Lattice QCD calculations of nucleon transverse momentum-dependent parton distributions (TMDs) at 170 MeV pion mass

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(Lattice TMD Collaboration)

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Gauge ensembles provided by:
RBC/UKQCD Collaboration

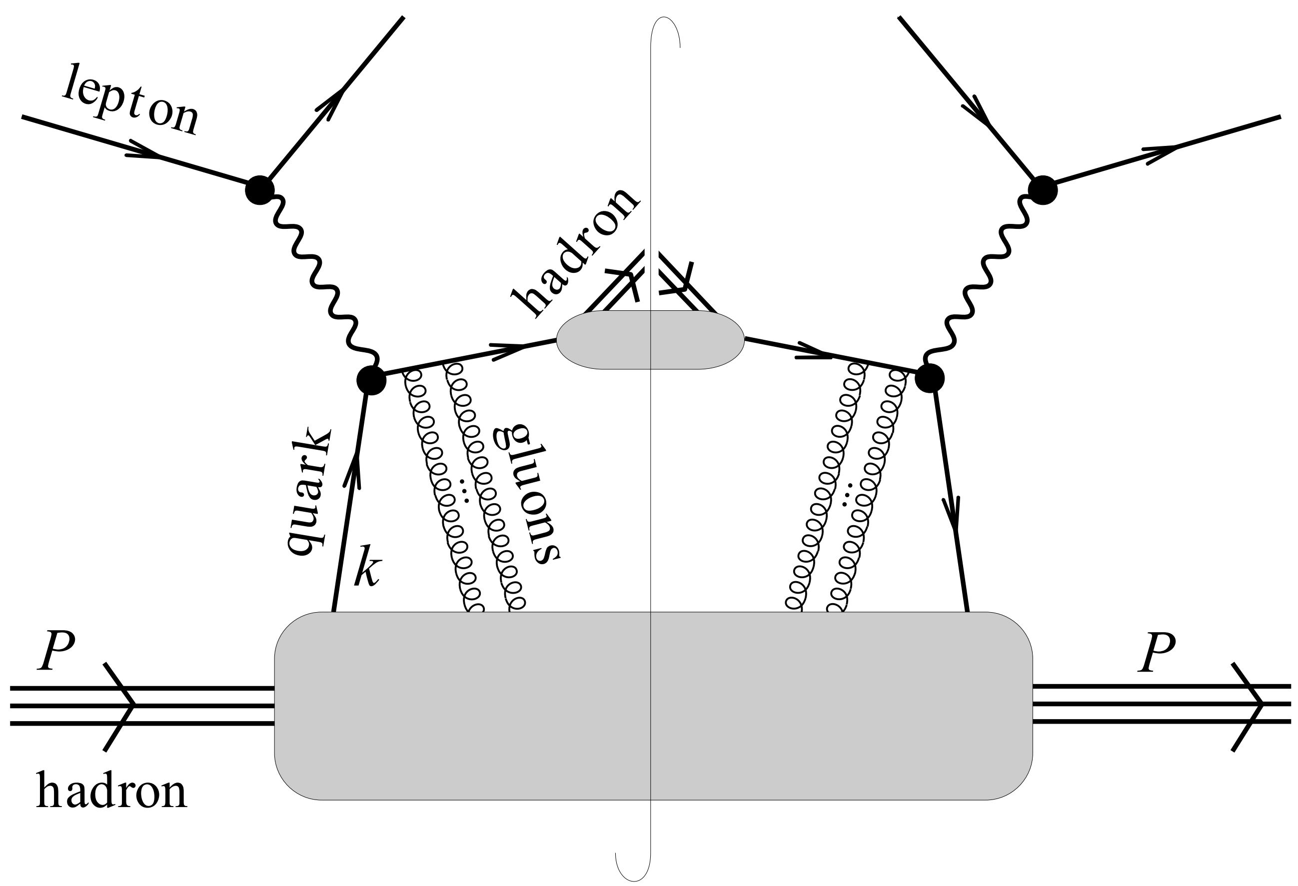
Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots) \equiv \frac{1}{2}\langle P,S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0,\ldots,b]\ q(b)\ |P,S\rangle$$

$$\Phi^{[\Gamma]}(x,k_T,P,S,\ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b\cdot P)}{(2\pi)P^+} \exp\left(ix(b\cdot P) - ib_T\cdot k_T\right) \frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots)}{\widetilde{\mathcal{S}}(b^2,\ldots)}\Bigg|_{b^+=0}$$

- "Soft factor" $\widetilde{\mathcal{S}}$ required to subtract divergences of Wilson line $\mathcal{U}$
- $\widetilde{\mathcal{S}}$ is typically a combination of vacuum expectation values of Wilson line structures
- Here, will consider only ratios in which soft factors cancel

Gauge link structure motivated by SIDIS



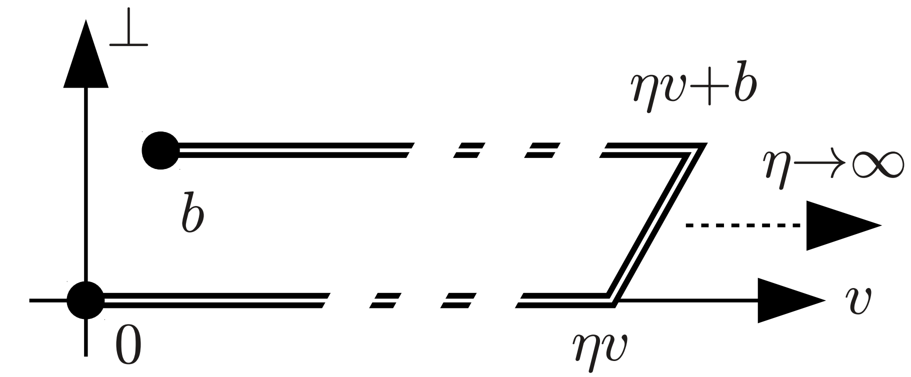
$l + H(P) \longrightarrow l' + h(P_h) + X$

Gauge link structure:

In matrix element $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv$

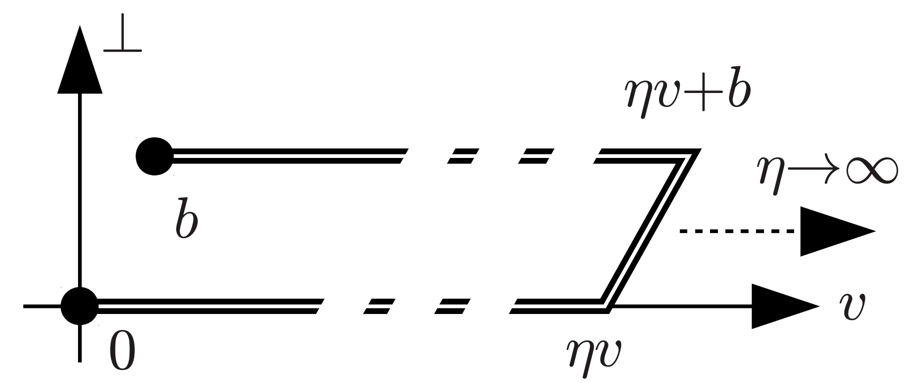
$\frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \ldots, b]\ q(b)\ |P, S\rangle$

Staple-shaped gauge link $\mathcal{U}[0, \eta v, \eta v + b, b]$



incorporates SIDIS final state effects

Gauge link structure motivated by SIDIS



Beyond tree level: Rapidity divergences suggest taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes v space-like. Parametrize in terms of Collins-Soper parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for $\hat{\zeta} \to \infty$. Perturbative evolution equations for large $\hat{\zeta}$.

Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2} \langle P, S| \ \bar{q}(0) \ \Gamma \ \mathcal{U}[0, \eta v, \eta v + b, b] \ q(b) \ |P, S\rangle$$

$$\Phi^{[\Gamma]}(x, k_T, P, S, \ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} \exp\left(ix(b \cdot P) - ib_T \cdot k_T\right) \left. \frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots)}{\widetilde{\mathcal{S}}(b^2, \ldots)} \right|_{b^+=0}$$

- "Soft factor" $\widetilde{\mathcal{S}}$ required to subtract divergences of Wilson line $\mathcal{U}$
- $\widetilde{\mathcal{S}}$ is typically a combination of vacuum expectation values of Wilson line structures
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Decomposition of Φ into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[\frac{\epsilon_{ij} k_i S_j}{m_H} f_{1T}^{\perp}\right] \text{odd}$$

$$\Phi^{[\gamma^+ \gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_H} g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_H^2} h_{1T}^{\perp} + \frac{\Lambda k_i}{m_H} h_{1L}^{\perp} + \left[\frac{\epsilon_{ij} k_j}{m_H} h_1^{\perp}\right] \text{odd}$$

TMD Classification

All leading twist structures:

H ↓ \ q →	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

← Boer-Mulders (T-odd)

↑ Sivers (T-odd)

Decomposition of $\widetilde{\Phi}$ into amplitudes

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}} = \widetilde{A}_{2B} + i m_H \epsilon_{ij} b_i S_j \widetilde{A}_{12B}$$

$$\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+\gamma^5]}_{\text{unsubtr.}} = -\Lambda \widetilde{A}_{6B} + i[(b\cdot P)\Lambda - m_H(b_T\cdot S_T)]\widetilde{A}_{7B}$$

$$\frac{1}{2P^+}\widetilde{\Phi}^{[i\sigma^{i+}\gamma^5]}_{\text{unsubtr.}} = i m_H \epsilon_{ij} b_j \widetilde{A}_{4B} - S_i \widetilde{A}_{9B} - i m_H \Lambda b_i \widetilde{A}_{10B} + m_H[(b\cdot P)\Lambda - m_H(b_T\cdot S_T)] b_i \widetilde{A}_{11B}$$

(Decompositions analogous to work by Metz et al. in momentum space)

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \ldots) \equiv \int d^2k_T \, \exp(ib_T \cdot k_T) f(x, k_T^2, \ldots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \ldots) \equiv n! \left(-\frac{2}{m_H^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \ldots)$$

In limit $|b_T| \to 0$, recover k_T-moments:

$$\tilde{f}^{(n)}(x, 0, \ldots) \equiv \int d^2k_T \left(\frac{k_T^2}{2m_H^2} \right)^n f(x, k_T^2, \ldots) \equiv f^{(n)}(x)$$

ill-defined for large k_T, so will not attempt to extrapolate to $b_T = 0$, but give results at finite $|b_T|$.

In this study, only consider first x-moments (accessible at $b \cdot P = 0$), rather than scanning range of $b \cdot P$:

$$f^{[1]}(k_T^2, \ldots) \equiv \int_{-1}^{1} dx \, f(x, k_T^2, \ldots)$$

$\longrightarrow$ Bessel-weighted asymmetries (Boer, Gamberg, Musch, Prokudin, JHEP 1110 (2011) 021)

Relation between Fourier-transformed TMDs and invariant amplitudes $\widetilde{A}_i$

Invariant amplitudes directly give selected x-integrated TMDs in Fourier (b_T) space (here, showing examples relevant for Sivers shift), up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = -2\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

Generalized shifts

Form ratios in which soft factors, (Γ-independent) multiplicative renormalization factors cancel

Sivers shift:

$$\langle k_y \rangle_{TU} \equiv m_H \frac{\tilde{f}_{1T}^{\perp[1](1)}}{\tilde{f}_1^{[1](0)}} = \left. \frac{\int dx \int d^2k_T \, k_y \Phi^{[\gamma^+]}(x, k_T, P, S, \ldots)}{\int dx \int d^2k_T \, \Phi^{[\gamma^+]}(x, k_T, P, S, \ldots)} \right|_{S_T=(1,0)}$$

Average transverse momentum of unpolarized ("U") quarks in a hadron polarized in the orthogonal transverse ("T") direction; normalized to the number of valence quarks. "Dipole moment" in $b_T^2 = 0$ limit, "shift".

Issue: k_T-moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero* b_T^2,

$$\langle k_y \rangle_{TU}(b_T^2, \ldots) \equiv m_H \frac{\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)}$$

(remember singular $b_T \to 0$ limit corresponds to taking k_T-moment). "Generalized shift".

Generalized shifts from amplitudes

Now, can also express this in terms of invariant amplitudes:

$$\langle k_y \rangle_{TU}(b_T^2, \ldots) \equiv m_H \frac{\tilde{f}_{1T}^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = -m_H \frac{\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

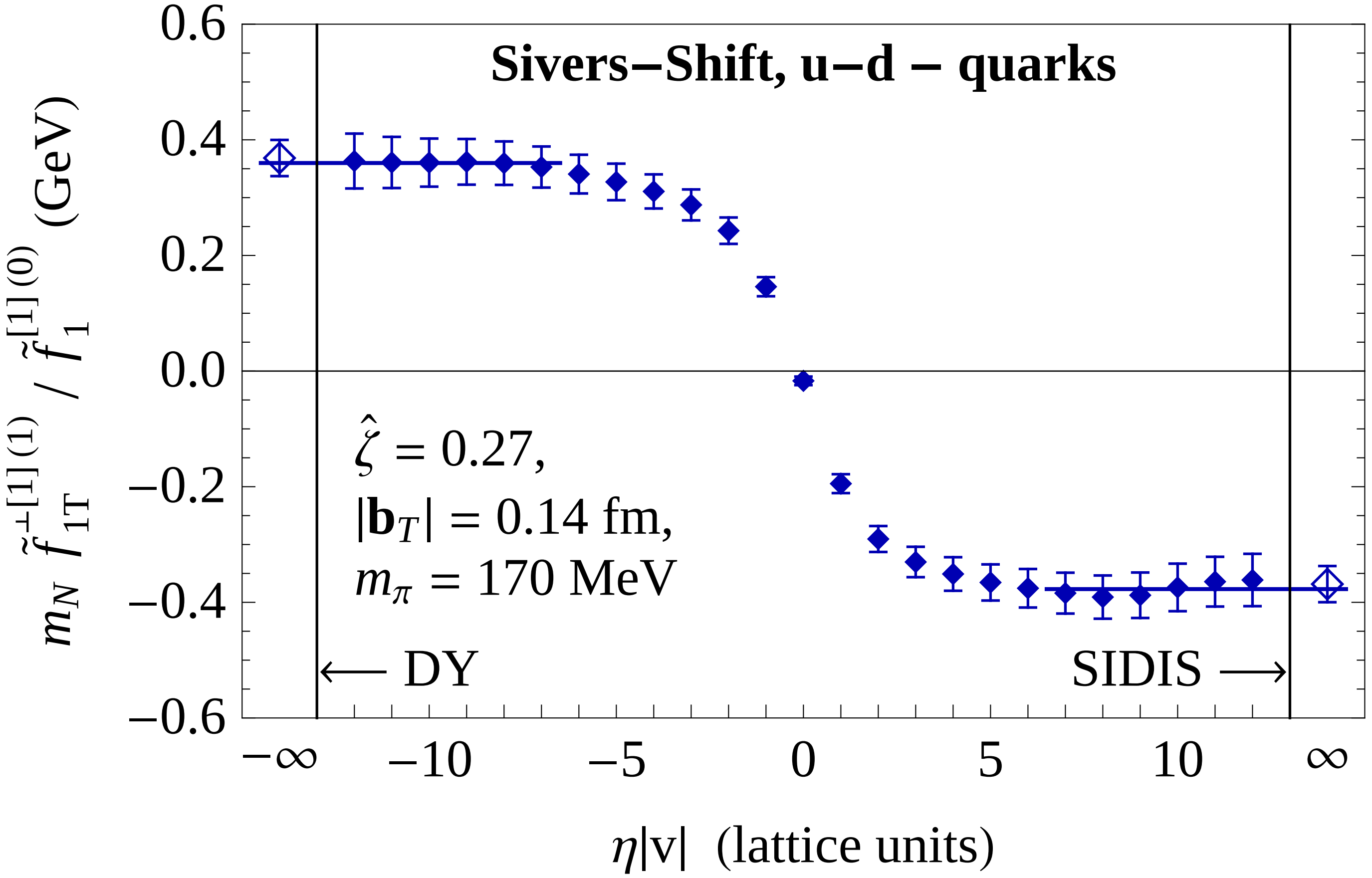
Analogously for generalized Boer-Mulders shift, transversity and g_{1T} worm gear shift

Lattice setup

- Evaluate directly $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$
- Euclidean time: Place entire operator at one time slice, i.e., b, ηv purely spatial
- Since generic b, v space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of $\widetilde{A}_i$ invariants permits direct translation of results back to original frame
- Form desired ratios of $\widetilde{A}_i$ invariants
- Extrapolate $\eta \to \infty$, $\hat{\zeta} \to \infty$ numerically.
- Here: Use RBC/UKQCD 2+1-flavor gauge ensemble with $a \approx 0.144\,\text{fm}$, $m_\pi = 170\,\text{MeV}$; compare with previous calculation on RBC/UKQCD 2+1-flavor gauge ensemble with $a \approx 0.084\,\text{fm}$, $m_\pi = 297\,\text{MeV}$.
- Use variety of P, b, ηv; here $b \perp P$, $b \perp v$ (lowest x-moment, kinematical choices/constraints)

Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



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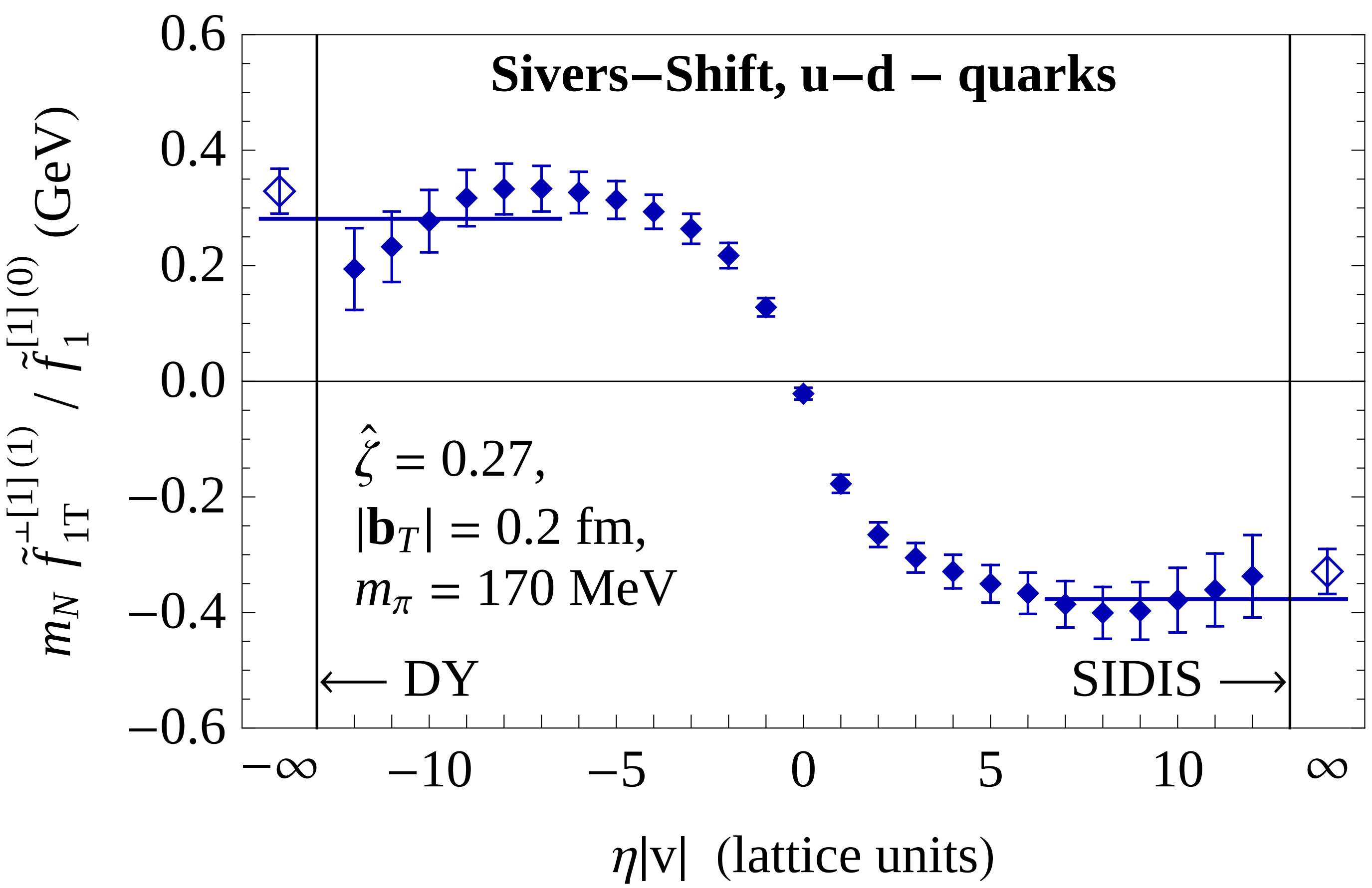

