Pion electromagnetic form factor from full Lattice QCD

Jonna Koponen
University of Glasgow
HPQCD collaboration*

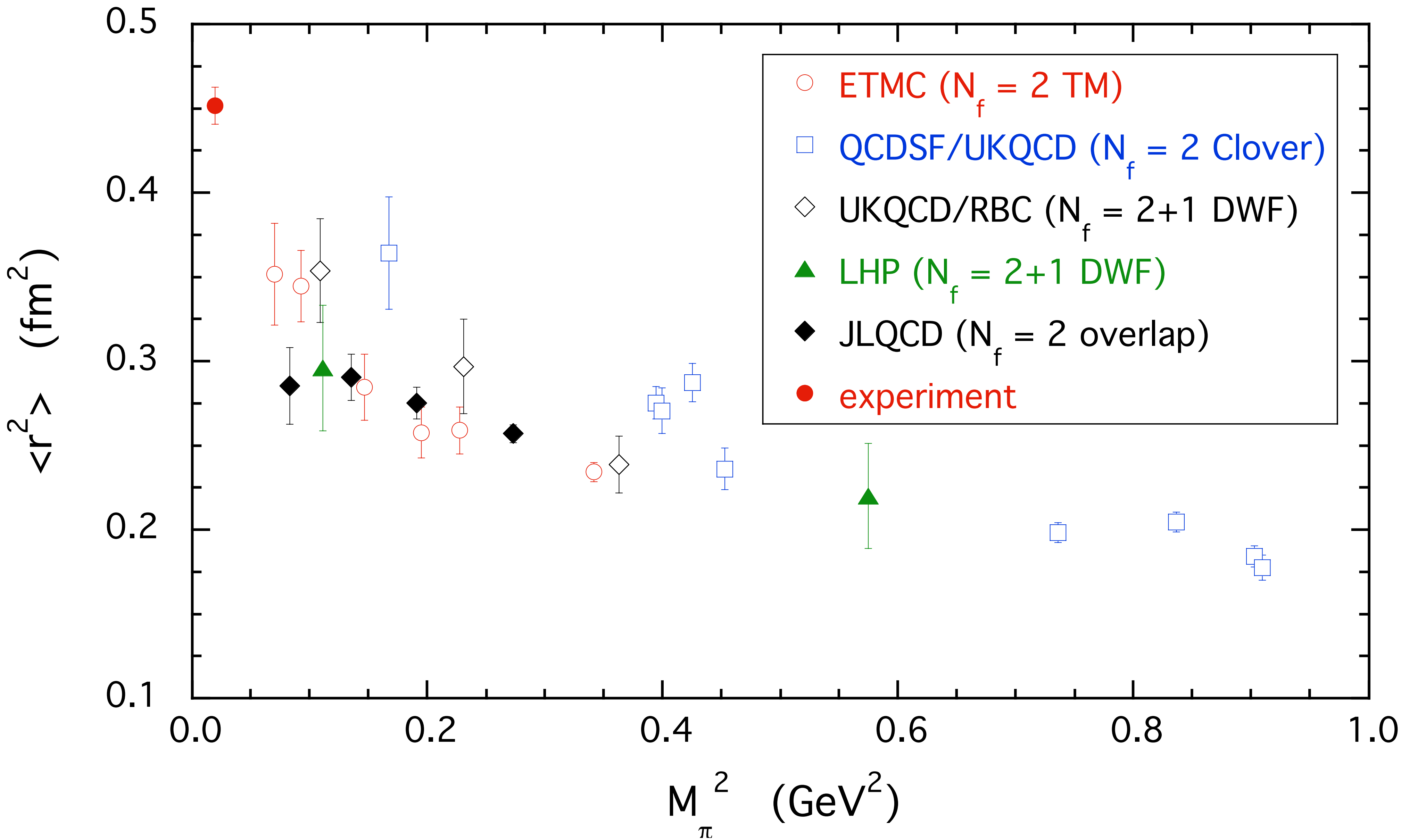
*F. Bursa, C. T. H. Davies, G. Donald, R. Dowdall and J. K.

Lattice 2015, Kobe, Japan

Motivation

- The electromagnetic form factor of the charged π meson parameterises the deviations from the behaviour of a point-like particle when struck by a photon
- These deviations arise from the internal structure of the π: constituent quarks and their strong interaction
- Can be calculated in QCD, but need fully nonperturbative treatment $\rightarrow$ use Lattice QCD
- Experimental determination from $\pi - e$ scattering
- Important to work at physical pion mass

Dependence on pion mass



ETMC, Phys. Rev. D79 (2009) 074506

Lattice configurations

- MILC n_f=*2+1+1* HISQ lattice configurations
- HISQ action for valence quarks
- quark masses tuned to physical masses
- large volumes: $(5.8\ \text{fm})^3$ for the coarse (*a=0.12* fm) and fine (*a=0.088* fm) lattices ($Lm_\pi \approx 4$)

Set	a/fm	am_l	am_s	am_c	m_π/MeV	$L/a \times L_t$	N_{conf}
1	0.15	0.00235	0.0647	0.831	133	32×48	1000
2	0.12	0.00184	0.0507	0.628	133	48×64	1000
3	0.088	0.00120	0.0363	0.432	128	64×96	223

