Lattice study of the Higgs-Yukawa model in and beyond the Standard Model

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date:July 15, 2015

work based on : *D. Y.-J. Chu et al. in preparation
*D. Y.-J. Chu, K. Jansen, B. Knippschild, C. -J. D. Lin, and A. Nagy, Phys. Lett. **B744**, 146 (2015).
*D. Y.-J. Chu el al. work in progress.

Outline

Finite-size Scaling Analysis near mean-field fixed point.

⇨ Logarithmic corrections near mean-field fixed point.

Finite Temperature with a dim-6 operator.

⇨ Properties of physics beyond standard model.

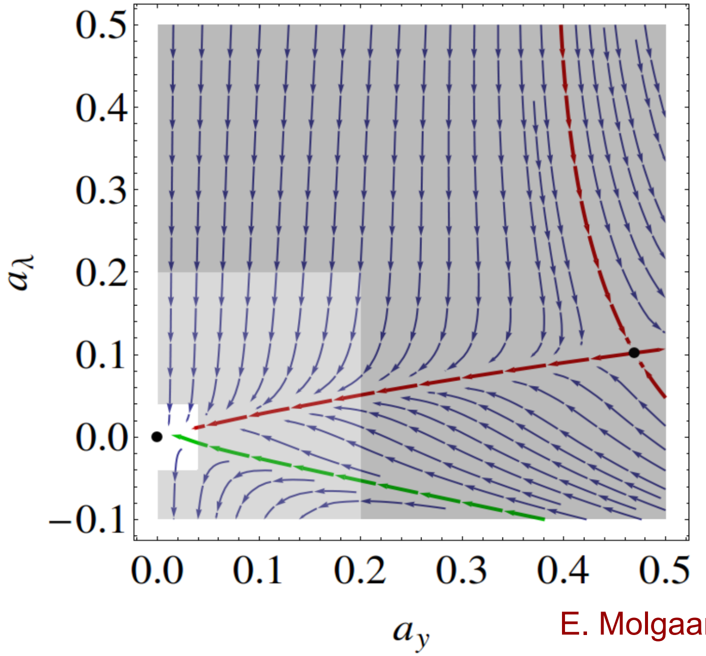
Conclusion and Outlook.

Finite size scaling analysis near mean field fixed point

Motivation

Analytic control of the simulation data.

Triviality of Higgs-Yukawa sector in Standard Model?



Perturbation theory predicts non-trivial fixed points.
(2-loop order for y, 1-loop order for λ , $\overline{MS}$ scheme.)

Logarithmic scaling near the mean-field fixed point in 4 dimension.

$$a_\lambda \equiv \frac{\lambda}{(4\pi)^2} \qquad a_y \equiv \frac{y^2}{(4\pi)^2}$$

E. Molgaard and R. Shrock (2014).

Finite size scaling analysis

Lattice data : $Y = y^2$ $\zeta_{\mathcal{M}}(l,L) = \exp\left(\int_L^l \gamma_{\mathcal{M}}(\rho)\,\mathrm{d}\log\rho\right)$

$1 = a < l < L$

UV IR

$$\mathcal{M}[m^2(1),\lambda(1),Y(1);1,L] = Z_{\mathcal{M}}(a,l)\mathcal{M}[m^2(l),\lambda(l),Y(l);l,L]$$

RG change of scale

$$= Z_{\mathcal{M}}(a,l)\zeta_{\mathcal{M}}(l,L)\mathcal{M}[m^2(L),\lambda(L),Y(L);L,L]$$

rescaling

$$= Z_{\mathcal{M}}(a,l)\zeta_{\mathcal{M}}(l,L)(L)^{-D_{\mathcal{M}}}\mathcal{M}[\underline{m^2(L)L^2,}\lambda(L),Y(L);1,1]$$

Note : The above analysis does not give the functional form of $\mathcal{M}$.

Finite size scaling analysis

Generic Higgs-Yukawa model with n scalars, Nf fermions.

Finite Volume ⇨ Dominance of scalar zero-mode.

