The hadronic vacuum polarization function with O(a)-improved Wilson fermions - an update

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Outline

1. Introduction and setup
2. Extended frequentist's method: study of systematic effects
3. Results and Conclusions

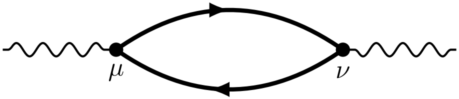
Determination of a_μ^{HLO}

The vacuum polarization tensor can be computed by

$$\Pi_{\mu\nu}(Q) = \int d^4x e^{iQx} \langle J_\mu(x) J_\nu(0)\rangle \,.$$

From Euclidean invariance and current conservation one finds

$$\Pi_{\mu\nu}(Q) = \left(g_{\mu\nu}Q^2 - Q_\mu Q_\nu\right) \Pi(Q^2).$$



The renormalized vacuum polarization function is given by

$$\widehat{\Pi}(Q^2) = 4\pi^2 \left(\Pi(Q^2) - \Pi(Q^2=0)\right) .$$

The anomalous magnetic moment of the muon is then given by the convolution integral:

$$a_\mu^{\mathrm{HLO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{Q^2} \, w(Q^2/m_\mu^2) \, \widehat{\Pi}(Q^2) \,,$$

where

$$w(r) = \frac{16}{r^2 \left(1+\sqrt{1+4/r}\right)^4 \sqrt{1+4/r}}$$

CLS-Ensembles with $N_f = 2$, update and changes to the setup

In our study we use $O(a)-$improved Wilson-fermions with $N_f = 2$ with partially twisted boundary conditions. The strange and charm quarks are partially quenched.

	L/a	β	$m_\pi L$	$a^{(1)}$ [fm]	m_π [MeV]	N_{cfg}	N_{meas}	$N_{\mathrm{meas}}^{\mathrm{old},(2)}$
A3	32	5.2	6.1	0.0755	495	251	1004	532
A4	32	5.2	4.7	0.0755	381	400	1600	800
A5	32	5.2	4.0	0.0755	331	251	1004	432
B6	48	5.2	5.2	0.0755	280	306	1224	-
E5	32	5.3	4.7	0.0658	437	1000	4000	672
F6	48	5.3	5.0	0.0658	311	300	1200	804
F7	48	5.3	4.2	0.0658	265	250	1000	820
G8	64	5.3	3.9	0.0658	185	205	3280	-
N5	48	5.5	5.2	0.0486	441	347	1388	644
N6	48	5.5	4.0	0.0486	340	559	2236	-
O7	64	5.5	4.2	0.0486	268	149	2384	-

$^{(1)}$ We use the scale determined via f_K in [P. Fritzsch et al., Nucl. Phys. B **865** (2012) 397]
$^{(2)}$ Number of measurements in [M. Della Morte et al., JHEP **1203** (2012) 055]

- We use the Padé ansätze $[1,2]$, $[2,2]$.
- The number of twist angles is reduced from 3 to 1, in favor of more inversions per twist.
- The extended frequentist's method is used to estimate systematic errors. [W-M Yao et al 2006 J. Phys. G: Nucl. Part. Phys. 33 1; S. Durr et al., Science **322** (2008) 1224]

Extended frequentist's method

Estimation of systematic errors for large a number of variations

Step 1: The central value is given by the median of the central values of all variations, such as

- picking subsets of data: $m_\pi < 400$ MeV,...
- different fit ansätze: Padé [1,2], [2,2],...
- lattice artifacts,
- ....

The central 68% gives the systematic error $\rightarrow$ histogram of the data

Step 2: Compute for each bootstrap sample the median. The statistical error is given by the central 68% of these medians.

- The errors can be computed with weights (e.g. p-values of the fits).

Application to a_μ^{HLO}

Step 1: First determine $\widehat{\Pi}(Q^2)$ by fitting lattice data of $\Pi(Q^2)$ for each ensemble and all variations.

Step 2: Then compute a_μ^{HLO} from $\widehat{\Pi}(Q^2)$.

Step 3: Perform the continuum and chiral extrapolation for all variations of the previous step and all variations of the extrapolation.

