New results from lattice $\mathcal{N} = 4$ super Yang–Mills

David Schaich (Syracuse)

Lattice 2015, 18 July

arXiv:1410.6971, arXiv:1411.0166, arXiv:1505.03135 & more to come
with Simon Catterall, Poul Damgaard, Tom DeGrand and Joel Giedt

Brief review of motivations for lattice supersymmetry

- Much interesting physics in 4D supersymmetric gauge theories:
  dualities, holography, confinement, conformality, BSM, ...

- Lattice promises non-perturbative insights from first principles

Problem: Discrete spacetime breaks supersymmetry algebra

$$\left\{Q_\alpha^{\mathrm{I}}, \overline{Q}_{\dot\alpha}^{\mathrm{J}}\right\} = 2\delta^{\mathrm{IJ}}\sigma_{\alpha\dot\alpha}^{\mu}P_\mu \text{ where } \mathrm{I}, \mathrm{J} = 1, \cdots, \mathcal{N}$$

$\Longrightarrow$ Impractical fine-tuning generally required to restore susy,
especially for scalar fields (from matter multiplets or $\mathcal{N} > 1$)

Solution: Preserve (some subset of) the susy algebra on the lattice
Possible for $\mathcal{N} = 4$ supersymmetric Yang–Mills (SYM)

Brief review of $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM is a particularly interesting theory

—Context for development of AdS/CFT correspondence

—Testing ground for reformulations of scattering amplitudes

—Arguably simplest non-trivial field theory in four dimensions

Basic features:

- SU(N) gauge theory with four fermions Ψ^{I} and six scalars Φ^{IJ}, all massless and in adjoint rep.
- Action consists of kinetic, Yukawa and four-scalar terms with coefficients related by symmetries
- Supersymmetric: 16 supercharges Q_α^{I} and $\overline{Q}_{\dot\alpha}^{\mathrm{I}}$ with $\mathrm{I} = 1, \cdots, 4$
  Fields and Q's transform under global SU(4) $\simeq$ SO(6) R symmetry
- Conformal: β function is zero for any 't Hooft coupling λ

Topological twisting $\longrightarrow$ exact susy on the lattice

What is special about $\mathcal{N} = 4$ SYM

The 16 spinor supercharges Q^{I}_{α} and $\overline{Q}^{\mathrm{I}}_{\dot{\alpha}}$ fill a Kähler–Dirac multiplet:

$$\begin{pmatrix} Q^1_\alpha & Q^2_\alpha & Q^3_\alpha & Q^4_\alpha \\ \overline{Q}^1_{\dot{\alpha}} & \overline{Q}^2_{\dot{\alpha}} & \overline{Q}^3_{\dot{\alpha}} & \overline{Q}^4_{\dot{\alpha}} \end{pmatrix} = \mathcal{Q} + \mathcal{Q}_\mu \gamma_\mu + \mathcal{Q}_{\mu\nu}\gamma_\mu\gamma_\nu + \overline{\mathcal{Q}}_\mu \gamma_\mu \gamma_5 + \overline{\mathcal{Q}}\gamma_5$$

$$\longrightarrow \mathcal{Q} + \gamma_a \mathcal{Q}_a + \gamma_a \gamma_b \mathcal{Q}_{ab}$$

with $a, b = 1, \cdots, 5$

$\mathcal{Q}$'s transform with **integer spin** under "twisted rotation group"

$$\mathrm{SO}(4)_{tw} \equiv \mathrm{diag}\left[\mathrm{SO}(4)_{\mathrm{euc}} \otimes \mathrm{SO}(4)_R\right] \qquad \mathrm{SO}(4)_R \subset \mathrm{SO}(6)_R$$

This change of variables gives a susy subalgebra $\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0$

This subalgebra can be exactly preserved on the lattice

Formal supersymmetric lattice action

Directly transcribe twisted continuum action:

$$S = \frac{N}{2\lambda_{\text{lat}}} \mathcal{Q} \left(\chi_{ab}\mathcal{F}_{ab} + \eta \overline{\mathcal{D}}_a \mathcal{U}_a - \frac{1}{2}\eta d \right) - \frac{N}{8\lambda_{\text{lat}}} \epsilon_{abcde}\ \chi_{ab} \overline{\mathcal{D}}_c\ \chi_{de}$$

—Twisting reorganizes fermions $\Psi^I \longrightarrow \eta, \psi_a, \chi_{ab}$,
combines gauge & scalar fields into complexified links $\mathcal{U}_a, \overline{\mathcal{U}}_a$

—Complexification $\longrightarrow$ U(N) $=$ SU(N) $\otimes$ U(1) gauge invariance

—Nilpotent $\mathcal{Q}$ directly interchanges bosonic $\longleftrightarrow$ fermionic d.o.f.