Anisotropic lattice : $L_t = sL$

$$\phi_a(x) = \varphi_a + \chi_a(x)$$

The partition function :

$$Z \sim \int_0^\infty \mathrm{d}\varphi\, \varphi^{n-1} \exp\left[-sL^4\left(\frac{1}{2}m^2\varphi^2 + \lambda\varphi^4\right)\right] \det[M_B]^{-1}\det[M_F]$$

The determinants leads to :

1. RG corrections, 2. Volume dependent shift $\frac{A}{L^2}$ of m_c^2 , 3. Higher dimensional operators.

E. Brezin, J. Zinn-Justin (1985)

The setup of perturbation theory

Change of variable : $\varphi \to \left(sL^4\lambda\right)^{-1/4}\varphi$

$$Z \sim \int_0^\infty \mathrm{d}\varphi\,\varphi^{n-1}\exp\left[-\frac{1}{2}z\varphi^2-\varphi^4\right]$$

Scaling variable : $z=\sqrt{s}m^2L^2\lambda^{-1/2}$ RG corrected near mean-field fixed point.

Define integrals :

$$\bar{\varphi}_N(z)=\int_0^\infty \mathrm{d}\varphi\,\varphi^{N}\exp\left[-\frac{1}{2}z\varphi^2-\varphi^4\right]$$

$$\begin{aligned}
\bar{\varphi}_0(z) &= \frac{\pi}{8}\exp\left(\frac{z^2}{32}\right)\sqrt{|z|}\left[I_{-1/4}\left(\frac{z^2}{32}\right)-\mathrm{Sign}(z)I_{1/4}\left(\frac{z^2}{32}\right)\right],\\
\bar{\varphi}_1(z) &= \frac{1}{4}\exp\left(\frac{z^2}{16}\right)\left[1-\mathrm{Sign}(z)\mathrm{Erf}\left(\frac{|z|}{4}\right)\right],\\
\bar{\varphi}_{N+2}(z) &= -2\times\frac{\mathrm{d}\bar{\varphi}_N(z)}{\mathrm{d}z}.
\end{aligned}$$

RG Improvement

RG improvement :

$$-\rho\frac{\mathrm{d}\lambda(\rho)}{\mathrm{d}\rho} \equiv \beta_{\lambda\lambda^2}\lambda(\rho)^2 + \beta_{\lambda\lambda Y}\lambda(\rho)Y(\rho) + \beta_{\lambda Y^2}Y(\rho)^2,$$

$$-\rho\frac{\mathrm{d}Y(\rho)}{\mathrm{d}\rho} \equiv \beta_{YY^2}Y(\rho)^2,$$

$$\left[\frac{Y_l}{Y(L)}\right] = \left[1 - Y_l\beta_{YY^2}\log(l) + Y_l\beta_{YY^2}\log(L)\right]$$

$$-\rho\frac{\mathrm{d}m^2(\rho)}{\mathrm{d}\rho} \equiv 2\left[\gamma_\lambda\lambda(\rho) + \gamma_Y Y(\rho)\right]m^2(\rho),$$

$$-\rho\frac{\mathrm{d}\varphi(\rho)}{\mathrm{d}\rho} \equiv 2\delta_Y Y(\rho)\varphi(\rho).$$

Scaling Variable :

$$z = \sqrt{s}m^2L^2\lambda^{-1/2}$$

$$z = \left(\frac{2s\beta_{\lambda Y^2}}{Y_l}\right)^{1/2}\left(Y_l(\beta_+-\beta_-)\right)^{\frac{2\gamma_\lambda}{\beta_{\lambda\lambda^2}}}\left(m^2-m_c^2+\frac{A}{L^2}\right)L^2$$

$$\times\left(\frac{Y_l}{Y(L)}\right)^{\frac{1}{2}-\frac{2\gamma_Y}{\beta_{YY^2}}-\frac{\beta_-\gamma_\lambda}{\beta_{YY^2}\beta_{\lambda\lambda^2}}}\frac{\left\{B_+ - B_-\left[\frac{Y_l}{Y(L)}\right]^{\frac{\beta_+-\beta_-}{2\beta_{YY^2}}}\right\}^{\frac{1}{2}-\frac{2\gamma_\lambda}{\beta_{\lambda\lambda^2}}}}{\left\{\beta_-B_+ - \beta_+B_-\left[\frac{Y_l}{Y(L)}\right]^{\frac{\beta_+-\beta_-}{2\beta_{YY^2}}}\right\}^{\frac{1}{2}}}.$$