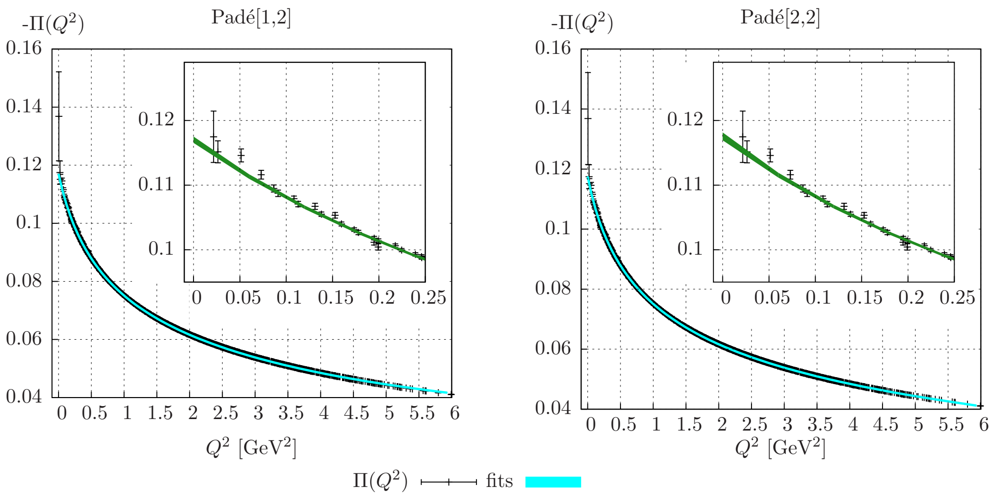
Extended frequentist's method: Step 1

Step 1: Determination of $\widehat{\Pi}(Q^2)$ by fitting lattice data of $\Pi(Q^2)$. We use correlated fits to subsets of the available data (Q^2-samples). For this step we consider the following variations:

- Padé [1,2] and [2,2],
- 30 and 40 points per Q^2-sample,
- and 1000 Q^2-samples for each ensemble.

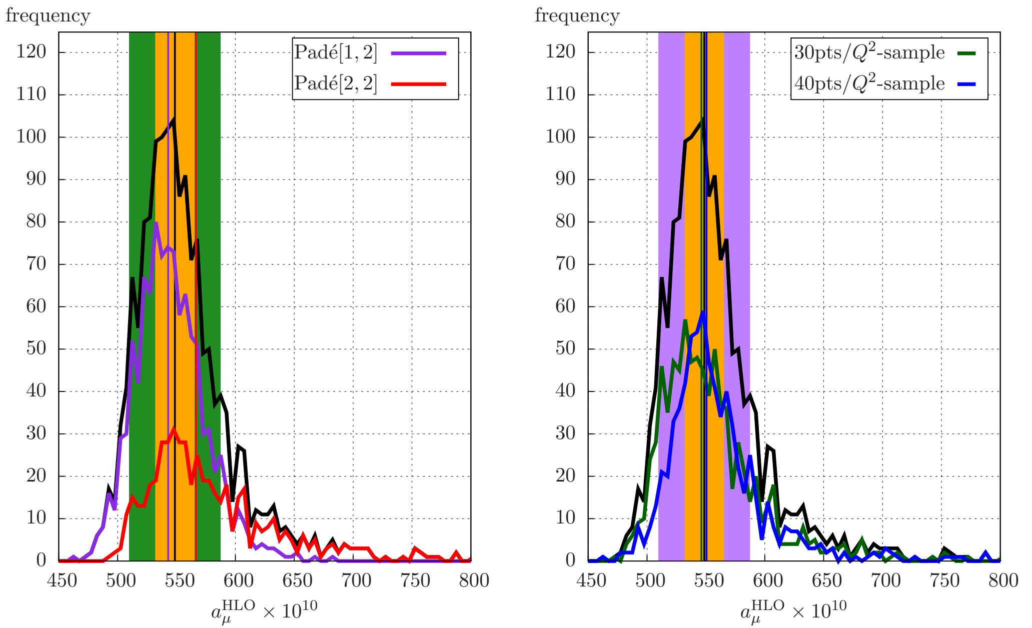
We do not include weights in this step.