—Susy ($\mathcal{Q}S = 0$) follows from $\mathcal{Q}^2 \cdot = 0$ and Bianchi identity

Not quite suitable for numerical calculations

Exact zero modes and flat directions must be regulated,
especially important in U(1) sector

New improved lattice action

arXiv:1505.03135

—Scalar potential $V = \frac{1}{2N\lambda_{\text{lat}}}\left(\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - N\right)^2$ lifts SU(N) flat directions

—Constraint on plaquette det. lifts U(1) zero mode & flat directions

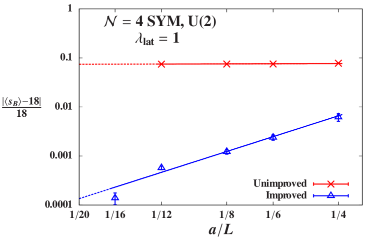
New development — supersymmetric plaquette det. deformation:

$$S = \frac{N}{2\lambda_{\text{lat}}}\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \quad\downarrow\quad - \frac{1}{2}\eta d\right) - \frac{N}{8\lambda_{\text{lat}}}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de} + \mu^2 V$$

$$\eta\left(\overline{\mathcal{D}}_a\mathcal{U}_a + G\sum_{\mathcal{P}}\left[\det\mathcal{P} - 1\right]\mathbb{I}_N\right)$$

Scalar potential **softly** breaks $\mathcal{Q}$,
much less than old non-susy det $\mathcal{P}$
($\sim$500$\times$ smaller lattice artifacts for $L = 16$)

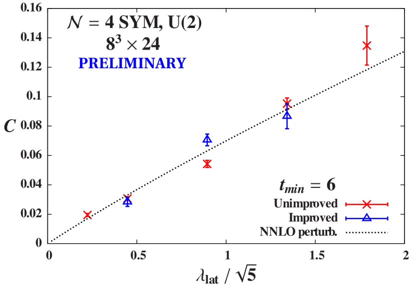
Effective $\mathcal{O}(a)$ improvement
since $\mathcal{Q}$ forbids all dim-5 operators

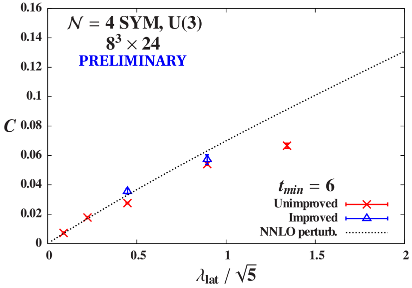


Brief update on the static potential

Previously reported Coulombic static potential $V(r)$ at all λ

Currently confirming and extending with improved action





Left: Agreement with perturbation theory for $N = 2$, $\lambda \lesssim 2$

Right: Tantalizing $\sqrt{\lambda}$-like behavior for $N = 3$, $\lambda \gtrsim 1$,
possibly approaching large-N AdS/CFT prediction $C(\lambda) \propto \sqrt{\lambda}$

Konishi operator scaling dimension

$\mathcal{N} = 4$ SYM is conformal at all λ
$\longrightarrow$ power-law decay for all correlation functions

The Konishi operator is the simplest conformal primary operator

$$\mathcal{O}_K = \sum_{\mathrm{I}} \mathrm{Tr}\left[\Phi^{\mathrm{I}}\Phi^{\mathrm{I}}\right] \qquad\qquad C_K(r) \equiv \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$$

There are many predictions for the scaling dim. $\Delta_K(\lambda) = 2 + \gamma_K(\lambda)$

- From weak-coupling perturbation theory,
  related to strong coupling by $\frac{4\pi N}{\lambda} \longleftrightarrow \frac{\lambda}{4\pi N}$ S duality
- From holography for $N \to \infty$ and $\lambda \to \infty$ but $\lambda \ll N$
- Upper bounds from the conformal bootstrap program

Only lattice gauge theory can access nonperturbative λ at moderate N

Konishi scaling dimension on the lattice

Extract scalar fields from polar decomposition of complexified links

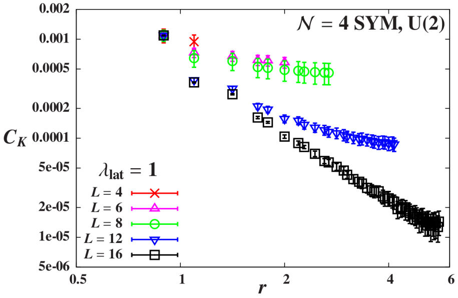
$$\mathcal{U}_a \simeq U_a \left(\mathbb{I}_N + \varphi_a\right) \qquad \widehat{\mathcal{O}}_K = \sum_a \text{Tr}\left[\varphi_a \varphi_a\right] \qquad \overline{\mathcal{O}}_K = \widehat{\mathcal{O}}_K - \left\langle \widehat{\mathcal{O}}_K \right\rangle$$

$$\overline{C}_K(r) = \overline{\mathcal{O}}_K(x+r)\overline{\mathcal{O}}_K(x) \propto r^{-2\Delta_K}$$

Obvious sensitivity to volume
as desired for conformal system

$\Longrightarrow$ finite-size scaling to find Δ_K,

$$\int r^{n+3}\, \overline{C}_K^{(L)}(r)\, dr \propto L^{4+n-2\Delta_K}$$



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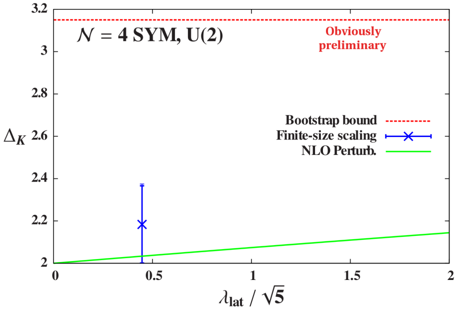


—Work in progress to add more points & reduce uncertainties
—Also carrying out complementary MCRG analyses. . .

