

# Studying near conformal behavior with four light flavors and eight flavors of variable mass

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Witzel and the LSD collaboration

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# Lattice Strong Dynamics Collaboration



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Two projects with the goal of exploring conformal or near conformal dynamics:

SU(3) with 4 light flavors and 8 flavors of variable mass

Richard Brower, Anna Hasenfratz, C.R., Evan Weinberg, Oliver Witzel

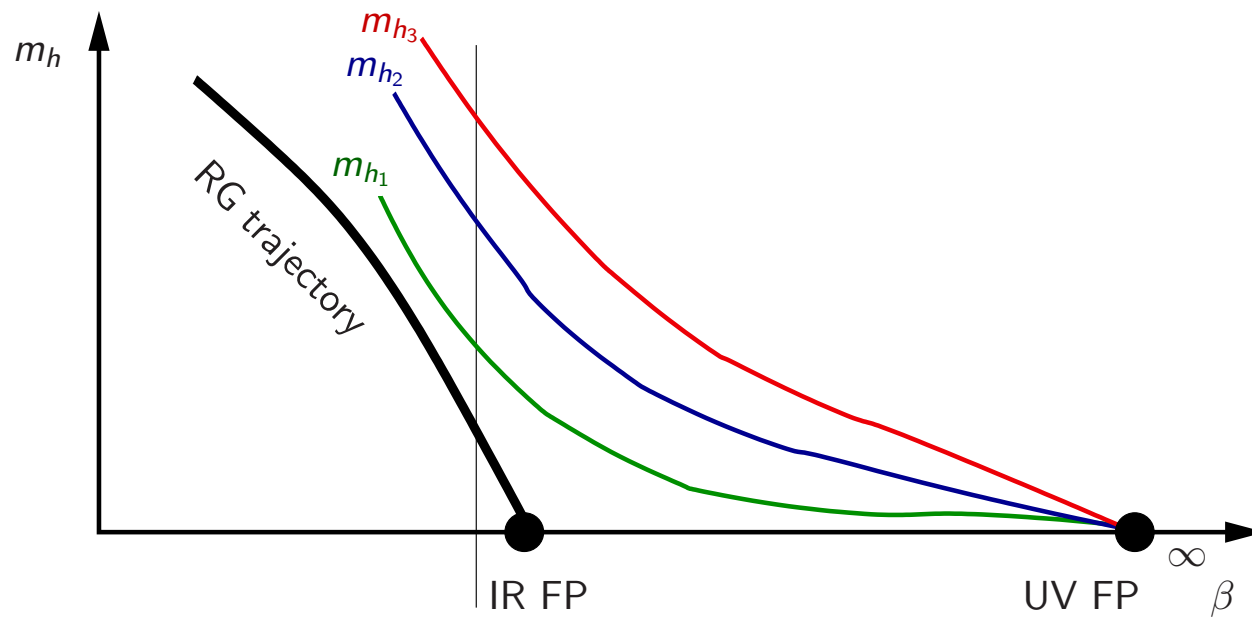
SU(3) with 8 flavors

the LSD collaboration and Anna Hasenfratz

Common aspects:

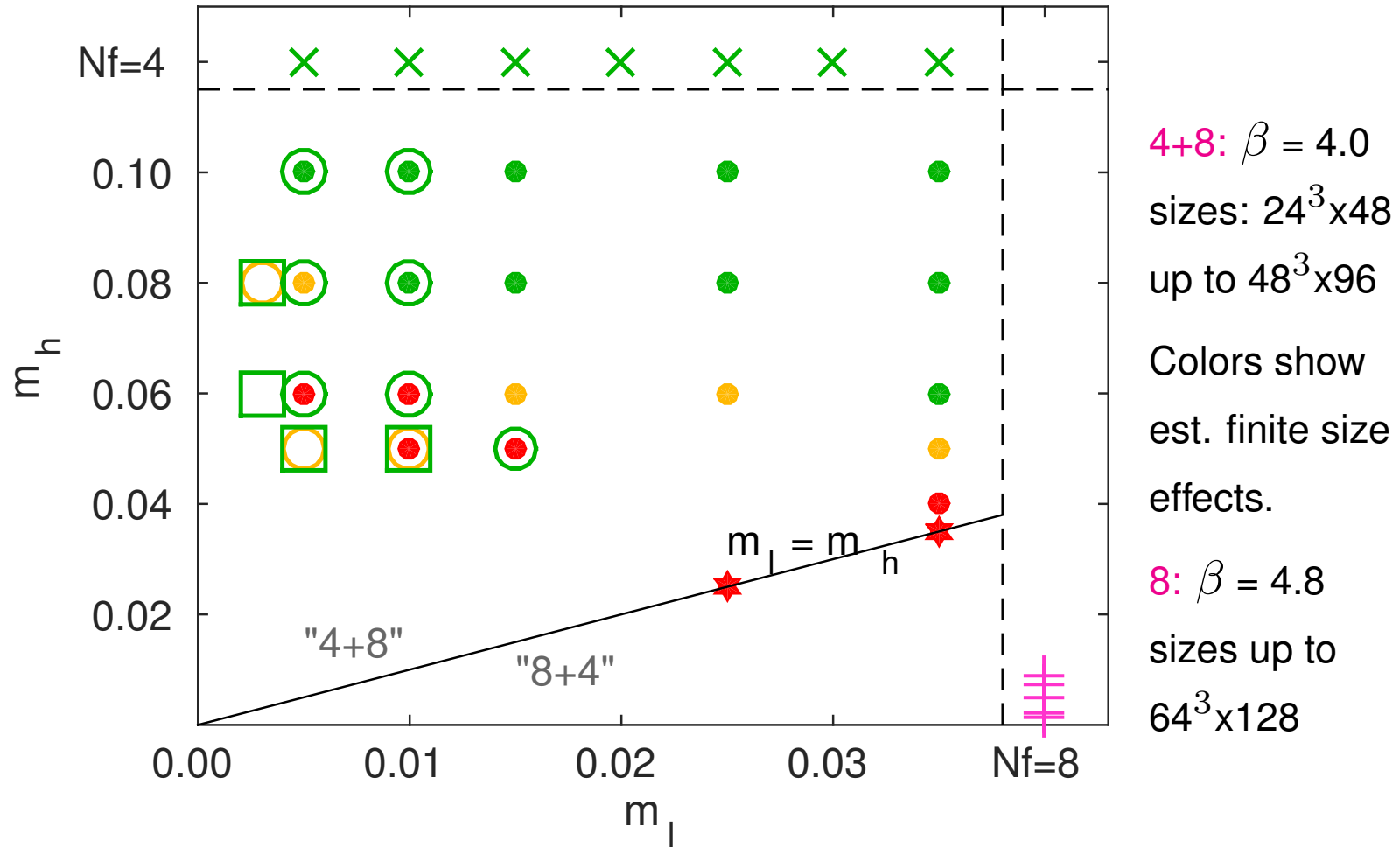
- fundamental adjoint gauge action with  $\beta_a = -\beta/4$
- nHYP smeared staggered Fermions
- most calculations performed with **FUEL**

The SU(3) theory with 4 light flavors and 8 flavors of variable mass interpolates between the QCD-like 4-fermion theory and the almost certainly conformal SU(3) with 12 fermions in the fundamental representation:

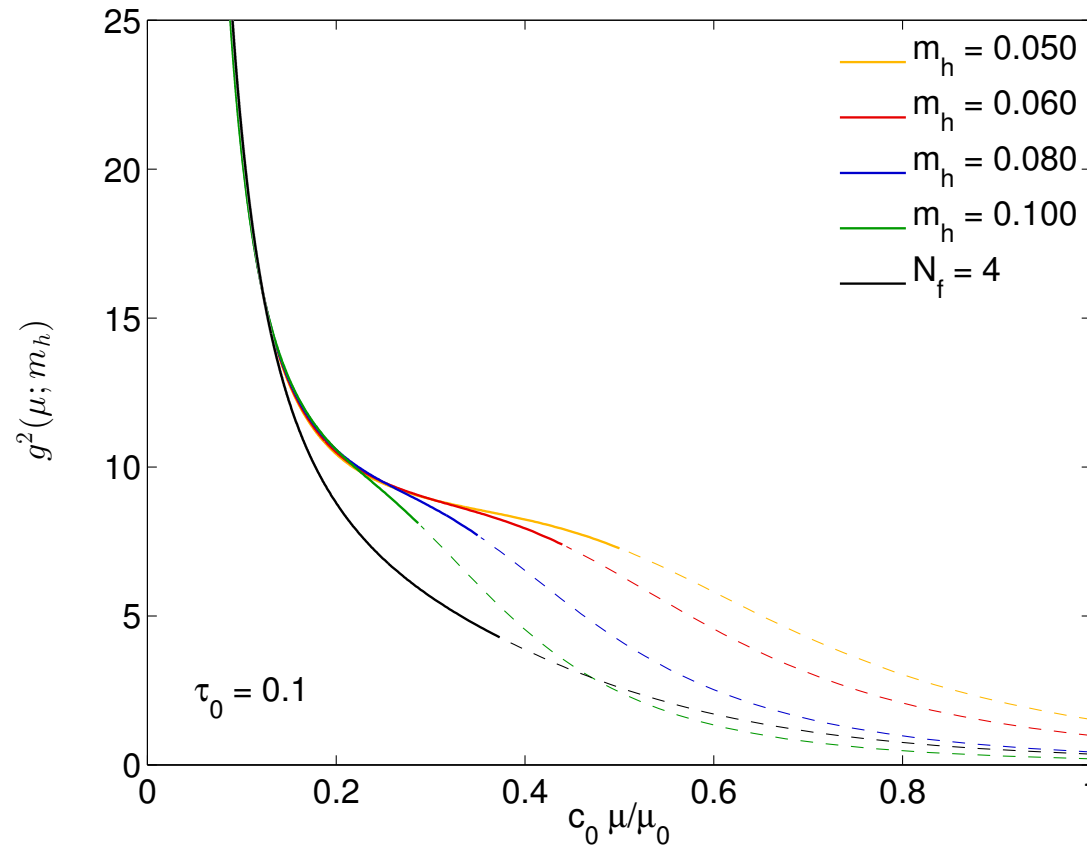


The SU(3) theory with 8 flavors in the fundamental representation may be a mass-broken conformal theory or a near conformal theory.