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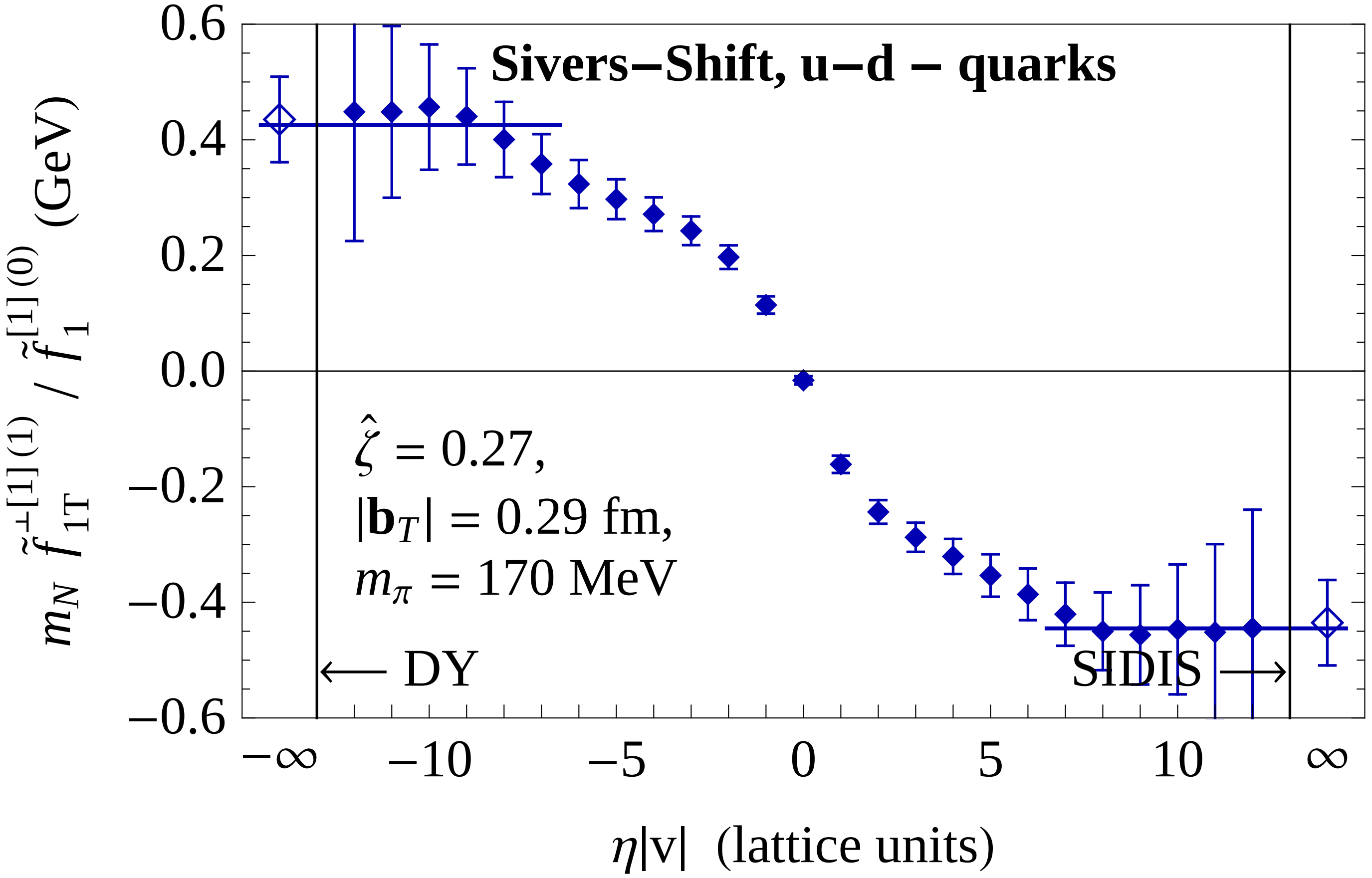

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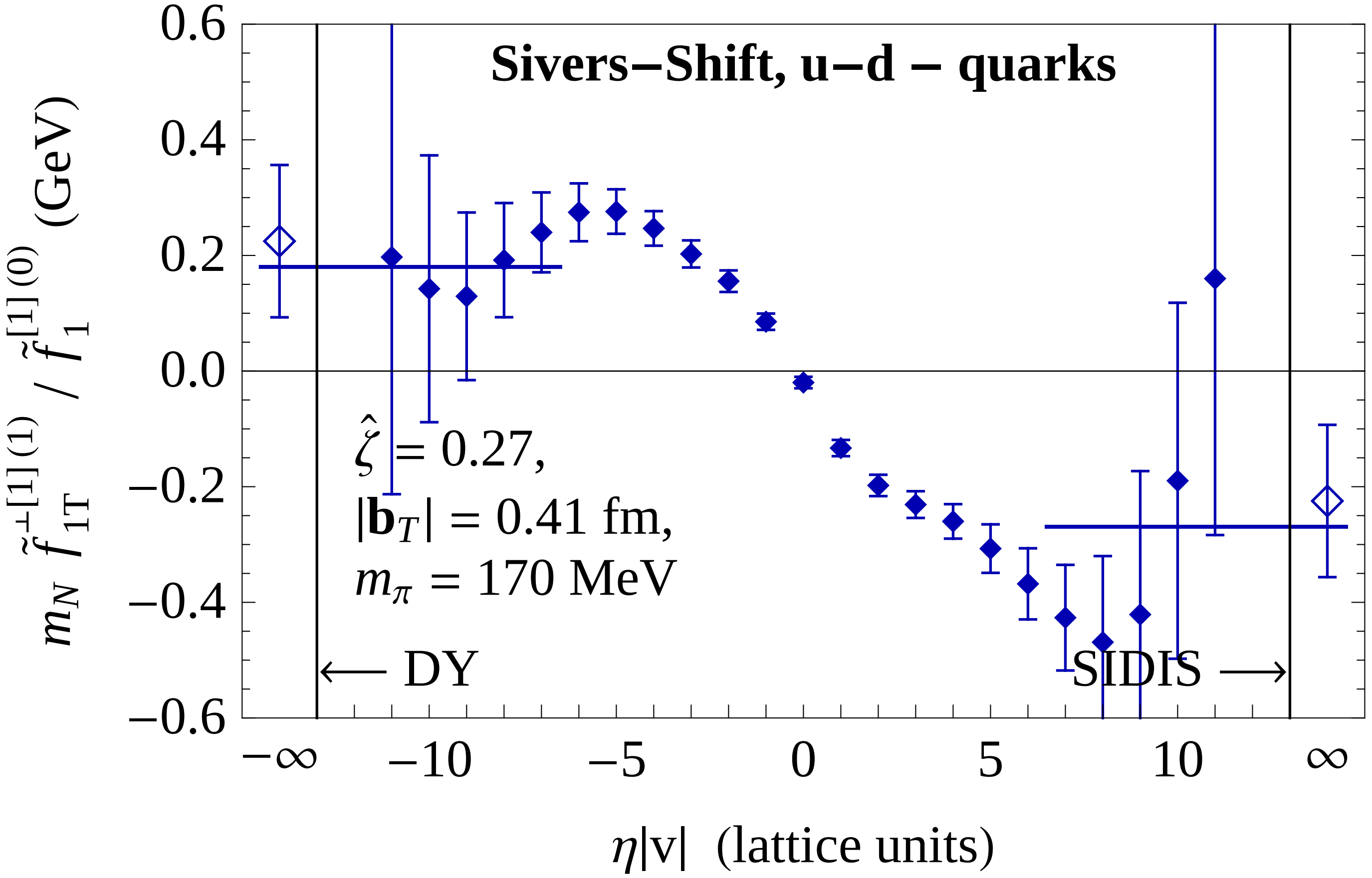

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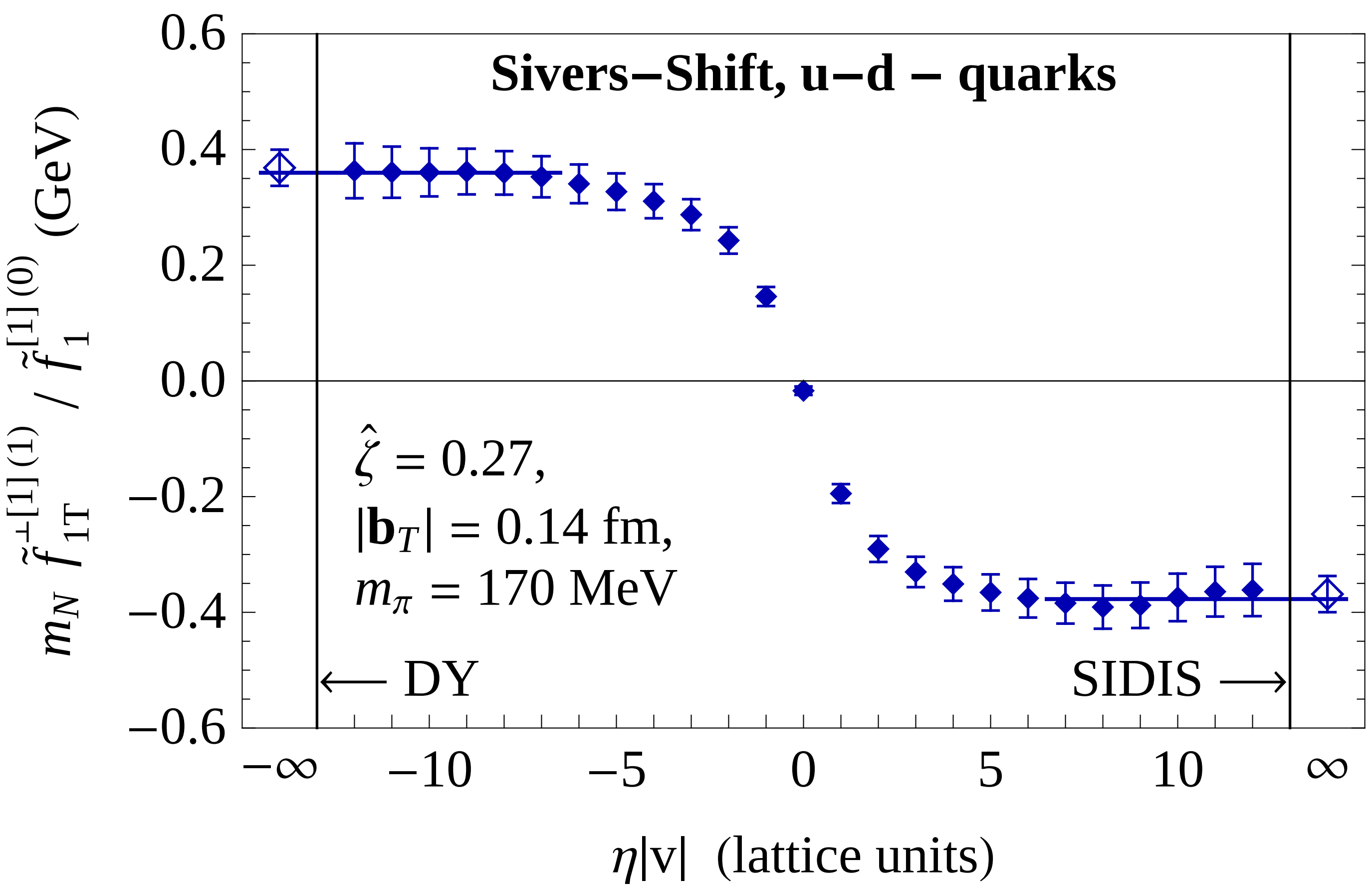

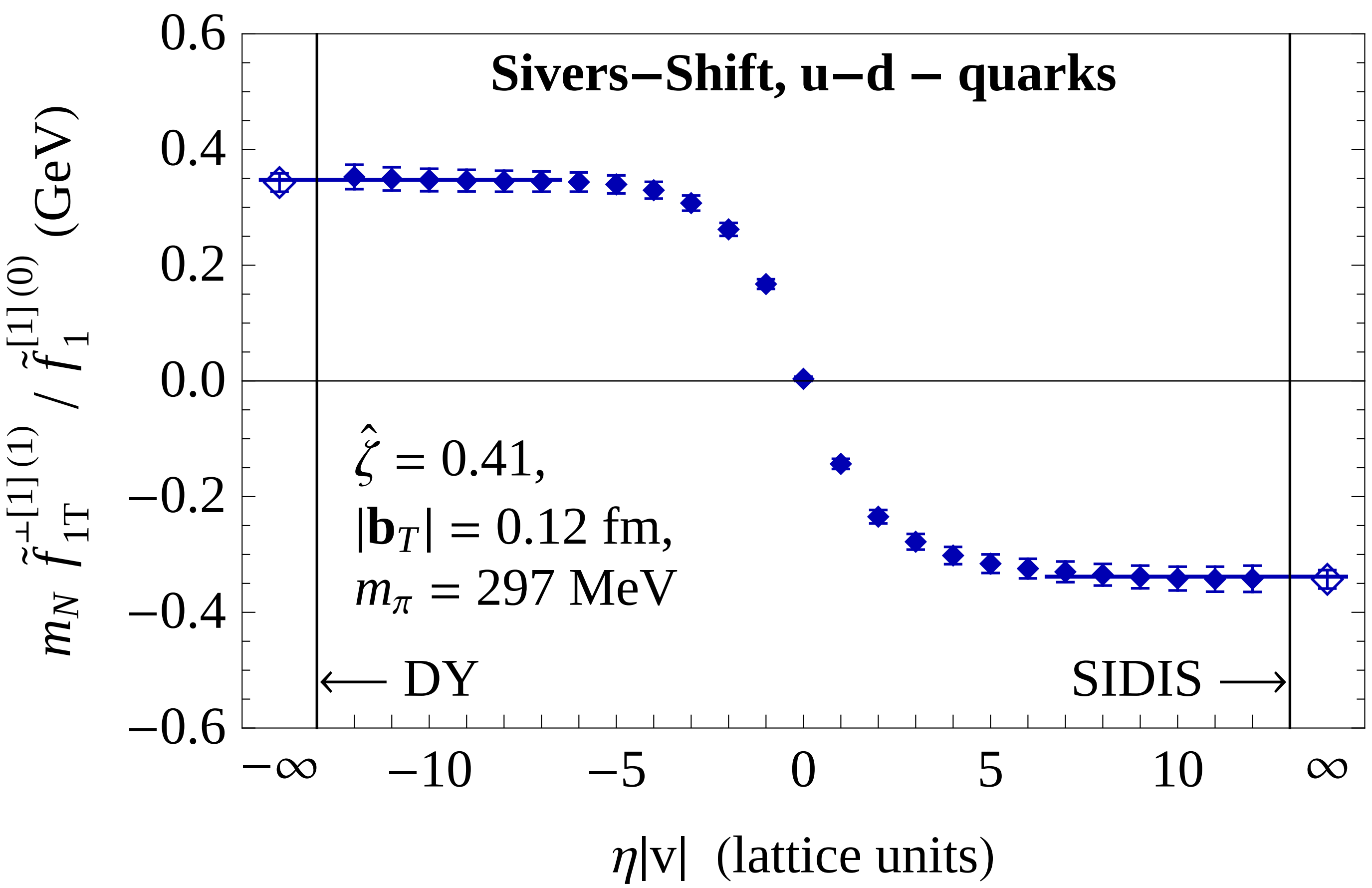

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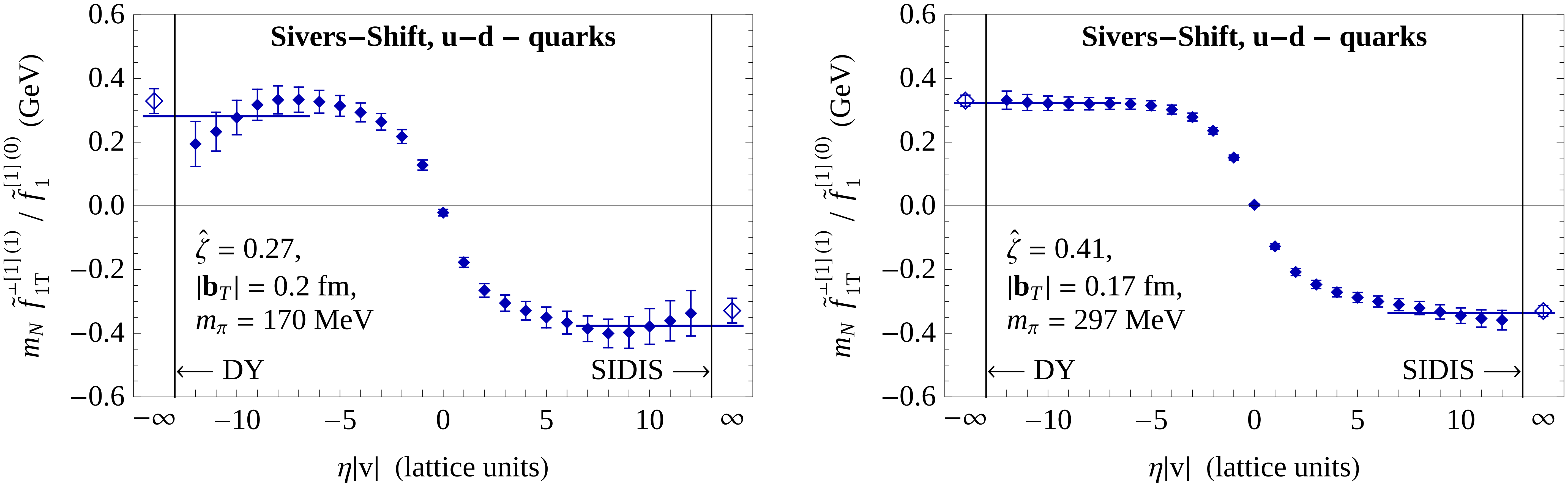