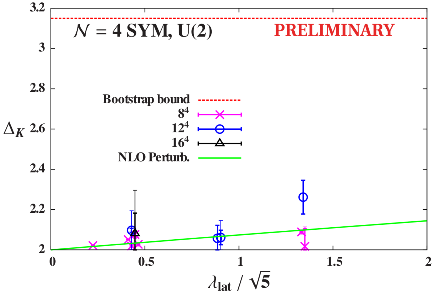
Konishi scaling dimension from Monte Carlo RG

Eigenvalues of MCRG stability matrix $\longrightarrow$ scaling dimensions

RG blocking parameter ξ set by matching plaquettes for L vs. $L/2$

Horizontally displaced points use different auxiliary couplings μ & G

Currently running larger λ_{lat} and larger $N = 3, 4$



Uncertainties from weighted histogram of results from...

- ⋆ 1 & 2 RG blocking steps
- ⋆ Blocked volumes 3^4 through 8^4
- ⋆ 1–5 operators in stability matrix

Revisiting the sign problem

Pfaffian can be complex for lattice $\mathcal{N} = 4$ SYM, $\quad \text{pf}\,\mathcal{D} = |\text{pf}\,\mathcal{D}|e^{i\alpha}$

Previously found $1 - \langle\cos(\alpha)\rangle \ll 1$, independent of lattice volume

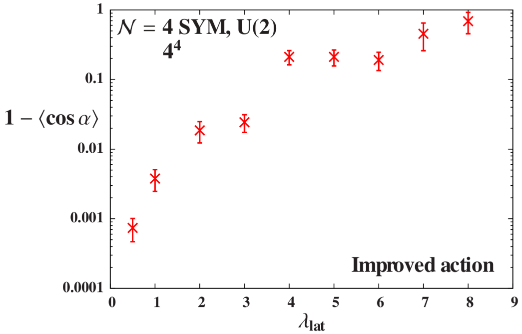
Now extending with improved action, which allows access to larger λ

Finding much larger phase fluctuations at stronger couplings

Parallel $\mathcal{O}(n^3)$ algorithm

Typical 4^4 measurement requires $\sim$60 hours, $\sim$4GB memory

Filling in more volumes & N



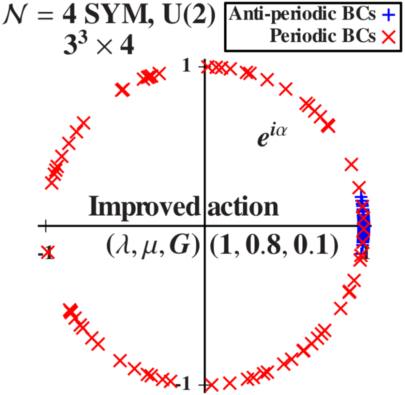
Two puzzles posed by the sign problem

- With periodic temporal boundary conditions for the fermions
  we have an obvious sign problem, $\left\langle e^{i\alpha}\right\rangle$ consistent with zero
- With anti-periodic BCs and all else the same $\left\langle e^{i\alpha}\right\rangle \approx 1$,
  phase reweighting not even necessary

Why such sensitivity to the BCs?

Also, other observables
are nearly identical
for these two ensembles

Why doesn't the sign problem
have observable effects?



Recapitulation

- Rapid progress in lattice $\mathcal{N} = 4$ SYM
- New improved action dramatically reduces lattice artifacts
- $N = 3$ static potential apparently approaching AdS/CFT prediction
- Promising initial results for Konishi anomalous dimension
- New information on origin and effects of sign problem

Advertisement: Public code for lattice $\mathcal{N} = 4$ SYM

so that the full improved action becomes

$$
\begin{aligned}
S_{\text{imp}} &= S'_{\text{exact}} + S_{\text{closed}} + S'_{\text{soft}} \qquad (3.10) \\
S'_{\text{exact}} &= \frac{N}{2\lambda_{\text{lat}}} \sum_n \text{Tr}\Bigg[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) - \chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{b]}(n) - \eta(n)\overline{\mathcal{D}}^{(-)}_a\psi_a(n) \\
&\qquad + \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a(n) + G\sum_{a\neq b}\left(\det\mathcal{P}_{ab}(n) - 1\right)\mathbb{I}_N\right)^2\Bigg] - S_{\text{det}} \\
S_{\text{det}} &= \frac{N}{2\lambda_{\text{lat}}} G \sum_n \text{Tr}\left[\eta(n)\right] \sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n)\right] \text{Tr}\left[\mathcal{U}_b^{-1}(n)\psi_b(n) + \mathcal{U}_a^{-1}(n+\widehat{\mu}_b)\psi_a(n+\widehat{\mu}_b)\right] \\
S_{\text{closed}} &= -\frac{N}{8\lambda_{\text{lat}}} \sum_n \text{Tr}\left[\epsilon_{abcde}\ \chi_{de}(n+\widehat{\mu}_a+\widehat{\mu}_b+\widehat{\mu}_c)\overline{\mathcal{D}}^{(-)}_c\chi_{ab}(n)\right], \\
S'_{\text{soft}} &= \frac{N}{2\lambda_{\text{lat}}}\mu^2 \sum_n \sum_a \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a(n)\overline{\mathcal{U}}_a(n)\right] - 1\right)^2
\end{aligned}
$$

The lattice action is obviously very complicated
(the fermion operator involves $\gtrsim$100 gathers)

To reduce barriers to entry our parallel code is publicly developed at
github.com/daschaich/susy

Evolved from MILC code, presented in arXiv:1410.6971

Thank you!

Thank you!

Collaborators

Simon Catterall, Poul Damgaard, Tom DeGrand and Joel Giedt

Funding and computing resources

Backup: Failure of Leibnitz rule in discrete space-time

Given that $\left\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\right\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu = 2i\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu$ is problematic,

why not try $\left\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\right\} = 2i\sigma^\mu_{\alpha\dot{\alpha}} \nabla_\mu$ for a discrete translation?