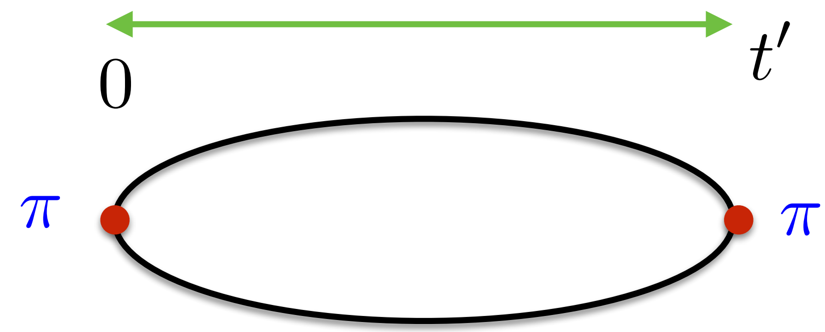
Form factors = 3pt amplitudes

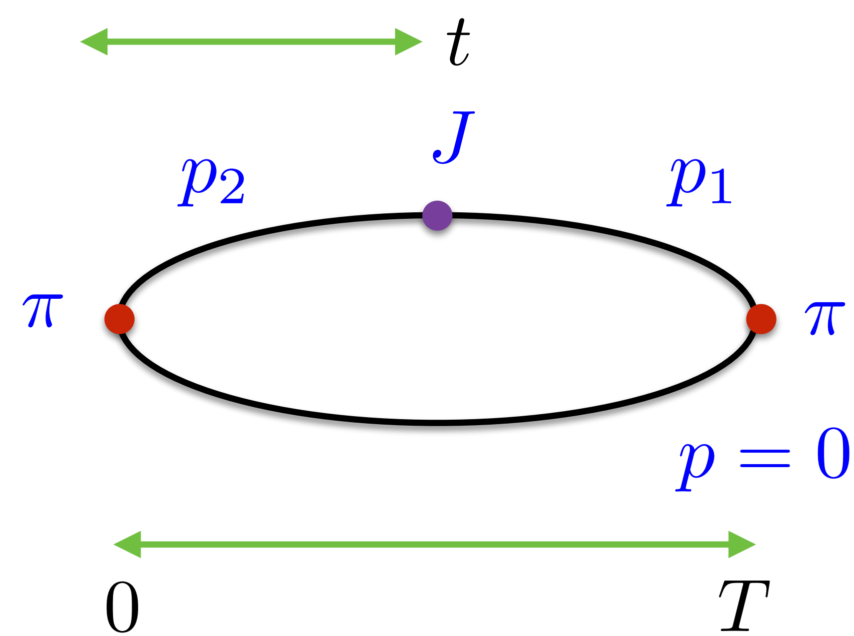
- Consider two currents, a 1-link spatial vector current and a scalar current
- Use a phase at the boundary to give a quark a momentum:

$$\Phi(x + \hat{e}_j L) = e^{i2\pi\theta_j}\Phi(x)$$
$$\rightarrow p_j = 2\pi\theta_j / L$$

- Tune θ to get the desired q^2 and extract $f_+(q^2)$ in the space-like (negative) region of q^2 near zero

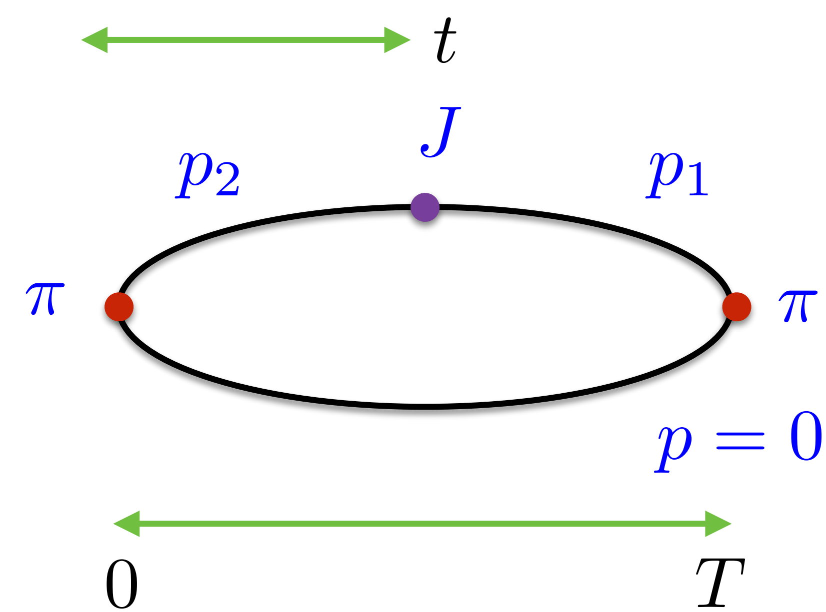
$$q^2 = (E_2 - E_1)^2 - (\vec{p}_2 - \vec{p}_1)^2$$





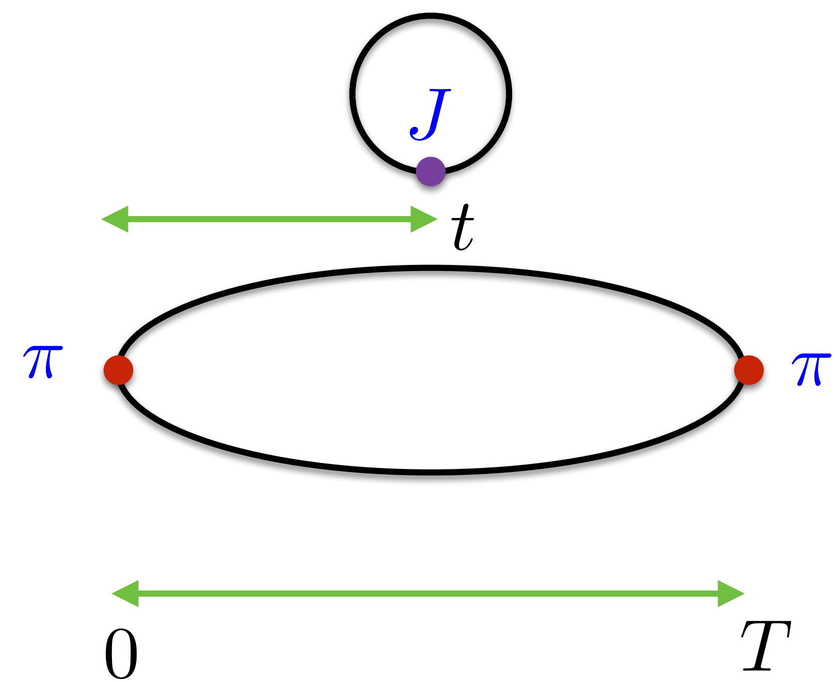
Connected and disconnected diagrams

- Vector current: Disconnected diagrams cancel due to charge conjugation and isospin symmetries



- Scalar current: For a full calculation need both connected and disconnected diagrams, where the disconnected piece

$$\langle\pi|S_{\bar{q}q}|\pi\rangle^{\mathrm{disc}} = \langle\pi(p)|\bar{q}q|\pi(p)\rangle - \langle\pi(p)|\pi(p)\rangle\langle\bar{q}q\rangle$$