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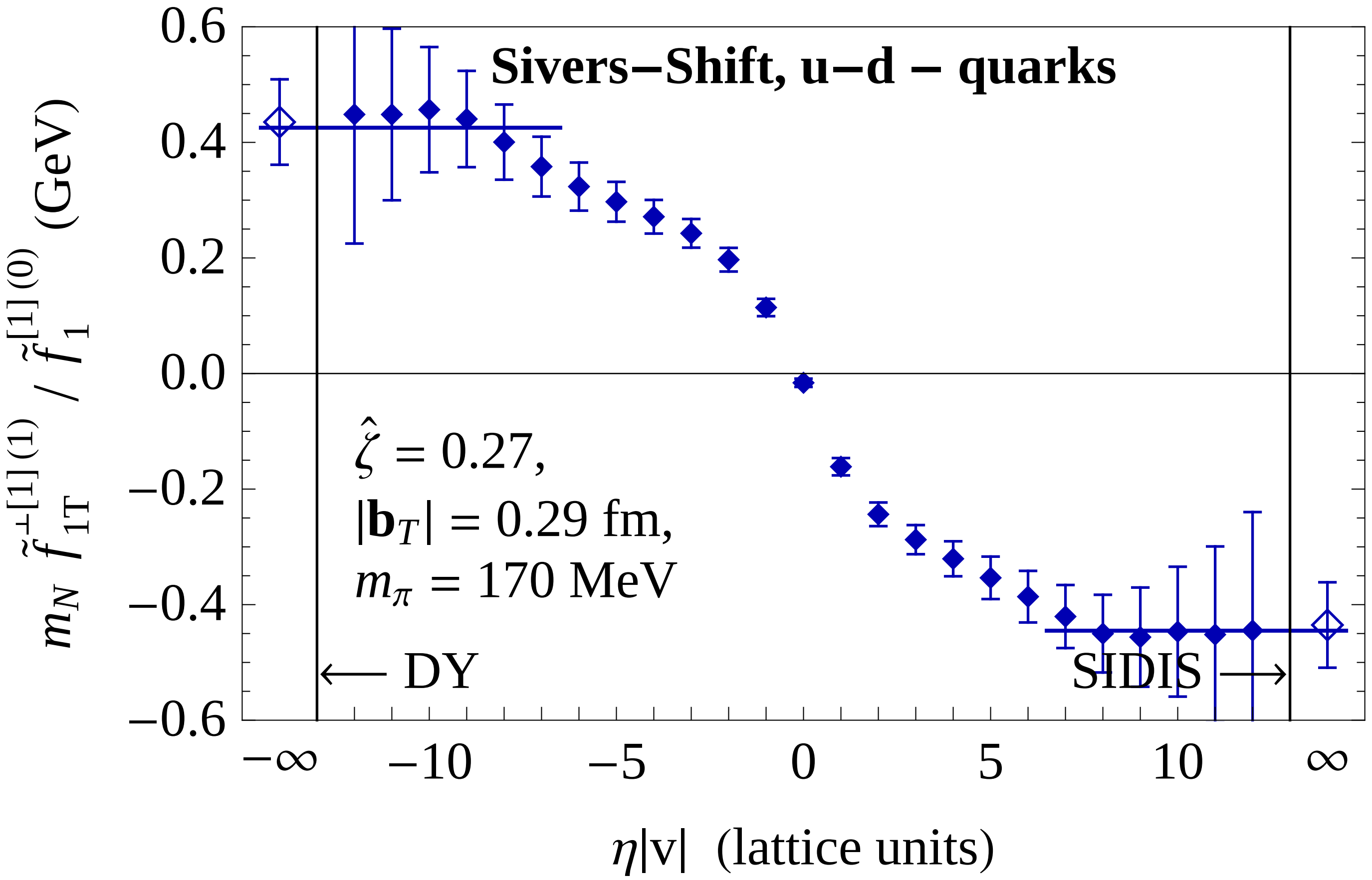

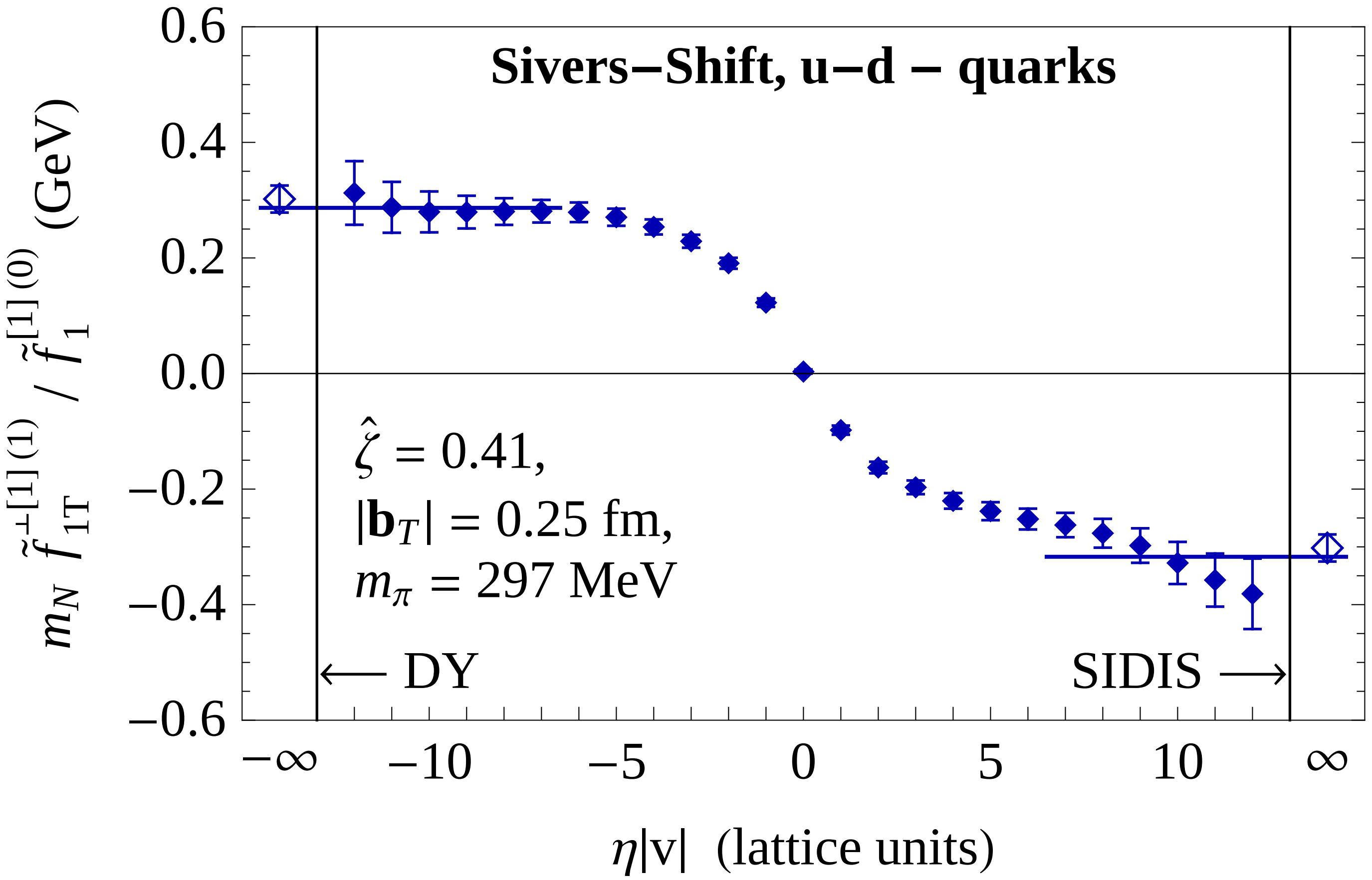

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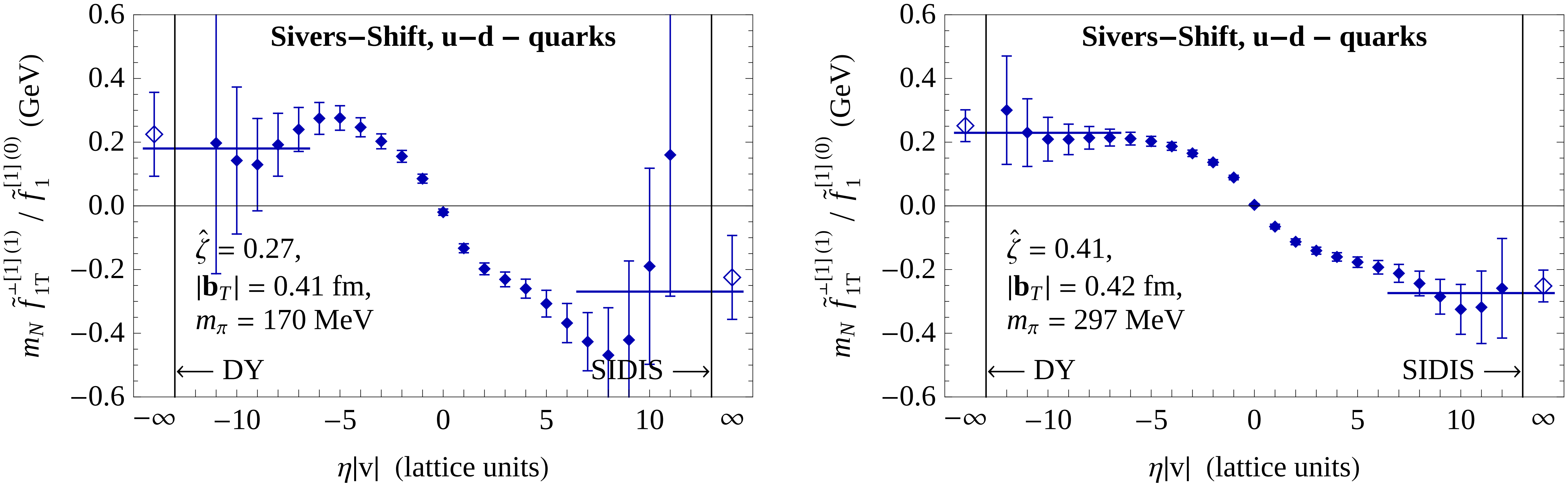
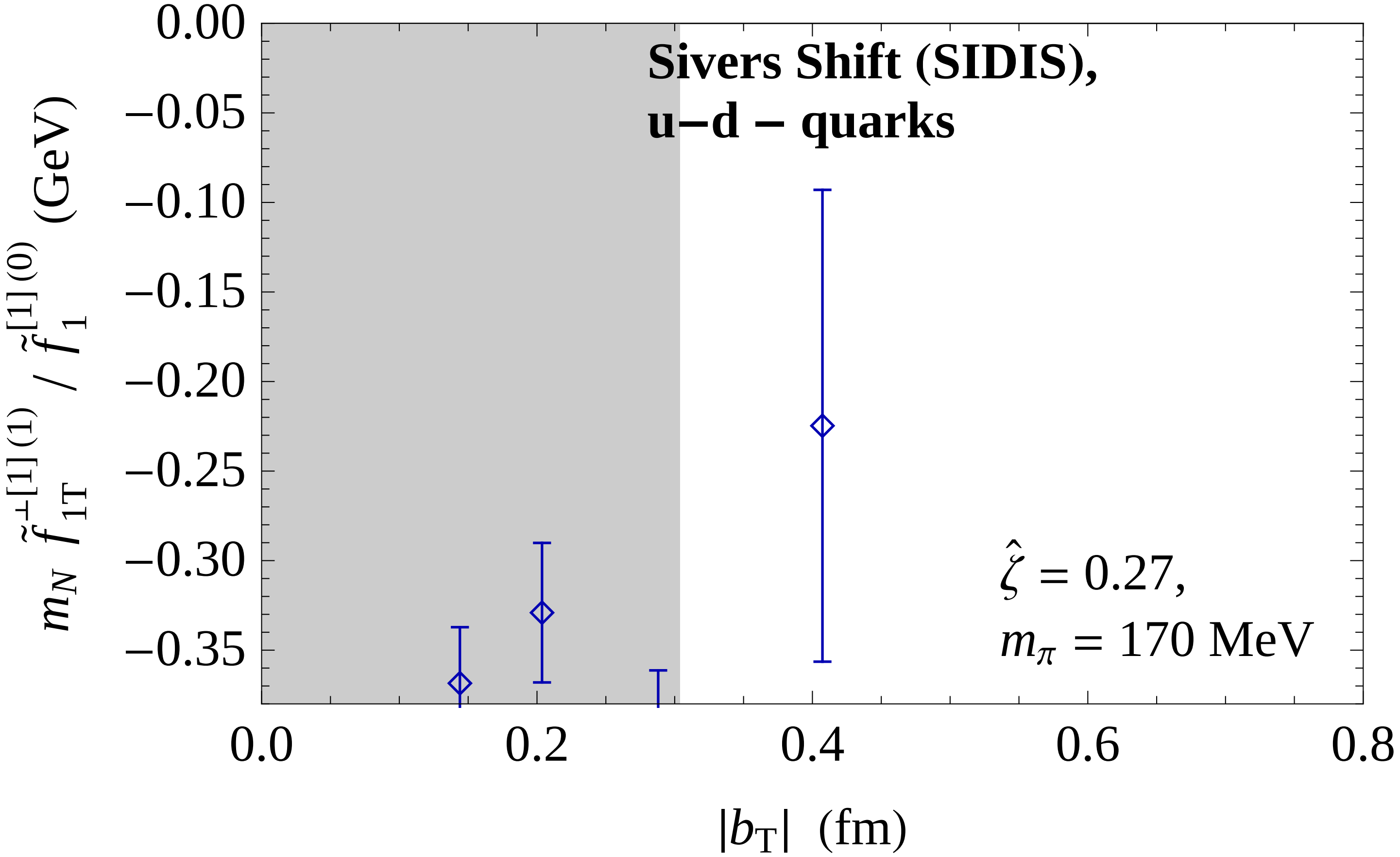
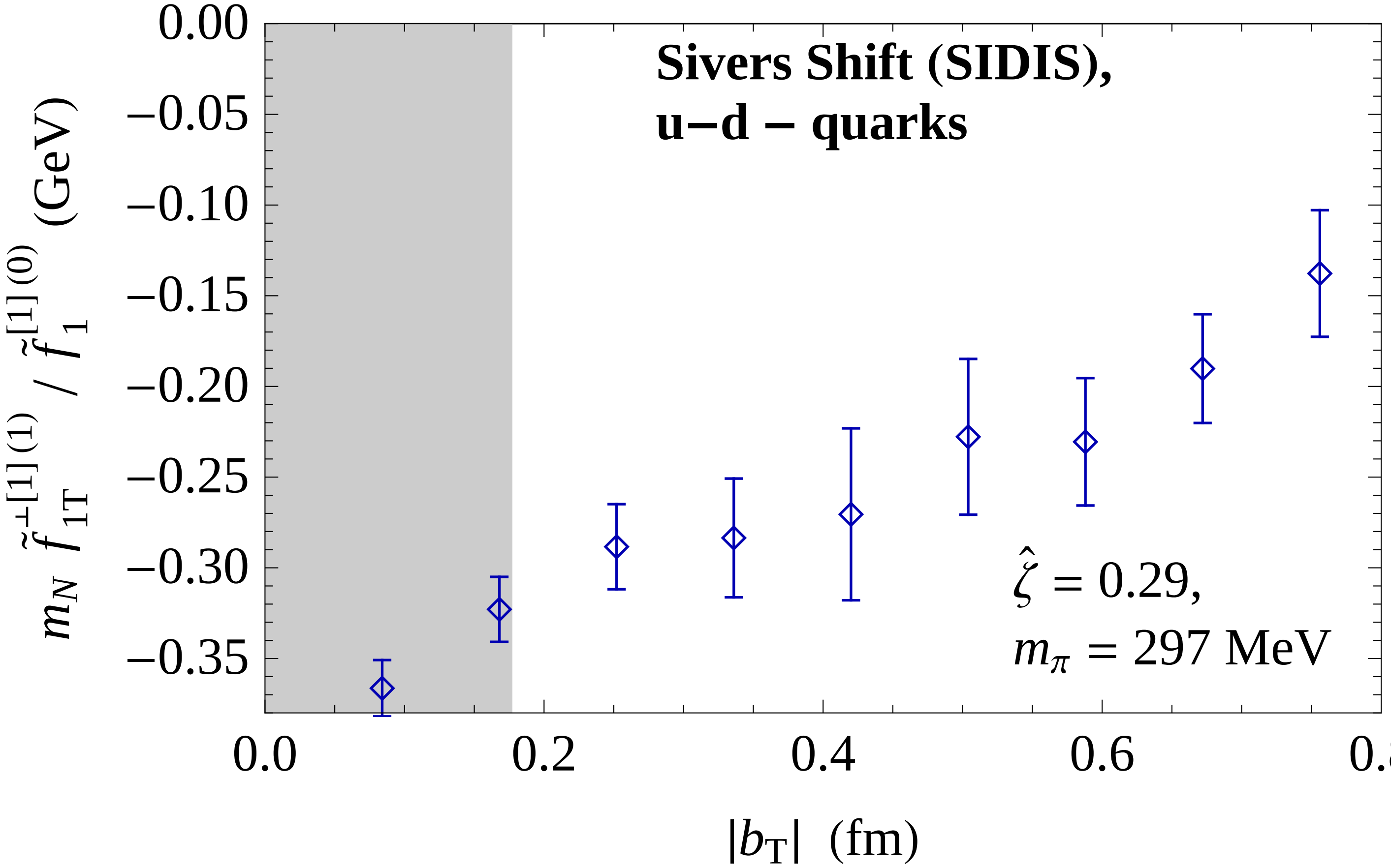


