# Chiral symmetry breaking, instantons, and monopoles

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# Introduction

- **We want to show that monopoles condensing in the QCD vacuum are closely related to instantons, and chiral symmetry breaking.**
- **We add monopoles by a monopole creation operator in SU(3) quenched configurations** [C. Bonati, et al., PRD 85 (2012) 065001]**. We use Overlap fermions as an analytical tool** [R. G. Edwards, et al., PRD 61 (2000) 074504; L. Giusti, et al., JHEP 11 (2003) 023; L. Del Debbio, et al., PRL 94 (2005) 032003; L. Del Debbio, et al., JHEP 02 (2004) 003]**.**

  **We have found three things as follows:**

  **(1) Monopoles make instantons** [A. Di Giacomo and M. H. PRD 91 (2015) 054512]**.**

  **(2) Chiral symmetry breaking is induced by monopoles** [A. Di Giacomo and M. H. arXiv: 1412.2704]**.**

  **(3) The chiral condensate increases by increasing the monopole charge without affecting the spectrum of the Overlap Dirac operator** [A Di Giacomo, M. H., and F. Pucci, Chiral Dynamics 2015, Pisa]**.**

# The purpose of this study

- In this study, we want to confirm using large lattices as follows:

  (1) One monopole charge makes one instanton.

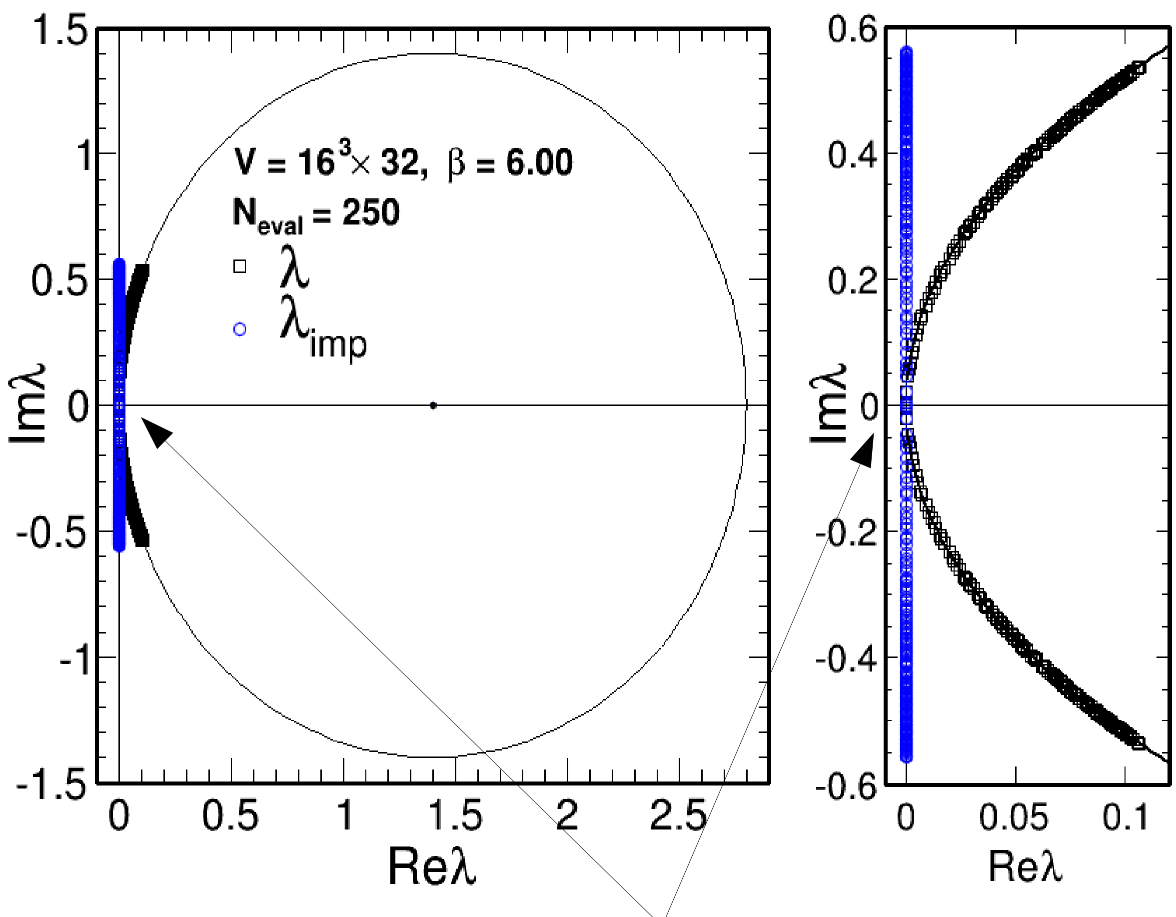
  (2) The chiral condensate decreases by increasing the monopole charge.

  (3) The pion (pseudo-scalar) mass increases by increasing the monopole charge.

  (4) The pion decay constant is not changed by the monopoles.

# Overlap fermions

- The Overlap fermion holds the exact chiral symmetry in the Lattice gauge theory [P. H. Ginsparg and K. G. Wilson PRD 25 (1982) 2649; N. Neuberger, PLB 427 (1998) 353].
- We compute the low-lying ***O***(60-100) eigenvalues **λ** of the Overlap Dirac operator which are on the circle, and the improved eigenvalues $\boldsymbol{\lambda}_{\mathbf{imp}}$ are on the imaginary axis [S. Capitani, et al., PLB 468 (1999) 150; L. Giusti, et al. Comp. Phys. Comm. 153 (2003) 31].
- We use the improved eigenvalues in this study.




**There is one zero mode.**

# Overlap fermions

- The improved eigenvalues $\boldsymbol{\lambda}^{imp}$ are computed from the improved Overlap Dirac operator $\boldsymbol{D}^{imp}$:

$$D^{\mathrm{imp}}(0) = \left(1 - \frac{a}{2\rho} D(0)\right)^{-1} D(0)$$

- The spectral density $\boldsymbol{\rho(\lambda)}$ is computed from the improved eigenvalues:

$$\rho(\lambda,\ V) = \frac{1}{V}\langle\sum_{\lambda}\delta(\lambda - \bar{\lambda})\rangle,\ \bar{\lambda} = \mathrm{Im}(\lambda^{imp})$$

- The number of zero modes of the plus chirality is $\boldsymbol{n_+}$. The number of zero modes of the minus chirality is $\boldsymbol{n_-}$.
- Topological charge $\boldsymbol{Q}$ is computed from the number of zero modes: $\mathbf{Q} = \mathbf{n}_+ - \mathbf{n}_-$

# Instantons

- The number of instantons of plus charges is $n_+$. The number of instantons of minus charges is $n_-$.
- However, **we never observed the numbers of zero modes of the plus chirality and the minus chirality in the same configuration at the same time.**
- In simulations, the observed number of zero modes is the **net number of zero modes**; **the topological charge *Q*.**
- **We have shown that the number of instantons $N_i$ is counted from the average square of the topological charge** [A. Di Giacomo and M. H. PRD 91 (2015) 054512]**:** $\mathbf{N_i = \langle Q^2 \rangle}$