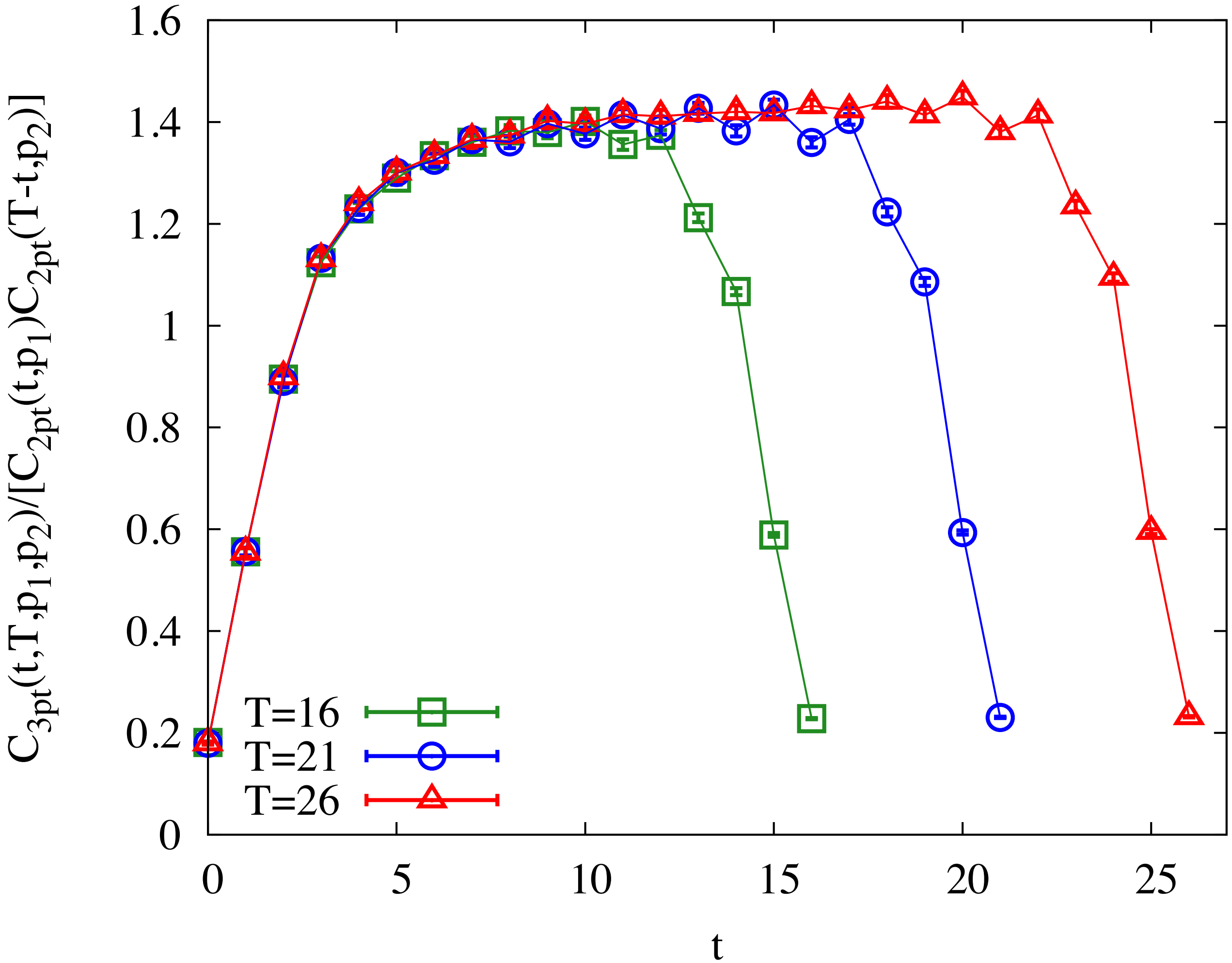
Disconnected diagrams

- $q^2=0$: disconnected piece is determined from the correlation of a π meson 2-point function (source at $t'=0$, sink at $t'=T$) with a scalar current (condensate) summed over timeslice at $t'=t$

- calculate the $\bar{q}q$ condensate by summing over a pseudoscalar meson 2-point function with valence quarks q: $\mathrm{Tr} M_{00}^{-1} = am_q \sum_n \mathrm{Tr} \left| M_{0n}^{-1} \right|^2$

- non-zero q^2: project onto non-zero lattice spatial momenta $2\pi(n_x,n_y,n_z)/L$