Here $\nabla_\mu \phi(x) = \frac{1}{a}\left[\phi(x + a\widehat{\mu}) - \phi(x)\right] = \partial_\mu \phi(x) + \frac{a}{2}\partial_\mu^2 \phi(x) + \mathcal{O}(a^2)$

Essential difference between ∂_μ and ∇_μ on the lattice, $a > 0$

$$\begin{aligned}\nabla_\mu \left[\phi(x)\chi(x)\right] &= a^{-1}\left[\phi(x + a\widehat{\mu})\chi(x + a\widehat{\mu}) - \phi(x)\chi(x)\right] \\ &= \left[\nabla_\mu \phi(x)\right]\chi(x) + \phi(x)\nabla_\mu \chi(x) + a\left[\nabla_\mu \phi(x)\right]\nabla_\mu \chi(x)\end{aligned}$$

We only recover the Leibnitz rule $\partial_\mu(fg) = (\partial_\mu f)g + f\partial_\mu g$ when $a \to 0$

$\Longrightarrow$ “Discrete supersymmetry” breaks down on the lattice

(Dondi & Nicolai, “Lattice Supersymmetry”, 1977)

Backup: Twisting $\longleftrightarrow$ Kähler–Dirac fermions

The Kähler–Dirac representation is related to the spinor $Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{I}}_{\dot{\alpha}}$ by

$$\begin{pmatrix} Q^1_\alpha & Q^2_\alpha & Q^3_\alpha & Q^4_\alpha \\ \overline{Q}^1_{\dot{\alpha}} & \overline{Q}^2_{\dot{\alpha}} & \overline{Q}^3_{\dot{\alpha}} & \overline{Q}^4_{\dot{\alpha}} \end{pmatrix} = \mathcal{Q} + \mathcal{Q}_\mu \gamma_\mu + \mathcal{Q}_{\mu\nu}\gamma_\mu\gamma_\nu + \overline{\mathcal{Q}}_\mu \gamma_\mu \gamma_5 + \overline{\mathcal{Q}}\gamma_5 \longrightarrow \mathcal{Q} + \gamma_a \mathcal{Q}_a + \gamma_a \gamma_b \mathcal{Q}_{ab}$$

with $a, b = 1, \cdots, 5$

The 4×4 matrix involves R symmetry transformations along each row and (euclidean) Lorentz transformations along each column

$\Longrightarrow$ Kähler–Dirac components transform under "twisted rotation group"

$$\mathrm{SO}(4)_{tw} \equiv \mathrm{diag}\left[\mathrm{SO}(4)_{\mathrm{euc}} \otimes \mathrm{SO}(4)_R\right]$$

$\uparrow$ only $\mathrm{SO}(4)_R \subset \mathrm{SO}(6)_R$

Backup: Twisted $\mathcal{N} = 4$ SYM fields and $\mathcal{Q}$

Everything transforms with **integer spin** under SO(4)$_{tw}$ — **no spinors**

$$Q^{\rm I}_{\alpha} \text{ and } \overline{Q}^{\rm I}_{\dot\alpha} \longrightarrow \mathcal{Q},\ \mathcal{Q}_a \text{ and } \mathcal{Q}_{ab}$$

$$\Psi^{\rm I} \text{ and } \overline{\Psi}^{\rm I} \longrightarrow \eta,\ \psi_a \text{ and } \chi_{ab}$$

$$A_\mu \text{ and } \Phi^{\rm IJ} \longrightarrow \mathcal{A}_a = (A_\mu, \phi) + i(B_\mu, \overline{\phi}) \text{ and } \overline{\mathcal{A}}_a$$

The twisted-scalar supersymmetry $\mathcal{Q}$ acts as

$$\mathcal{Q}\ \mathcal{A}_a = \psi_a \qquad\qquad \mathcal{Q}\ \psi_a = 0$$

$$\mathcal{Q}\ \chi_{ab} = -\overline{\mathcal{F}}_{ab} \qquad\qquad \mathcal{Q}\ \overline{\mathcal{A}}_a = 0$$

$$\mathcal{Q}\ \eta = d \qquad\qquad \mathcal{Q}\ d = 0$$

↖ bosonic auxiliary field with e.o.m. $d = \overline{\mathcal{D}}_a \mathcal{A}_a$

1. $\mathcal{Q}$ directly interchanges bosonic $\longleftrightarrow$ fermionic d.o.f.
2. The susy subalgebra $\mathcal{Q}^2 \cdot = 0$ is manifest

Backup: Lattice $\mathcal{N} = 4$ SYM fields and $\mathcal{Q}$

The lattice theory is very nearly a direct transcription

- Covariant derivatives $\longrightarrow$ finite difference operators
- Gauge fields $\mathcal{A}_a \longrightarrow$ gauge links $\mathcal{U}_a$

$$\mathcal{Q}\,\mathcal{A}_a \longrightarrow \mathcal{Q}\,\mathcal{U}_a = \psi_a \qquad\qquad \mathcal{Q}\,\psi_a = 0$$

$$\mathcal{Q}\,\chi_{ab} = -\overline{\mathcal{F}}_{ab} \qquad\qquad \mathcal{Q}\,\overline{\mathcal{A}}_a \longrightarrow \mathcal{Q}\,\overline{\mathcal{U}}_a = 0$$

$$\mathcal{Q}\,\eta = d \qquad\qquad \mathcal{Q}\,d = 0$$

- Formal lattice action retains same form as continuum action and remains supersymmetric, $\mathcal{Q}S = 0$