$$\beta_\pm = (\beta_{YY^2}-\beta_{\lambda\lambda Y}) \pm \sqrt{(\beta_{YY^2}-\beta_{\lambda\lambda Y})^2 - 4\beta_{\lambda\lambda^2}\beta_{\lambda Y^2}}$$

$$B_\pm = Y_l\beta_\pm - 2\lambda_l\beta_{\lambda\lambda^2}$$

The (Euclidean) model

$$S_{HY}[\phi, \bar{\psi}, \psi] = \int d^4x \left\{ \frac{1}{2}\partial_\mu \phi^\dagger \partial_\mu \phi + \frac{1}{2} m_0^2 \left(\phi^\dagger \phi\right) + \lambda \left(\phi^\dagger \phi\right)^2 \right\}$$

$$+ \int d^4x \left\{ \bar{t} \partial\!\!\!/ t + \bar{b} \partial\!\!\!/ b + \left[y \left(\bar{\psi}_L \phi b_R + \bar{\psi}_L \tilde{\phi} t_R \right) + h.c. \right] \right\}.$$

$$\psi = \begin{pmatrix} t \\ b \end{pmatrix}, \; \phi = \begin{pmatrix} \phi^2 + i\phi^2 \\ \phi^0 - i\phi^3 \end{pmatrix}, \; \tilde{\phi} = i\tau_2 \phi^*.$$

Note : degenerate Yukawa coupling.

Simulation : polynomial Hybrid Monte-Carlo, Overlap fermion. $s = 2$

$\lambda = 0.15 \quad y = 175/246$

Fit result - Binder's Cumulant

Formula : $$Q_L = 1 - \frac{\langle \varphi^4 \rangle}{3\langle \varphi^2 \rangle^2} = 1 - \frac{\bar{\varphi}_7(z) \times \bar{\varphi}_3(z)}{3\bar{\varphi}_5(z)^2}.$$

Case 1 : Mean field approximation,

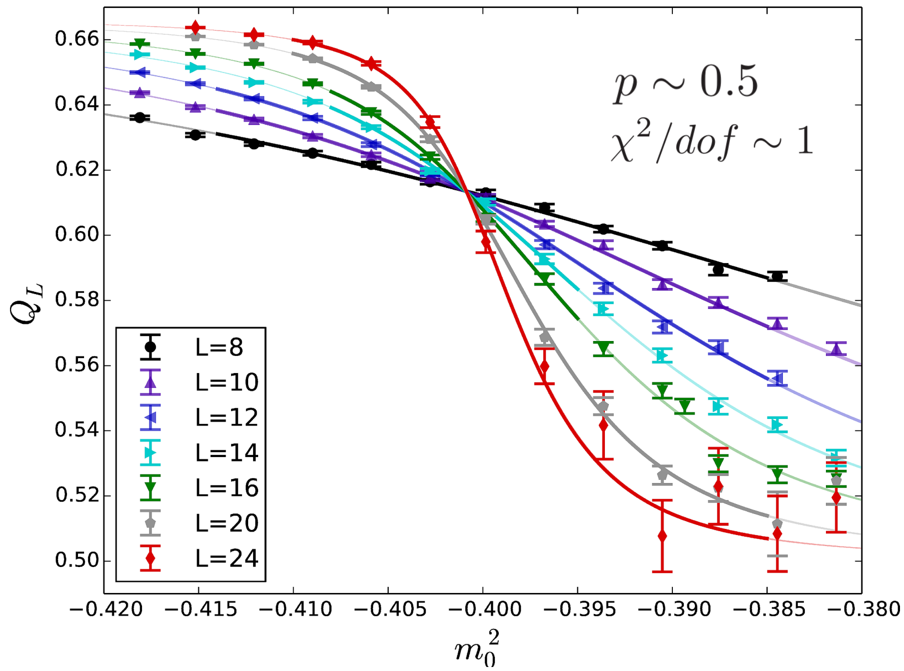
⇨ Fit does not converge. $z = \sqrt{s}\left(m^2 - m_c^2\right) L^2 \lambda^{-1/2}$