Below: ud for G8, $m_\pi = 185$ MeV, $a = 0.0658$ fm



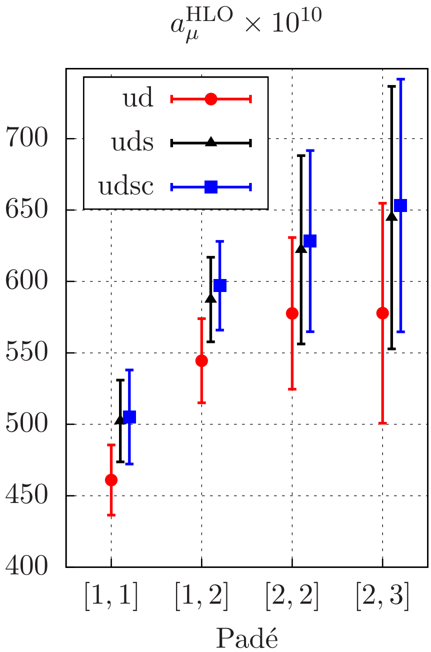
Extended frequentist's method: Step 2

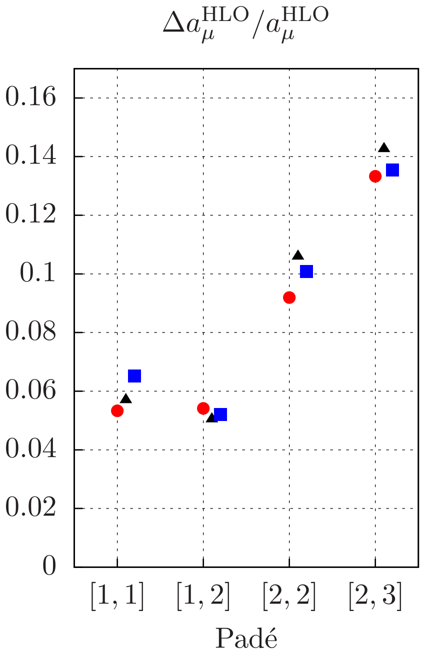
Step 2: Compute a_μ^{HLO} from $\widehat{\Pi}(Q^2)$ for each ensemble and all variations. In order to remove unphysical results we impose the mild cut $0 < a_\mu^{\mathrm{HLO}} < 10^{-6}$. Below: ud for G8, $m_\pi = 185$ MeV, $a = 0.0658$ fm



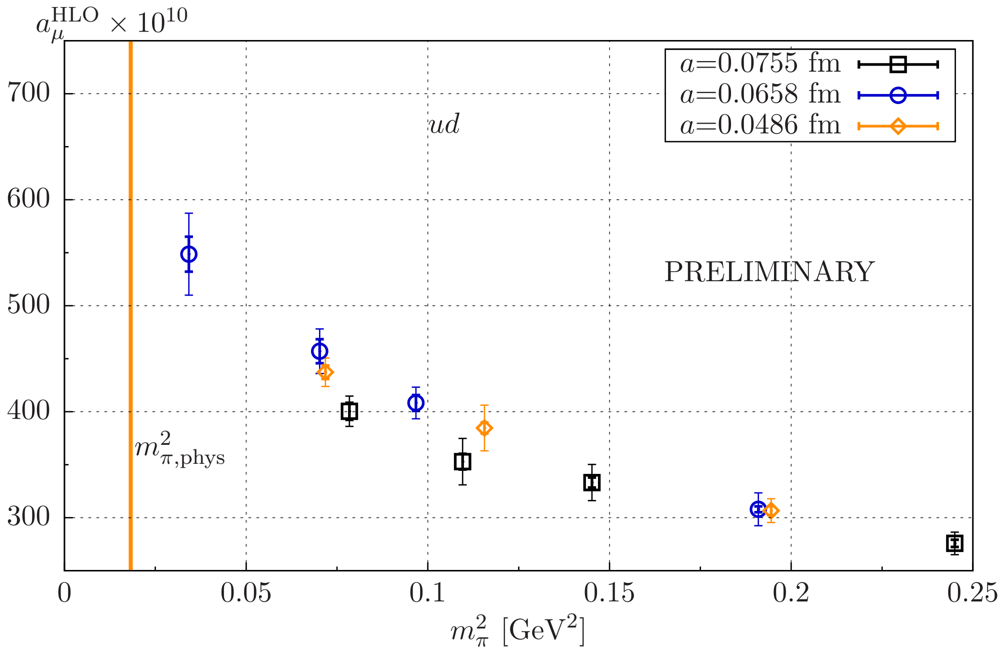
Extended frequentist's method: choice of variations

We test the variations of interest on G8 ($m_\pi = 185$ MeV, $a = 0.0658$ fm) for ud, uds, udsc.