## Simulation parameters



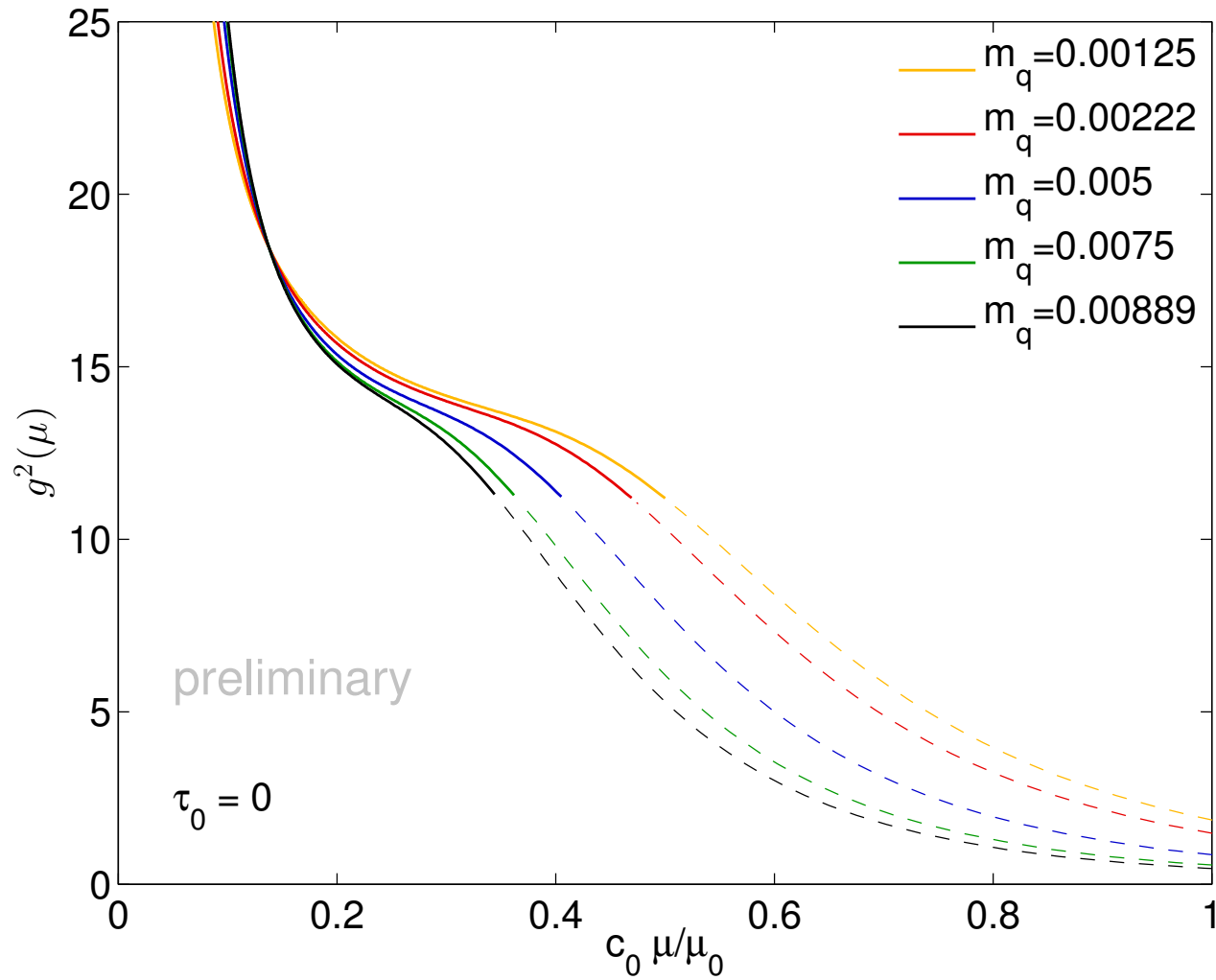
## The running coupling constant, 4+8:



From the gradient flow:  $g_{GF}^2(\mu = 1/\sqrt{8t}) = t^2 \langle E(t) \rangle / \mathcal{N}$ ,  $t$  flow time,  
 $E$  energy density;

$g_{GF}^2$  is also used to set the scale:  $g_{GF}^2(t = t_0) = 0.3/\mathcal{N}$

# The running coupling constant, 8 flavors:



## The isosinglet scalar state.

A viable theory for dynamical electroweak symmetry breaking should give origin to a  $0^{++}$  state of mass much lower than the other excitations in the spectrum, that could be identified with the Higgs.

Calculating the mass of the isosinglet scalar is however difficult because, contrary to the states which do not couple to the vacuum, it requires an accurate evaluation of disconnected diagrams.



## Chasing the elusive $0^{++}$

$0^{++}$  correlator:

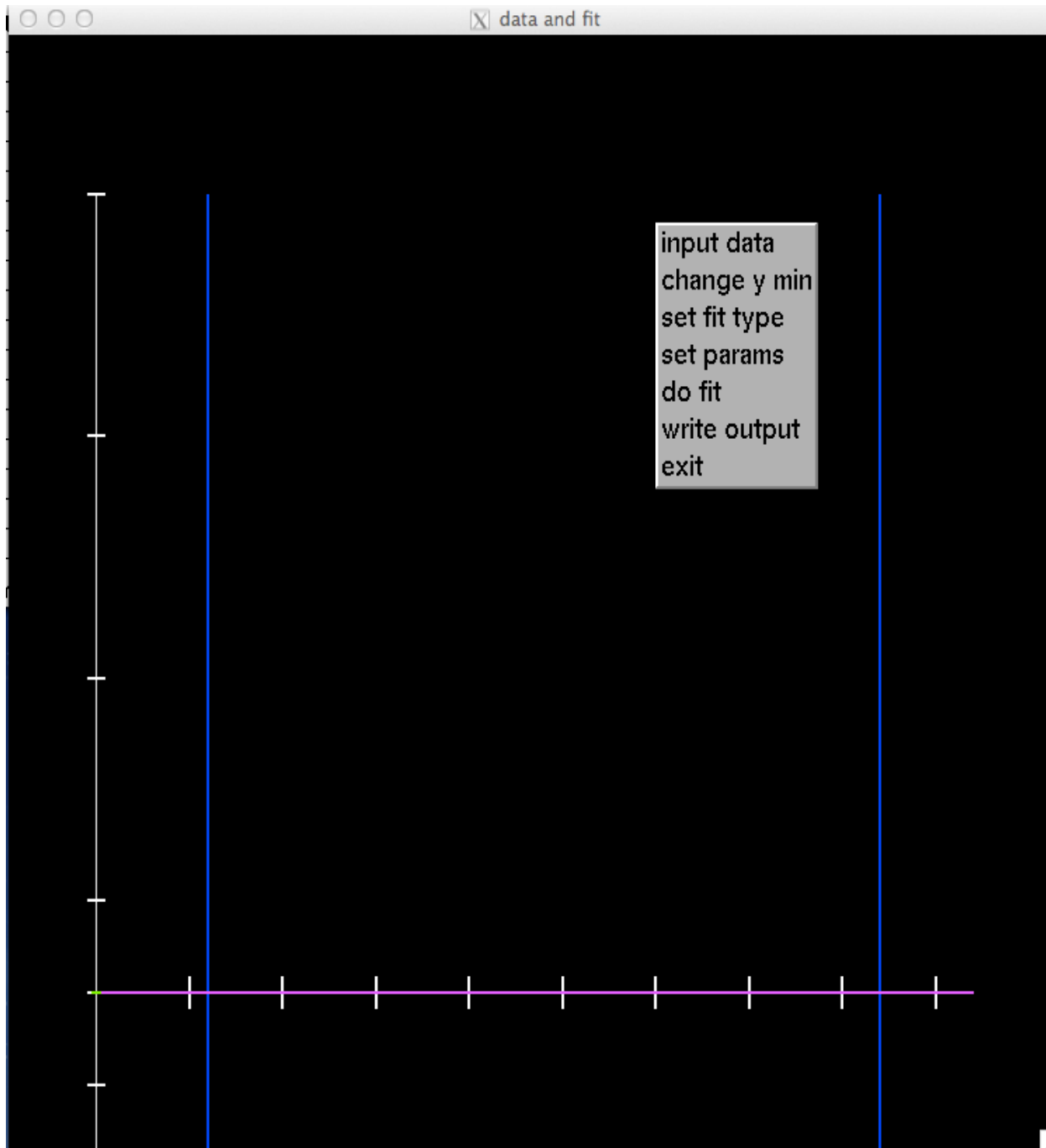
$$S(t) = (1/N_s) \sum_{\mathbf{x}, \mathbf{y}} \langle (\bar{\psi}\psi)(\mathbf{x}, 0) (\bar{\psi}\psi)(\mathbf{y}, t) \rangle - \mathbf{v.s.} = [2]D(t) - C(t)$$