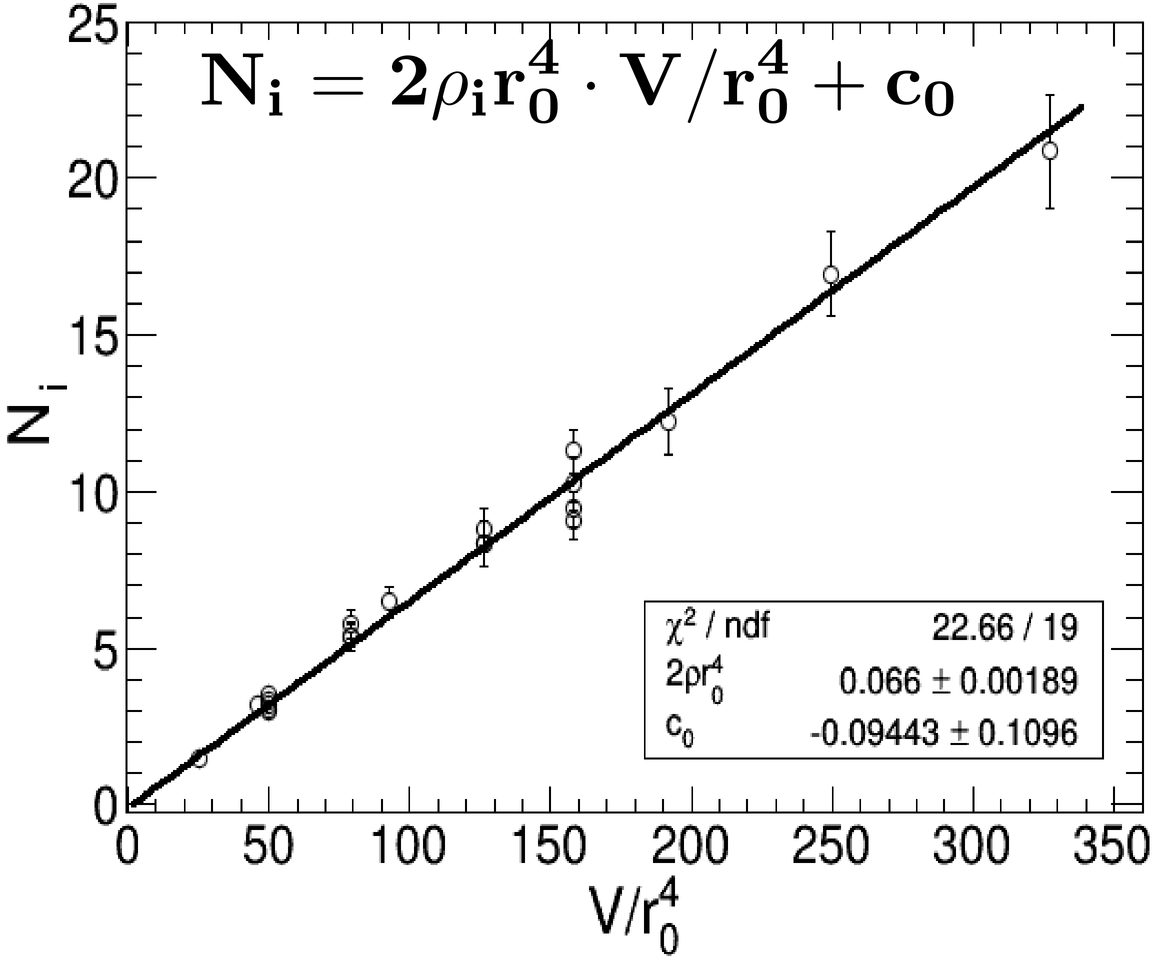
# Instantons

- There is a relation as follows: $\langle \mathbf{Q^2} \rangle = \mathbf{N_i} = \mathbf{2\rho_i V}$
- The instanton density $\boldsymbol{\rho_i}$ which is evaluated by fitting a linear function:

  $\rho_{\mathbf{i}} = \mathbf{8.0(2)} \times \mathbf{10^{-4}}\ [\mathrm{GeV}^{\mathbf{4}}]$
- The instanton liquid mode [E. V. Shuryak, NPB 203 (1982) 93]:

  $\mathbf{n_c} = \mathbf{8} \times \mathbf{10^{-4}}\ [\mathrm{GeV}^{\mathbf{4}}]$

  **Consistent!**
- The fitting values $\boldsymbol{c_0} \approx \mathbf{0}$, and $\chi^2$/d.o.f = 1.2.




All of the lattice spacings are computed from the analytic interpolation [S. Necco, et al., NPB 622 (2002) 328] ($r_0$ = 0.5 [fm]).

# Monopoles

- **We add a pair of a monopole and an anti-monopole by the monopole creation operator in quenched SU(3) configurations** [C. Bonati, et al. PRD 85 (2012) 065001]**.**

$$S + \Delta S = \sum_{n,\mu<\nu} \mathrm{Re}(1 - \bar{\Pi}_{\mu\nu}(n))$$

$$\bar{\Pi}_{i0}(t,\vec{n}) = \frac{1}{\mathrm{Tr}[I]} \mathrm{Tr}[U_i(t,\vec{n}) M_i^\dagger(\vec{n}+\hat{i})$$

$$\times U_0(t,\vec{n}+\hat{i}) M_i(\vec{n}+\hat{i}) U_i^\dagger(t+1,\vec{n}) U_0^\dagger(t,\vec{n})]$$

$$M_i(\vec{n}) = \exp(iA_i^0(\vec{n}-\vec{x})), (i = x,\ y,\ z)$$

$$(i)\ n_z - z \geqq 0$$

$$\begin{pmatrix} A_x^0 \\ A_y^0 \\ A_z^0 \end{pmatrix} = \begin{pmatrix} \frac{m_c}{2gr} \frac{\sin\phi(1+\cos\theta)}{\sin\theta} \lambda_3 \\ -\frac{m_c}{2gr} \frac{\cos\phi(1+\cos\theta)}{\sin\theta} \lambda_3 \\ 0 \end{pmatrix}$$

$$(ii)\ n_z - z < 0$$

$$\begin{pmatrix} A_x^0 \\ A_y^0 \\ A_z^0 \end{pmatrix} = \begin{pmatrix} -\frac{m_c}{2gr} \frac{\sin\phi(1-\cos\theta)}{\sin\theta} \lambda_3 \\ \frac{m_c}{2gr} \frac{\cos\phi(1-\cos\theta)}{\sin\theta} \lambda_3 \\ 0 \end{pmatrix}$$