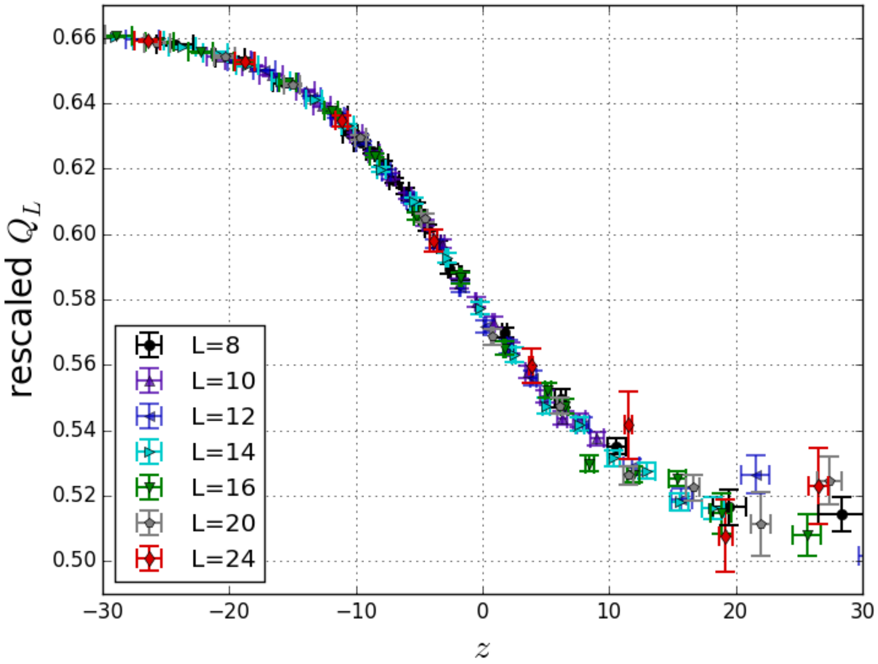
Case 2 : Include volume dependent shift of m_c^2,

⇨ Good fit. $$z = \sqrt{s}\left(m^2 - m_c^2 + \frac{A}{L^2}\right) L^2 \lambda^{-1/2}$$

Fit result - Binder's Cumulant

Case 3 : Full consideration of logs $z[\lambda_l, Y_l, A, m_c^2]$,





Finite temperature with a dim-6 operator

$$S = S_{HY} + \int \mathrm{d}^4 x \lambda_6 (\phi^\dagger \phi)^3.$$

$$a^2 m^2 = \frac{1 - 8\kappa^2\lambda - 8\kappa}{\kappa}$$

Motivation

Assuming triviality of Higgs-Yukawa sector in Standard Model.

Scale of new physics unkown.

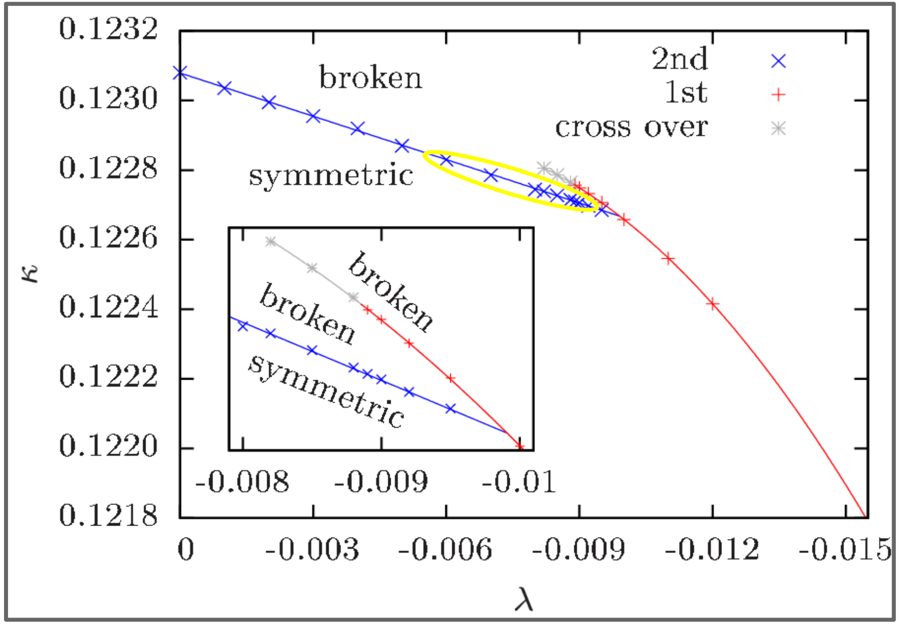
A dim-6 operator as representative of new physics.

Strong first-order electroweak phase transition from one-loop effective potential.