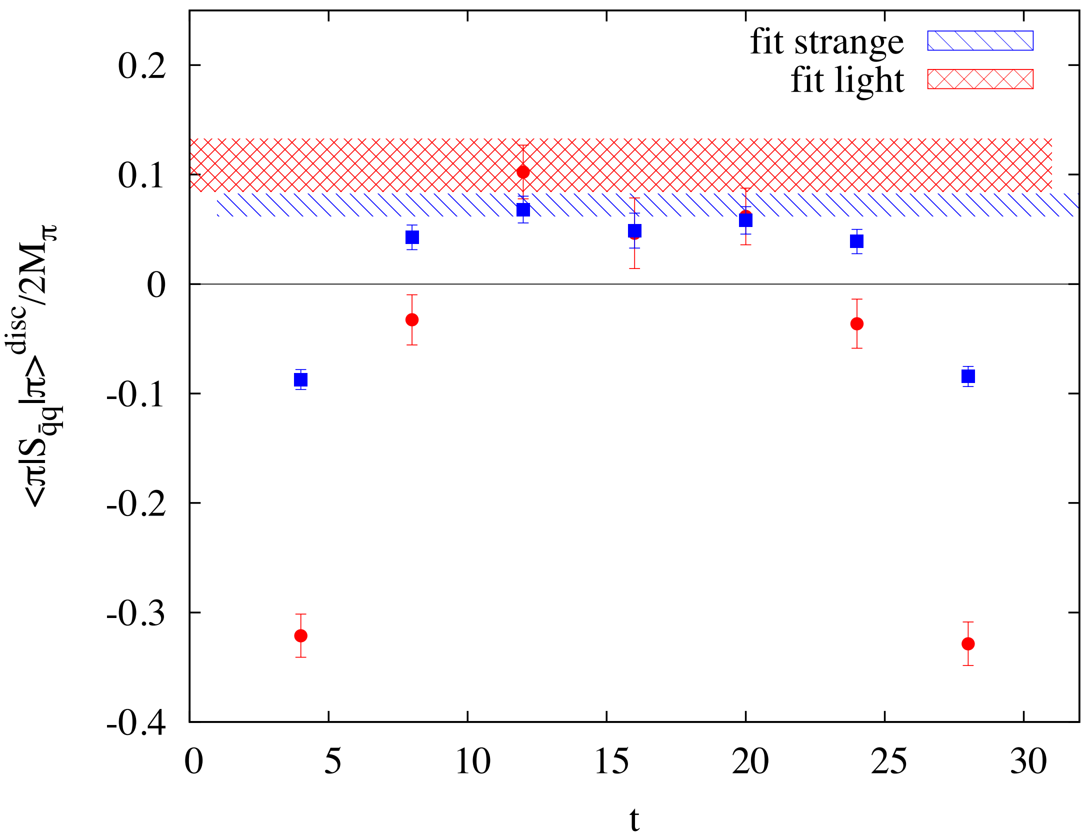
Fitting the correlators



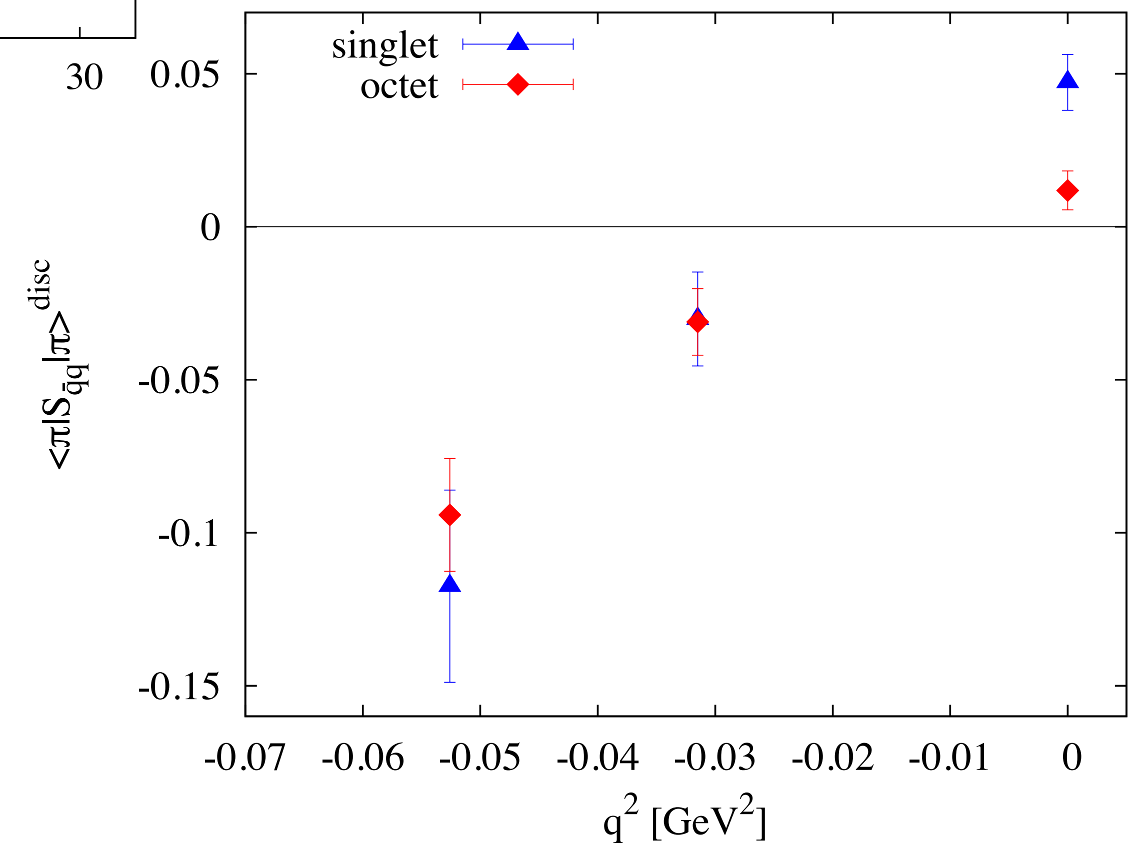
- Fit 2-point and 3-point correlators simultaneously
- Multi-exp fits to reduce systematical errors from the excited states

- Use Bayesian priors to constrain fit parameters
- Fit all q^2 values simultaneously to take into account the correlations

Disconnected pieces



- Include light and strange quarks in the loop



- two flavour combinations:

$$S_{\text{singlet}} = 2\bar{l}l + \bar{s}s$$
$$S_{\text{octet}} = 2\bar{l}l - 2\bar{s}s$$

Scalar and vector form factors

- Relate matrix elements and 3-point amplitudes:

$$\langle\pi(\vec{p}_1)|J|\pi(\vec{p}_2)\rangle = Z\sqrt{4E_0(\vec{p}_1)E_0(\vec{p}_2)}J_{0,0}(\vec{p}_1,\vec{p}_2)$$

- Calculate form factors from the matrix elements. Need renormalisation constant Z for the vector current: demand that $f_+(0)=1$

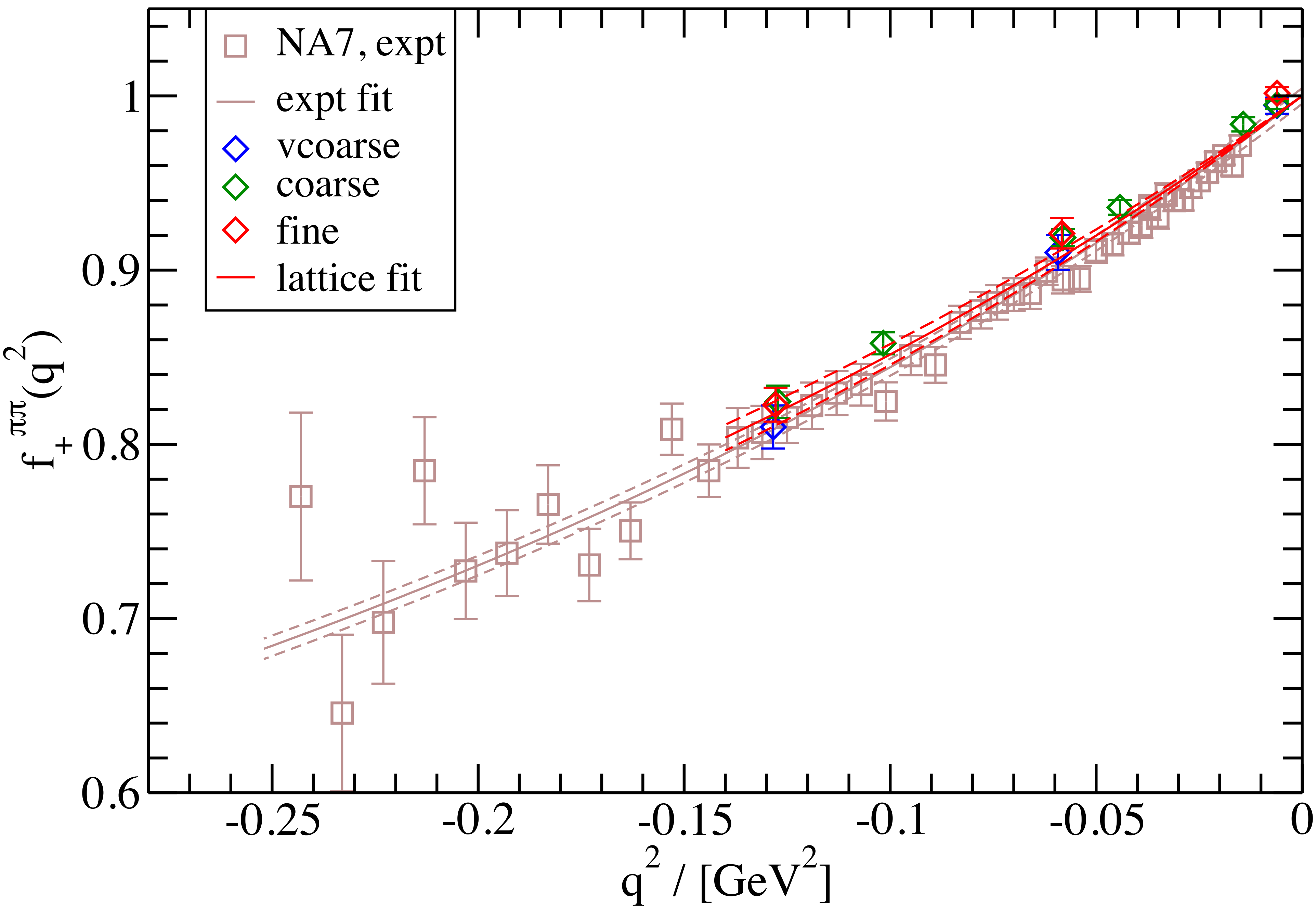
$$\langle\pi(\vec{p}_1)|V_i|\pi(\vec{p}_2)\rangle = f_+(q^2)(\vec{p}_1+\vec{p}_2)_i$$