Geometrical formulation facilitates discretization

η live on lattice sites $\qquad$ ψ_a live on links

χ_{ab} connect opposite corners of oriented plaquettes

Orbifolding / dimensional deconstruction produces same lattice system

Backup: A_4^* lattice with five links in four dimensions

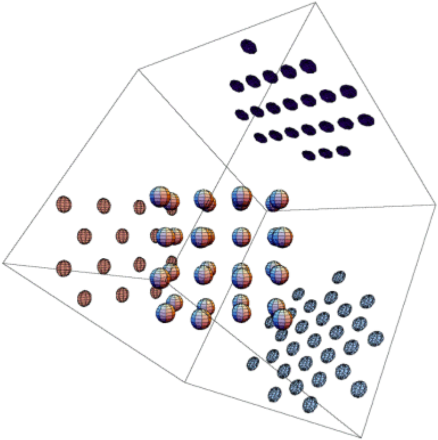
$A_a = (A_\mu, \phi)$ may remind you of dimensional reduction

On the lattice we want to treat all five $\mathcal{U}_a$ symmetrically
to obtain $S_5 \longrightarrow \text{SO}(4)_{tw}$ symmetry

—Start with hypercubic lattice
in 5d momentum space

—**Symmetric** constraint $\sum_a \partial_a = 0$
projects to 4d momentum space

—Result is A_4 lattice
$\longrightarrow$ dual A_4^* lattice in real space

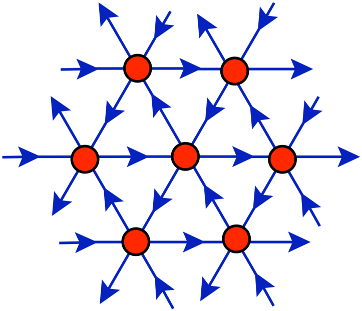


Backup: A_4^* lattice point group symmetry

—Can picture A_4^* lattice
as 4d analog of 2d triangular lattice

—Preserves S_5 point group symmetry

—Basis vectors are non-orthogonal
and linearly dependent

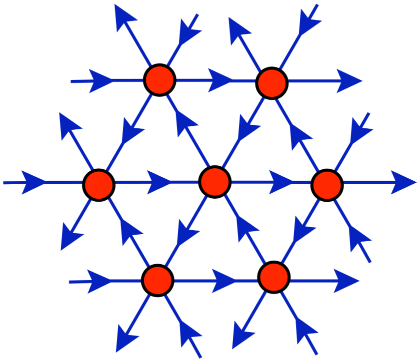


S_5 irreps precisely match onto irreps of twisted $\text{SO}(4)_{tw}$

$$\mathbf{5} = \mathbf{4} \oplus \mathbf{1}: \quad \mathcal{U}_a \longrightarrow A_\mu + iB_\mu, \quad \phi + i\overline{\phi}$$
$$\psi_a \longrightarrow \psi_\mu, \quad \overline{\eta}$$
$$\mathbf{10} = \mathbf{6} \oplus \mathbf{4}: \quad \chi_{ab} \longrightarrow \chi_{\mu\nu}, \quad \overline{\psi}_\mu$$

Backup: Hypercubic representation of A_4^* lattice

In the code it is very convenient to represent the A_4^* lattice
as a hypercube with a backwards diagonal



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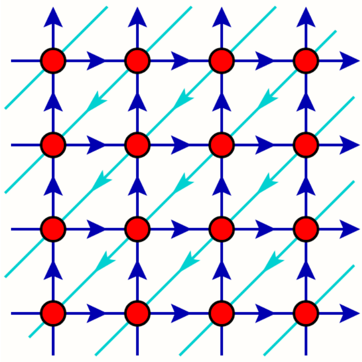


Backup: More on flat directions

1. Complex gauge field $\Longrightarrow$ U(N) = SU(N) $\otimes$ U(1) gauge invariance
   U(1) sector decouples only in continuum limit

2. $\mathcal{Q}\,\mathcal{U}_a = \psi_a \Longrightarrow$ gauge links must be elements of algebra
   Resulting flat directions required by supersymmetric construction
   but must be lifted to ensure $\mathcal{U}_a = \mathbb{I}_N + \mathcal{A}_a$ in continuum limit

We need to add two deformations to regulate flat directions

SU(N) scalar potential $\propto \mu^2 \sum_a \left(\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - N\right)^2$

U(1) plaquette determinant $\sim G \sum_{a \neq b} \left(\det \mathcal{P}_{ab} - 1\right)$

Scalar potential **softly** breaks $\mathcal{Q}$ supersymmetry

↖susy-violating operators vanish as $\mu^2 \to 0$

Plaquette determinant can be made $\mathcal{Q}$-invariant $\longrightarrow$ improved action

Backup: One problem with flat directions

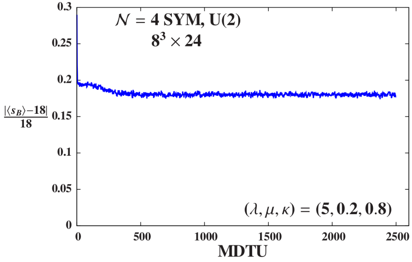
Gauge fields $\mathcal{U}_a$ can move far away from continuum form $\mathbb{I}_N + \mathcal{A}_a$