with

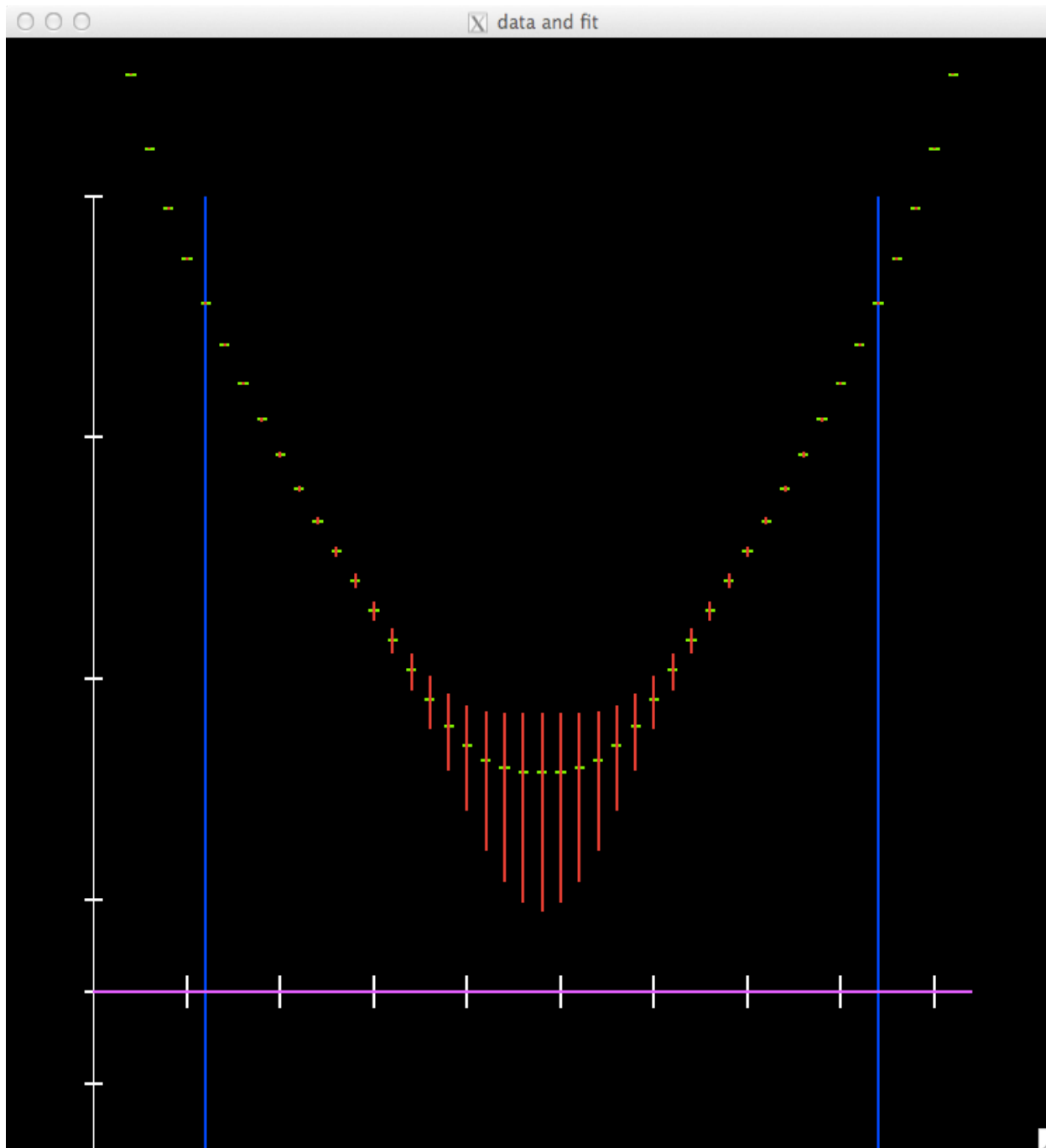
$$D(t) = (1/N_s) \langle \sum_{\mathbf{x}} \langle (\bar{\psi}\psi)(\mathbf{x}, 0) \rangle_U \sum_{\mathbf{y}} \langle (\bar{\psi}\psi)(\mathbf{y}, t) \rangle_U \rangle - \mathbf{v.s.}$$

$$C(t) = (1/N_s) \langle \sum_{\mathbf{x}, \mathbf{y}} \langle \psi(\mathbf{x}, 0) \bar{\psi}(\mathbf{y}, t) \rangle_U \langle \psi(\mathbf{y}, t) \bar{\psi}(\mathbf{x}, 0) \rangle_U \rangle$$

The evaluation of  $D(t)$  is problematic: requires very good statistics and inspection of the data.



An interactive fitting program allows the user to enter data, set initial parameters, choose a fit type, and perform a fit.



4 plus 8 simula-  
tion:

$$\beta = 4.0,$$

$$m_\ell = 0.01$$

$$m_h = 0.08,$$

$$24^3 \times 48,$$

$0^{++}$  correlator  
( $D - C$ ),

averaged over  
2000 configura-  
tions.

A remark on the representation of the data:

rather than a logarithmic representation:

$$y = \log[f(x)]$$

I use

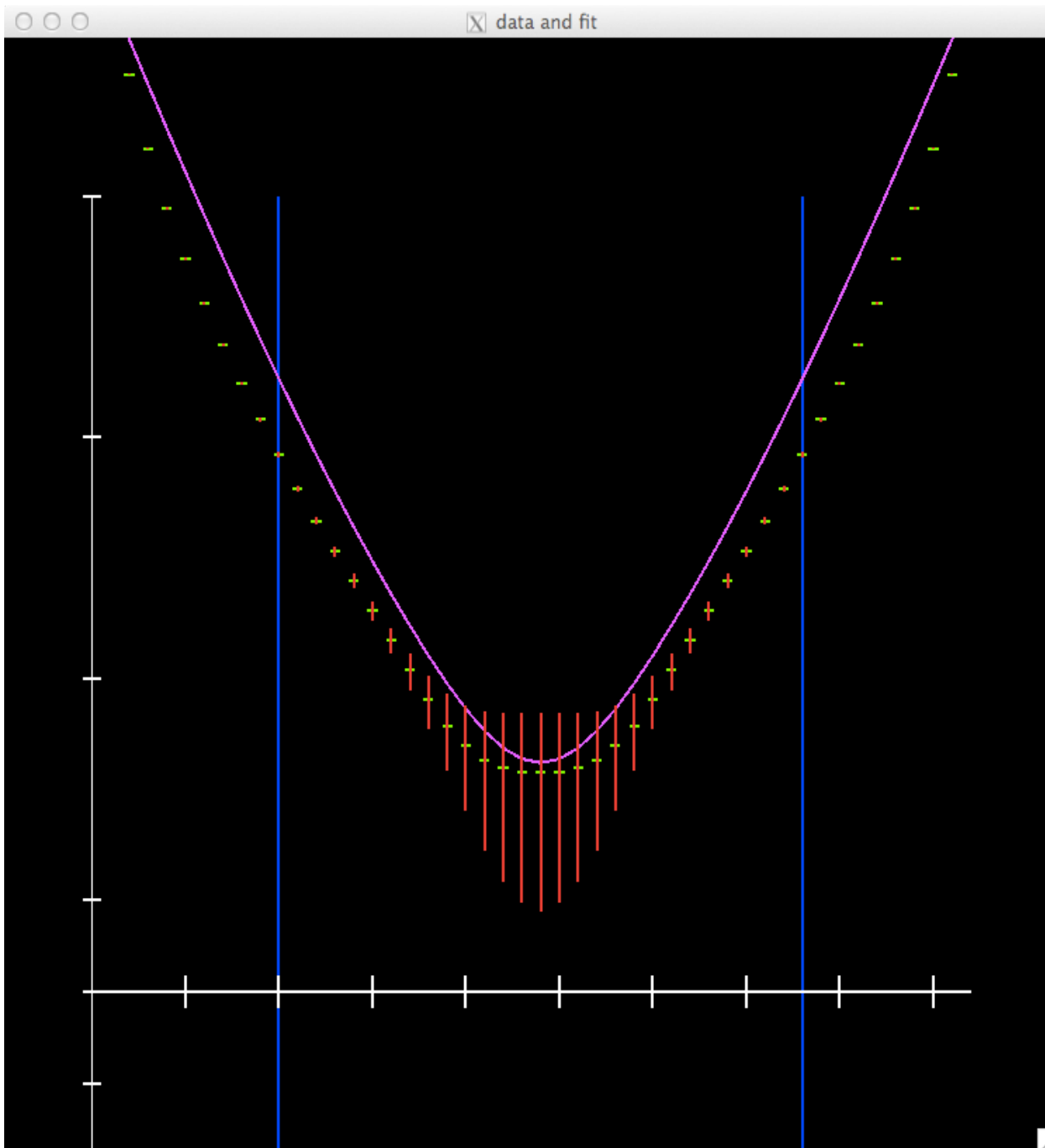
$$y = \operatorname{arcsinh}[\alpha f(x)]$$

$\alpha$  being a suitable scaling constant.

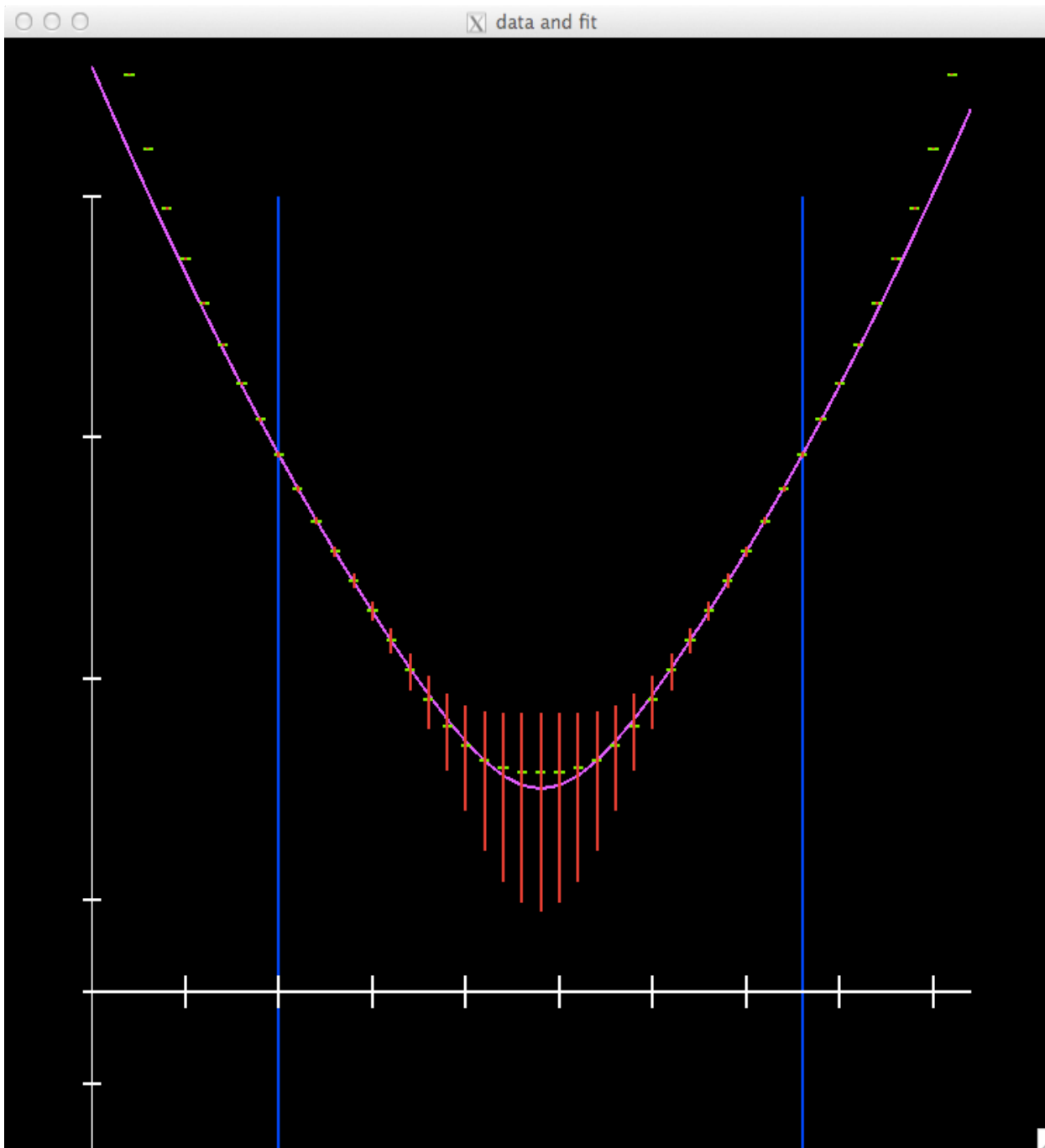
$$\operatorname{arcsinh}(x) = \log(x + \sqrt{1 + x^2})$$

is an odd function which behaves like  $\pm \log(\pm x)$  for large positive or negative  $x$  and interpolates linearly between the two domains.