- From Feynman-Hellmann theorem:

$$\langle\pi(\vec{p}_1)|S|\pi(\vec{p}_2)\rangle = Af_0(q^2), A_{\mathrm{conn}} = \frac{\partial M_\pi^2}{2\partial m_l}$$

- Scalar current is absolutely normalised, but we need to calculate the coefficient A - treat the scalar current as requiring a Z factor and set $f_0(0)=1$

Results: vector form factor



Continuum extrapolation

- Fit the form factors to the pole form

$$f(q^2) = \frac{1}{1 + c_{a^2}(\Lambda a)^2 + c_{a^4}(\Lambda a)^4 - \langle r^2\rangle_V q^2/6}$$

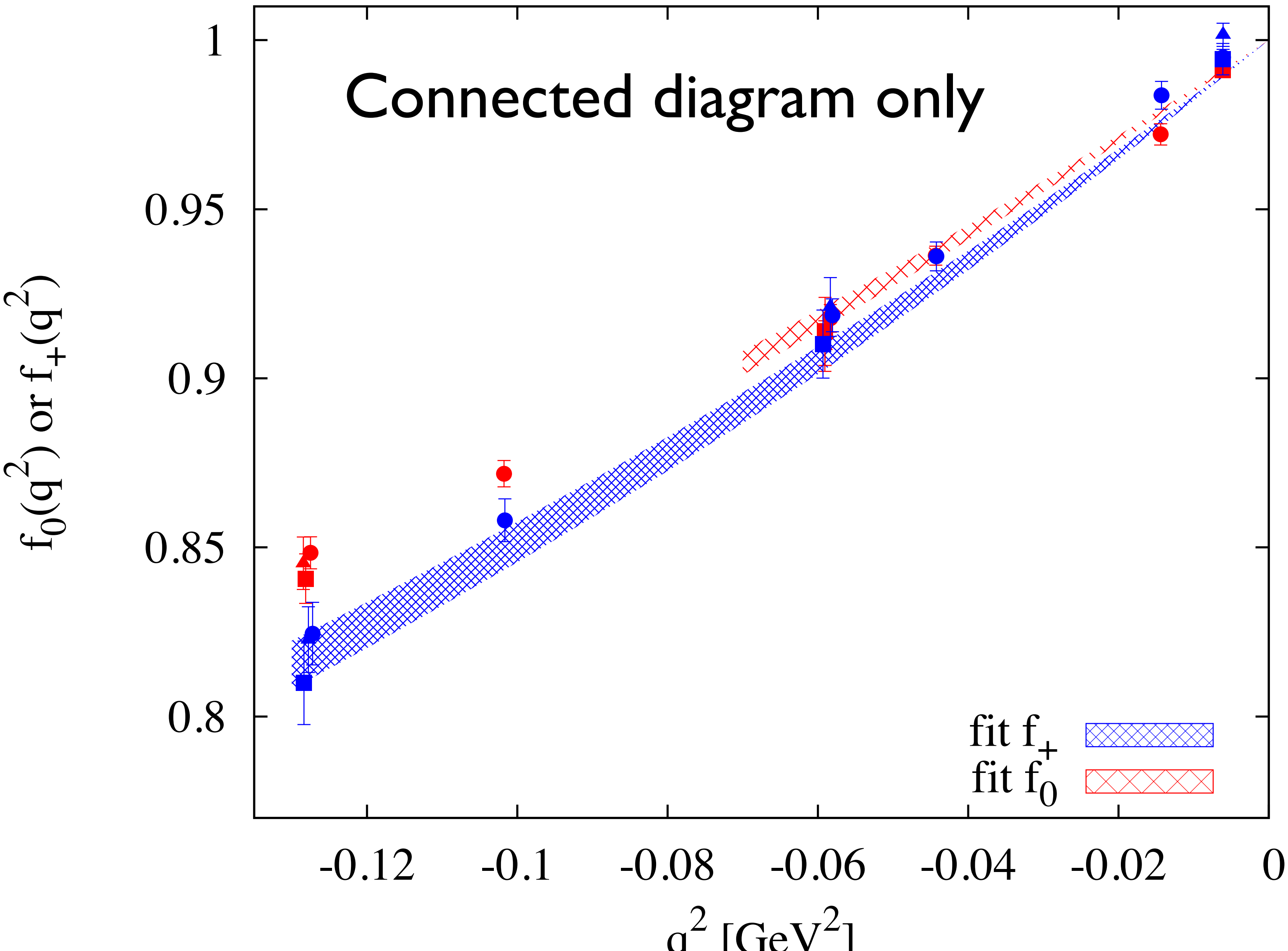
$$\langle r^2\rangle_V = \langle r^2\rangle_{V,0}\left[1 + b_{a^2}(\Lambda a)^2 + b_{a^4}(\Lambda a)^4 + \frac{b_{\text{sea}}\delta m_{\text{sea}}}{10 m_{s,\text{phys}}}\right] - \frac{1}{\Lambda_\chi^2}\ln\left[\frac{m_\pi^2}{m_{\pi,\text{phys}}^2}\right], \text{ where } \Lambda = 500 \text{ MeV } \& \ \Lambda_\chi = 1.16 \text{ GeV}$$