$$g = \sqrt{\frac{6}{\beta}}\ :\ \text{Electric charge in SU(3)}$$

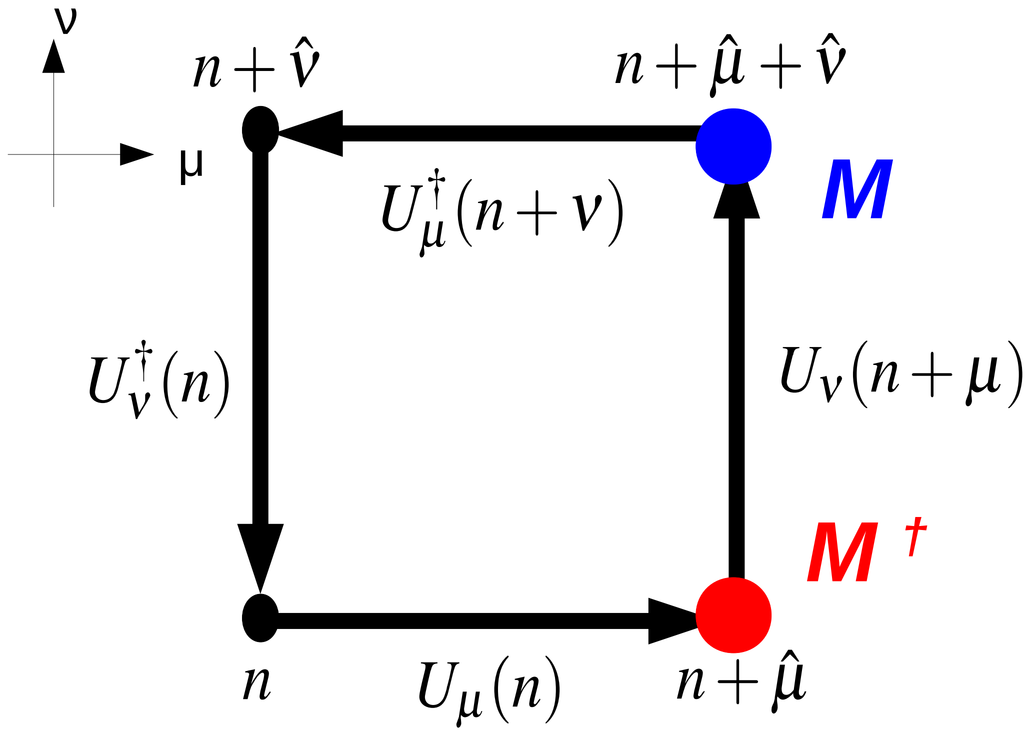
$$\mathbf{m_c}\ :\ \textbf{Monopole charge}$$

**We add the pair of monopoles varying monopole charges $m_c$ = 0, 1 , 2, 4, 5.**

# Monopoles

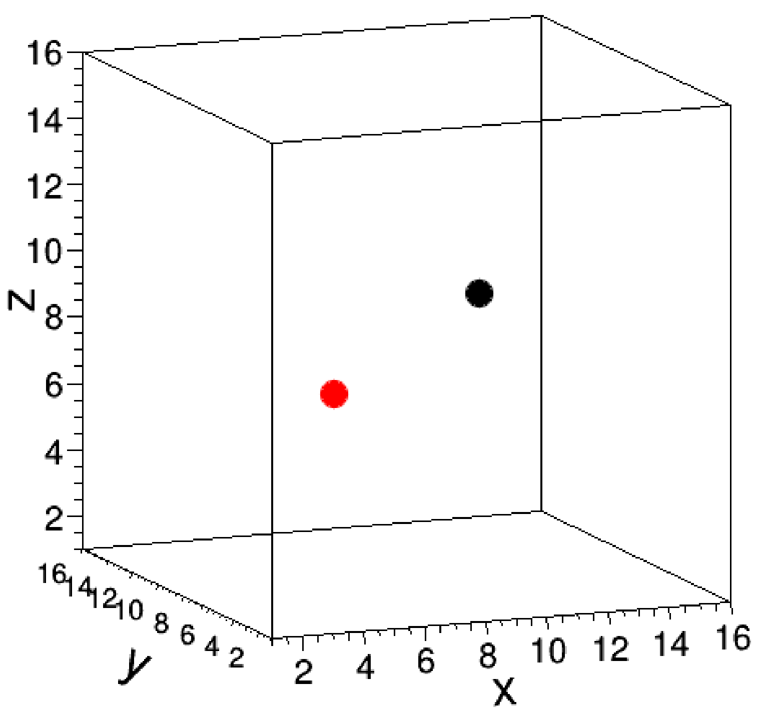
- Ref. C. Bonati, et al., PRD 85 (2012) 065001.

**Plaquette action**



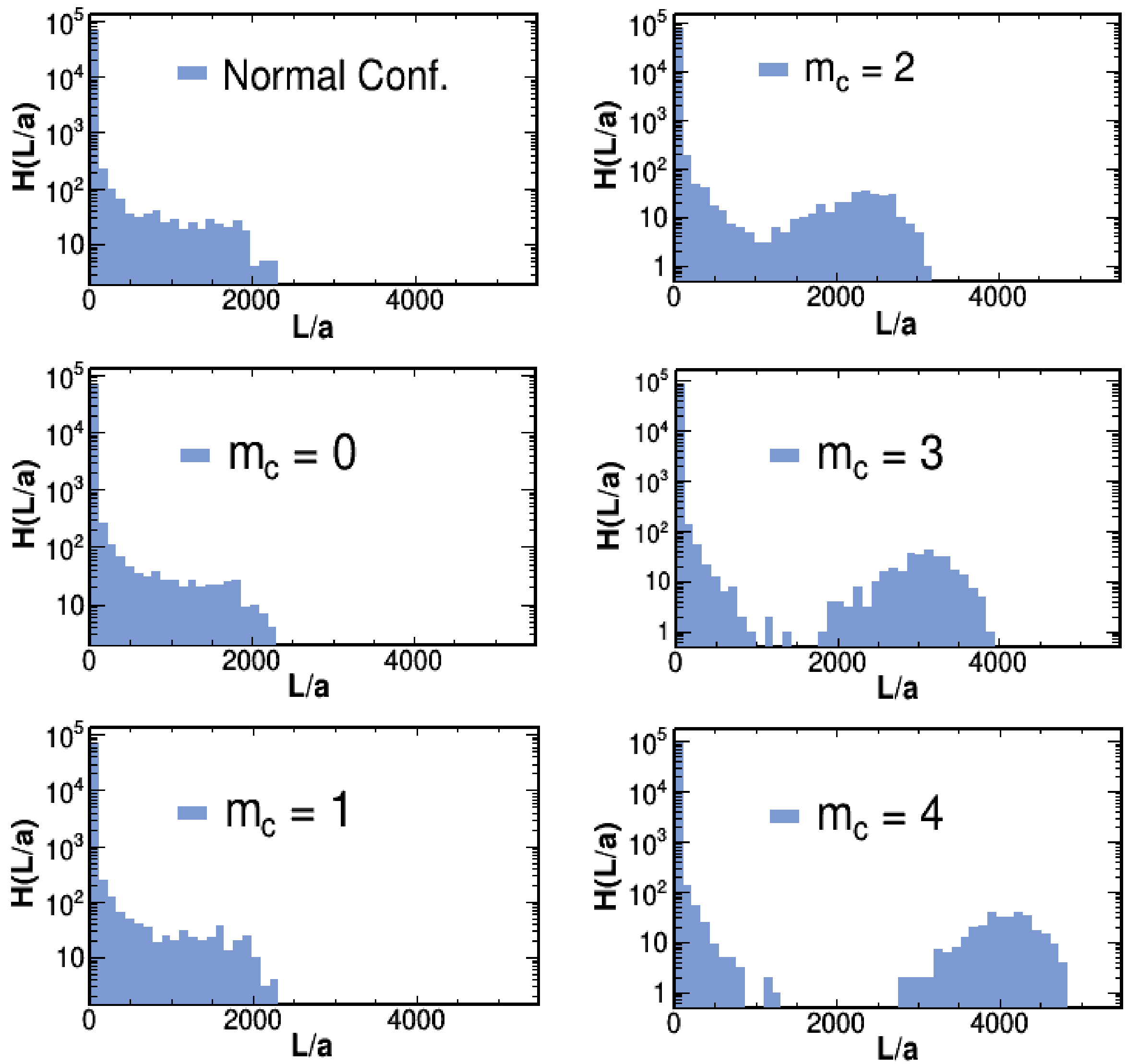

- **The locations of the monopole and the anti-monopole.**

$$\mathbf{V = 16^3 \times 32}$$



# Monopoles

- We measure the length of the monopole loops [A. Bode, et al., hep-lat/9312006; DIK collaboration, PRD 70 (2004) 074511].
- **This figure shows that the monopole loops *L* become long with monopole charges $m_c$ increasing.**
- **The monopole creation operator makes only the long monopole loops.**




The lattice is $V = 16^3\text{x}32$, $\beta = 6.00$.