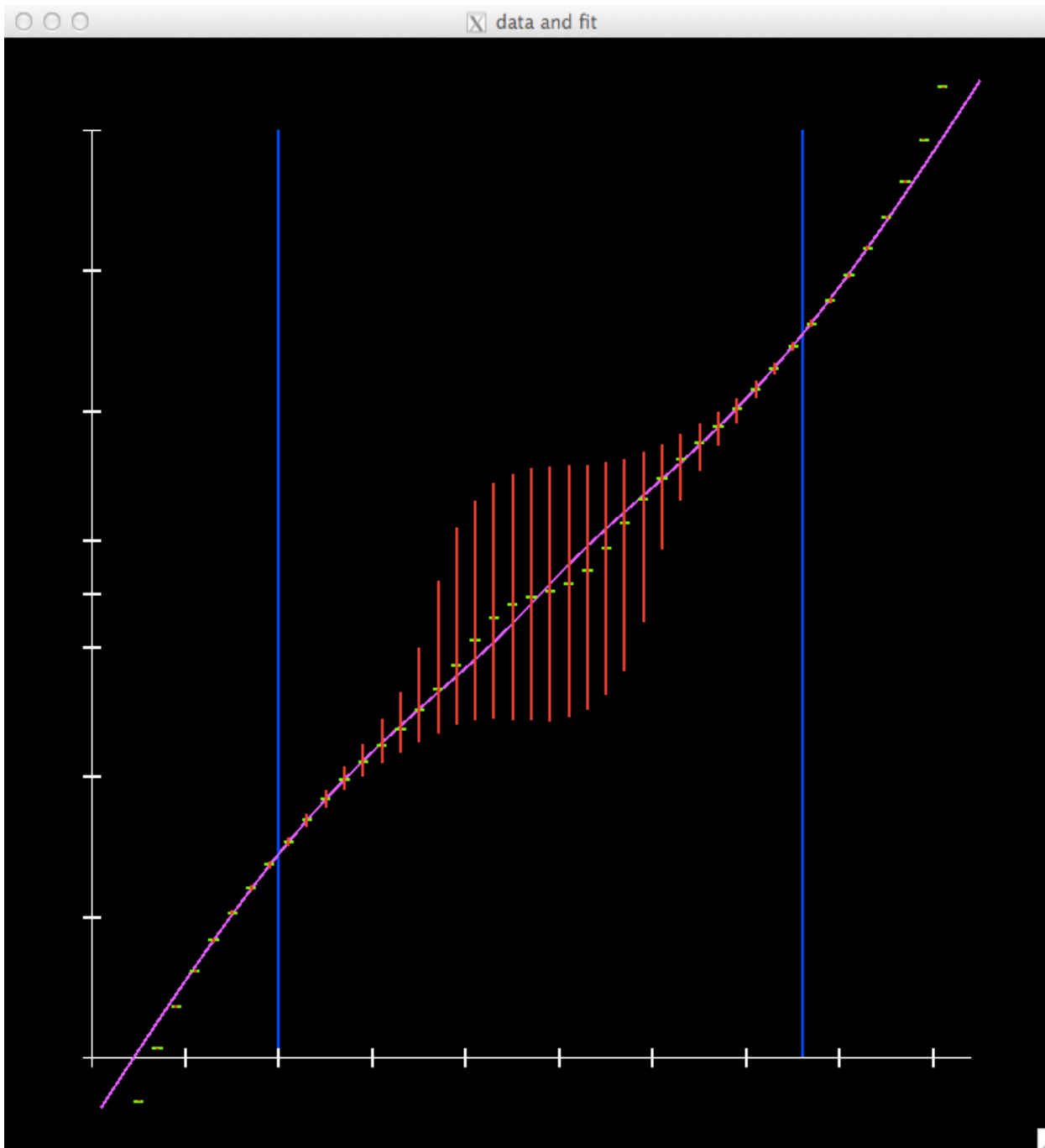
The use of this representation is very useful for negative data or data (errors) approaching zero.



Choosing an initial set of parameters for a two mass fit..



With  
gap=0.5E-3  
the fit over  
t=10-38 gives  
 $m_0=0.249$ ,  
 $c_0=0.412E-3$ ,  
 $m_1=0.450$ ,  
 $c_1=0.014E-3$ .



The dependence on the gap can be eliminated by fitting the time differences, or allowing an extra parameter for the vacuum subtraction.

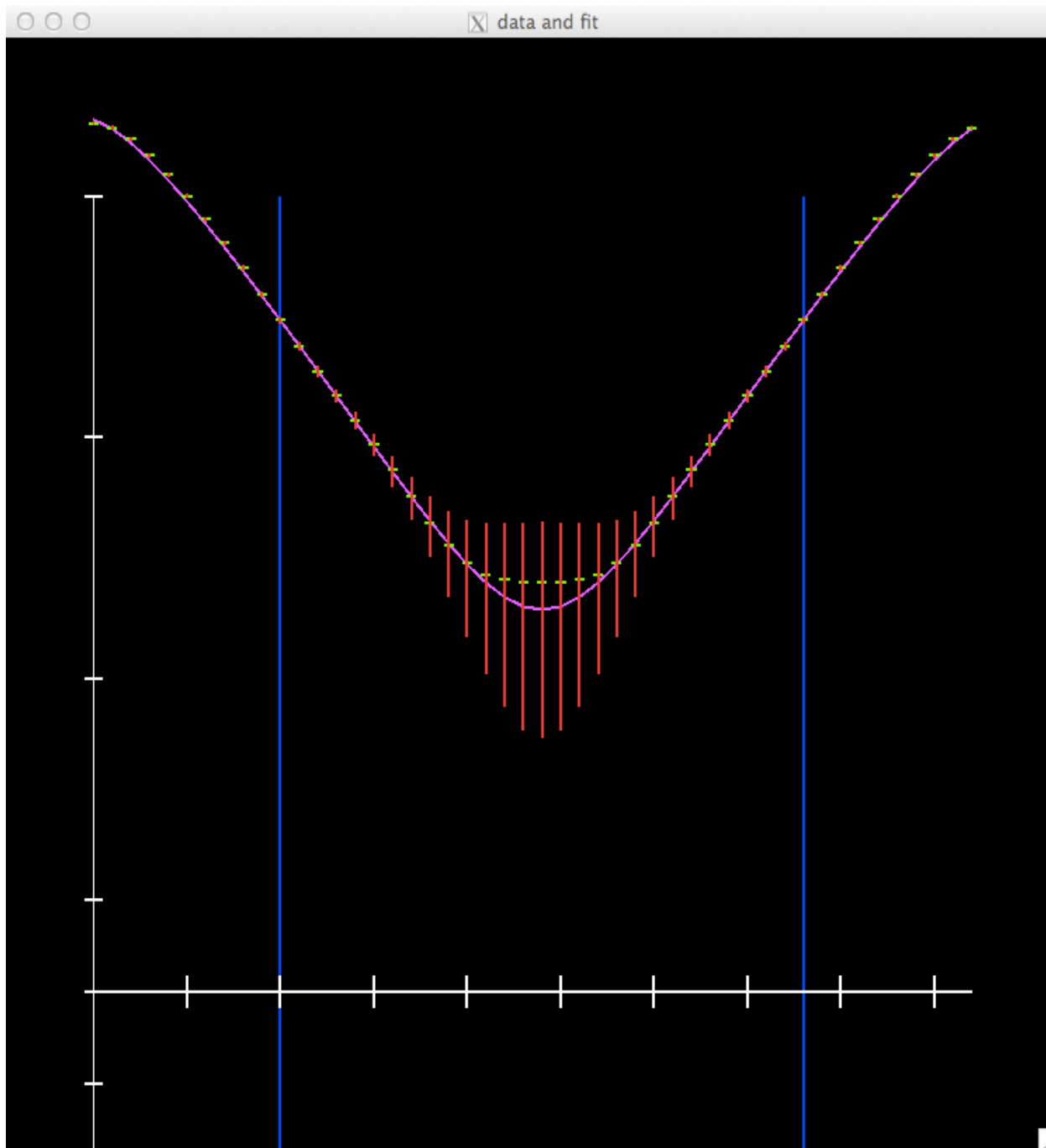
Here we get:

$$m_0=0.211,$$

$$c_0'=0.144E-3,$$

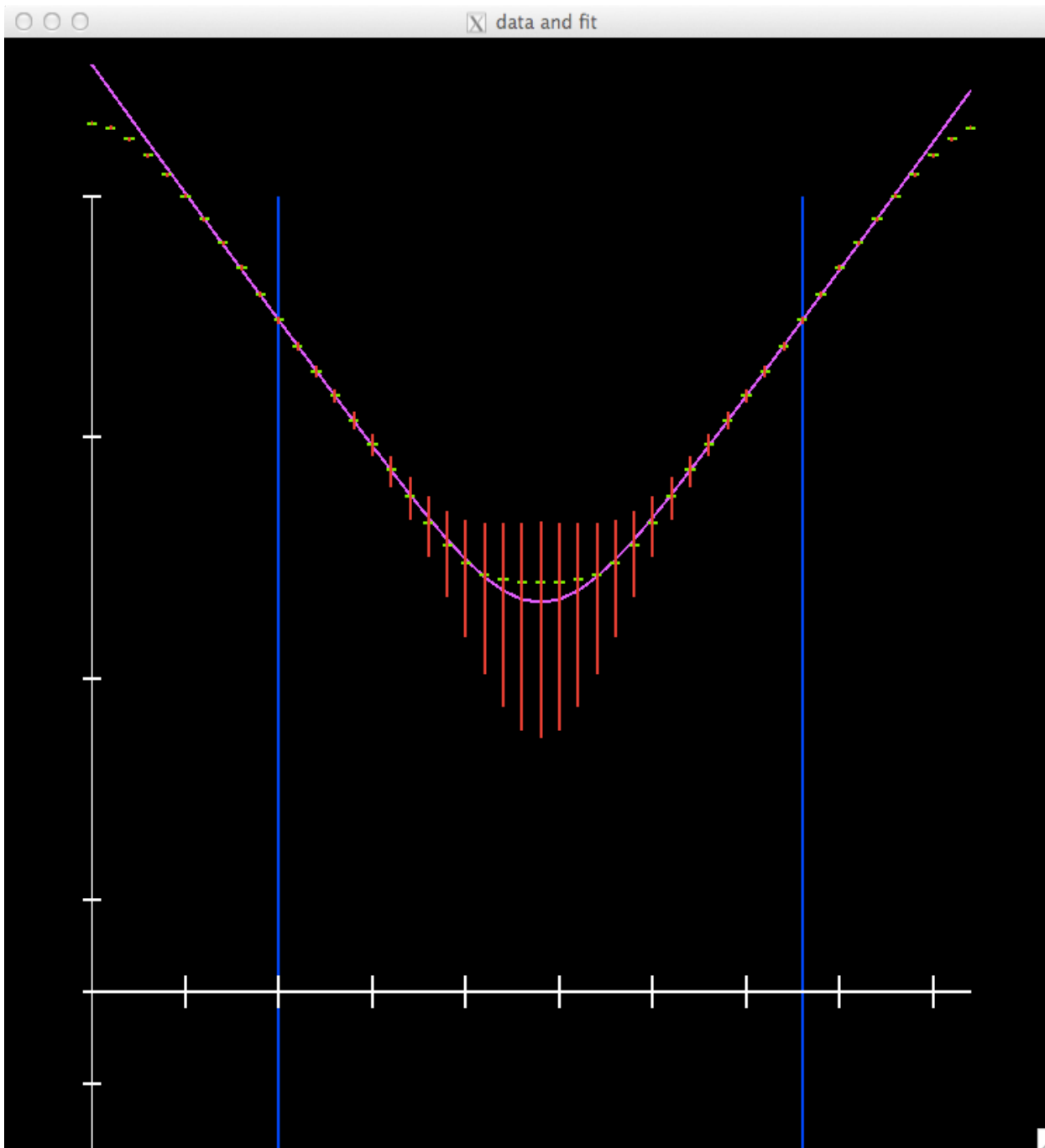
$$m_1=0.485,$$

$$c_1'=0.005E-3.$$



The disconnected correlator  $D(t)$  must be used with caution because of the unphysical behavior for smaller  $t$ . The fit here gives  $m_0=0.251$ ,  $c_0=0.386E-3$ ,  $m_1=0.475$ ,  $c_1=-0.001E-3$ . Notice the negative curvature for small  $t$ .