Results: Sivers shift

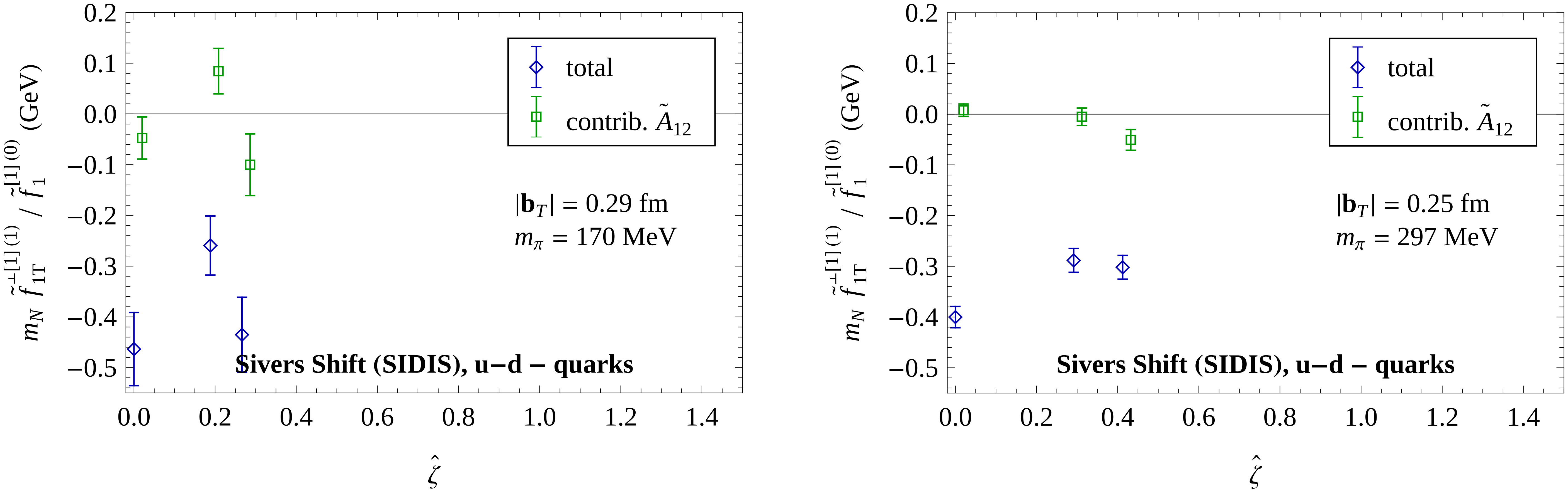
Dependence of SIDIS limit on $|b_T|$





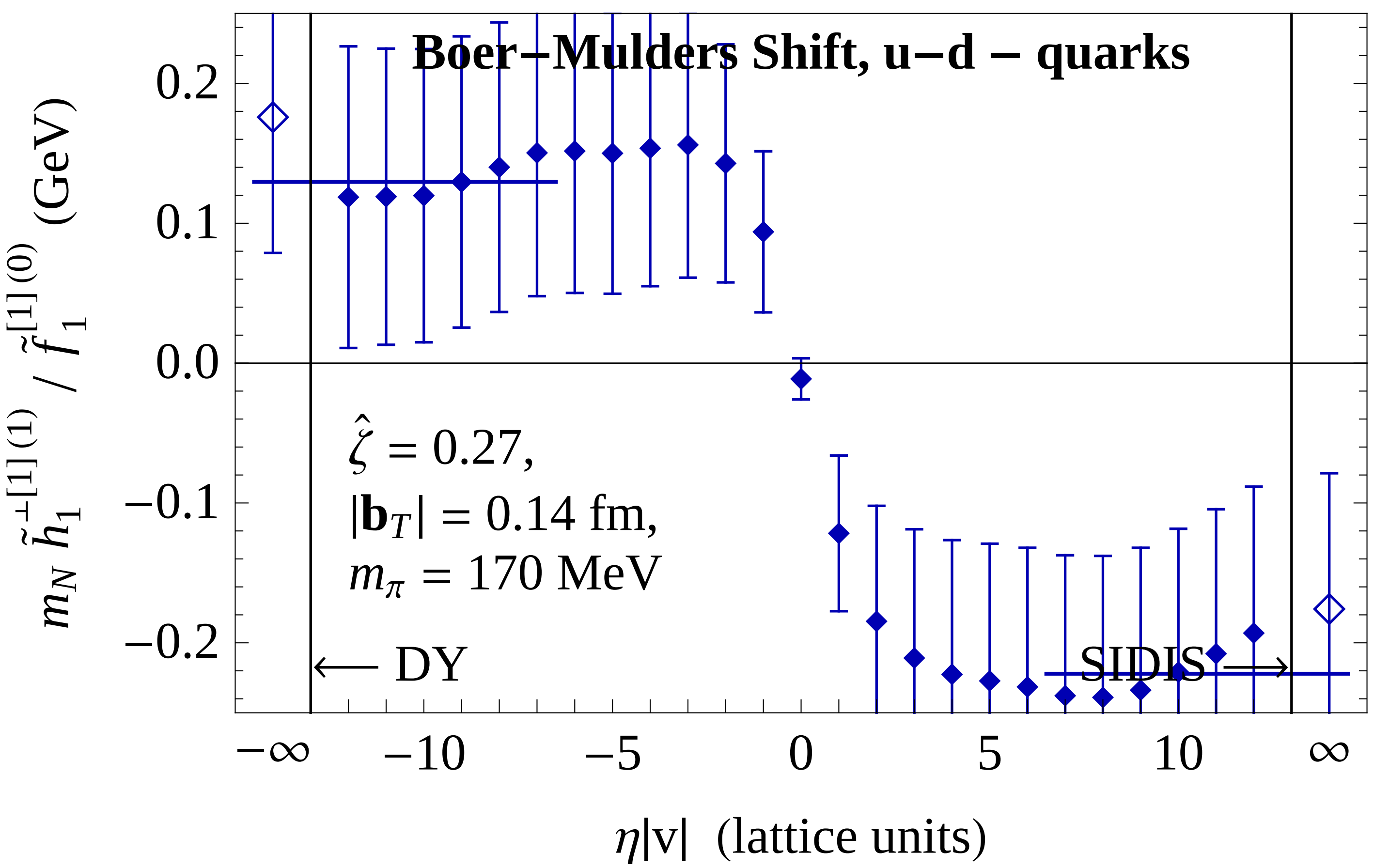
Results: Sivers shift

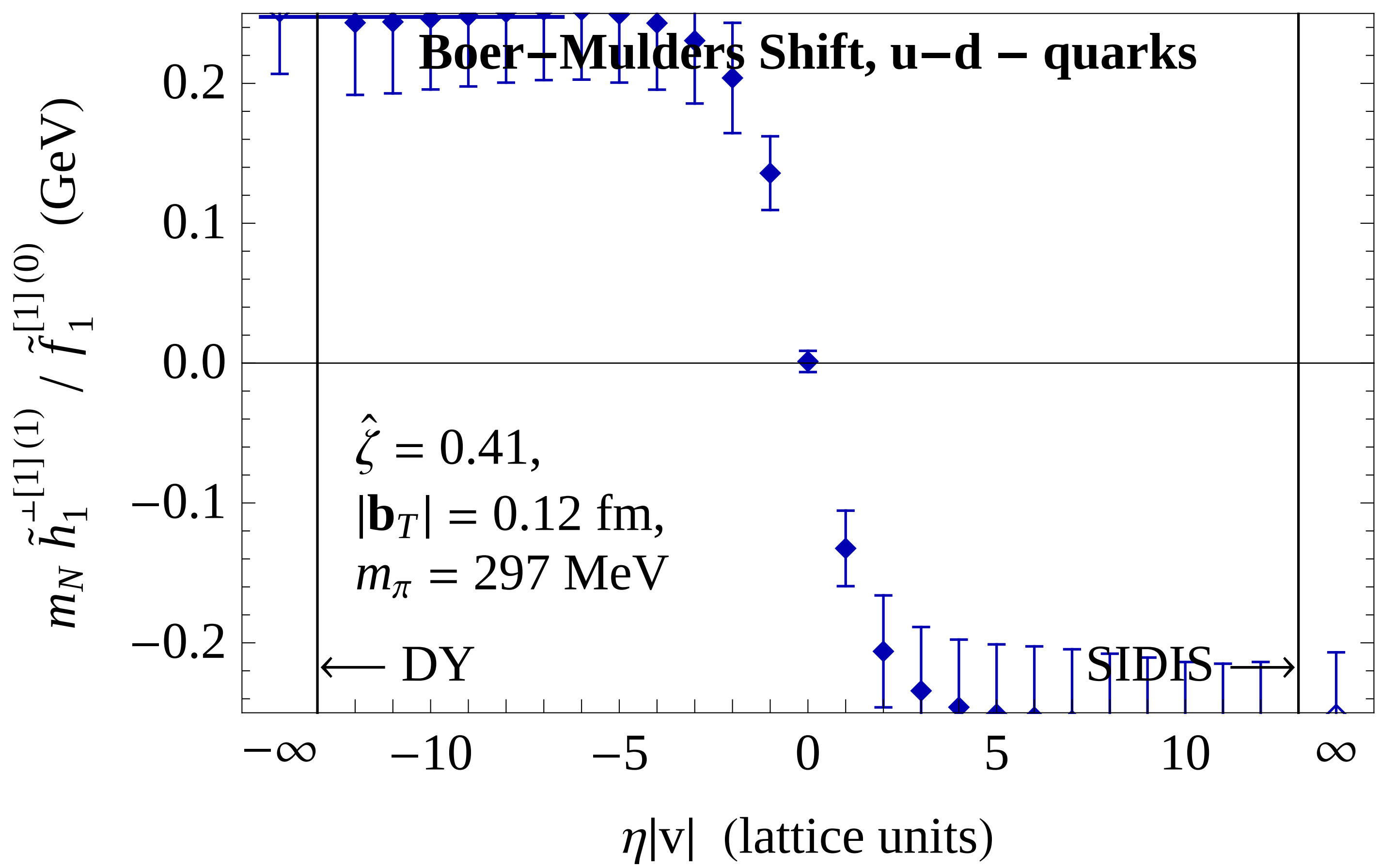
Dependence of SIDIS limit on $\hat{\zeta}$



Results: Boer-Mulders shift

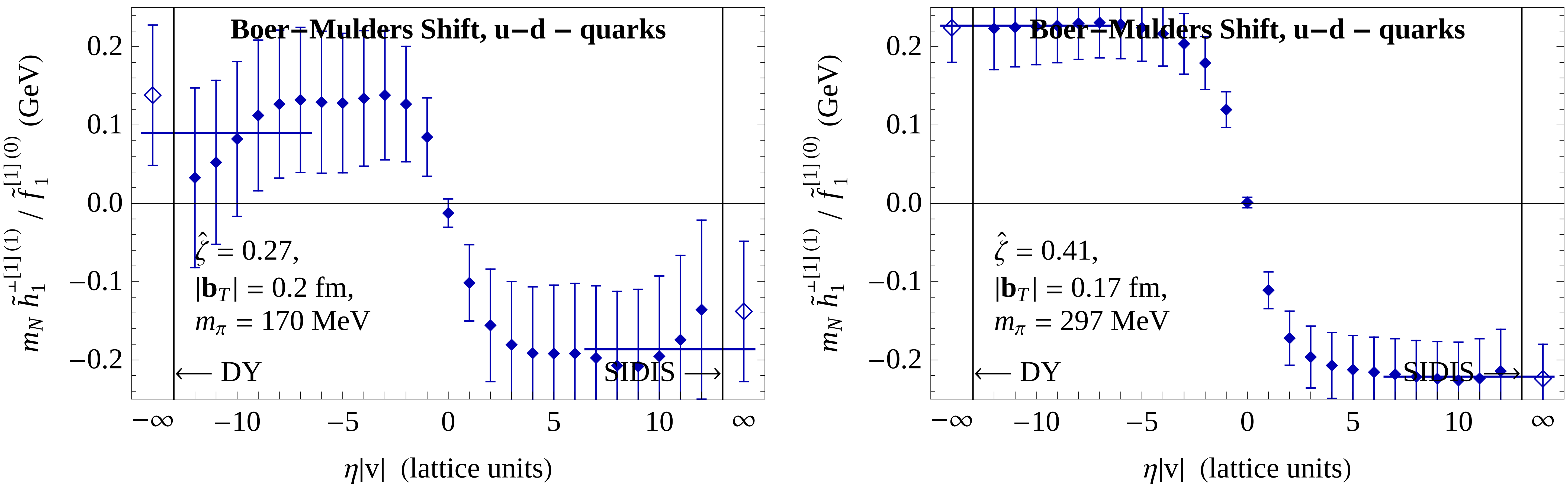
Dependence on staple extent; sequence of panels at different $|b_T|$





Results: Boer-Mulders shift

Dependence on staple extent; sequence of panels at different $|b_T|$



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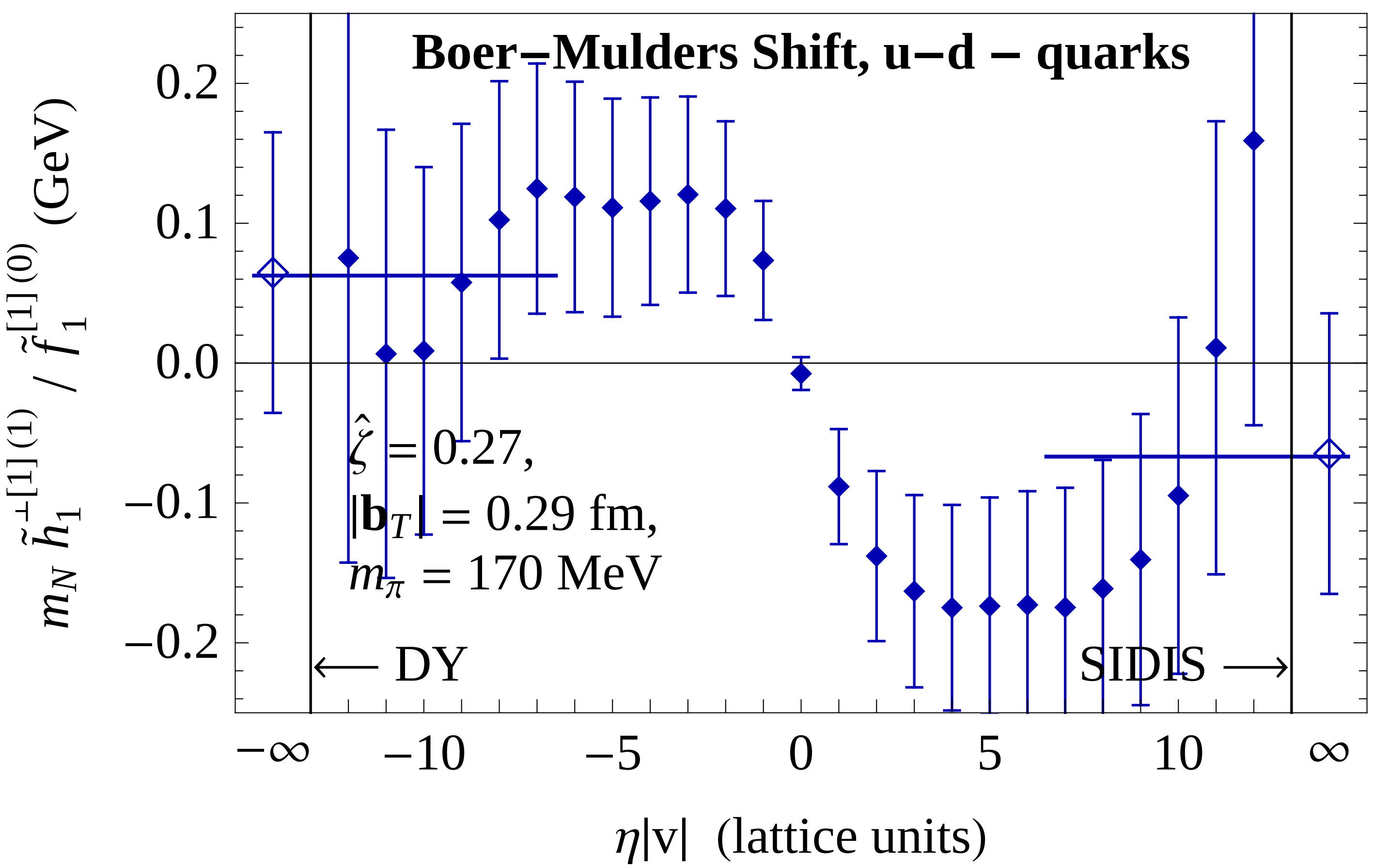

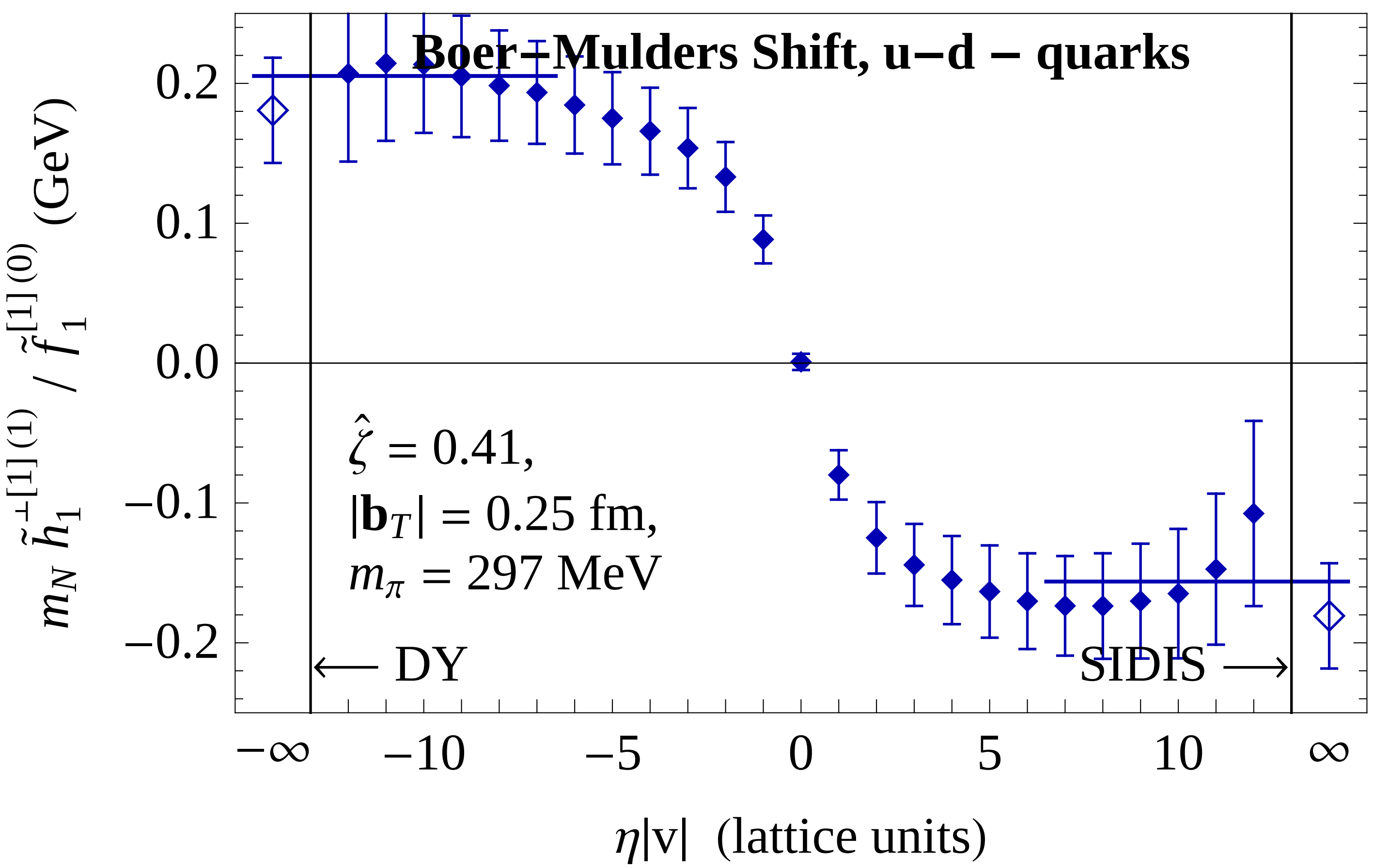

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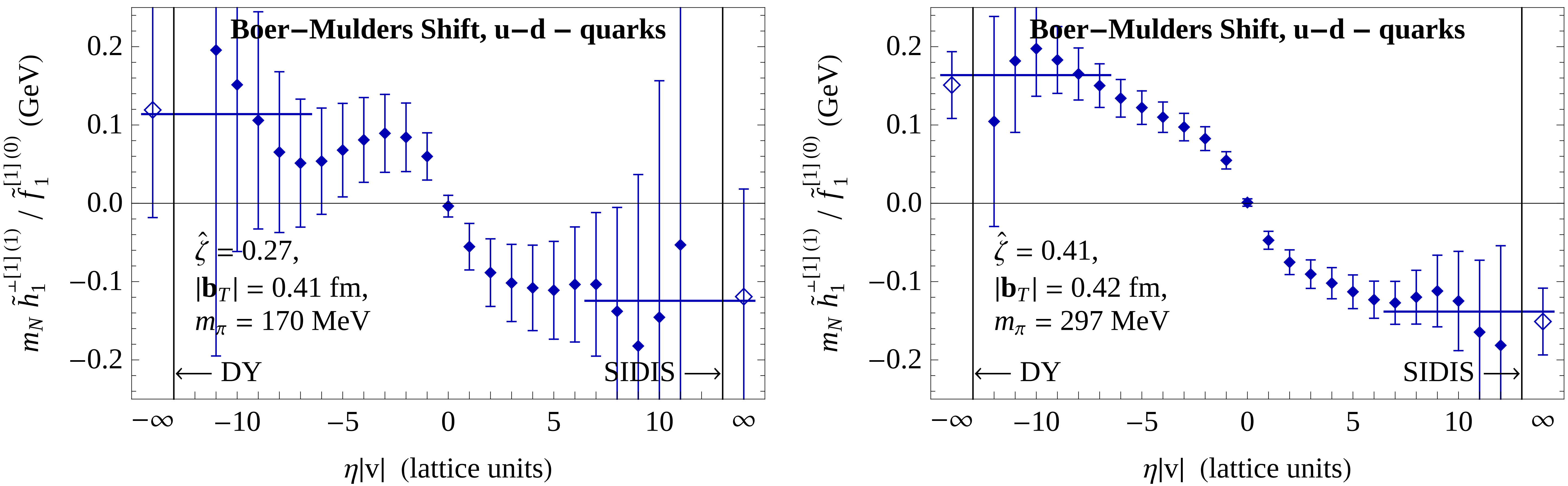