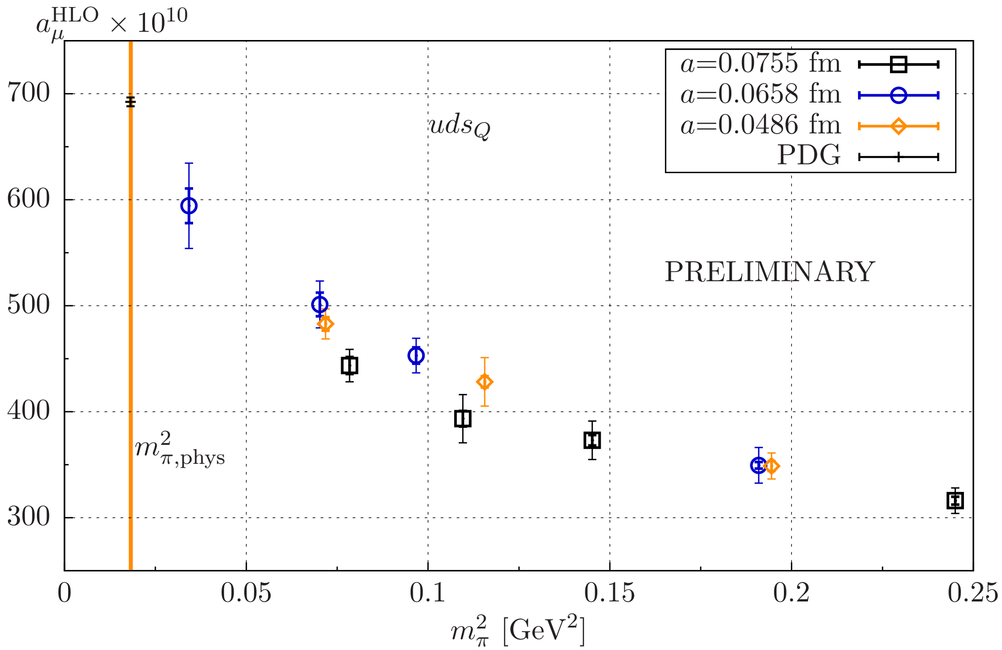




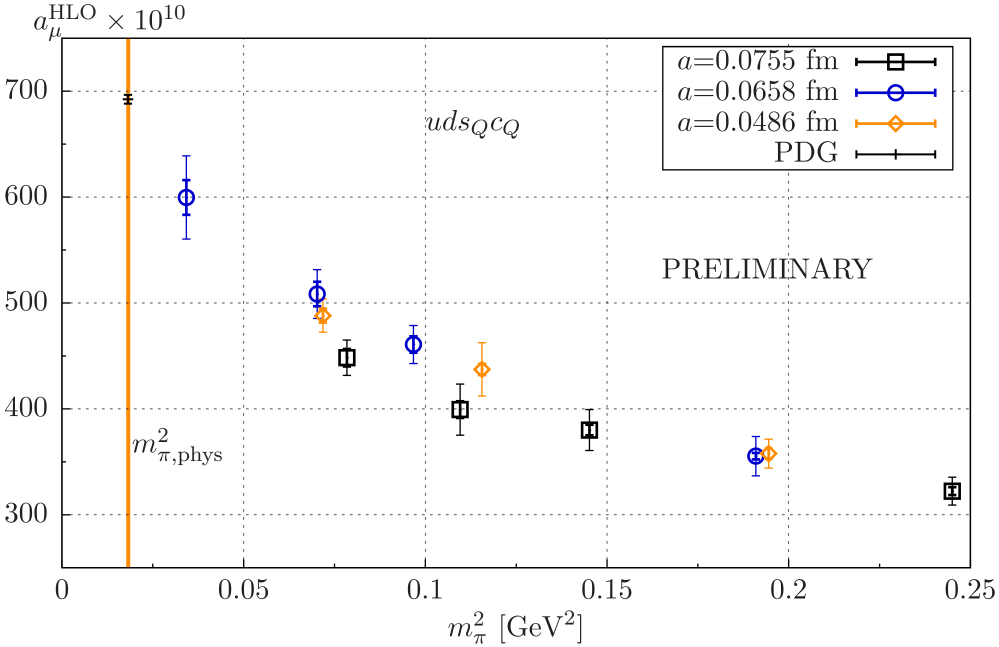
Extended frequentist's method: Step 2



Extended frequentist's method: Step 2



Extended frequentist's method: Step 2



Extended frequentist's method: Step 3

Step 3: For the continuum and chiral extrapolation we consider the following variations:

Type A:
- $c_1 + c_2 m_\pi^2 + c_3 m_\pi^2 \log(m_\pi^2) + c_4 a$
- $c_1 + c_2 m_\pi^2 + c_3 m_\pi^2 \log(m_\pi^2) + c_4 a$ with $m_\pi < 400$ MeV
- $c_1 + c_2 m_\pi^2 + c_3 m_\pi^2 \log(m_\pi^2)$ with $a < 0.07$ fm
- $c_1 + c_2 m_\pi^2 + c_3 m_\pi^2 \log(m_\pi^2)$ with $a < 0.07$ fm and $m_\pi < 400$ MeV