C. Grojean, G. Servant, and J. D. Wells. (2005)

Zero temperature phase structure

First-order phase transition expected: $\lambda_6 = 0.001 \quad y = 175/246$



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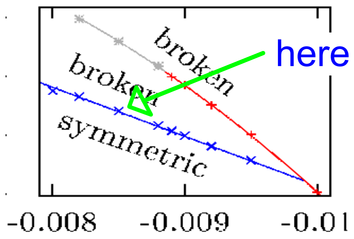
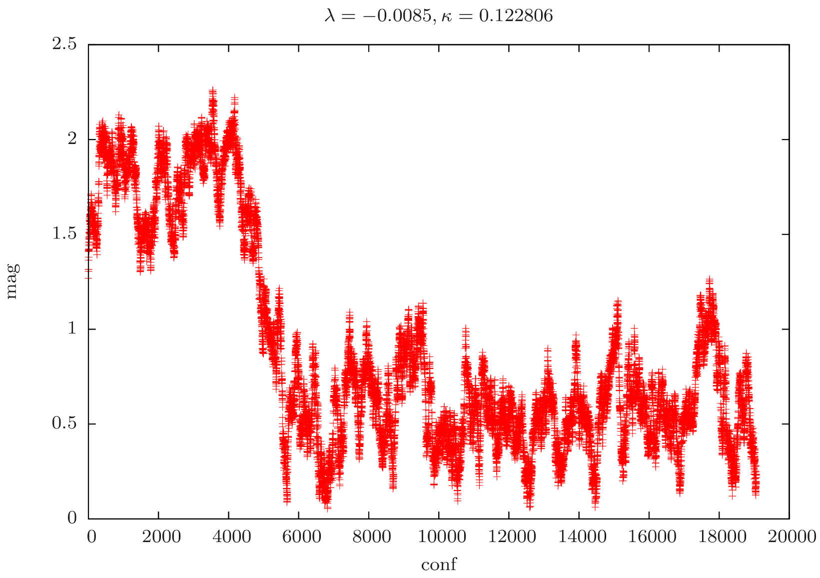
Finite temperature simulation performed on the border of first-order line.

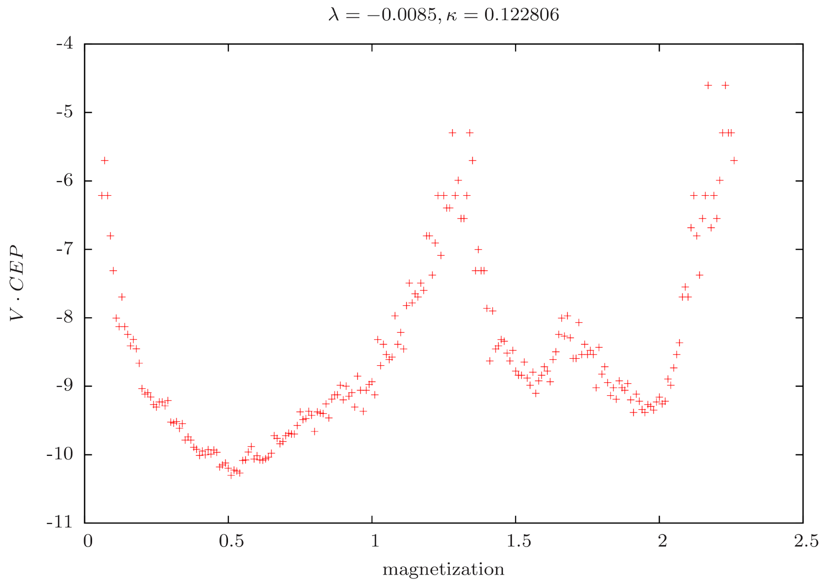
Simulation in Finite Temperature

Simulation performed on :

L_t = 4, L = 12, 16, 20; L_t = 6, L = 12, 16, 20, 24, 32







A 1-st order phase transition between two broken phases.

Conclusion and Outlook

Predicted logarithmic dependence near mean-field fixed point agrees with data.

- ⇨ Can be used to identify the type of fix point.
- ⇨ Large bare Yukawa and quartic couplings?

Second order in zero-temperature turns first order in finite temperature.

- ⇨ Only seen in the presence of fermion.
- ⇨ Large bare λ_6 simulation?