# Instantons and monopoles

- **We count the number of zero modes, and compute the number of instantons.**
- **We quantitatively show how many monopole charges make instantons.**
- The simulation parameters are in Table as below.

| $\beta$ | $V$ | $V/r_0^4$ | $D$ | Kinds of conf | $N_{conf}$ |
|---|---|---|---|---|---|
| 6.00 | $14^4$ | 46.28 | - | Normal conf | O(2000) |
| | | | 8 | $m_c = 0 - 4$ | O(2000) |
| | $14^3 \times 28$ | 92.55 | - | Normal conf | O(400) |
| | | | 8 | $m_c = 0 - 4$ | O(400) |
| | $16^3 \times 32$ | 157.8 | - | Normal conf | O(500) |
| | | | 8 | $m_c = 0 - 5$ | O(500) |
| 5.93 | $14^3 \times 28$ | 157.8 | - | Normal conf | O(500) |
| | | | 7 | $m_c = 0 - 4$ | O(500) |
| 6.05 | $18^3 \times 32$ | 157.8 | - | Normal conf | O(500) |
| | | | 9 | $m_c = 0 - 4$ | O(500) |

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| | | | 7 | [illegible] | [illegible] |
| 6[illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
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**1. Finite lattice volume effects**

# Instantons and monopoles

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- **We quantitatively show how many monopole charges make instantons.**
- The simulation parameters are in Table as below.

**2. The continuum limit**

| $\beta$ | $V$ | $V/r_0^4$ | $D$ | Kinds of conf | $N_{conf}$ |
|---|---|---|---|---|---|
| 6.00 | $14^4$ | 46.28 | - | Normal conf | O(2000) |
| | | [illegible] | [illegible] | [illegible] | O(2000) |
| | 1[illegible] | [illegible] | [illegible] | [illegible] | O(400) |
| | | | 8 | $m_c = 0 - 4$ | O(400) |
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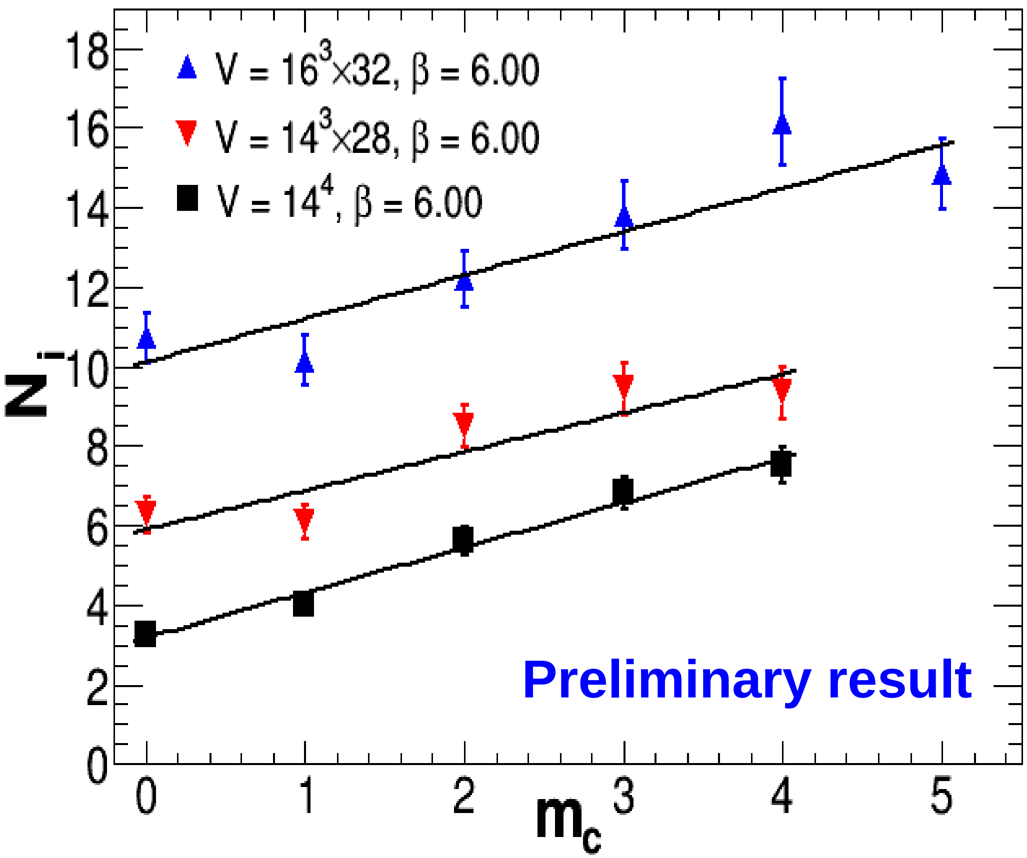
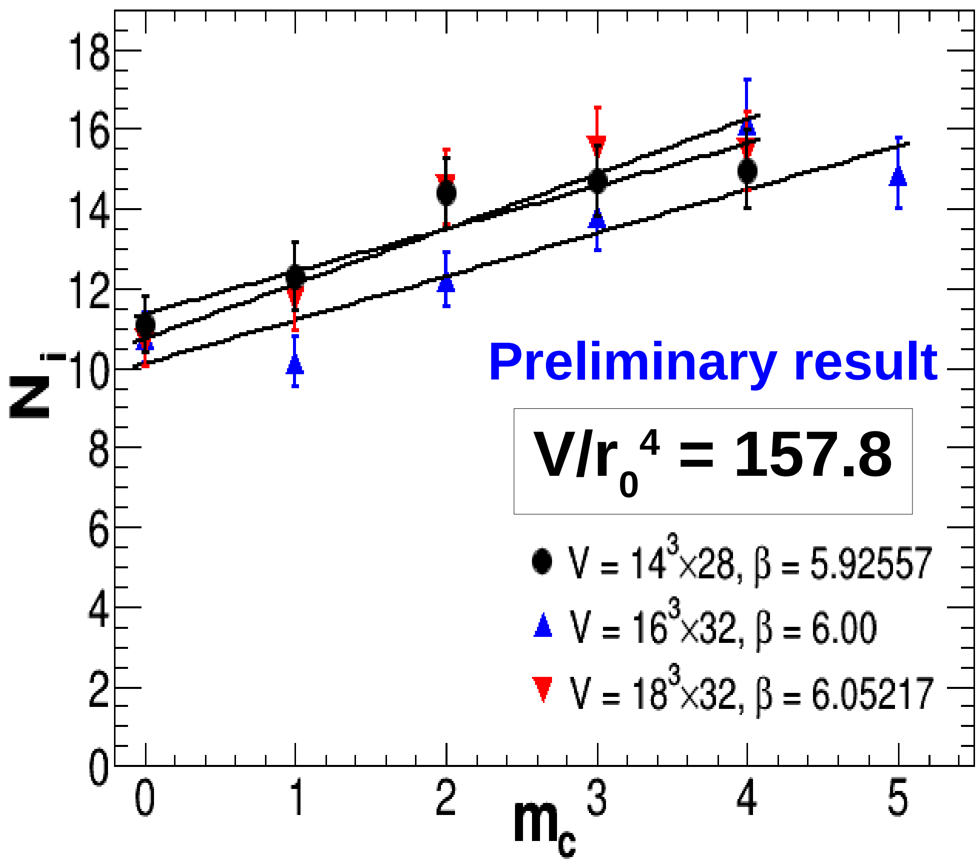
# Instantons and monopoles