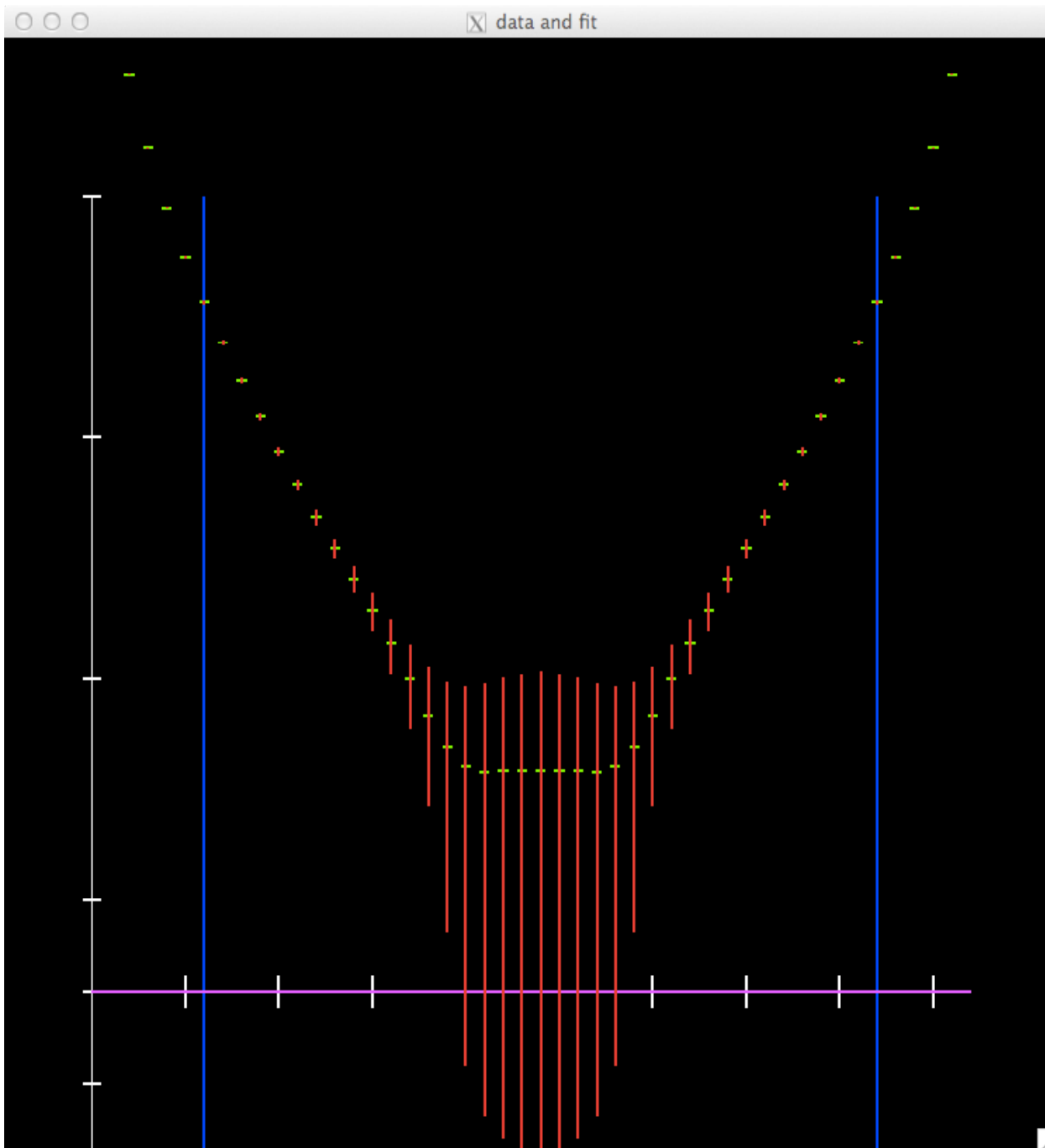
Pushing the fit  
further we get

$$m_0=0.242,$$

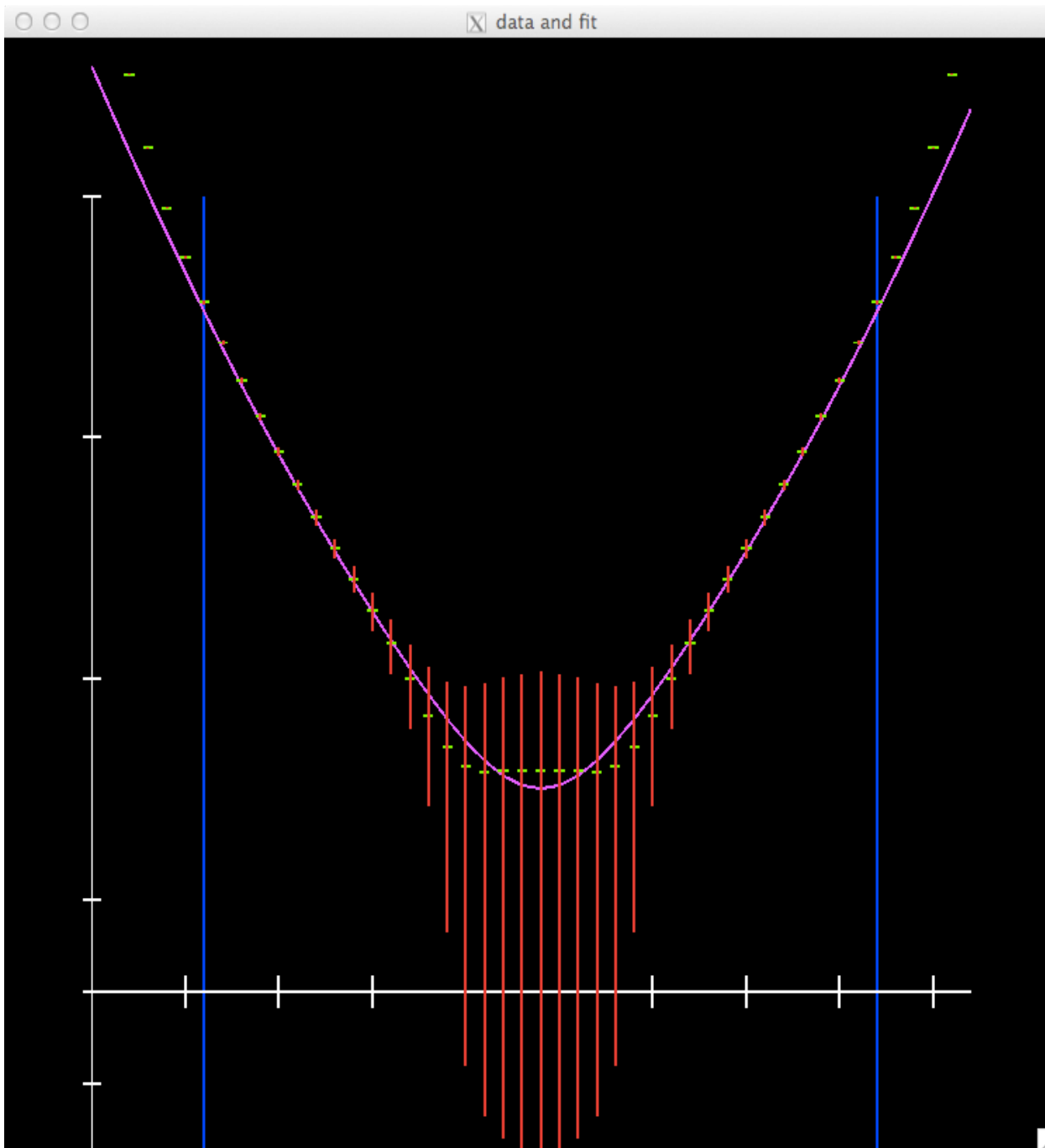
$$c_0=0.041E-3,$$

$$m_1=0.475,$$

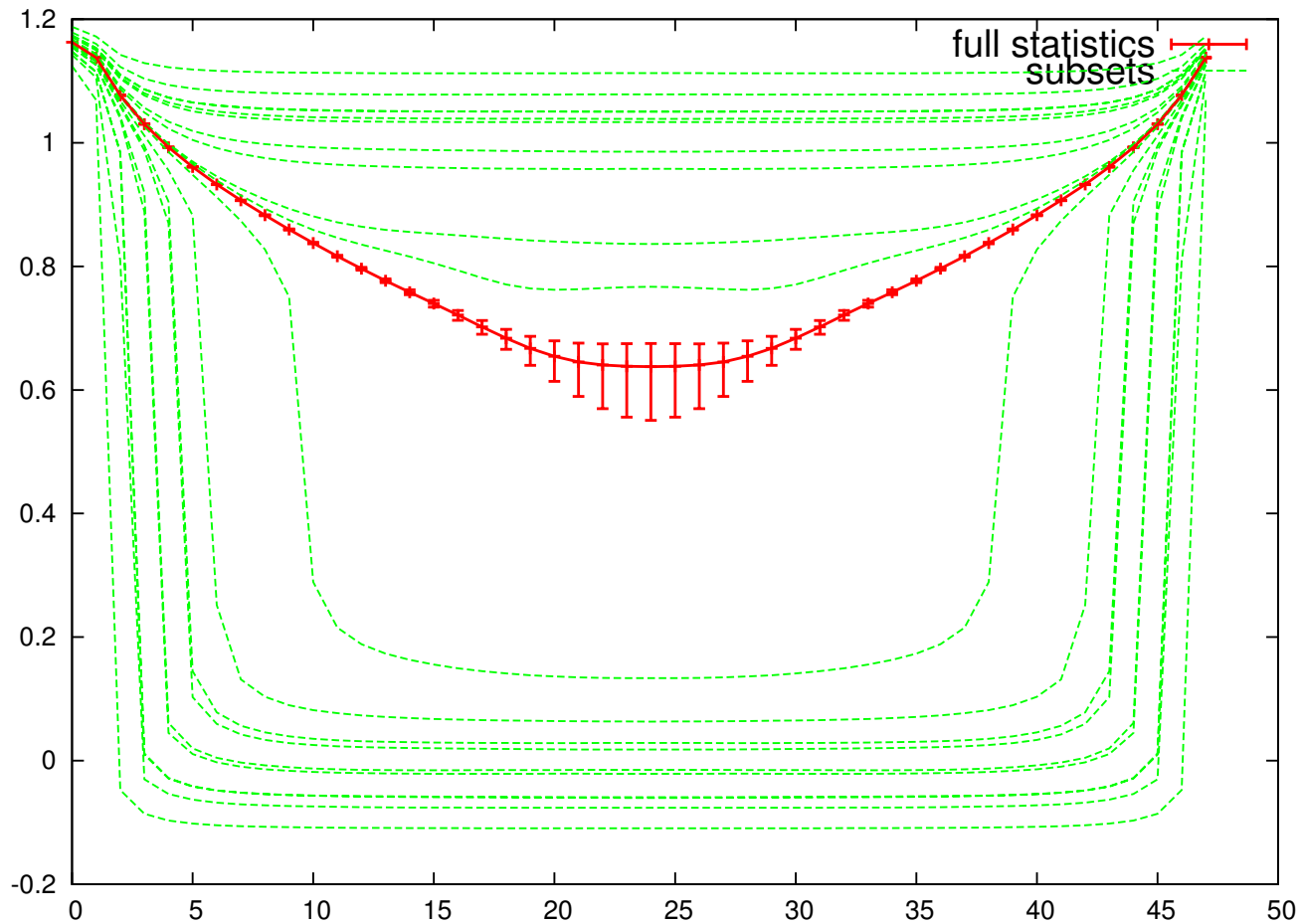
$$c_1=0.004E-3.$$



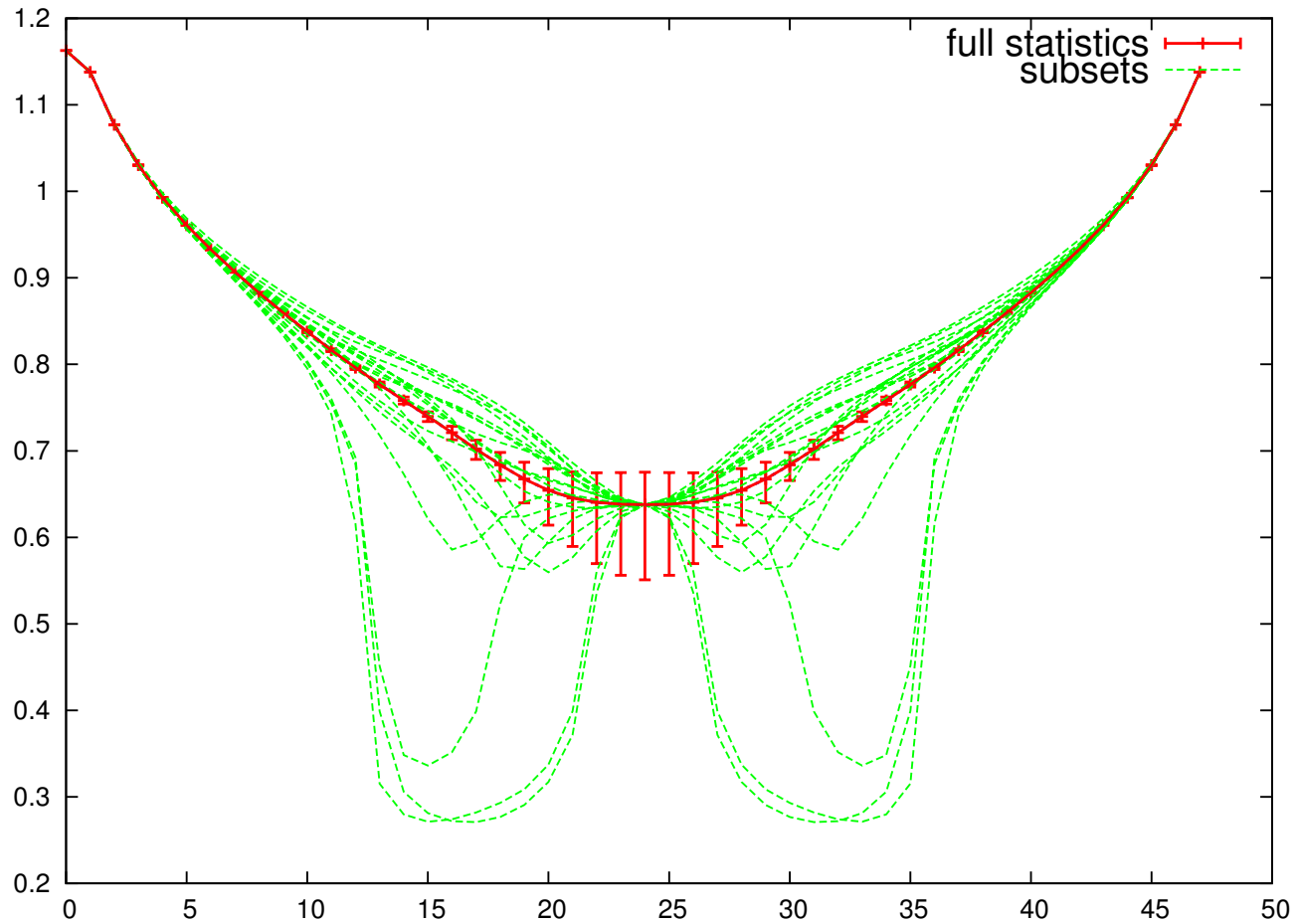
The average over the first 1000 configurations shows a very poor behavior in the center of the lattice.



The average over the first 1000 configurations shows a very poor behavior in the center of the lattice. The line shows the fit obtained with 2000 configurations.

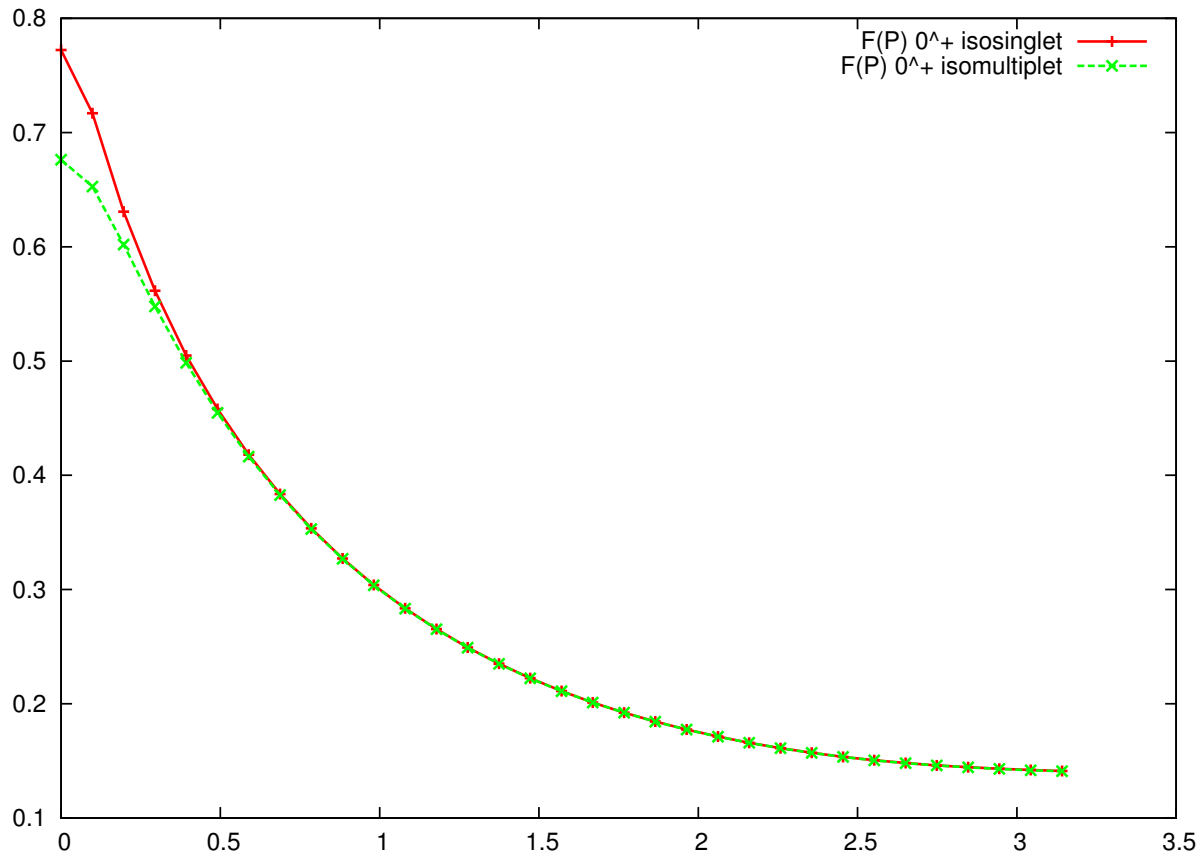


Taking averages over 20 groups of 100 consecutive configurations shows the huge fluctuations of the correlator, here mostly due to the fluctuation of the vacuum values.