- The slope at $q^2=0$ gives the mean square of the charge radius:

$$\langle r^2\rangle_V = -6\frac{\mathrm{d}f_+(q^2)}{\mathrm{d}q^2}\bigg|_{q^2=0}$$

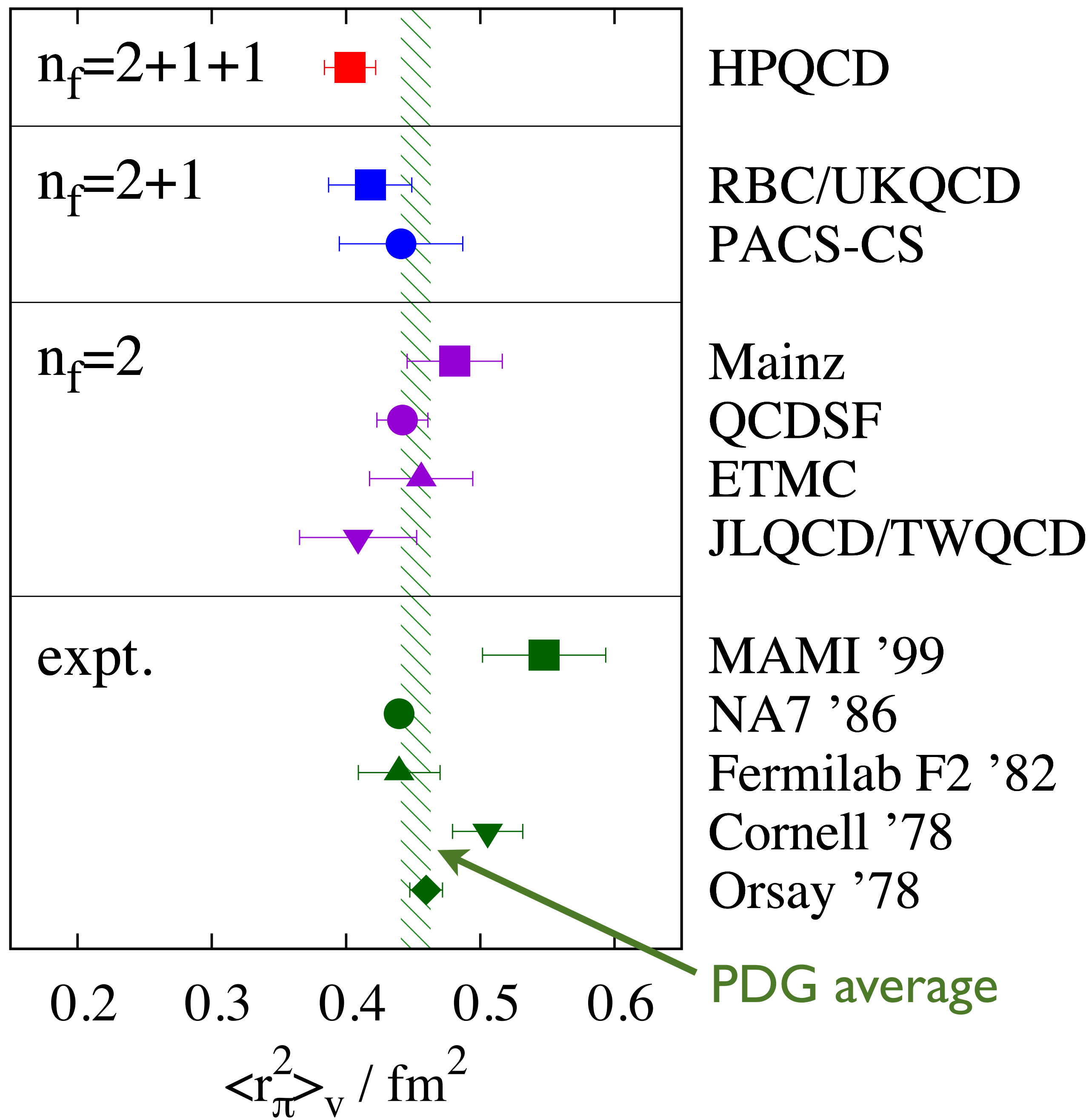
- In the case of scalar current the log is multiplied by 6