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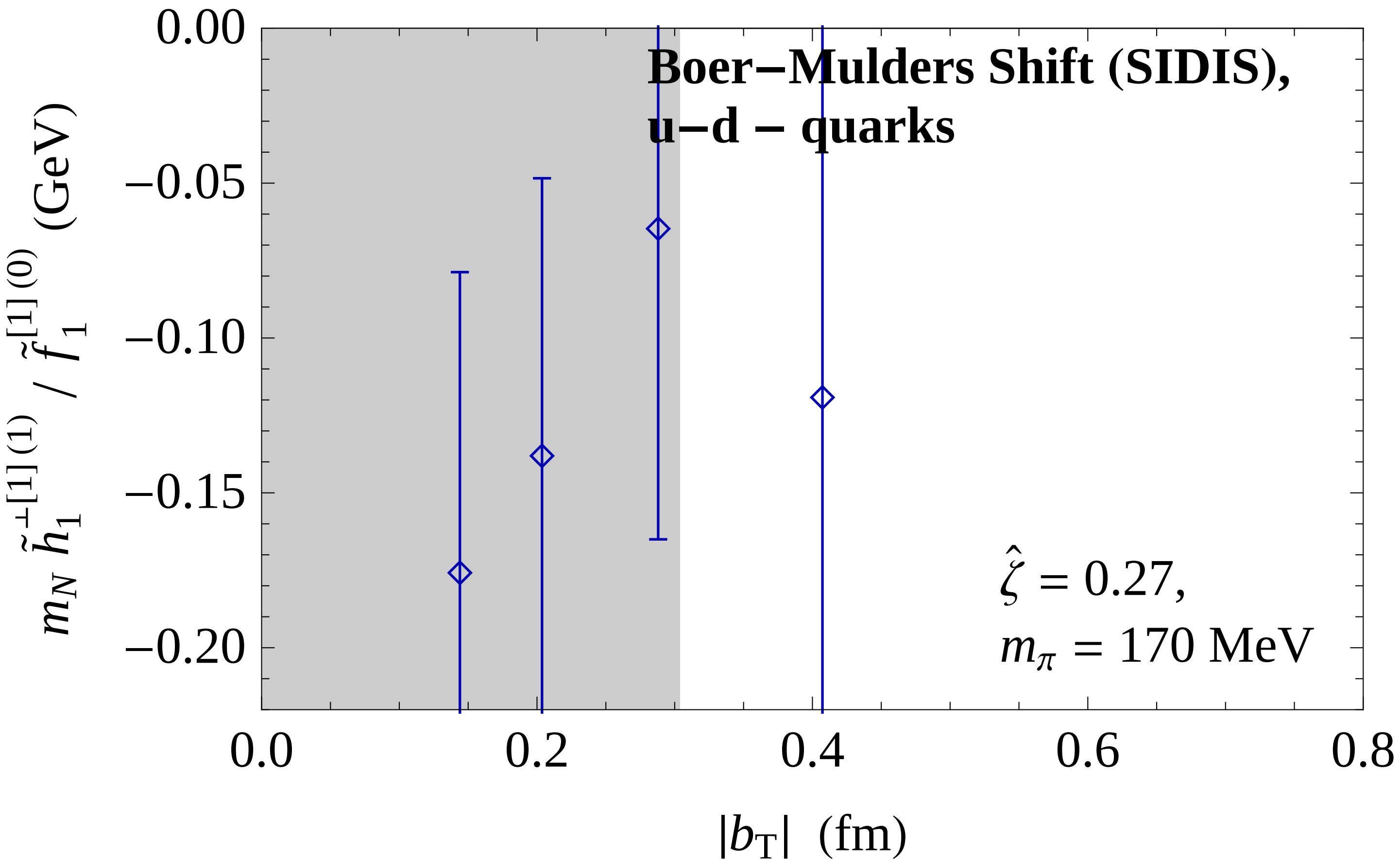
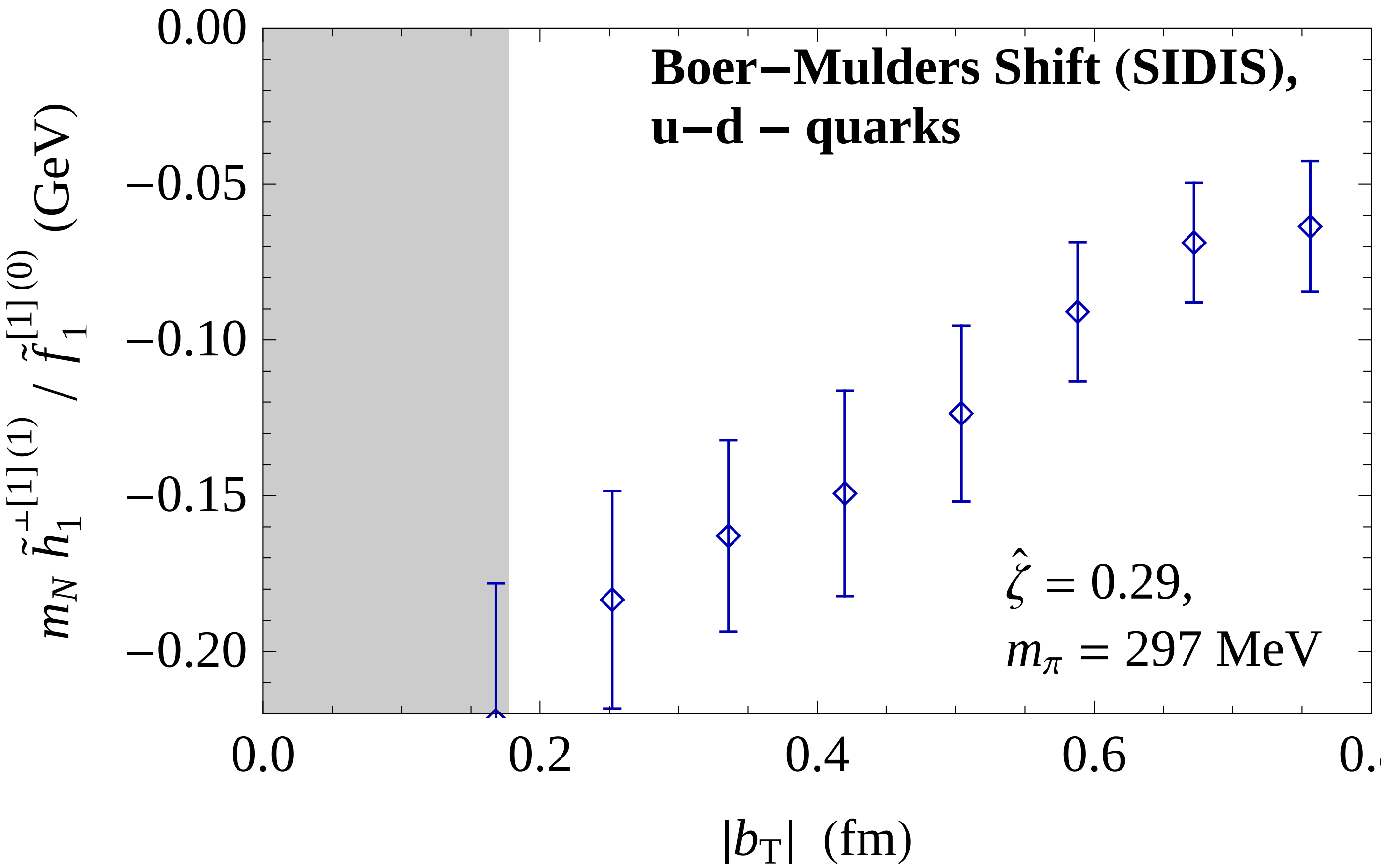
Dependence of SIDIS limit on $|b_T|$





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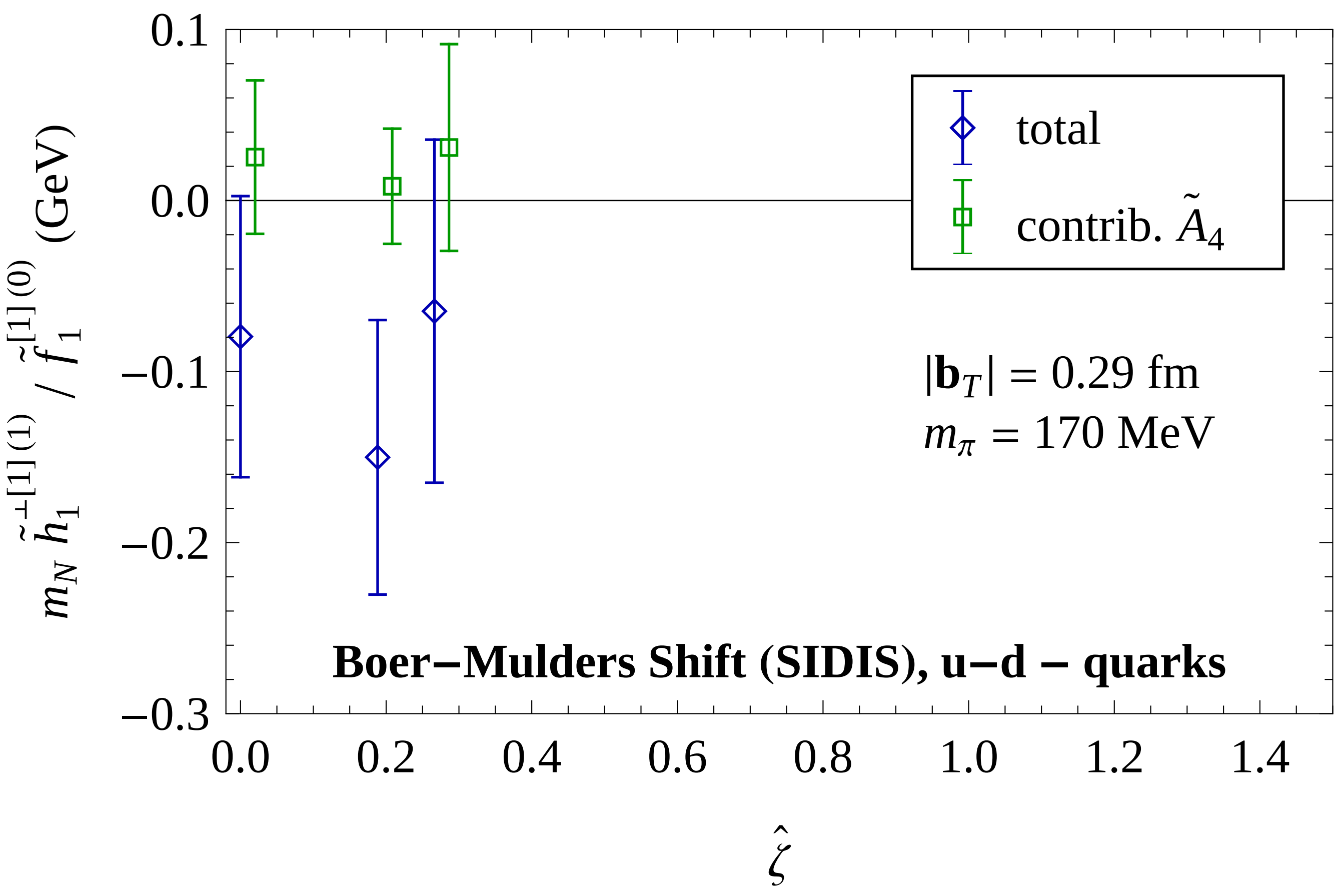
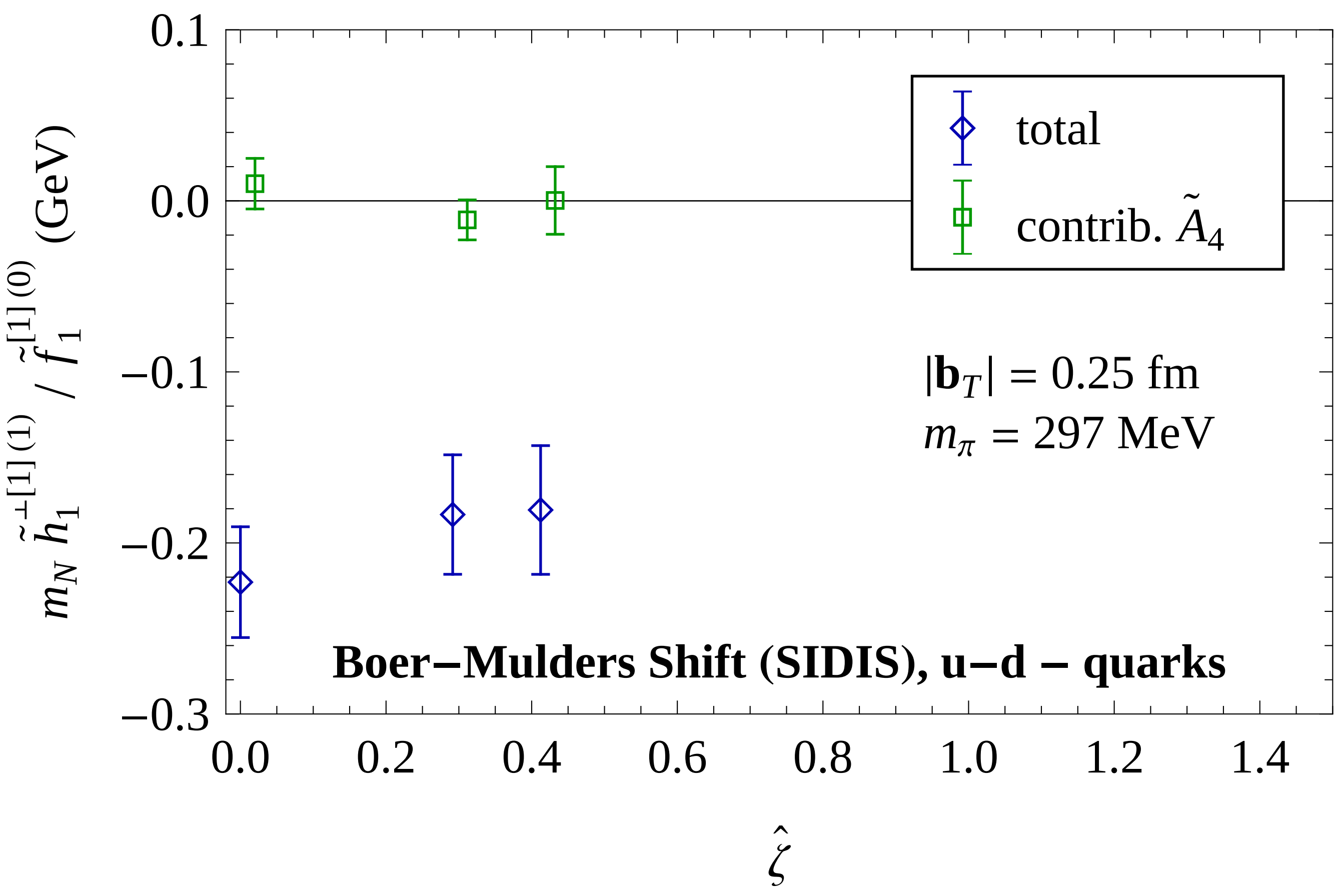
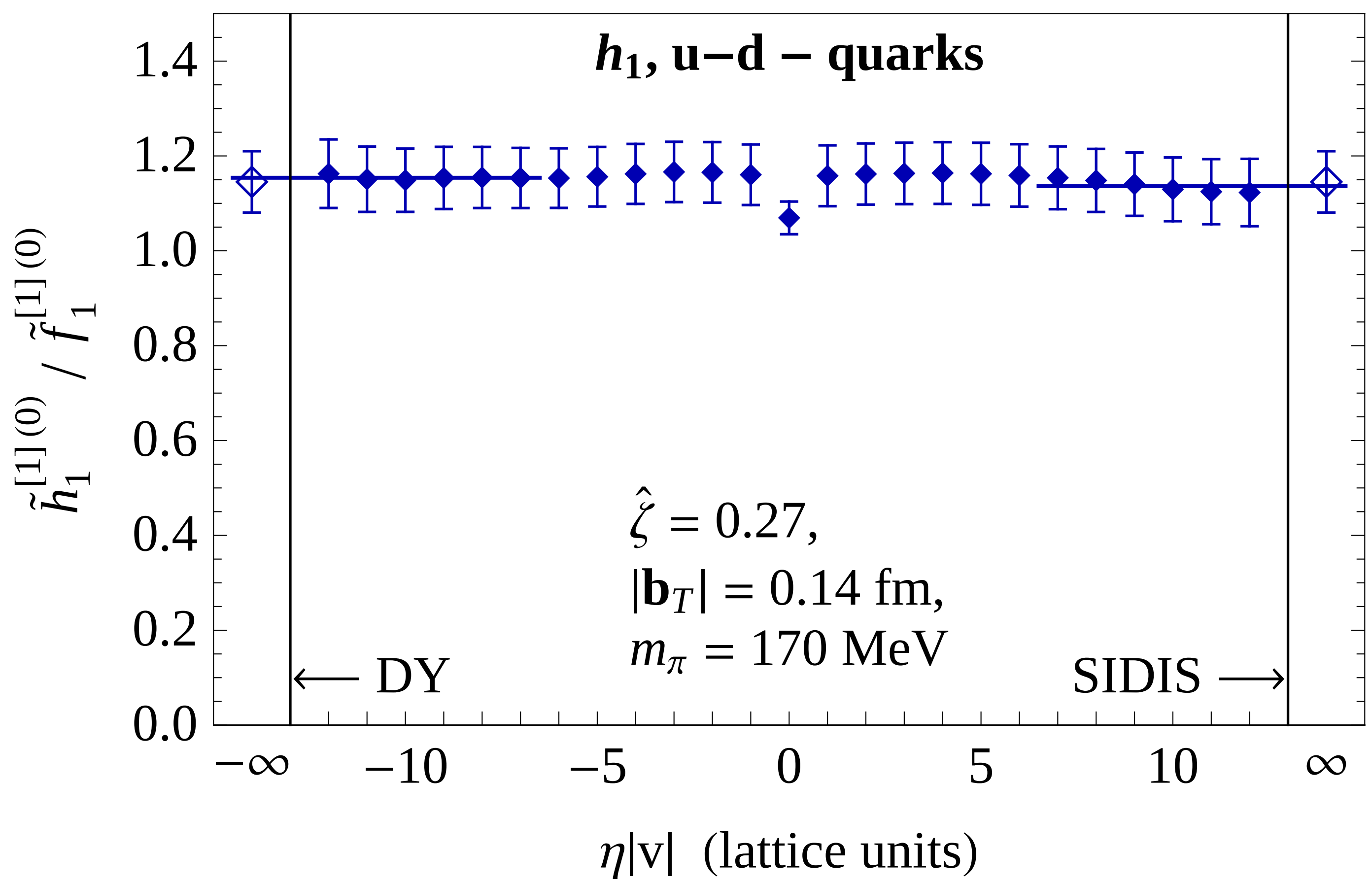
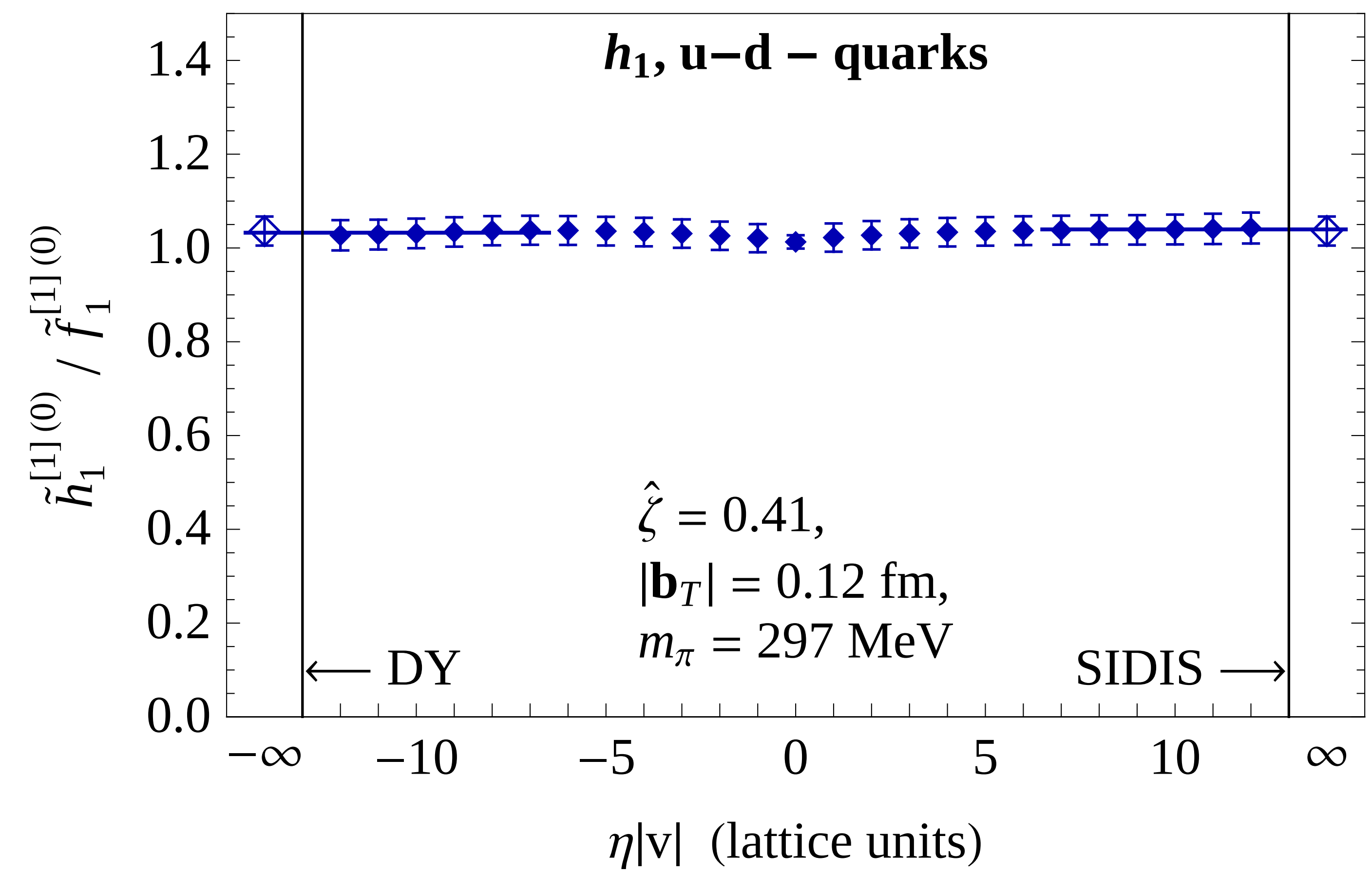