**1. Finite lattice volume effects**

**2. The continuum limit**



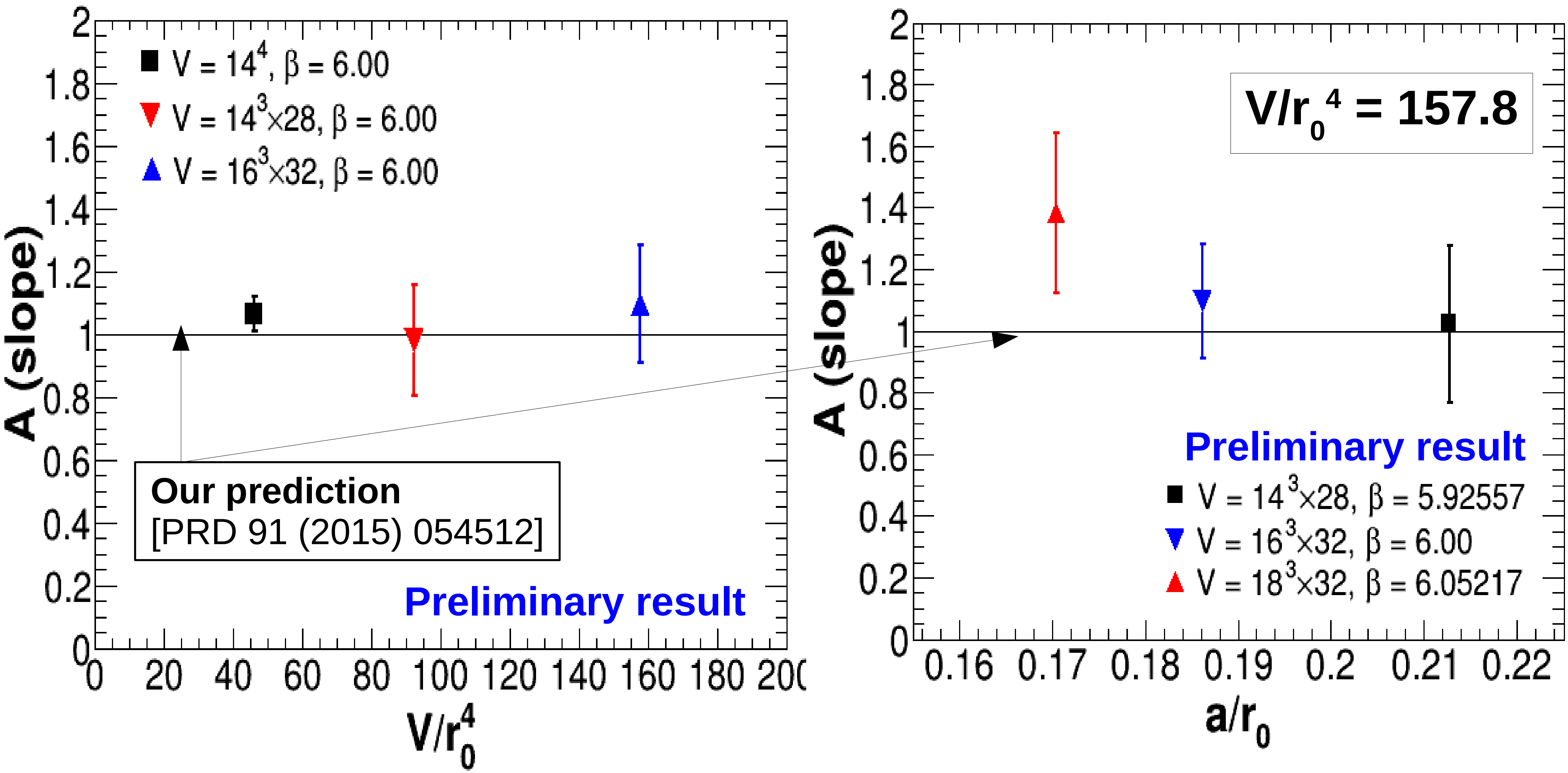




- We fit a linear function: $\mathbf{N_i = A \cdot m_c + B}$
- We evaluate the slop A and the intercept B.

# Instantons and monopoles

**1. Finite lattice volume effects**

**2. The continuum limit**



# Instantons and monopoles

- **The quantitative relation between the number of monopoles and the number of instantons.**