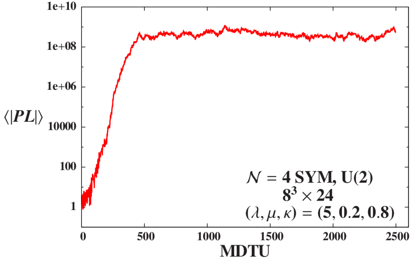
if $N\mu^2/(2\lambda_{\text{lat}})$ becomes too small

Example for two-color $(\lambda_{\text{lat}}, \mu, \kappa) = (5, 0.2, 0.8)$ on $8^3 \times 24$ volume

Left: Bosonic action is stable $\sim$18% off its supersymmetric value

Right: Polyakov loop wanders off to $\sim 10^9$

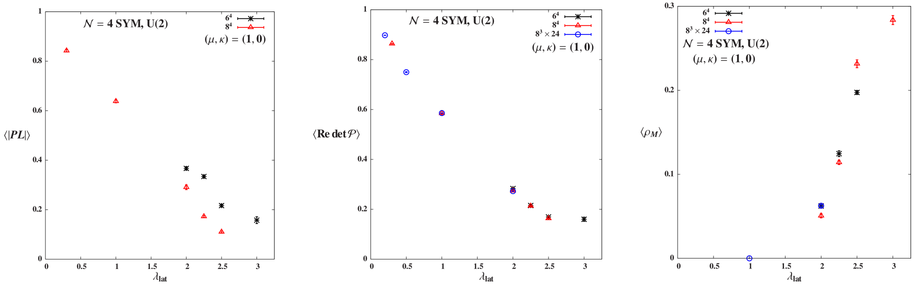




Backup: Another problem with U(1) flat directions

Flat directions in U(1) sector can induce transition to confined phase

This lattice artifact is not present in continuum $\mathcal{N} = 4$ SYM



Around the same $\lambda_{\text{lat}} \approx 2\ldots$

Left: Polyakov loop falls towards zero

Center: Plaquette determinant falls towards zero

Right: Density of U(1) monopole world lines becomes non-zero

Backup: Soft susy breaking

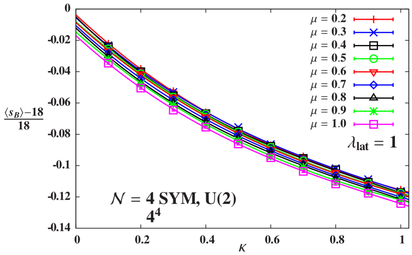
The unimproved action directly adds to the lattice action

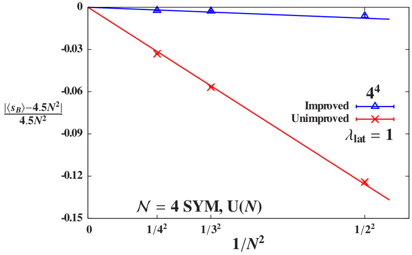
$$S_{soft} = \frac{N}{2\lambda_{\text{lat}}}\mu^2 \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - 1\right)^2 + \kappa \left|\det \mathcal{P}_{ab} - 1\right|^2$$

Both terms explicitly break $\mathcal{Q}$ but $\det \mathcal{P}_{ab}$ effects dominate

Left: The breaking is **soft** — guaranteed to vanish as $\mu, \kappa \longrightarrow 0$

Right: Soft $\mathcal{Q}$ breaking also suppressed $\propto 1/N^2$





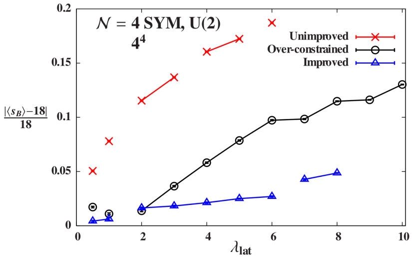
Backup: More on supersymmetric constraints

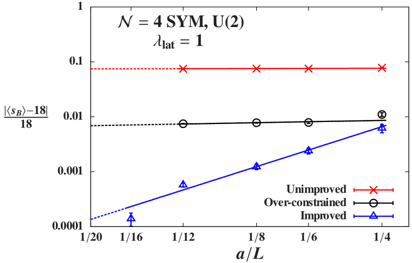
Improved action from arXiv:1505.03135
imposes $\mathcal{Q}$-invariant plaquette determinant constraint

Basic idea: Modify the equations of motion $\longrightarrow$ moduli space

$$d(n) = \overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(n) \longrightarrow \overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(n) + G\sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n) - 1\right]$$

Produces much smaller violations of $\mathcal{Q}$ Ward identity $\langle s_B\rangle = 9N^2/2$



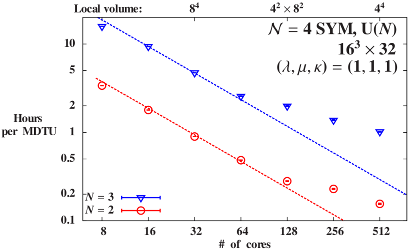


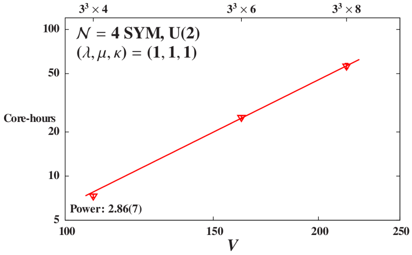
Backup: Code performance—weak and strong scaling

Results from arXiv:1410.6971 using the unimproved action

Left: Strong scaling for U(2) and U(3) $16^3 \times 32$ RHMC

Right: Weak scaling for $\mathcal{O}(n^3)$ pfaffian calculation (fixed local volume)
$n \equiv 16N^2L^3N_T$ is number of fermion degrees of freedom