Results: Transversity

Dependence on staple extent; sequence of panels at different $|b_T|$





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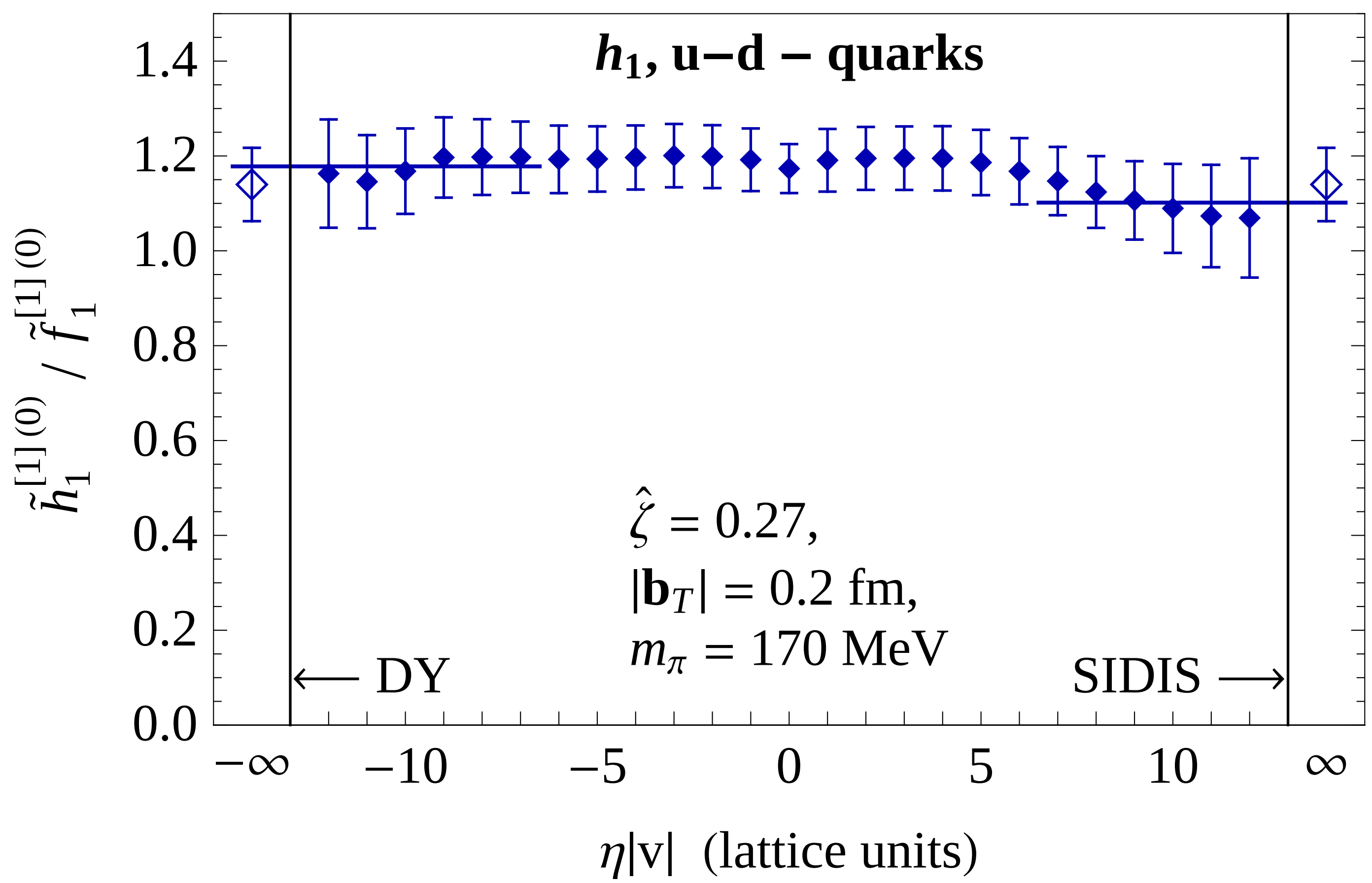

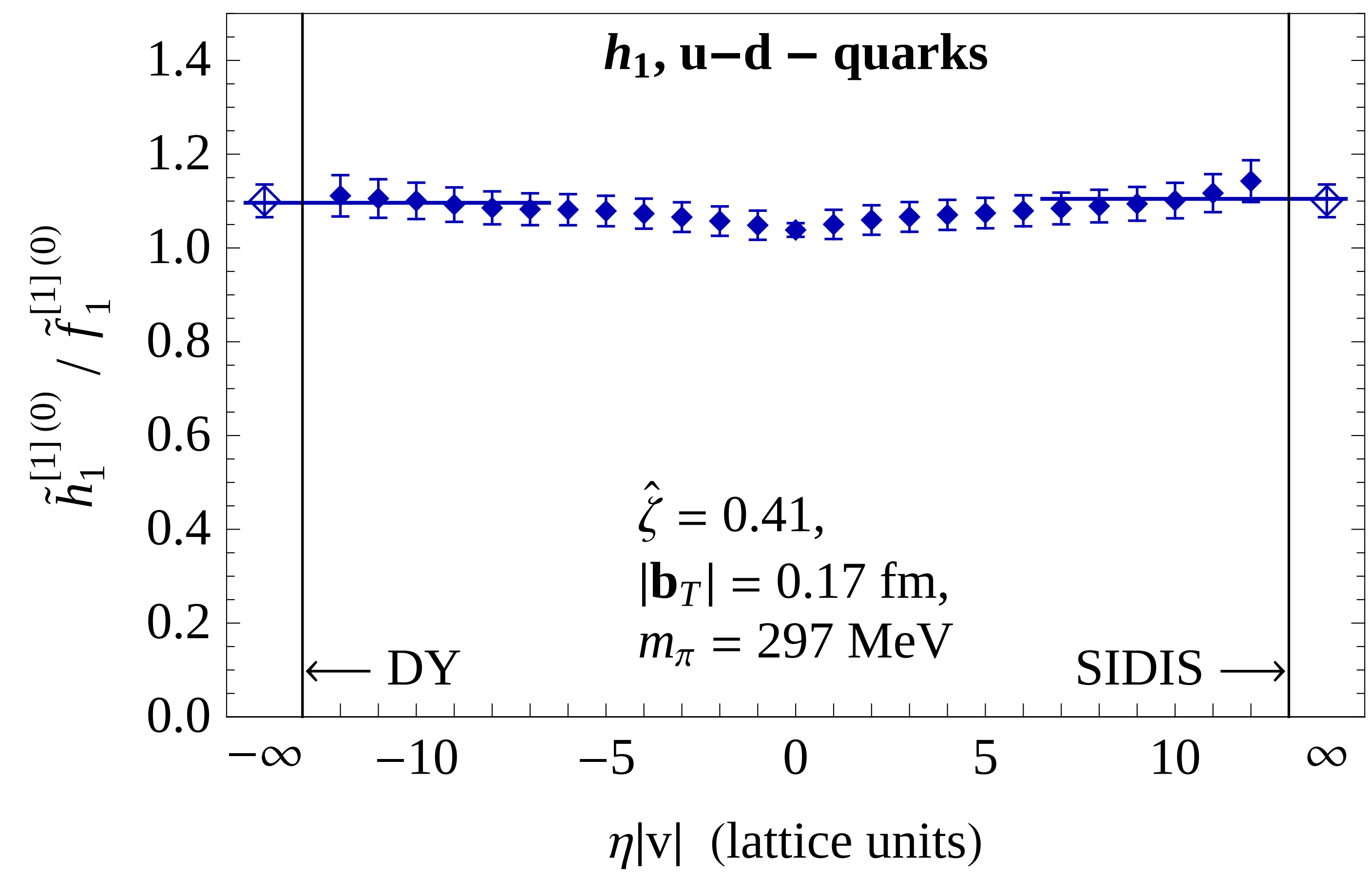

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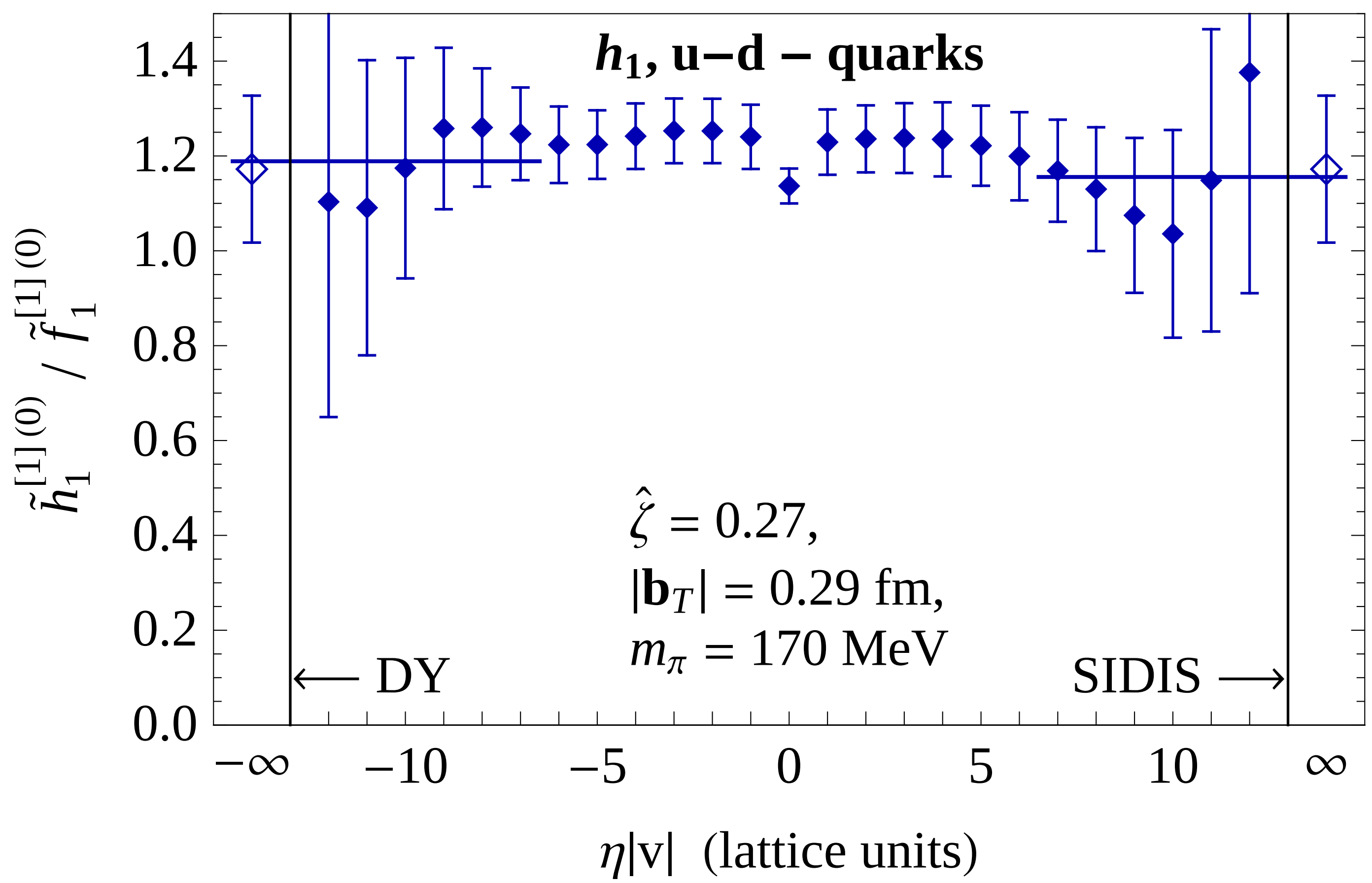