Results: scalar form factor

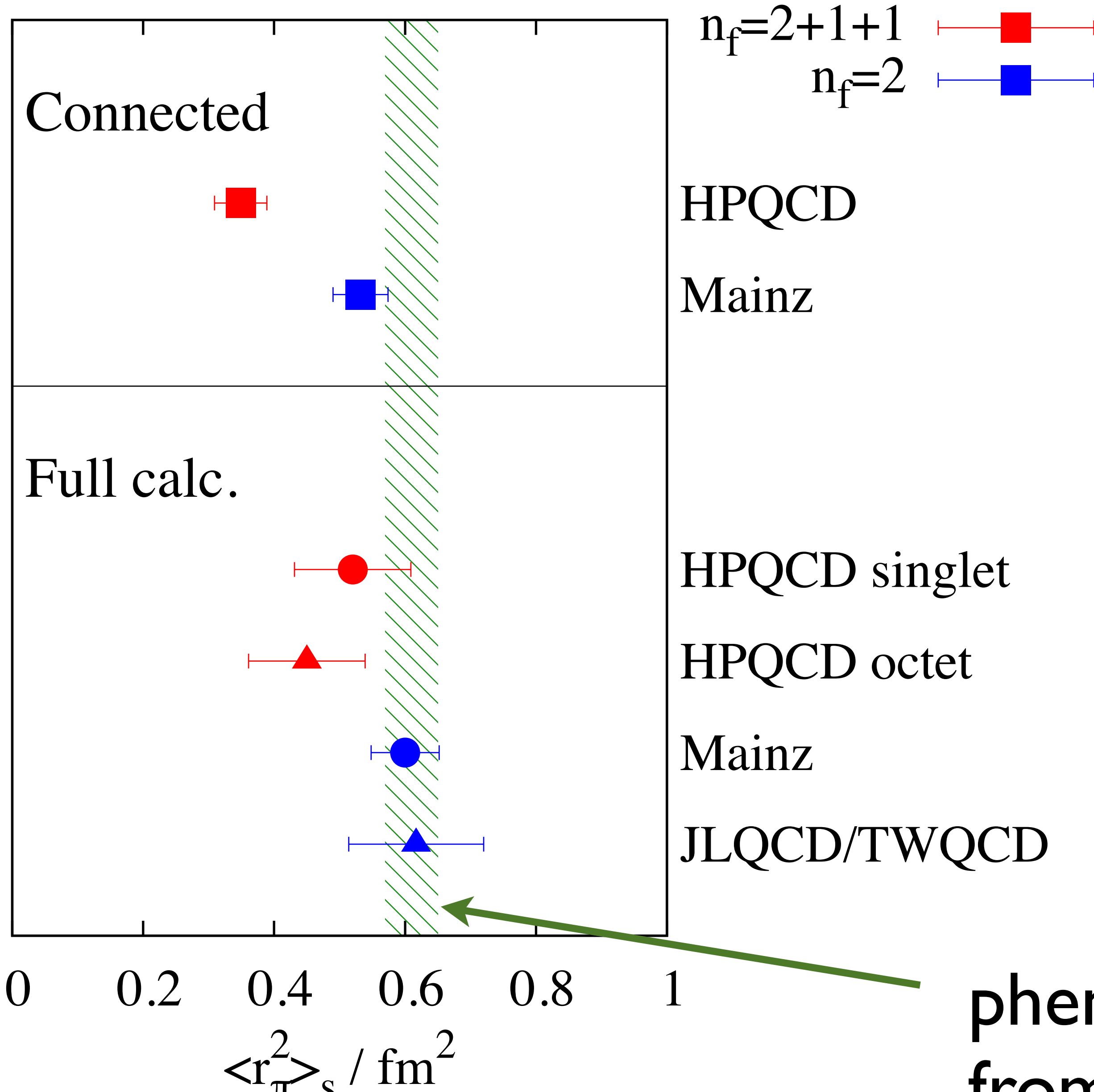


Vector mean square radius

- HPQCD:
  3 lattice spacings, physical pion mass
- other lattice calculations: smallest pion masses 240-400 MeV, some use only one lattice spacing



Scalar mean square radius



- HPQCD:

$$S_{\text{singlet}} = 2\bar{l}l + \bar{s}s$$
$$S_{\text{octet}} = 2\bar{l}l - 2\bar{s}s$$

- other lattice calculations: two flavours, including disconnected pieces

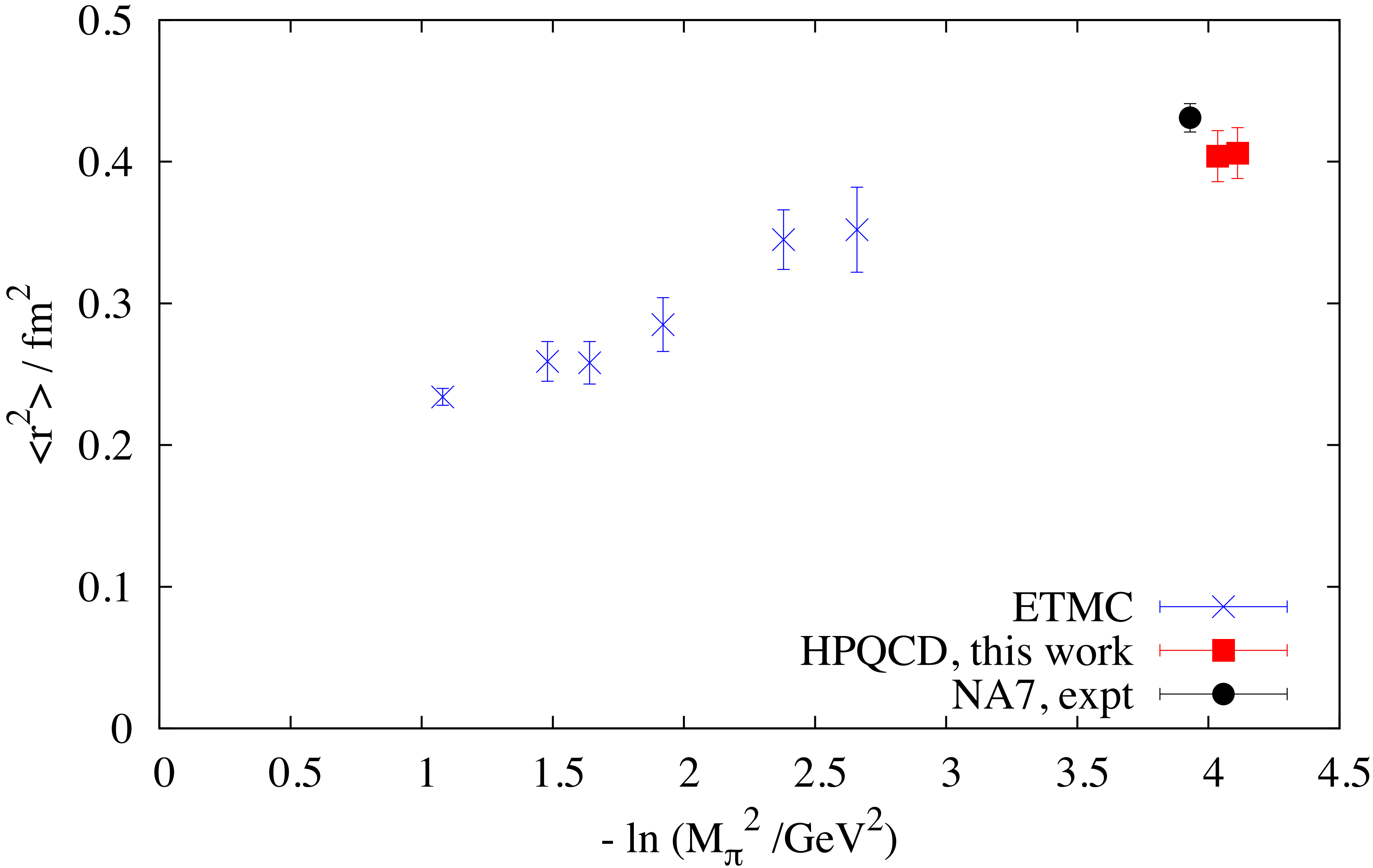
phenomenological result from $\pi\pi$ scattering + χPT

Errors: Finite volume and taste

- We do the calculation in a finite box, size $(4.8)^3$ - $5.8)^3$ fm^3
- Finite volume effects are small for masses and form factors, but can be large for $\langle r^2 \rangle$
- the use of staggered quarks means we have different taste pions (with different masses) in the chiral loops

