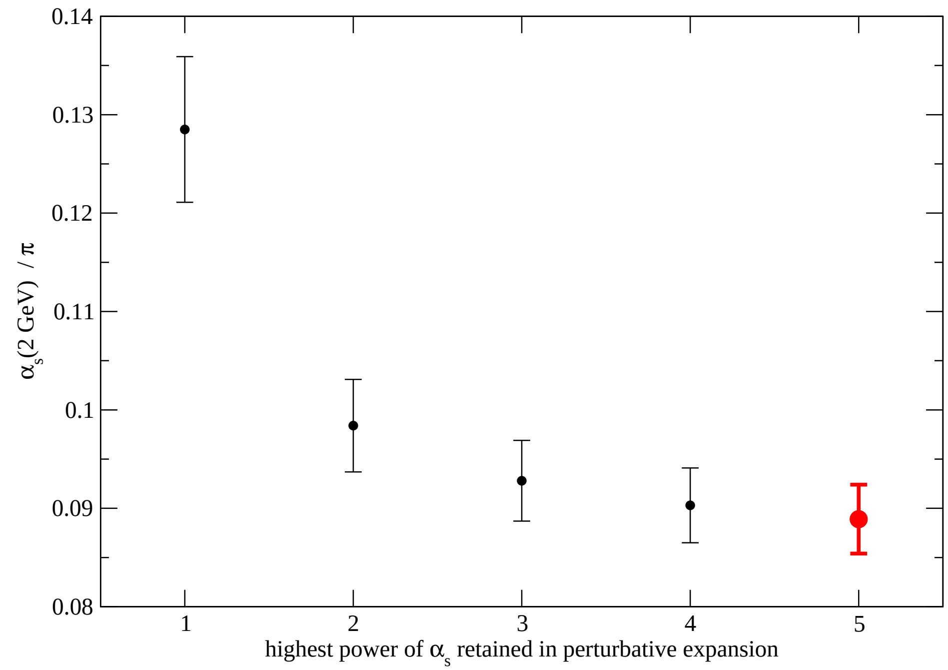


(The thick red data point is our default value that reappears in several graphs.)

varying $Q^2_{\rm ref}$ varying perturbative order

2-parameter fits to $\Pi_{\rm lat}(Q^2) = \Pi(Q^2) + c_1 a^2 \hat{Q}^2$ on fine lattice





$$\Delta(Q^2, Q^2_{\rm ref}) \equiv -4\pi^2 \left(\frac{\Pi_{\rm lat}(Q^2) - \Pi_{\rm lat}(Q^2_{\rm ref})}{\ln(\hat{Q}^2/\hat{Q}^2_{\rm ref})} \right) - 1$$

The estimated coefficient c^A_{50} appears at α_s^5. Our result is insensitive to its precise value.