Type B:
- $c_1 + c_2 m_\pi^2 + c_3 m_\pi^4 + c_4 a$
- $c_1 + c_2 m_\pi^2 + c_3 m_\pi^4 + c_4 a$ with $m_\pi < 400$ MeV
- $c_1 + c_2 m_\pi^2 + c_3 m_\pi^4$ with $a < 0.07$ fm
- $c_1 + c_2 m_\pi^2 + c_3 m_\pi^4$ with $a < 0.07$ fm and $m_\pi < 400$ MeV

Type C:
- $c_1 + c_2 m_\pi^2 + c_3 a$ for m_V-rescaled data [arXiv:1206.3113]
- $c_1 + c_2 m_\pi^2 + c_3 a$ for m_V-rescaled data with $m_\pi < 400$ MeV

$$w\left(\frac{Q^2}{m_\mu^2}\right) \longrightarrow w\left(\frac{Q^2}{m_\mu^2}\left(\frac{M_\rho^{\mathrm{phys}}}{M_V}\right)^2\right)$$

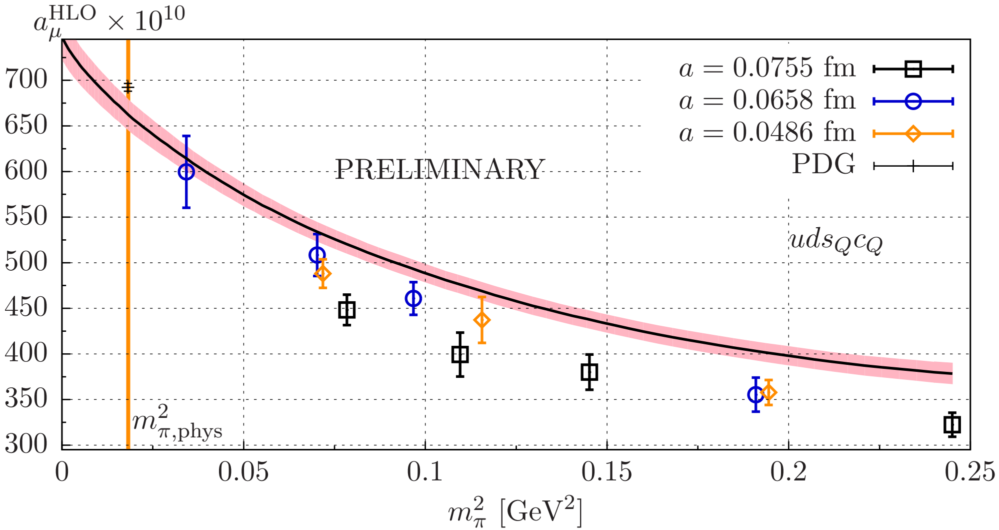
In this step we use the p-value of the fit as a weight in the extended frequentist's method.

Example for the χ-extrapolation

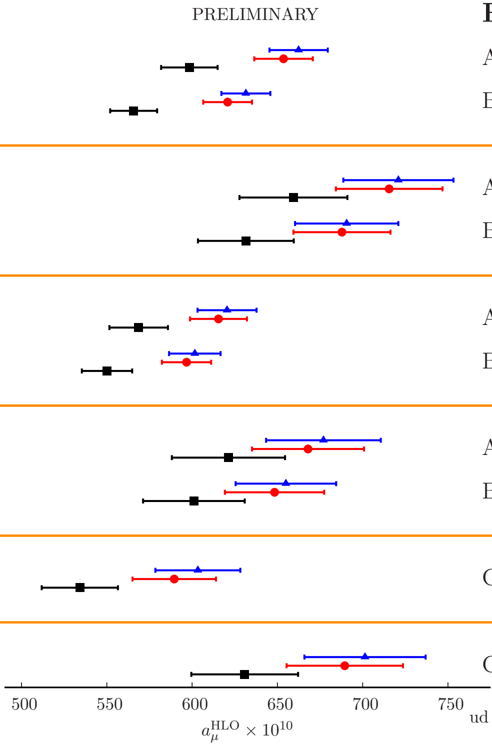
As an example we show the result for fit a A-type fit including lattice artifacts of $O(a)$:

$$c_1 + c_2 m_\pi^2 + c_3 m_\pi^2 \log(m_\pi^2) + c_4 a$$

Note: The fit function is evaluated in the continuum limit and only statistical errors for the curve are shown, while the points include systematic effects.