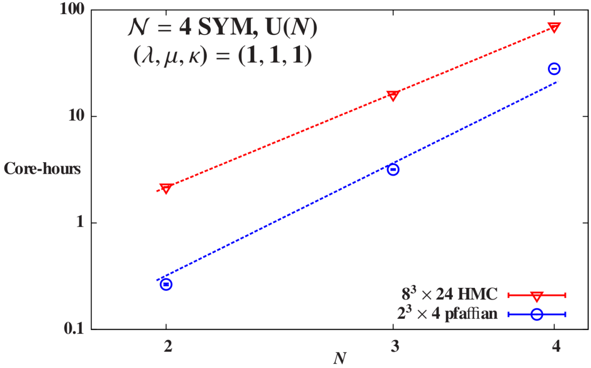
Both plots on log–log axes with power-law fits

Backup: Numerical costs for 2, 3 and 4 colors

Red: Find RHMC cost scaling $\sim N^5$ (recall adjoint fermion d.o.f. $\propto N^2$)

Blue: Pfaffian cost scaling consistent with expected N^6

Additional factor of $\sim 2\times$ from improved action, but same scaling

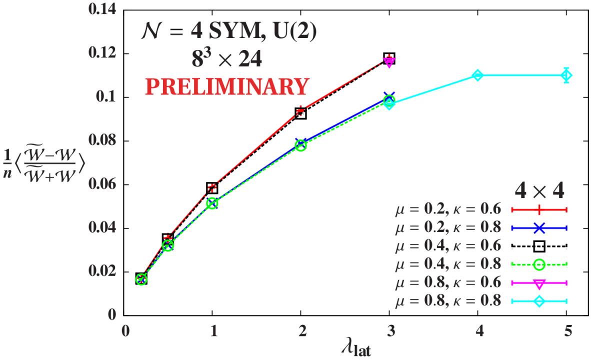


Backup: Restoration of $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ supersymmetries

Restoration of the other 15 $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ in the continuum limit follows from restoration of R symmetry (motivation for A_4^* lattice)

Modified Wilson loops test R symmetries at non-zero lattice spacing

Results from arXiv:1411.0166 to be revisited with the improved action

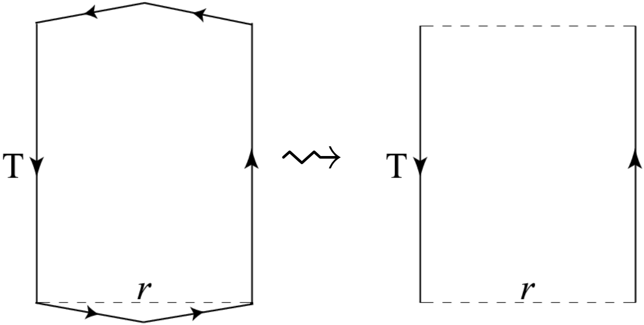


Backup: $\mathcal{N} = 4$ static potential from Wilson loops

Extract static potential $V(r)$ from $r \times T$ Wilson loops

$$W(r, T) \propto e^{-V(r)\,T} \qquad\qquad V(r) = A - C/r + \sigma r$$

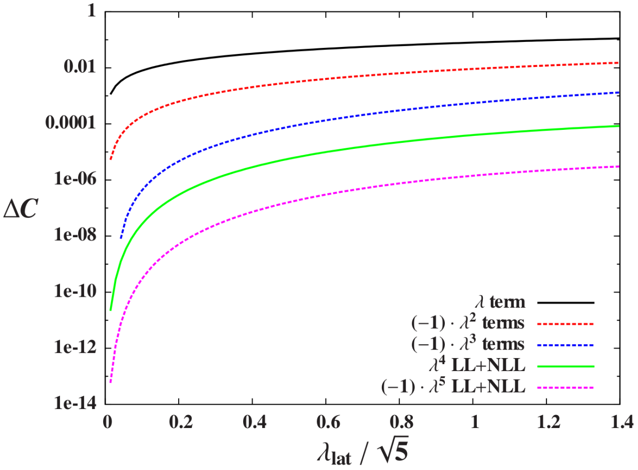
Coulomb gauge trick from lattice QCD reduces A_4^* lattice complications



Backup: Perturbation theory for Coulomb coefficient

For range of λ_{lat} currently being studied

perturbation theory for the Coulomb coefficient is well behaved

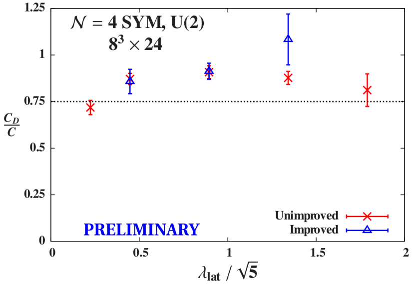


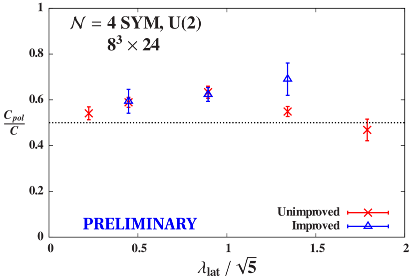
Backup: More tests of the U(2) static potential

Left: Projecting Wilson loops from U(2) $\longrightarrow$ SU(2)