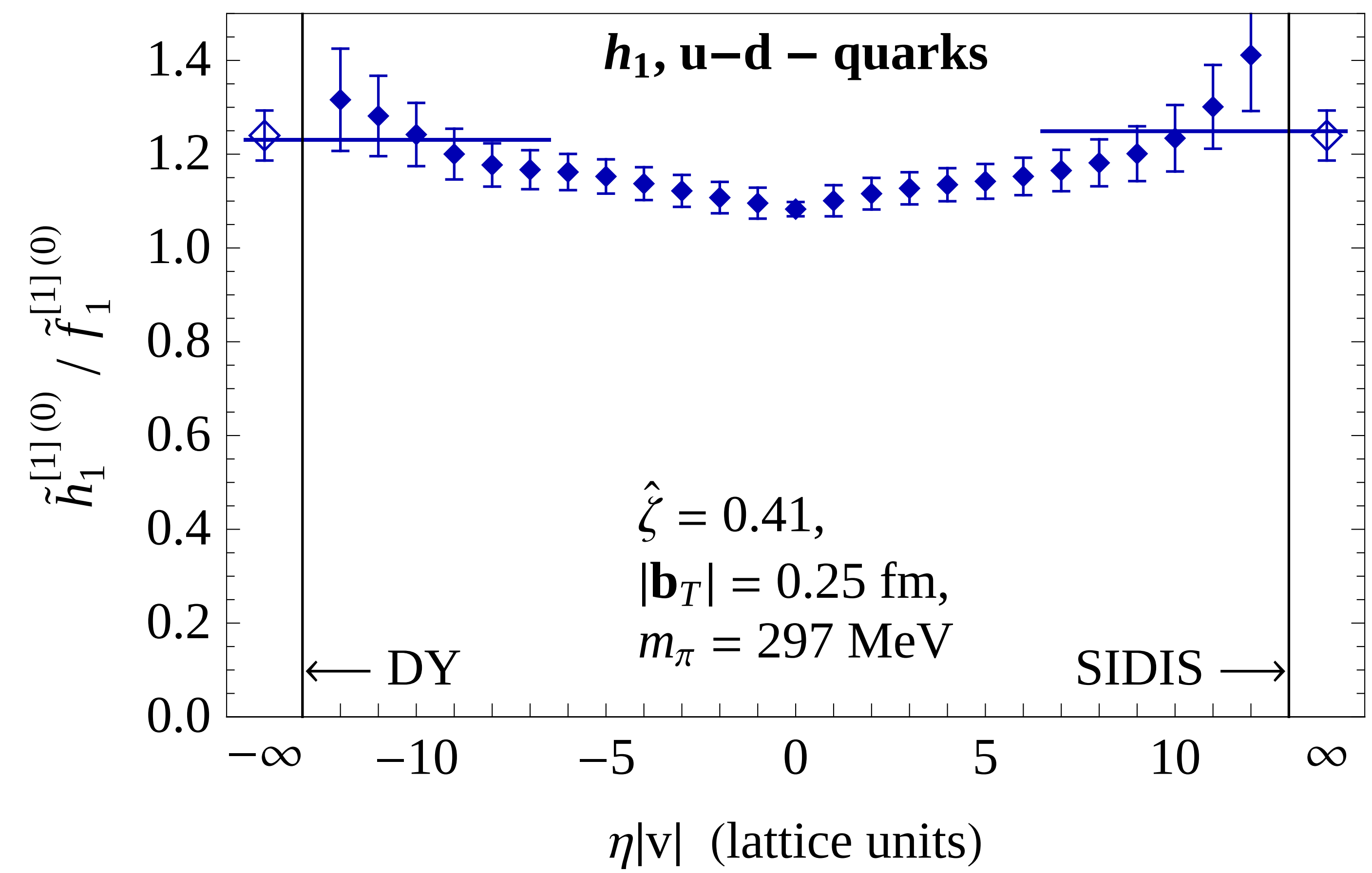

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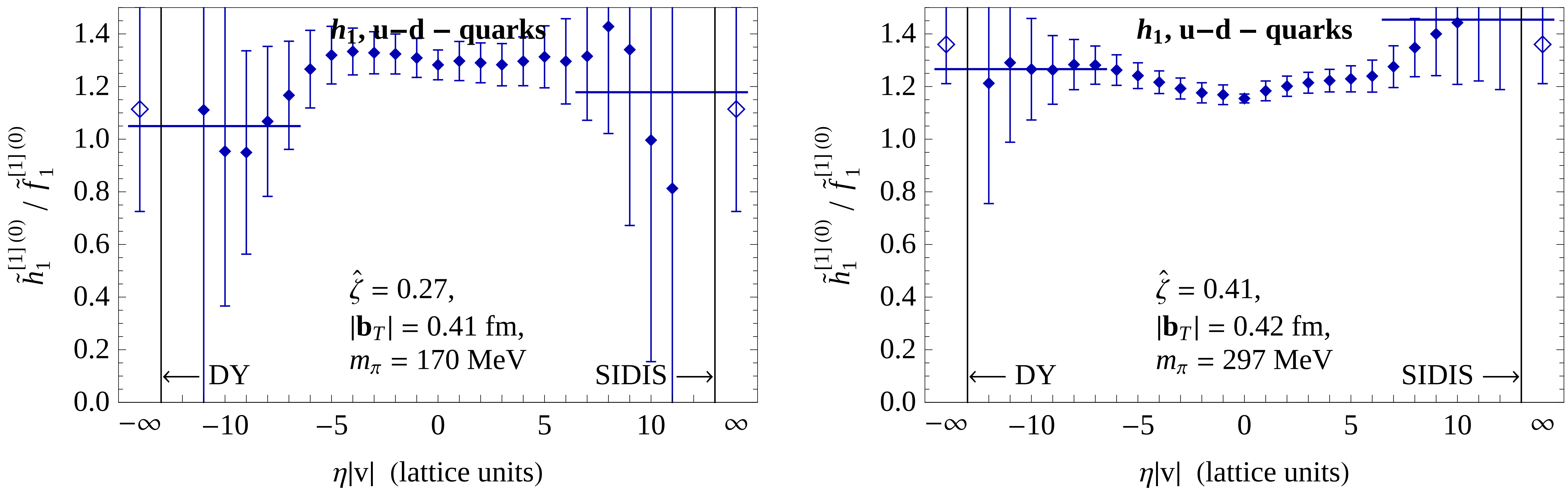
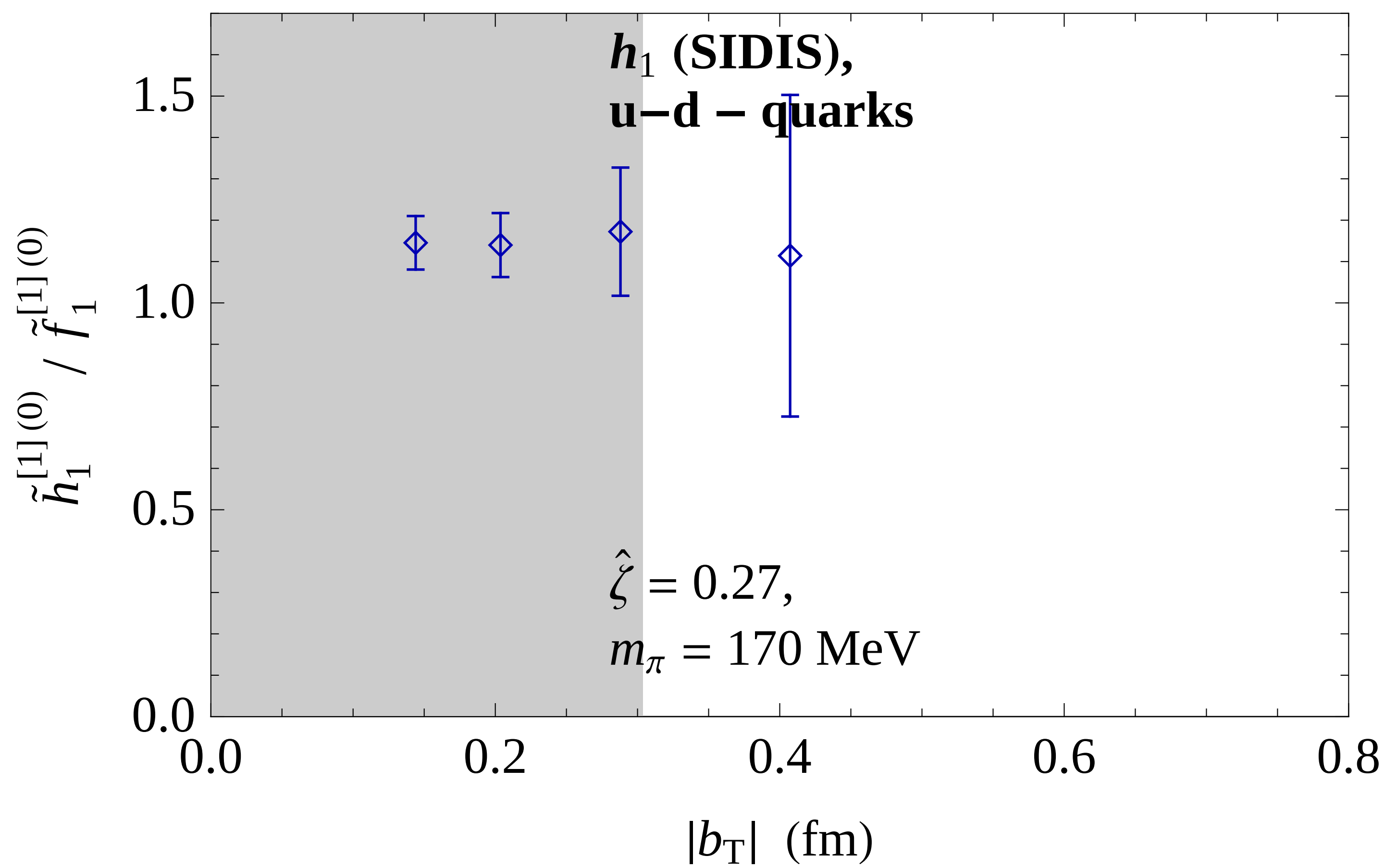
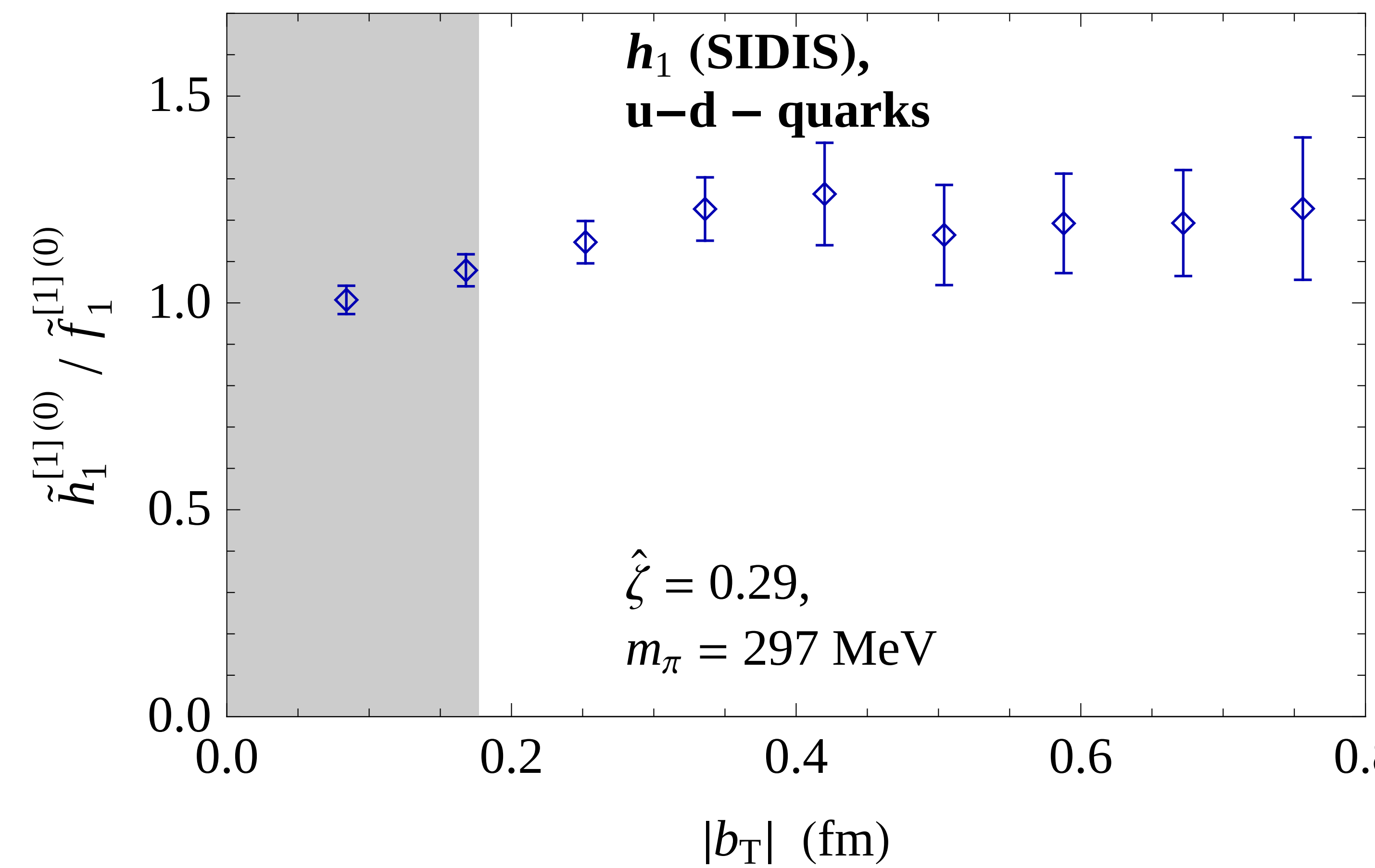


Results: Transversity

Dependence of SIDIS/DY limit on $|b_T|$





Results: Transversity

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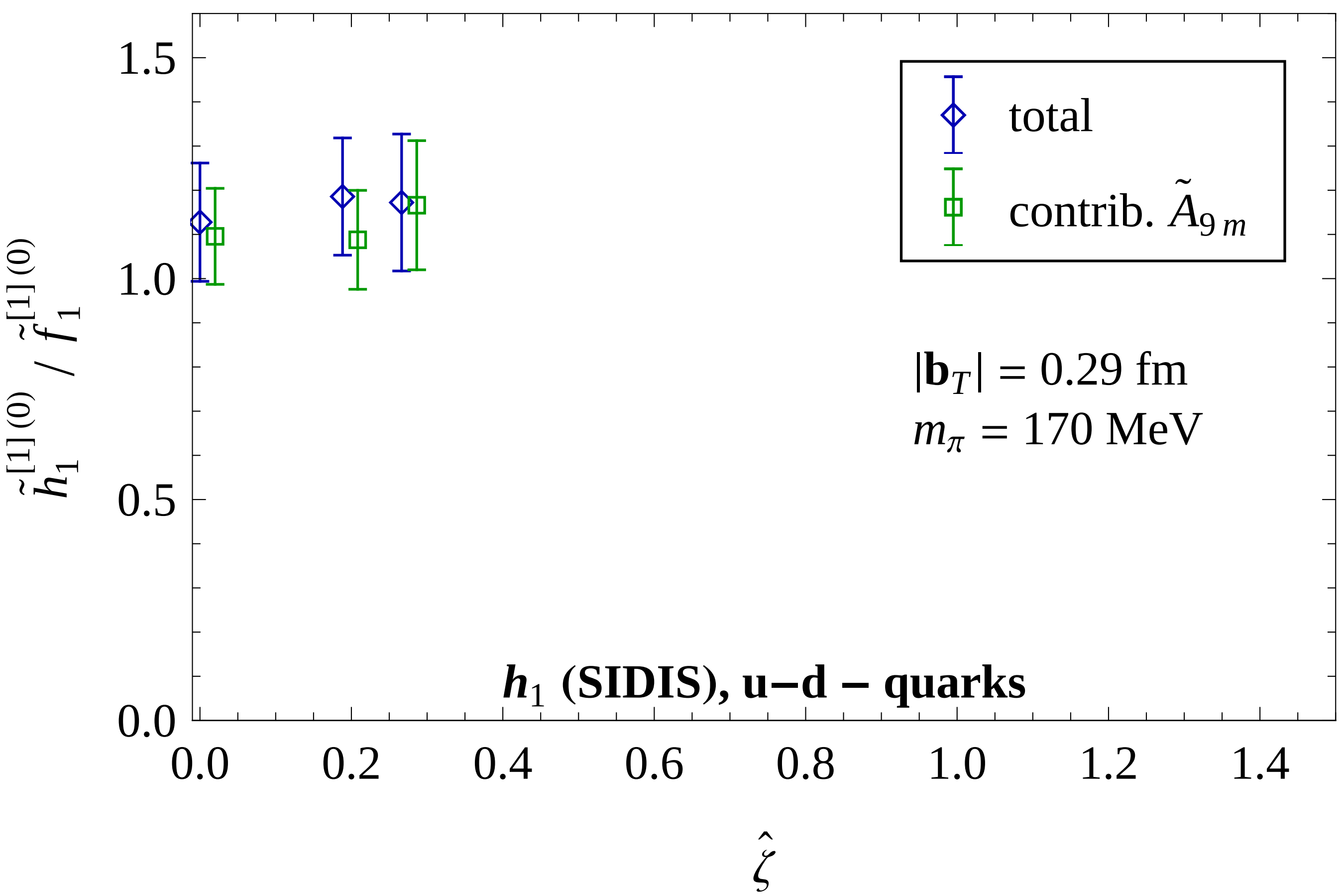
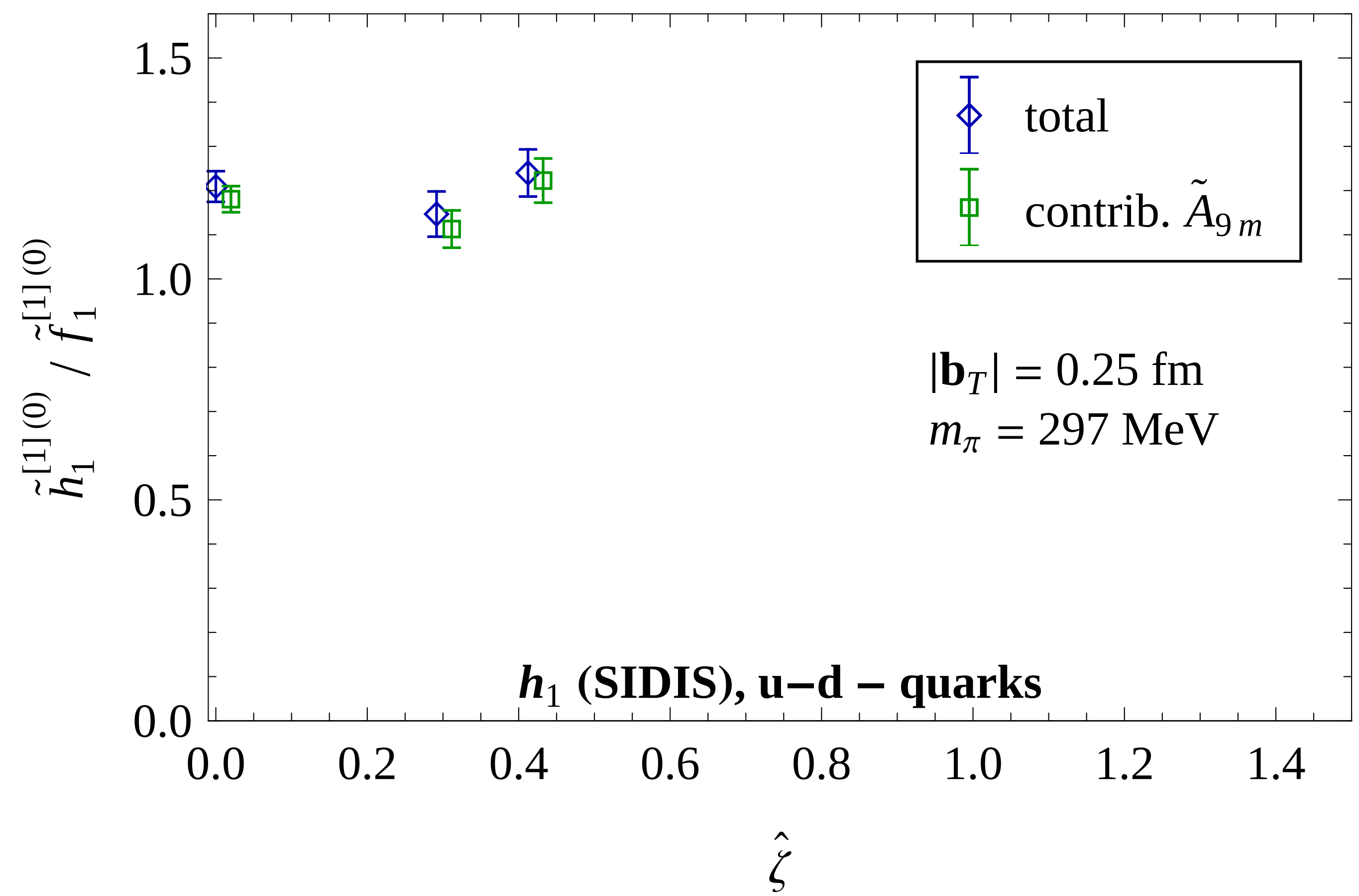
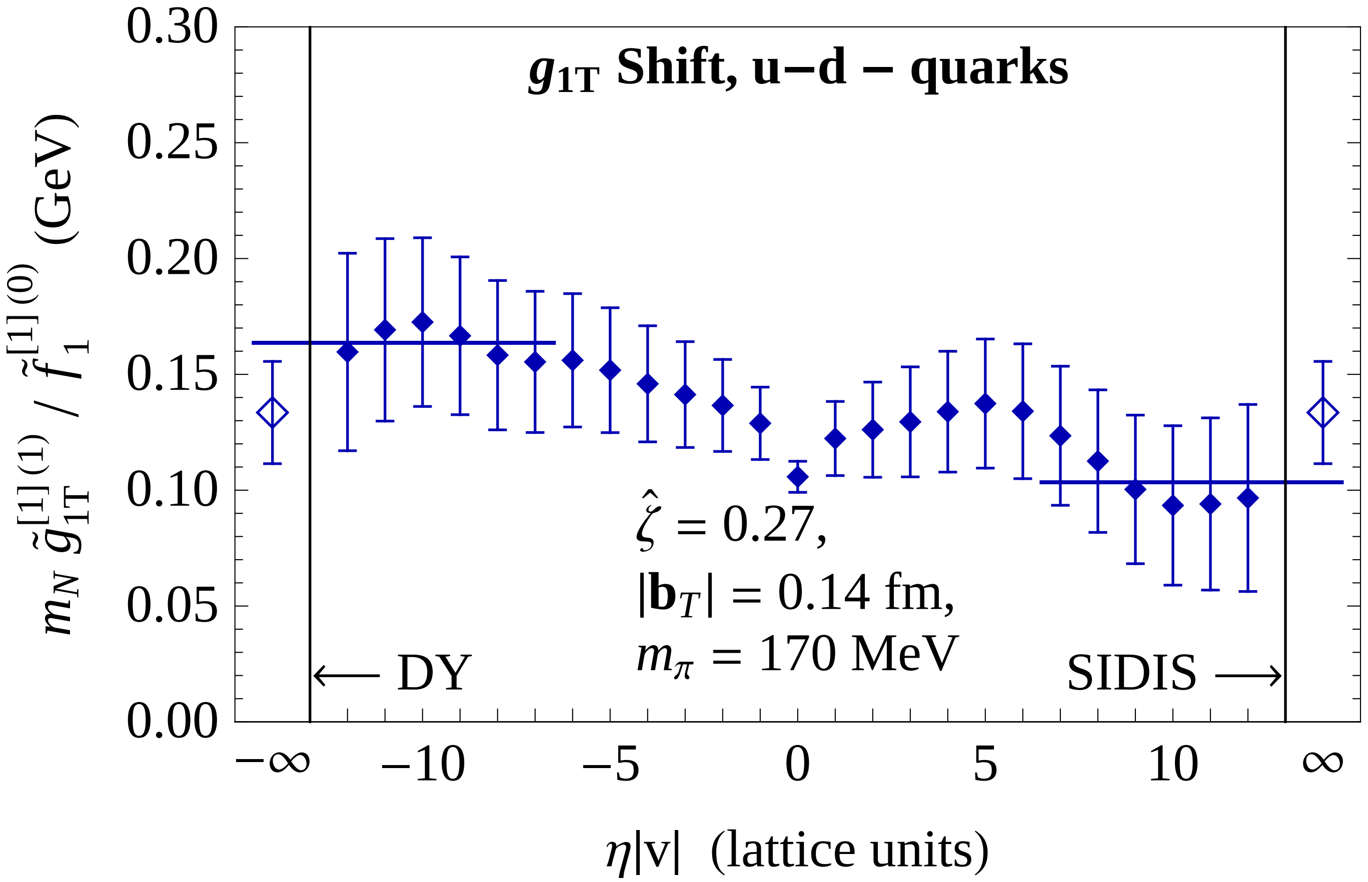
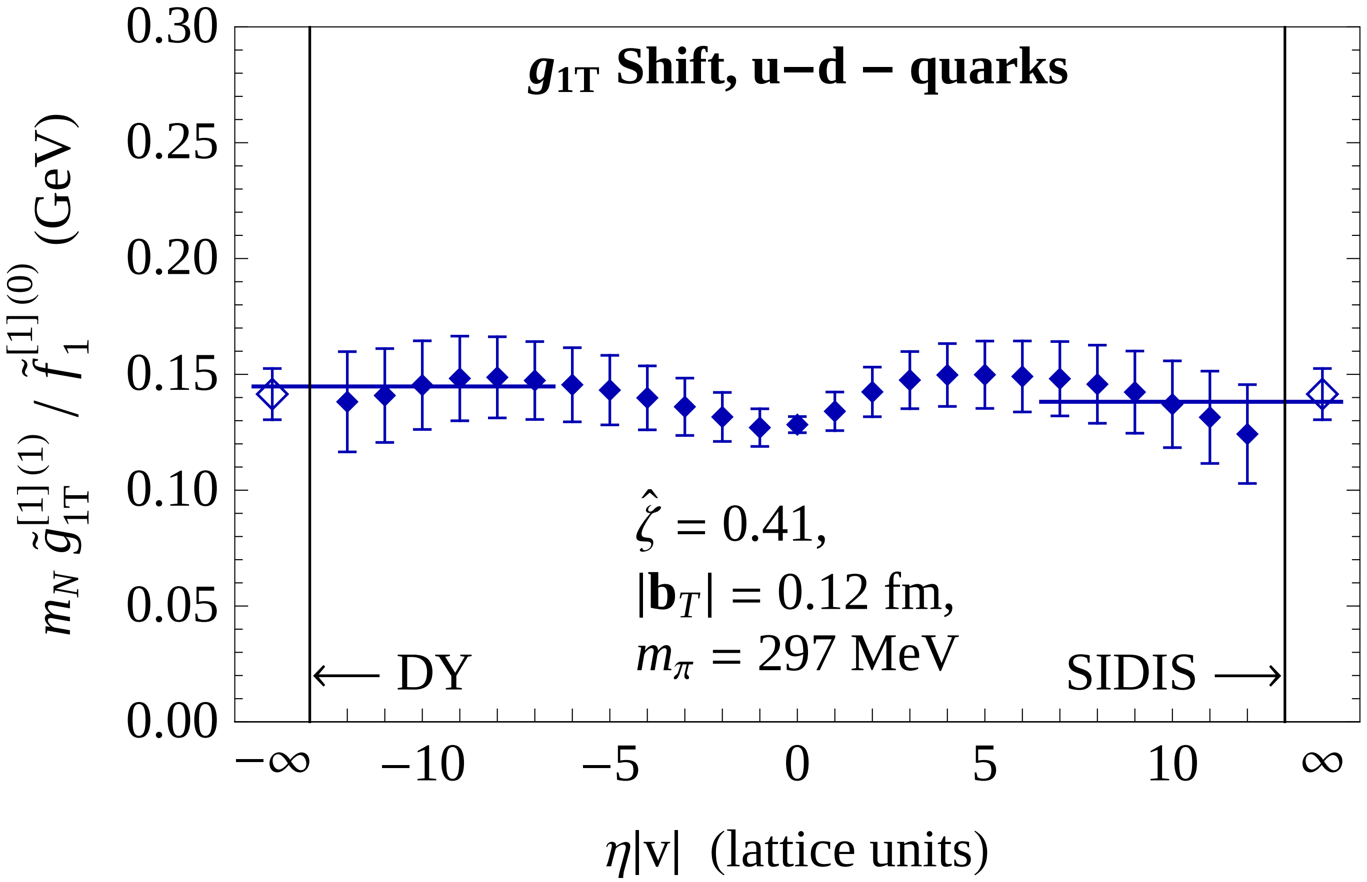