Preliminary results for the various extrapolations



Fit type:

A: $c_0 + c_1 m_\pi^2 + c_2 m_\pi^2 \log(m_\pi^2) + c_3 a$

B: $c_0 + c_1 m_\pi^2 + c_2 m_\pi^4 + c_3 a$

A: $c_0 + c_1 m_\pi^2 + c_2 m_\pi^2 \log(m_\pi^2) + c_3 a,\ m_\pi < 400$ MeV

B: $c_0 + c_1 m_\pi^2 + c_2 m_\pi^4 + c_3 a,\ m_\pi < 400$ MeV

A: $c_0 + c_1 m_\pi^2 + c_2 m_\pi^2 \log(m_\pi^2),\ a < 0.07$ fm

B: $c_0 + c_1 m_\pi^2 + c_2 m_\pi^4,\ a < 0.07$ fm

A: $c_0 + c_1 m_\pi^2 + c_2 m_\pi^2 \log(m_\pi^2),\ a < 0.07$ fm, $m_\pi <$400 MeV

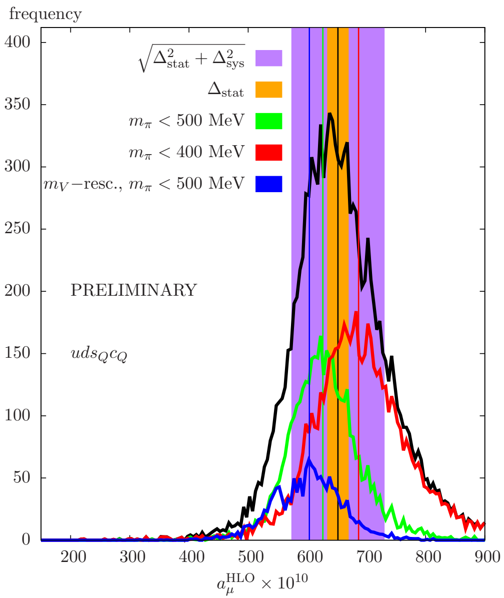
B: $c_0 + c_1 m_\pi^2 + c_2 m_\pi^4,\ a < 0.07$ fm, $m_\pi <$400 MeV