But even removing the vacuum fluctuation, by fixing the central value of the correlator, still shows very large fluctuations.

## The correlators in momentum space



We may obtain complementary information on the spectrum from the correlators in momentum space:

$$F(p) = \sum_t S(t) e^{ipt}$$

$(36^3 \times 64, \beta = 4, m_\ell = 0.003, m_h = 0.08, 1014 \text{ measurements})$

## The correlators in momentum space, cont'd

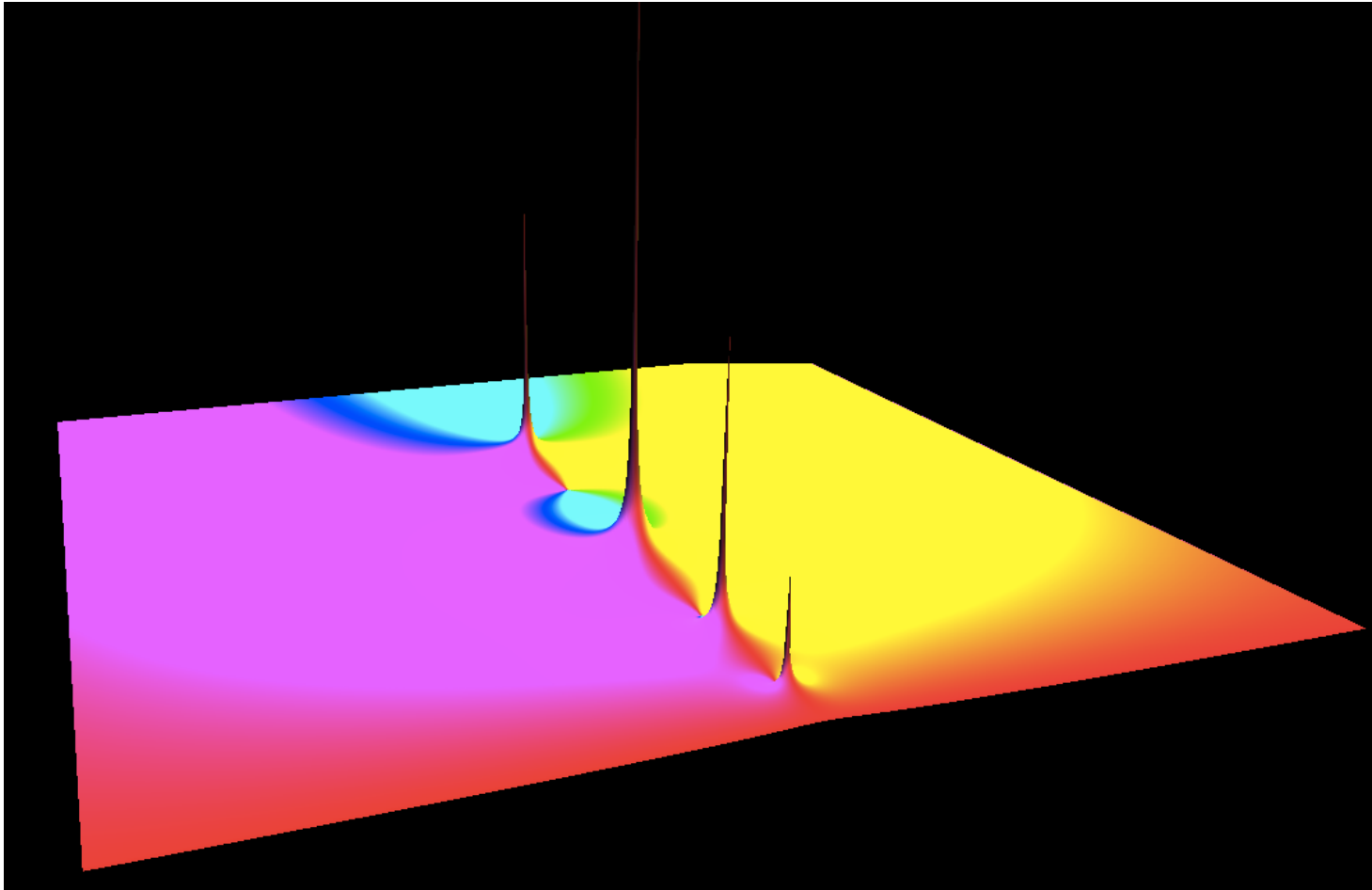
$$F(p) = \sum_t S(t) e^{ipt}$$

In the continuum limit and infinite volume  $F(p)$  has a pole at  $p = im$ ,  $m$  being the lowest state's mass.

Fit  $F(p)$  to

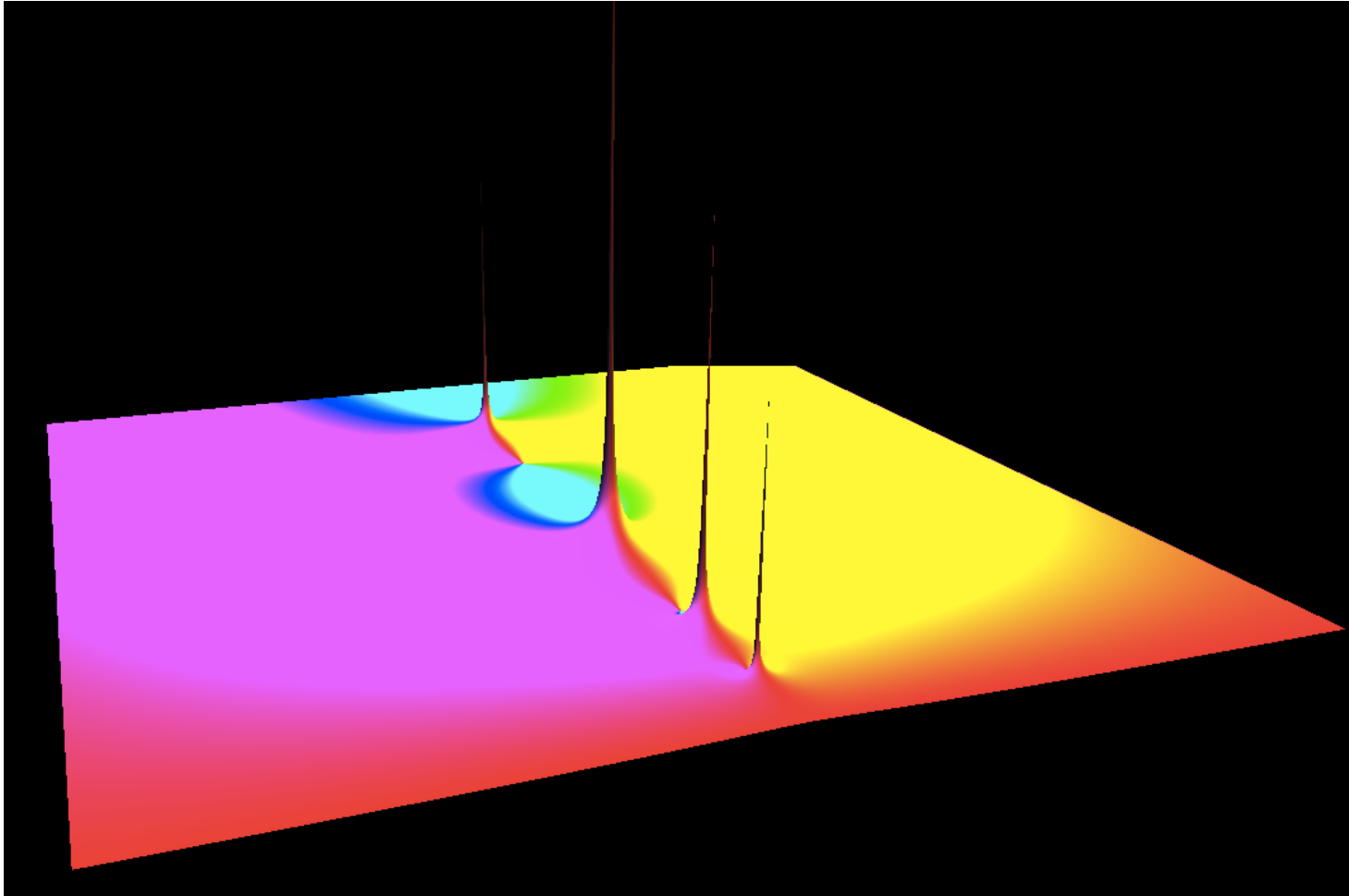
$$\frac{1 + a_0 \cos p + a_1 \cos^2 p + a_2 \cos^3 p + a_3 \cos^4 p}{b_0 + b_1 \cos p + b_2 \cos^2 p + b_3 \cos^3 p + b_4 \cos^4 p + b_5 \cos^5 p}$$

and continue to complex  $p$ .