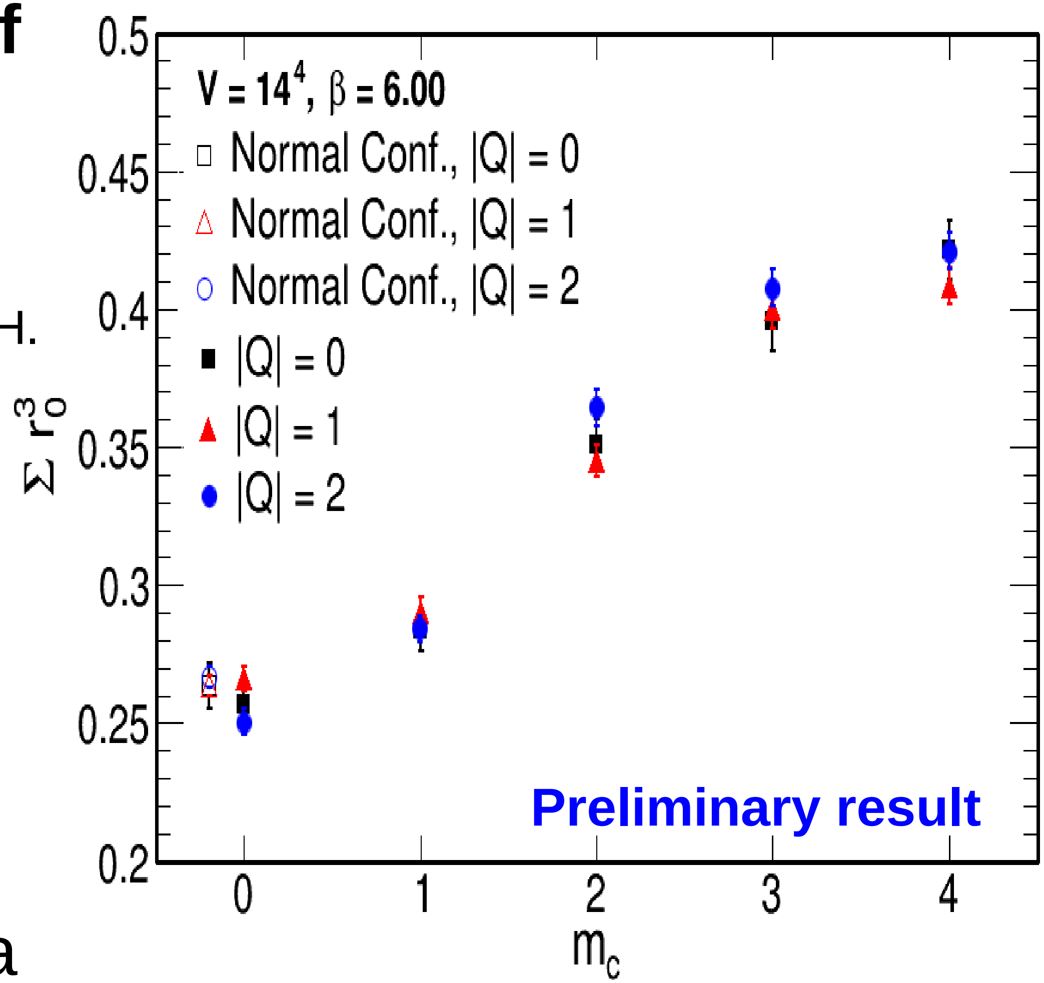
**Preliminary results**

| $\beta$ | $V$ | $A$ | $B$ | $B_{\mathbf{Pre}}$ | $\chi^2/d.o.f.$ |
|---|---|---|---|---|---|
| 6.00 | $14^4$ | 1.06(5) | 3.15(10) | 3.19(10) | 31.6/3.0 |
| | $14^3 \times 28$ | 0.98(18) | 5.9(4) | 6.5(5) | 7.0/3.0 |
| | $16^3 \times 32$ | 1.09(19) | 10.1(5) | 9.1(6) | 7.1/4.0 |
| 5.93 | $14^3 \times 28$ | 1.0 (3) | 11.5(6) | 11.2(7) | 1.7/3.0 |
| 6.05 | $18^3 \times 32$ | 1.4(3) | 10.7(5) | 9.4(6) | 2.4/3.0 |

**One monopole charge makes one instanton.**

# Chiral symmetry breaking and monopoles

- **We compare the spectrum of the Overlap Dirac operator with the prediction from Random Matrix Theory** [S. M. Nishigaki, et al., PRD 58 (1998) 087704; P. H. Damgaard, et al., hep-th/0006111 (PRD 63 (2001) 045012); R. G. Edwards, et al., PRL 82 (1999) 4188; L. Giusti, et al., JHEP 11 (2003) 023]**.**
- **The chiral condensate Σ increases by increasing the monopole charges without affecting the spectrum of eigenvalues** (**Σ** is defined as a positive value in this computation)**.**




**A. Di Giacomo, M. H., and F. Pucci,**
Chiral Dynamics 2015, Pisa

# The chiral condensate

- **Fermion propagator**

$$S_{\rm F}(y-x)_{\alpha\beta,ab} \equiv \sum_i \frac{\psi_{\alpha a i}(x)\psi^\dagger_{\beta b i}(y)}{\lambda_i^{imp}+m_q}$$

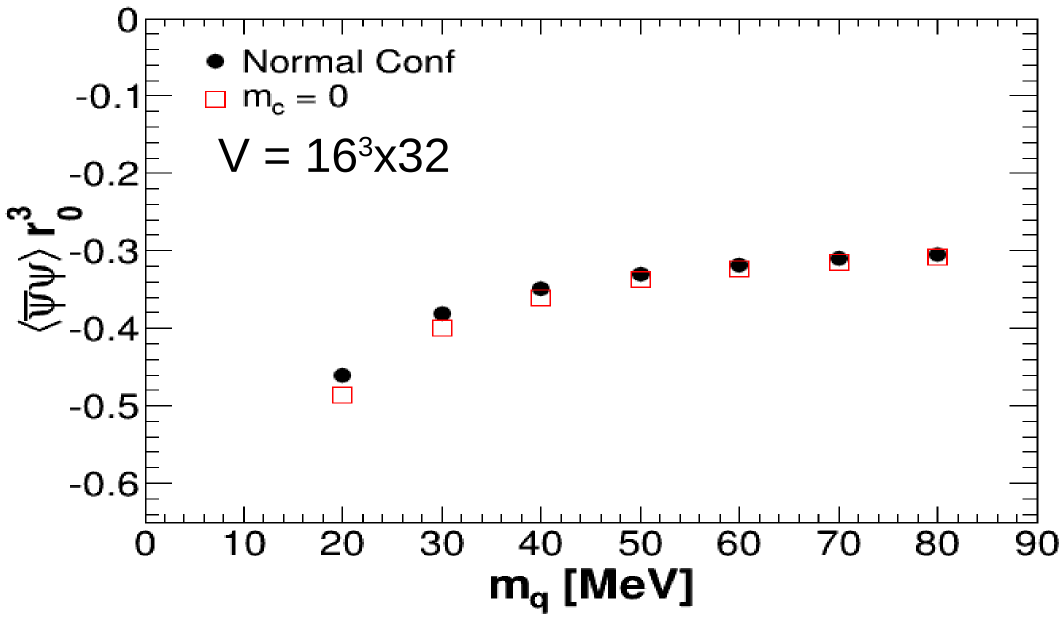
$\mathbf{m_q}$ **: Valence quark mass**

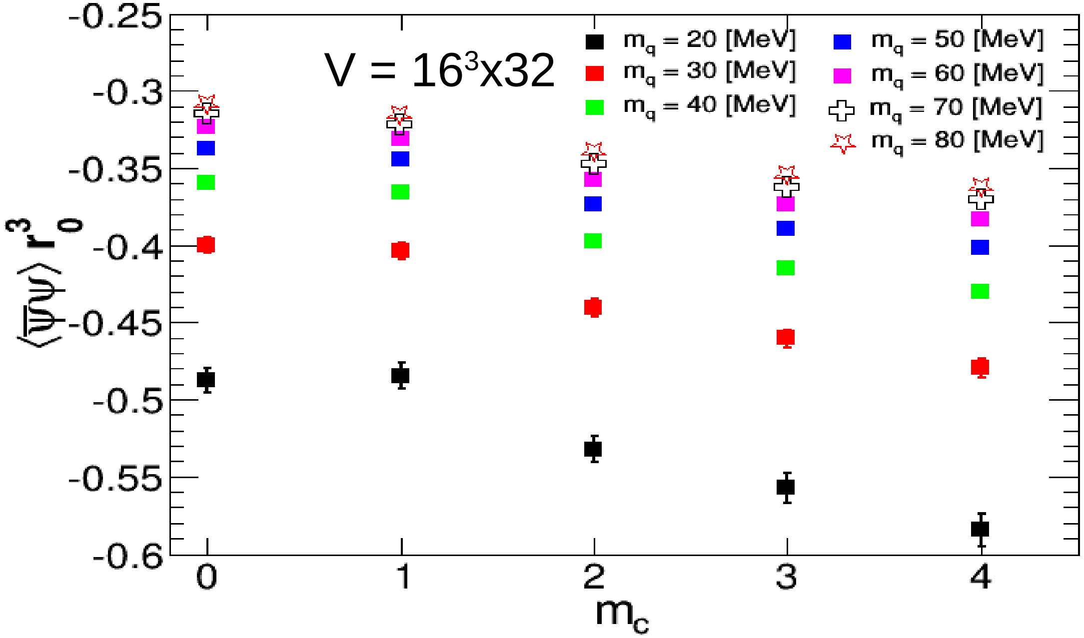
- **Chiral condensate**

$$\langle\bar\psi\psi\rangle \equiv \langle {\rm tr} S_{\rm F}(x-x)\rangle$$
$$= -\frac{1}{V}\sum_x\left(\sum_i \frac{\psi_i^\dagger(x)\psi_i(x)}{\lambda_i^{imp}+m_q}\right)$$
$$= -\frac{1}{V}\sum_i \frac{1}{\lambda_i^{imp}+m_q}$$