C: $c_0 + c_1 m_\pi^2 + c_2 a,\ m_V$-rescaling

C: $c_0 + c_1 m_\pi^2 + c_2 a,\ m_V$-rescaling, $m_\pi < 400$ MeV

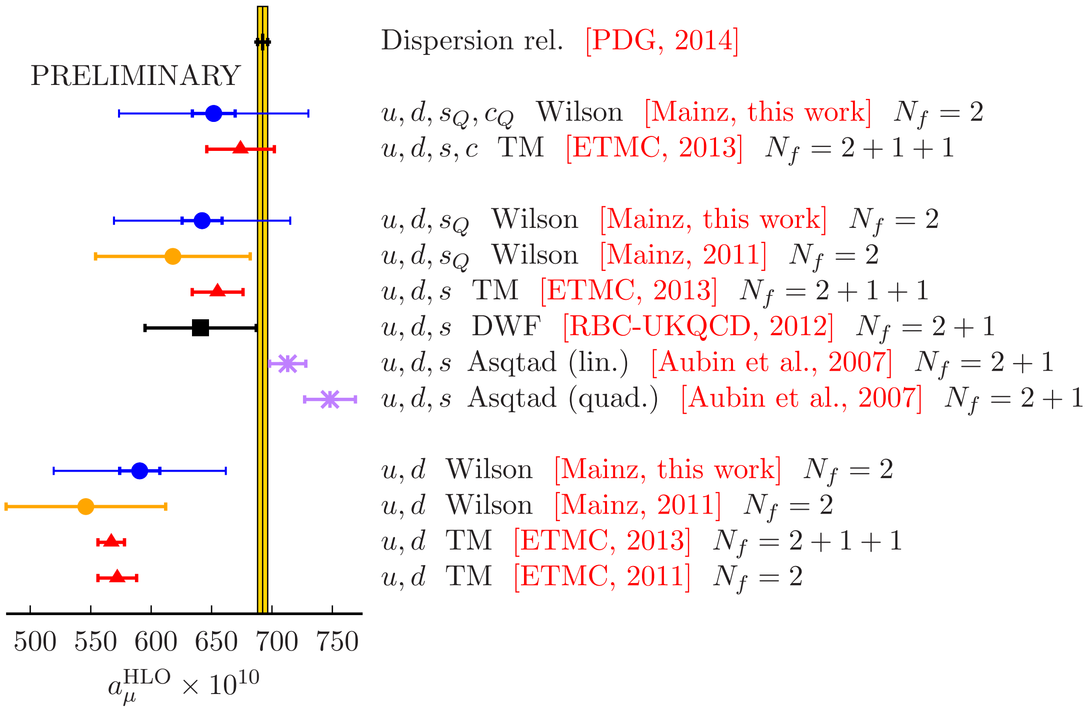
Preliminary error budget

To estimate the sources of the systematic error, we compute the median of different subgroups of the extrapolations. The estimate for each contribution to the systematic error is then given by the standard deviation of these medians. Below we show the subgroups based on the type of the fit function used for the chiral extrapolation.



Flavor	Label	rel. contr.
ud	$\chi-$extrapolation	46.7%
	Lattice artifacts	10.5%
	Q^2-sampling	31.5%
	Padé	10.9%
	Points/Q^2-sample	0.4%
uds	$\chi-$extrapolation	42.0%
	Lattice artifacts	13.8%
	Q^2-sampling	30.9%
	Padé	12.4%
	Points/Q^2-sample	0.9%
udsc	$\chi-$extrapolation	39.8%
	Lattice artifacts	14.7%
	Q^2-sampling	34.0%
	Padé	10.6%
	Points/Q^2-sample	1.0%

Comparison of results



Conclusions and Outlook

Conclusions

- We presented the application of the extended frequentist's method to the determination of a_μ^{HLO} to estimate systematic errors for a large number of variations.
- The method shows that our results are dominated by systematic effects.
  $\Rightarrow$ Currently systematic effects due to picking subsets of the VPF data are estimated conservatively: investigation in progress.
- We investigated different contributions to the systematic errors:
  - Q^2 behaviour of VPF: 2 Padé ansätze, correlations in Q^2
  - several types of chiral extrapolations,
  - lattice artifacts.

Outlook

- We are investigating different methods to pick subsets of the VPF data for the fits and estimating the covariance matrix.
- Alternatives to this method are the calculation of a_μ^{HLO} using the Adler function and the mixed representation method.
- We plan to use the mixed representation method to compute a_μ^{HLO} on the $N_f = 2+1$ ensembles produced in the CLS effort.

Thank you for your attention.

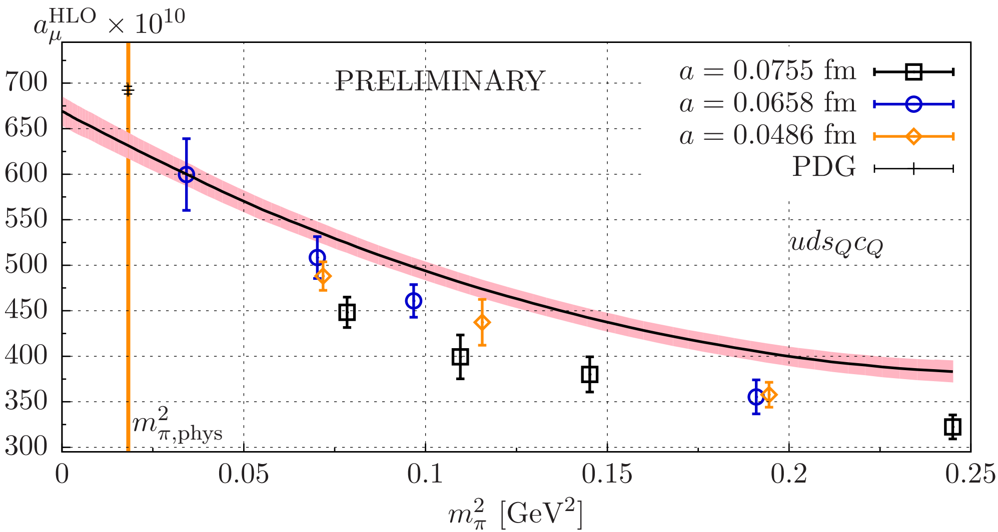
Backup

Example for the χ-extrapolation: B-type fit

Example for B-type fit including lattice artifacts of $O(a)$:

$$c_1 + c_2 m_\pi^2 + c_3 m_\pi^4 + c_4 a$$

Note: The fit function is evaluated in the continuum limit and only statistical errors for the curve are shown, while the points include systematic effects.