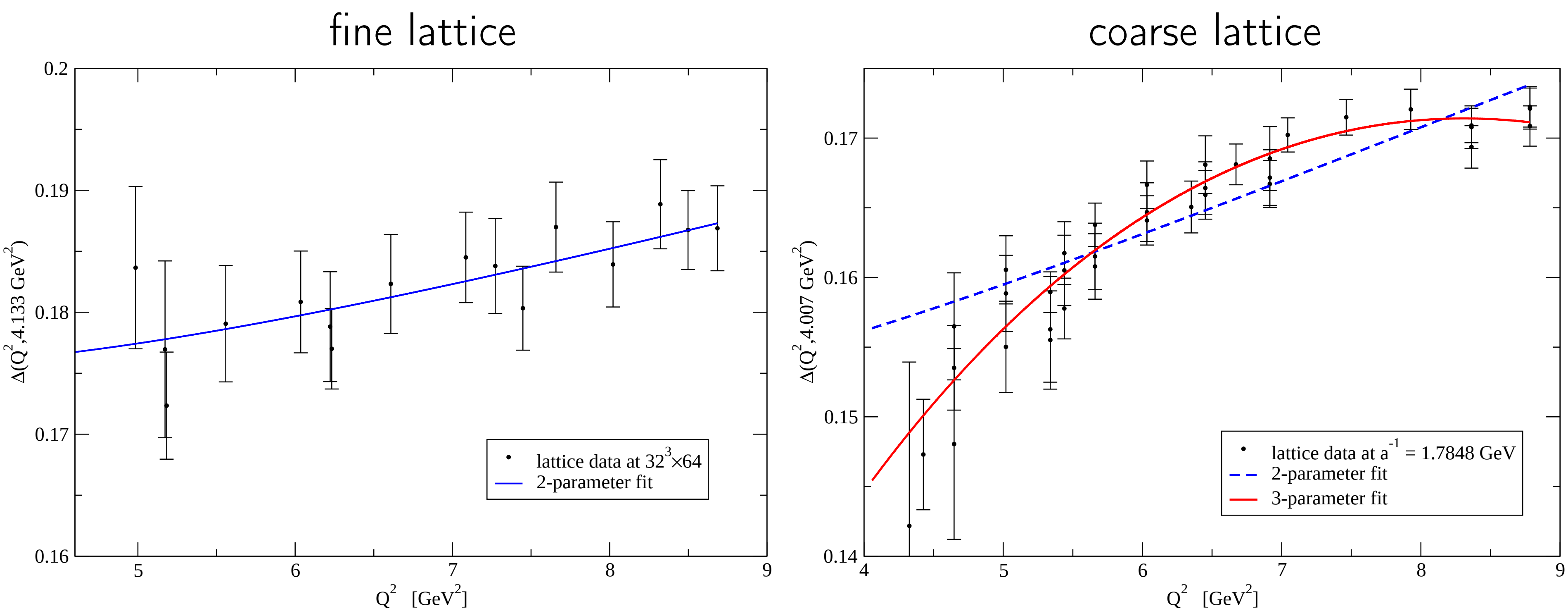
(The thick red data point is our default value that reappears in several graphs.)

attempting the coarse lattice

Results displayed so far were for $32^3{\times}64$ with $a^{-1} = 2.38$ GeV.

We should perform a similar analysis for $24^3{\times}64$ with $a^{-1} = 1.78$ GeV but

- a two-parameter fit (α_s and c_1) does not describe the data well.
- a three-parameter fit (α_s, c_1 and c_2) allows a huge error bar for α_s.



Conclusions: A two-parameter fit is not sufficient for the coarse lattice.

The statistical precision is not presently available to get α_s from the 3-parameter fit.

numerical result for α_s

The analysis reported here gives

$$\frac{\alpha_s(2\,\mathrm{GeV})}{\pi} = 0.0889 \pm 0.0035$$

and running to the τ mass (alternate analysis of same $\Pi(Q^2)$) gives

$$\alpha_s^{(3)}(m_\tau) = 0.296 \pm 0.013$$

which is in excellent agreement with results that use τ decay data from experiment. In particular, the recent continuum finite-energy sum rule (FESR) analysis of the 2013/14 corrected and updated ALEPH hadronic tau decay data arrived at

$$\alpha_s^{(3)}(m_\tau) = \begin{cases} 0.296 \pm 0.010 & \text{[fixed-order perturbation theory]} \\ 0.310 \pm 0.014 & \text{[contour-improved perturbation theory]} \end{cases}$$

Boito, Golterman, Maltman, Osborne and Peris, Nucl. Part. Phys. Proc. 260, 134 (2015)

Running our result to m_Z in the 5-flavor theory gives $\alpha_s^{(5)}(m_Z) = 0.1155 \pm 0.0018$.

For comparison, the FLAG Working Group result is $\alpha_s^{(5)}(m_Z) = 0.1184 \pm 0.0012$.

FLAG Working Group, Eur. Phys. J. C74, 2890 (2014).

summary

We have presented a new implementation to obtain α_s from vacuum polarization at short distances.

It avoids reliance on the OPE and the dangers of an alternating-sign series.

Our method needs $\Pi(Q^2)$ for many off-axis lattice directions.

Our numerical results are competitive with the determination of α_s from tau decay.

For the future: data from finer lattices would allow a study of the continuum limit.