Normalization factor:

$$\sum_x \psi_i^\dagger(x)\psi_i(x) = 1$$





D. Galletly, et al. Phys. Rev. D 75 (2007) 073015
H. Neff, et al. Phys. Rev. D 64 (2001) 114509

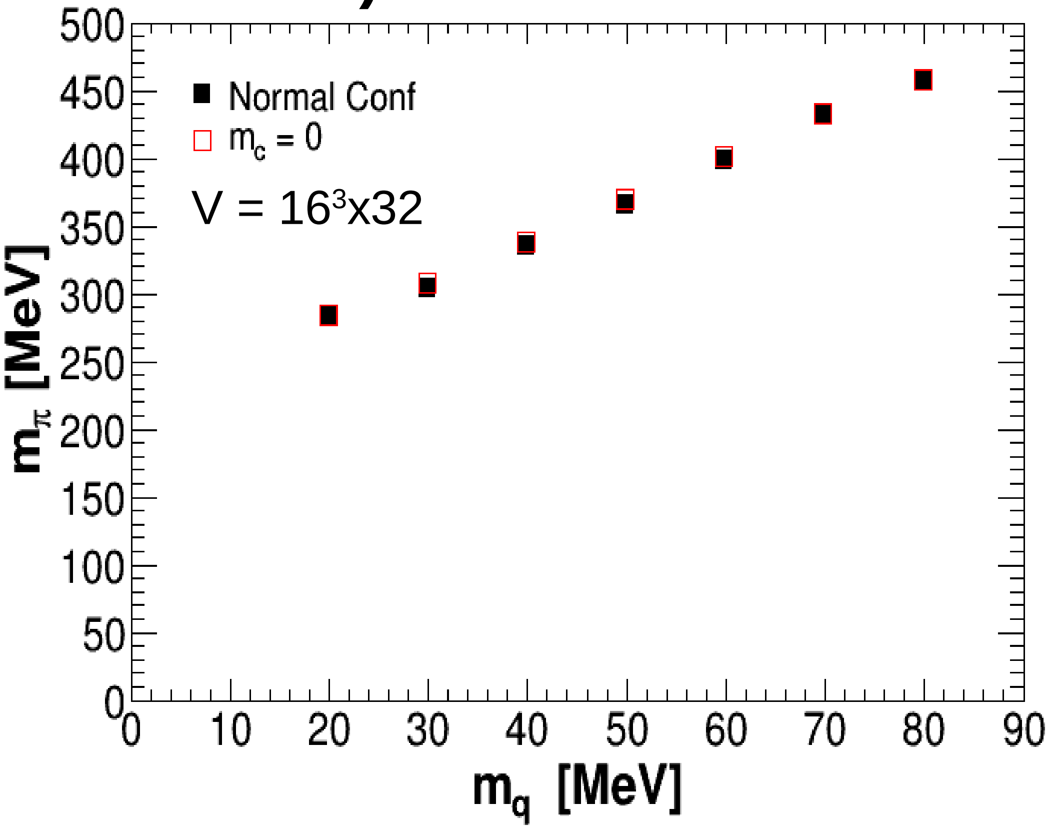
# Pseudo-scalar meson mass

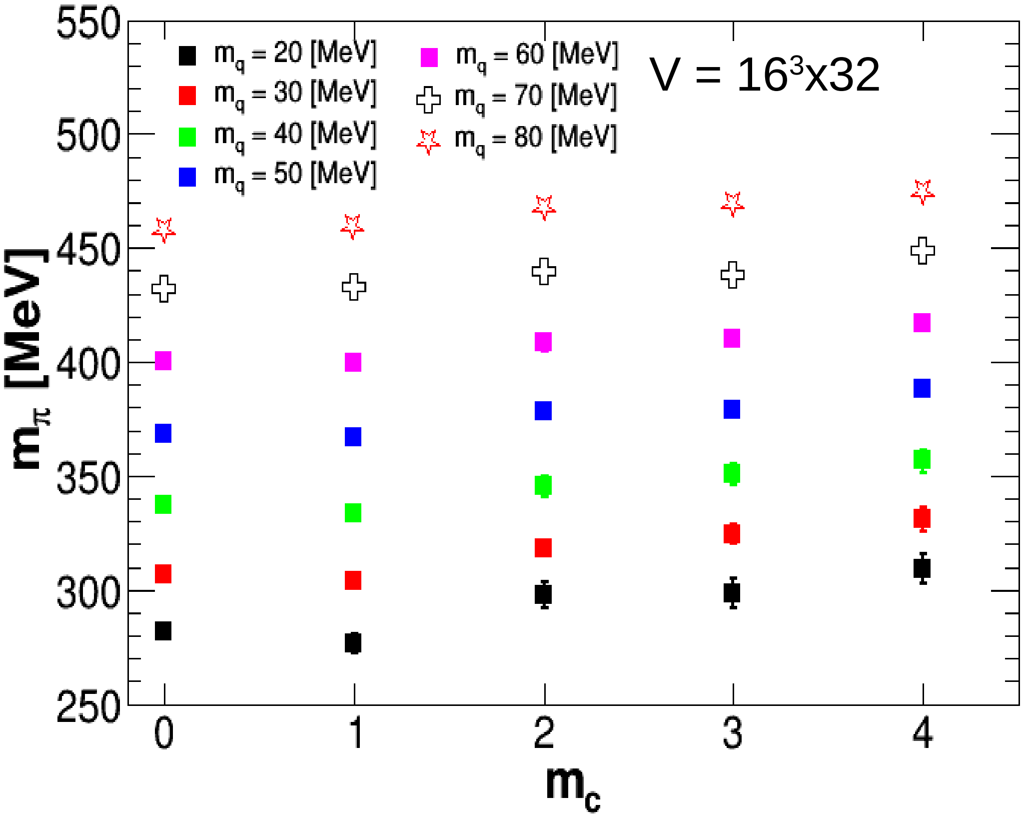
- **Pseudo-scalar correlation function**

T. DeGrand, et al. Comp. Phys. Com. 159 (2004) 185
D. Galletly, et al. Phys. Rev. D 75 (2007) 073015

$$C_{\rm ps}(\Delta t) \equiv \frac{1}{V}\sum_t \left[\sum_{ab}\sum_{ij}\sum_{\vec{x}}\sum_{\vec{y}} \frac{\left(\psi^\dagger_{ai}(\vec{x},t)\gamma_5\psi_{bj}(\vec{x},t)\right)\left(\psi^\dagger_{bj}(\vec{y},t+\Delta t)\gamma_5\psi_{ai}(\vec{y},t+\Delta t)\right)}{\left(\lambda_i^{imp}+m_q\right)\left(\lambda_j^{imp}+m_q\right)}\right]$$