	$\langle r^2 \rangle_V$	$\langle r^2 \rangle_S^{\mathrm{conn}}$	$\langle r^2 \rangle_S^{\mathrm{singlet}}$
statistics/fitting	4.5	5	15
isospin/electromagnetism	0.5	3	3
finite volume	1.5	10	8
total	4.8	12	17

Dependence on pion mass



Summary

- Full Lattice QCD calculation of the pion vector electromagnetic form factor
  - physical pion mass
  - can choose the q^2 range
  - determine the charge radius:
    our result is $\langle r_\pi^2 \rangle_V = 0.403(18)(6)\ \mathrm{fm}^2$
- Compare with experiment - get good agreement
- We also calculate the scalar form factor, including disconnected pieces, finding the scalar radii

$$\langle r_\pi^2 \rangle_{S,\mathrm{singlet}} = 0.52(8)(4)\ \mathrm{fm}^2$$

$$\langle r_\pi^2 \rangle_{S,\mathrm{octet}} = 0.45(8)(4)\ \mathrm{fm}^2$$

$$\langle r_\pi^2 \rangle_{S,\mathrm{conn}} = 0.35(2)(4)\ \mathrm{fm}^2$$

Thank you!

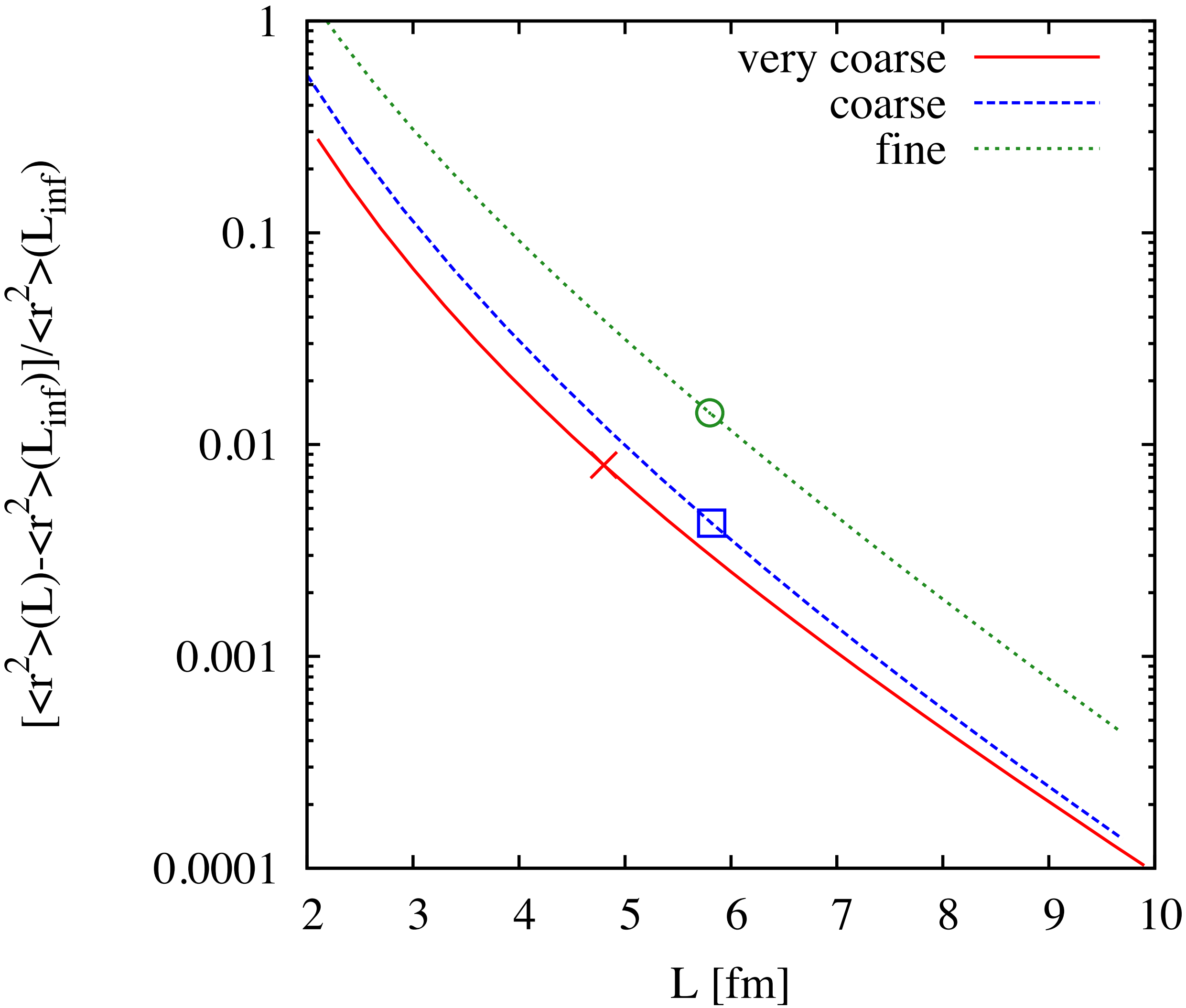
References

- **UKQCD/RBC**: P. Boyle et al., JHEP 07 (2008) 112
- **ETMC**: R. Frezzotti et al., PRD 79 (2009) 074506
- **Mainz**: B. Brandt et al., JHEP 11 (2013) 34; V. Gülpers et al., PRD 89 (2014) 094503 and arXiv:1507.01749
- **JLQCD/TWQCD**: S. Aoki et al., PRD 80 (2009) 034508
- **QCDSF/UKQCD**:
  D. Brömmel et al., Eur. Phys. J. C51 (2007) 335
- **PACS-CS**: O. Nguyen et al., JHEP 04 (2011) 122
- **NA7**: S. Amendolia et al., Nucl. Phys. B277 (1986) 168
- **ππ scattering**:
  G. Colangelo et al., Nucl. Phys. B603 (2001) 125
- **PDG**: K. Olive et al., Chin. Phys. C38 (2014) 090001

Spare slides

Errors: Finite volume and taste

- Estimate errors using chiral perturbation theory



$$\langle r^2\rangle = -\frac{6l_6}{f^2} + \frac{12}{N_s^3 N_t f^2}\sum_k \sin^2(ak_4)\left[\cos(ak_3)D^3(k) - 4\sin^2(ak_3)D^4(k)\right]$$

$$D(k) = \frac{1}{a^2x_\pi^2 + 2\sum_\mu(1-\cos(ak_\mu))}, \quad x_\pi = \sqrt{2Bm_q}$$

B. Borasoy and R. Lewis, PRD 71 (2005) 014033