$\Longrightarrow$ factor of $\frac{N^2-1}{N^2} = 3/4$

Right: Unitarizing links removes scalars $\Longrightarrow$ factor of $1/2$





Some results slightly above expected factors,

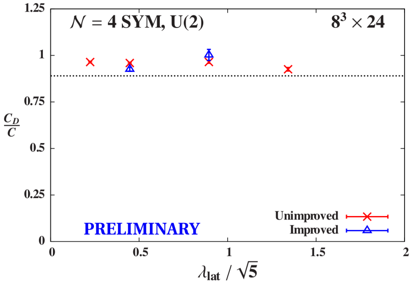
may be related to non-zero auxiliary couplings μ and κ / G

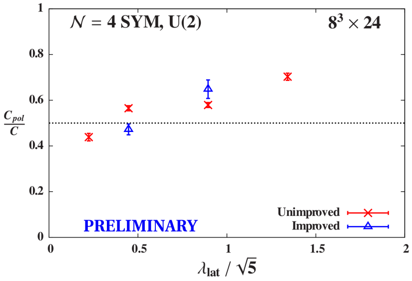
Backup: More tests of the U(3) static potential

Left: Projecting Wilson loops from U(3) $\longrightarrow$ SU(3)

$\Longrightarrow$ factor of $\frac{N^2-1}{N^2} = 8/9$

Right: Unitarizing links removes scalars $\Longrightarrow$ factor of $1/2$





Some results slightly above expected factors,

may be related to non-zero auxiliary couplings μ and κ / G

Backup: Smearing for Konishi analyses

—As in glueball analyses, operator basis enlarged through smearing

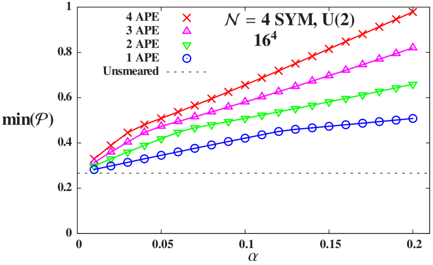
—Use APE-like smearing $(1-\alpha)$— $+\ \frac{\alpha}{8}\sum\sqcap$,

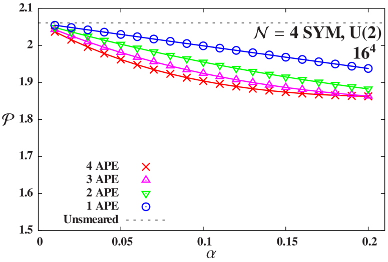
with staples built from unitary parts of links but no final unitarization

(unitarized smearing — e.g. stout — doesn't affect Konishi)

—Average plaquette is stable upon smearing (**right**)

even though minimum plaquette steadily increases (**left**)





Backup: Scaling dimensions from Monte Carlo RG

Write system as (infinite) sum of operators $\mathcal{O}_i$ with couplings c_i

Couplings c_i flow under RG blocking transformation R_b

n-times-blocked system is $H^{(n)} = R_b H^{(n-1)} = \sum_i c_i^{(n)} \mathcal{O}_i^{(n)}$

Consider linear expansion around fixed point $H^\star$ with couplings $c_i^\star$

$$c_i^{(n)} - c_i^\star = \sum_j \left. \frac{\partial c_i^{(n)}}{\partial c_j^{(n-1)}} \right|_{H^\star} \left(c_j^{(n-1)} - c_j^\star \right) \equiv \sum_j T_{ij}^\star \left(c_j^{(n-1)} - c_j^\star \right)$$

$T_{ij}^\star$ is the stability matrix

Eigenvalues of $T_{ij}^\star \longrightarrow$ scaling dimensions of corresponding operators

Backup: The sign problem

In lattice gauge theory we compute operator expectation values

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int [d\mathcal{U}][d\overline{\mathcal{U}}]\, \mathcal{O}\, e^{-S_B[\mathcal{U},\overline{\mathcal{U}}]}\, \text{pf}\, \mathcal{D}[\mathcal{U},\overline{\mathcal{U}}]$$

$\text{pf}\, \mathcal{D} = |\text{pf}\, \mathcal{D}| e^{i\alpha}$ can be complex for lattice $\mathcal{N} = 4$ SYM
$\longrightarrow$ Complicates interpretation of $\left[e^{-S_B}\, \text{pf}\, \mathcal{D}\right]$ as Boltzmann weight

Instead absorb $e^{i\alpha}$ into phase-quenched (pq) observables $\mathcal{O} e^{i\alpha}$
and reweight using $Z = \int e^{i\alpha}\, e^{-S_B}\, |\text{pf}\, \mathcal{D}| = \left\langle e^{i\alpha} \right\rangle_{pq}$

$$\langle \mathcal{O} \rangle_{pq} = \frac{1}{\mathcal{Z}_{pq}} \int [d\mathcal{U}][d\overline{\mathcal{U}}]\, \mathcal{O}\, e^{-S_B}\, |\text{pf}\, \mathcal{D}| \qquad \langle \mathcal{O} \rangle = \frac{\left\langle \mathcal{O} e^{i\alpha} \right\rangle_{pq}}{\left\langle e^{i\alpha} \right\rangle_{pq}}$$

Sign problem: This breaks down if $\left\langle e^{i\alpha} \right\rangle_{pq}$ is consistent with zero

Backup: Pfaffian phase volume dependence

No indication of a sign problem at $\lambda_{\text{lat}} = 1$ with anti-periodic BCs

- Results from arXiv:1411.0166 using the unimproved action
- Fluctuations in pfaffian phase don't grow with the lattice volume
- Insensitive to number of colors $N = 2, 3, 4$