- **We fit a single exponential $C(\Delta t) = A\cdot\exp(-m_\pi\cdot\Delta t)$, and evaluate the pion mass. The fitting range is fixed at $\chi^2$/d.o.f ≈ 1 ( $\chi^2$/d.o.f = 0.6 ~ 1.6).**

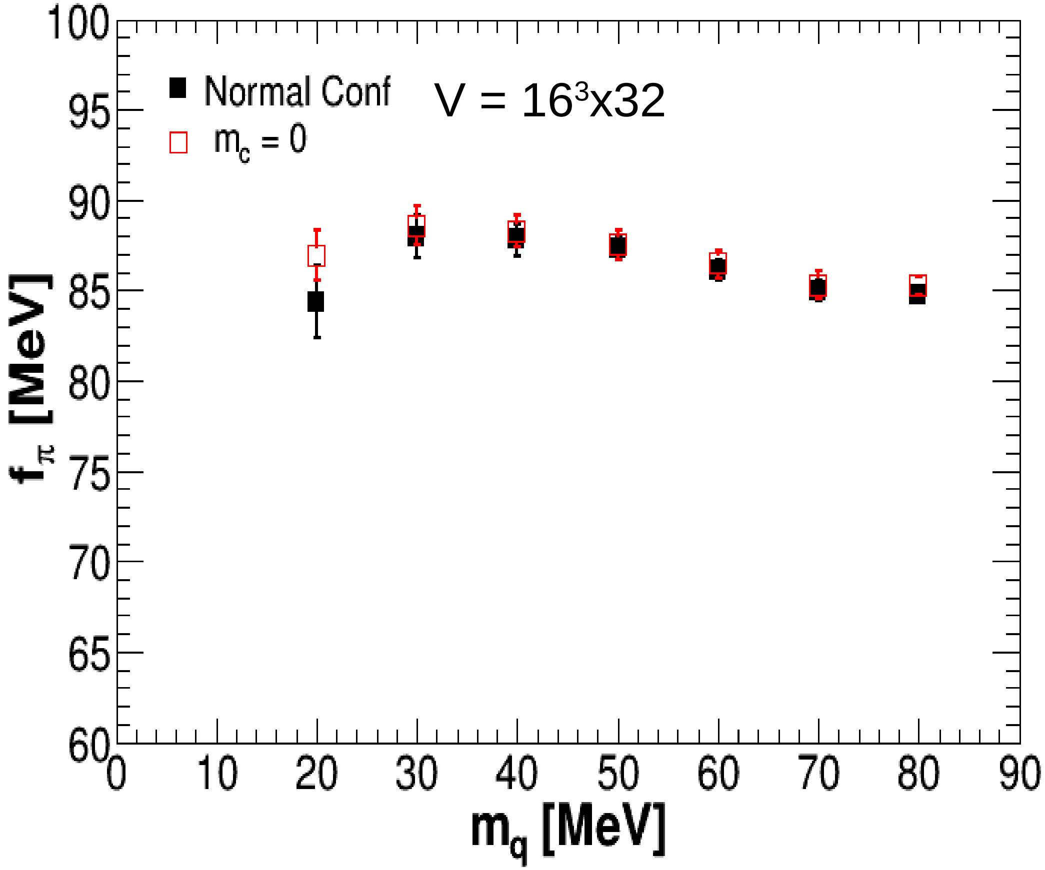


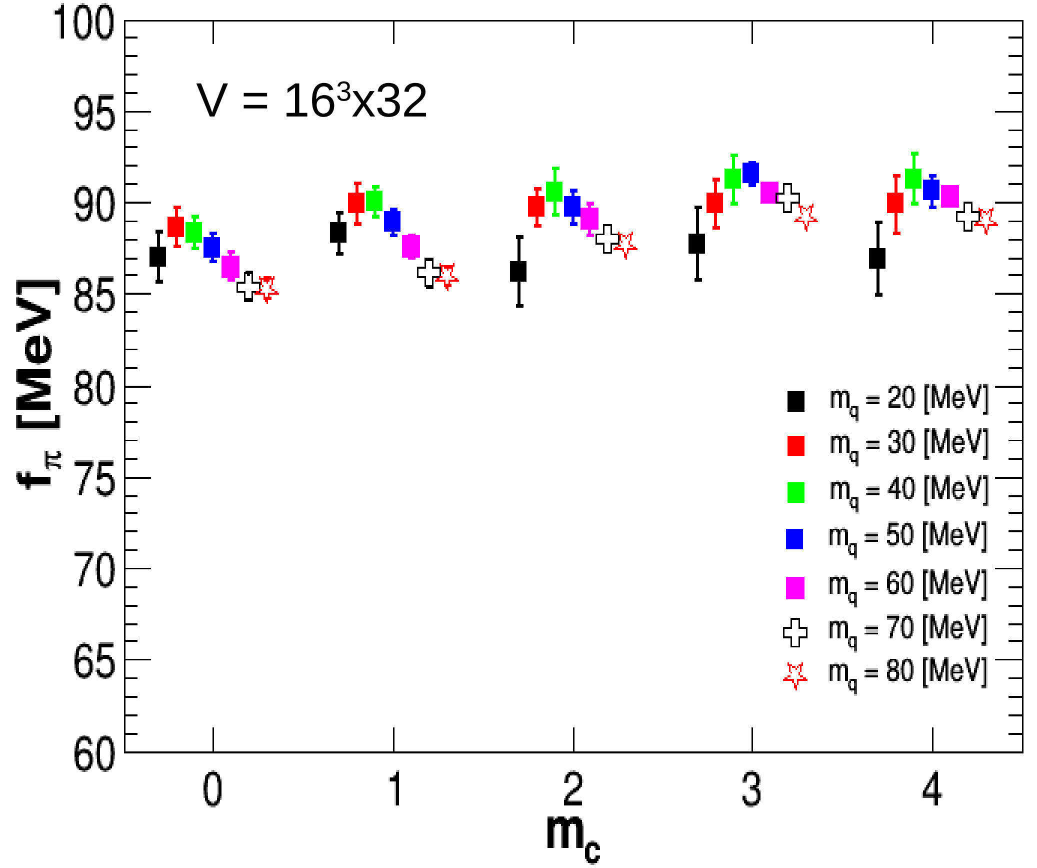



# The pion decay constant

- We compute the pion decay constant from the Gell-Mann, Oakes, and Renner (GMOR) relation [Phys. Rev. 175, (1968) 2195].

$$\frac{m_u + m_d}{2}\langle\bar{\psi}\psi\rangle = f_\pi^2 m_\pi^2,\ \ f_\pi = \frac{\sqrt{m_q\langle\bar{\psi}\psi\rangle}}{m_\pi},\ \ (m_q = m_u \sim m_d)$$



# Conclusions

**We confirmed:**

- **Monopoles and anti-monopoles are successfully added to configurations by the monopole creation operator.**
- **One monopole charge makes one instanton.**
- **The spectral density increase with monopole charges increasing.**
- **The chiral condensate decreases by increasing the monopole charges.**
- **The pion (pseudo-scalar) mass slightly increase by increasing the monopole charge.**
- **The pion decay constant is not changed by increasing the monopole charges.**

# Acknowledgment


- The simulations have been performed on, SX-ACE, SX-8, SX-9, and PC clusters at RCNP and CMC at the University of Osaka, and SR16000 at YITP at the University of Kyoto. We really appreciate their technical supports and the computational time.

# Thank you for attention!