Results: g_{1T} worm gear shift

Dependence on staple extent; sequence of panels at different $|b_T|$





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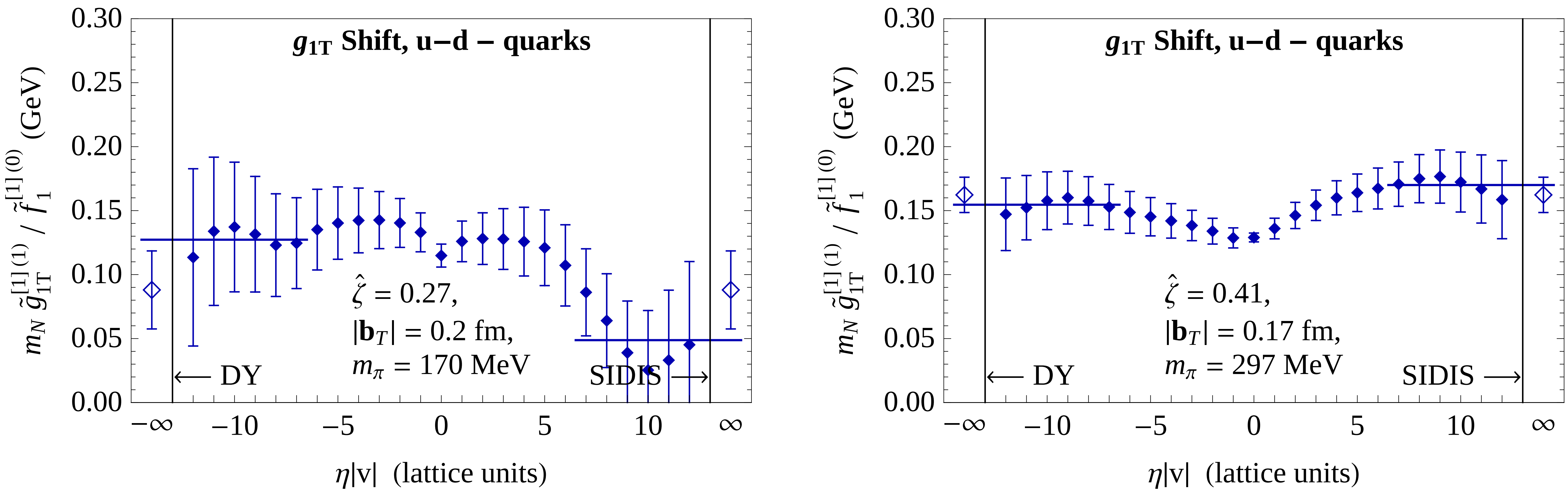

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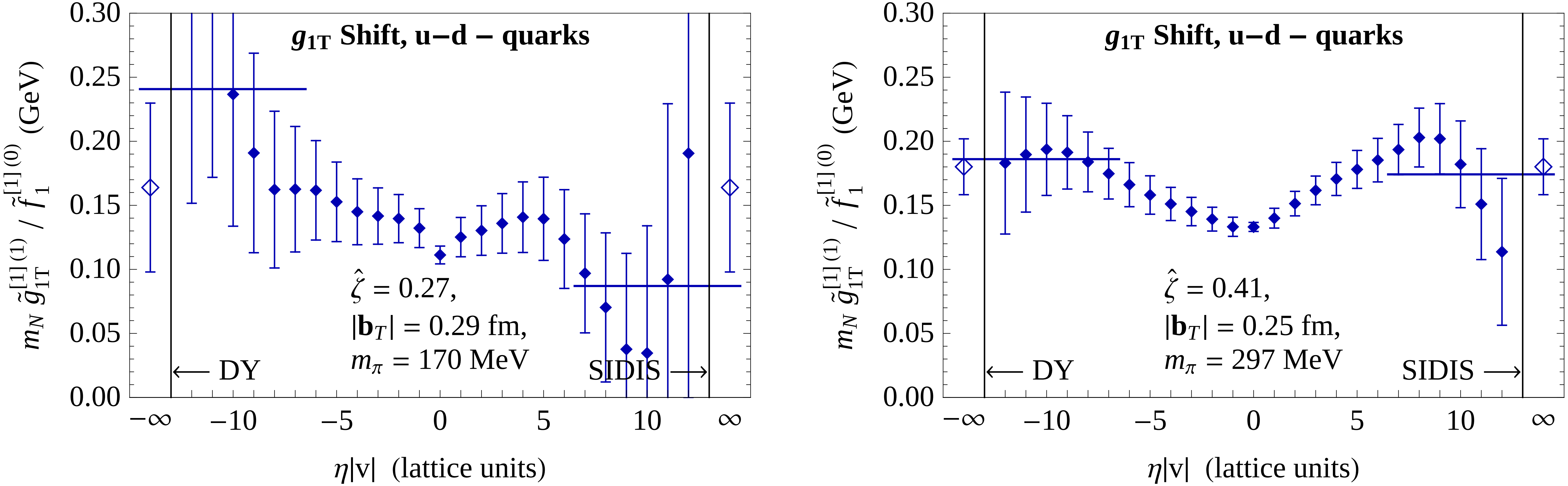

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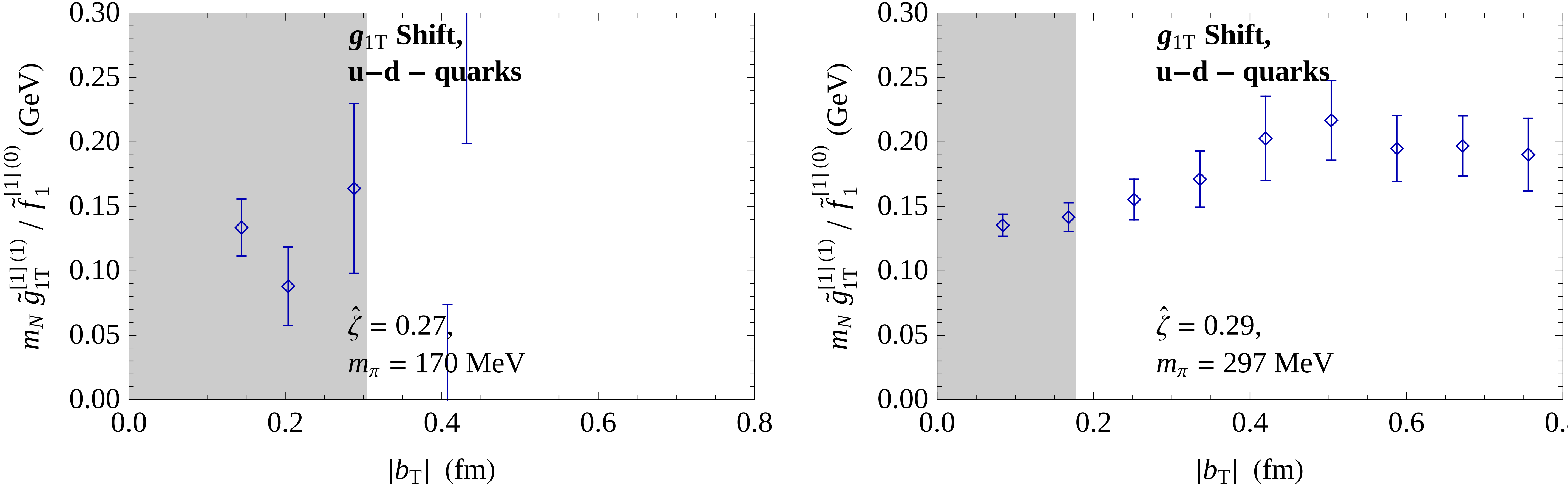
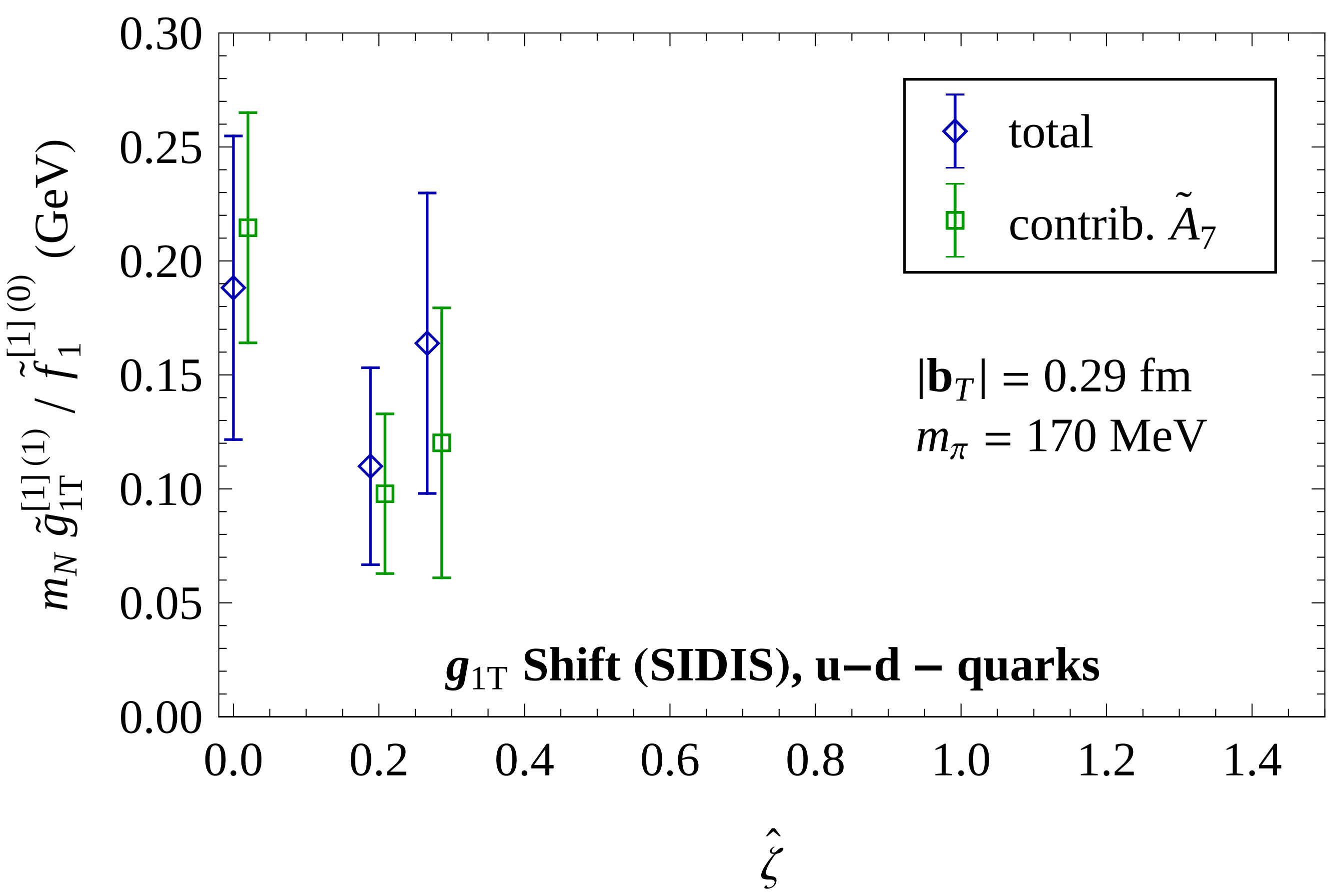
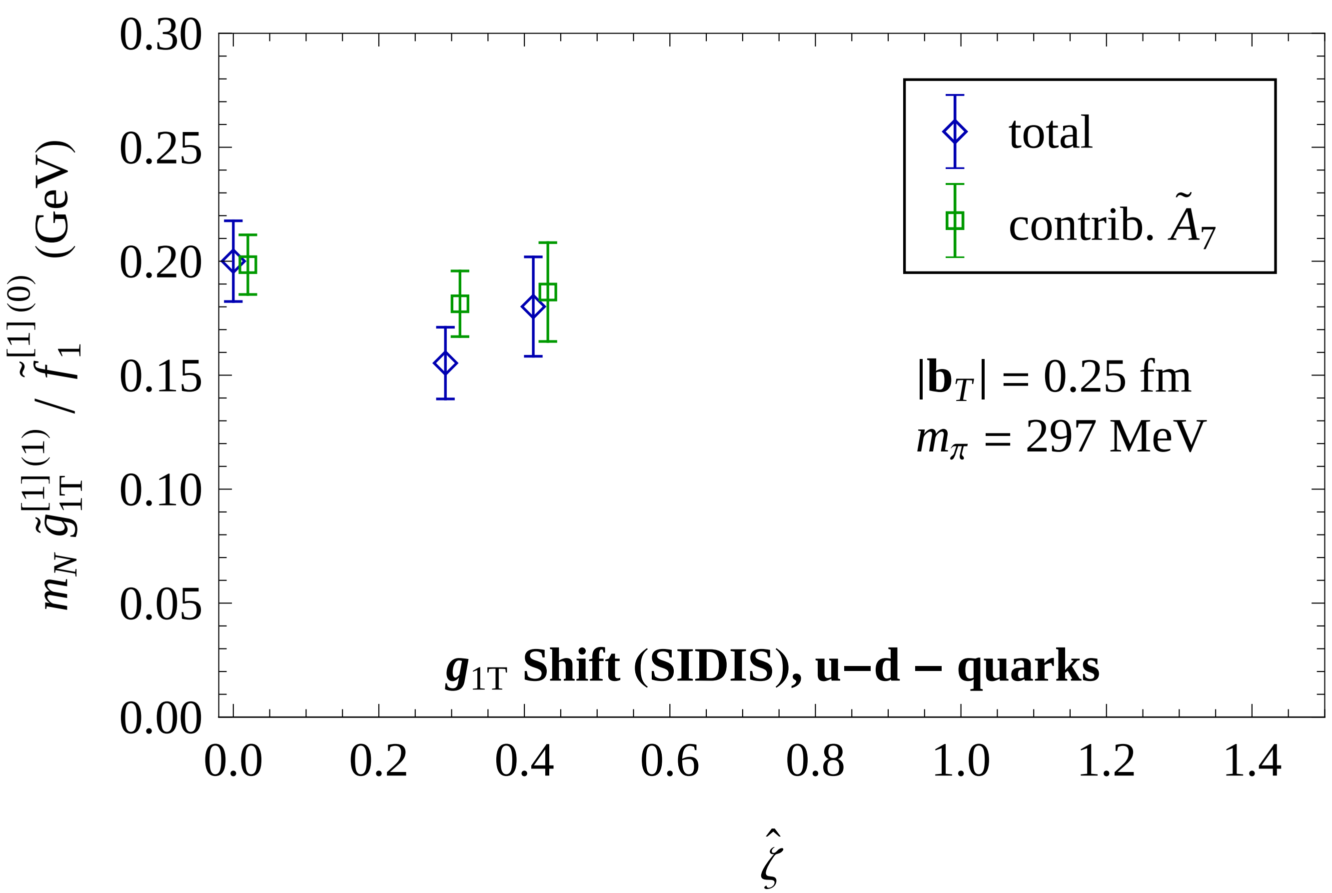
Dependence of SIDIS/DY limit on $|b_T|$



Results: g_{1T} worm gear shift

Dependence of SIDIS/DY limit on $\hat{\zeta}$





Conclusions and Outlook

- Continued exploration of TMDs using bilocal quark operators with staple-shaped gauge link structures; focus on lighter pion mass.
- To avoid soft factors, multiplicative renormalization constants, considered appropriate ratios of Fourier-transformed TMDs ("shifts").
- These observables show no statistically significant variation under the considered change of pion mass, within the (substantial) uncertainties
- In preparation: High-statistics calculation with clover ensemble (furnished by K. Orginos and collaborators) at 190 MeV pion mass.
- Generalization to mixed transverse momentum / transverse position observables (Wigner functions) underway, to directly access quark orbital angular momentum.