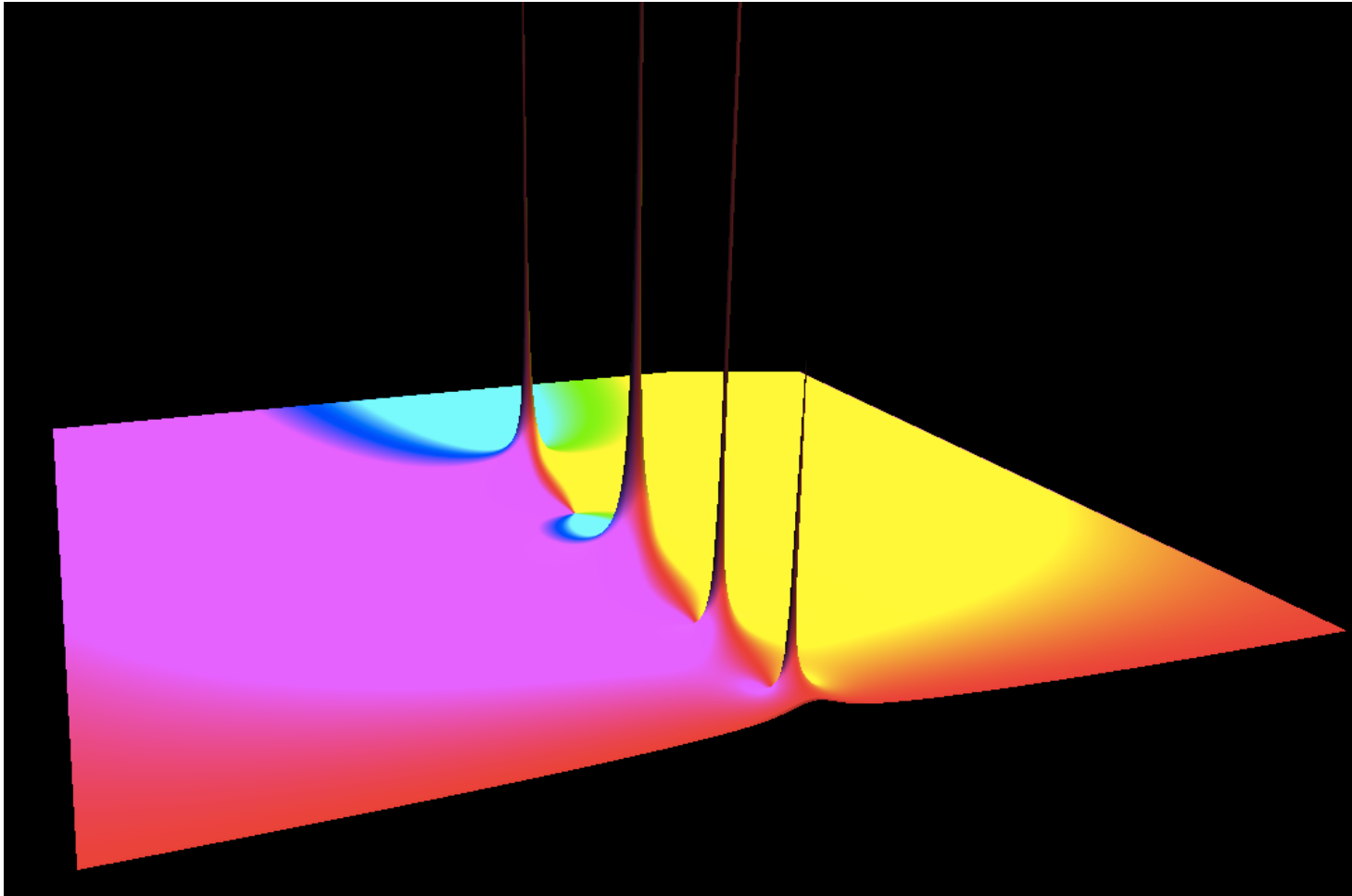


$0^+$  isosinglet correlator in complex momentum space. Color represents the complex phase. The pole closest to the real axis is at  $p = 0.194v$ .

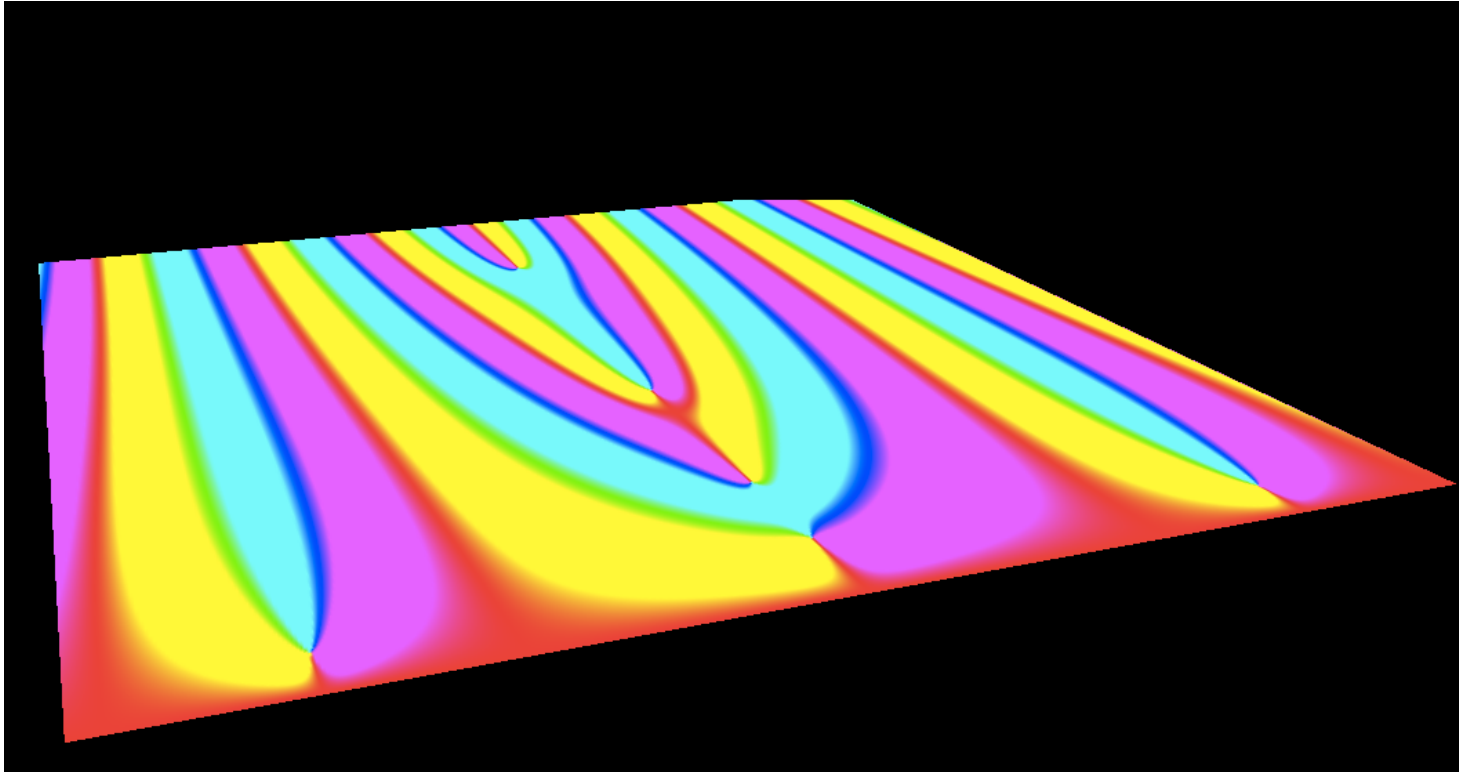




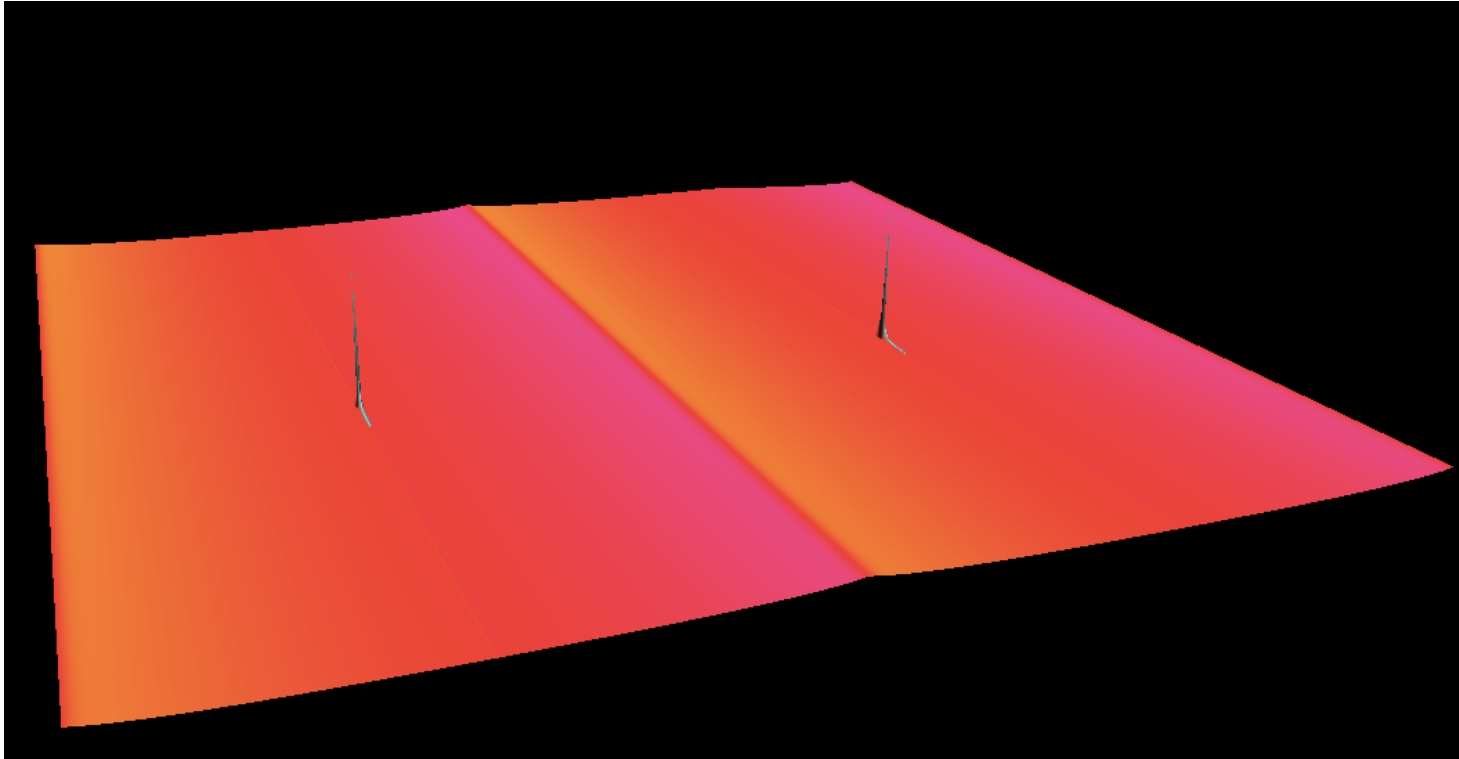
$0^+$  isomultiplet correlator in complex momentum space. The pole closest to the real axis is at  $p = 0.263 i$ .



Pseudoscalar correlator in complex momentum space. The pole closest to the real axis is at  $p = 0.120 i$ .



How about the  $0^+$  isomultiplet parity partner? The phase of the denominator shows zeros for  $\text{Real}(p) = \pm\pi$ .



Teeny tiny poles: the numerator also has a zero nearby, so the strength of the poles is weak.