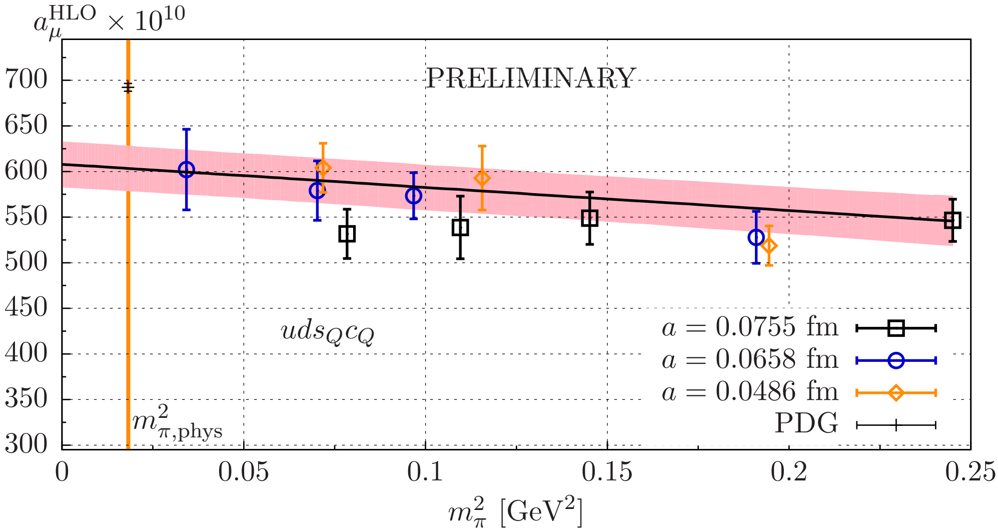


Example for the χ-extrapolation: C-type fit

Example for C-type fit including lattice artifacts of $O(a)$ to m_V-rescaled data:

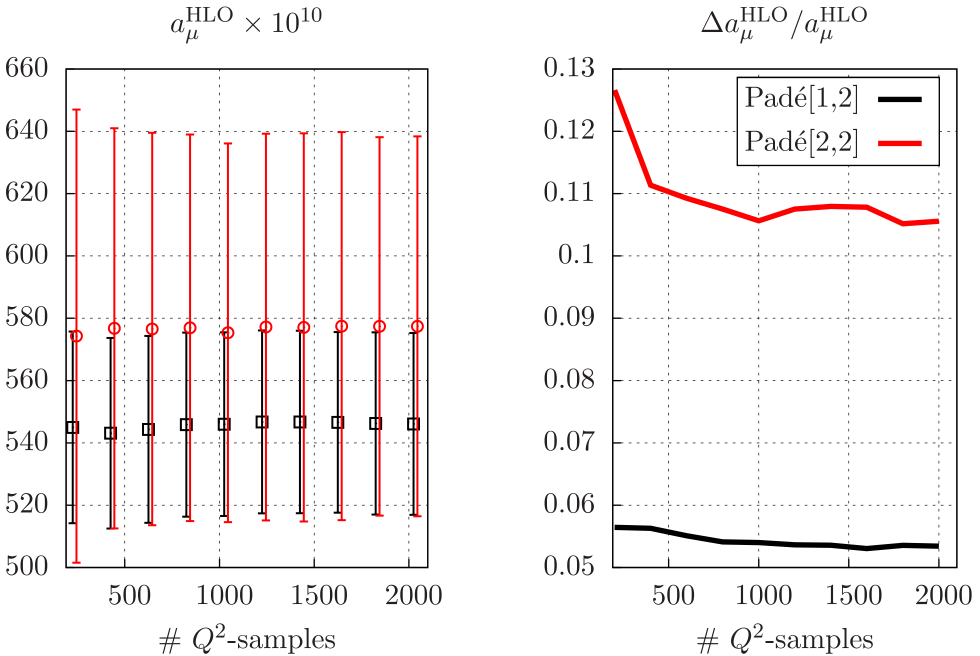
$$c_1 + c_2 m_\pi^2 + c_3 a$$

Note: The fit function is evaluated in the continuum limit and only statistical errors for the curve are shown, while the points include systematic effects.



The extended frequentist's method: choice of variations

The number of Q^2-samples is chosen such that the relative error is stable. Below: G8 ($m_\pi = 185$ MeV, $a = 0.0658$ fm) for ud.



Comparison of weighted and unweighted results on a single ensemble

We find that for a single ensemble the results for unweighted data agree withing errors with the weighted. Below: G8 ($m_\pi = 185$ MeV, $a = 0.0658$ fm).