Anatomy of SU(3) flux tubes at finite temperature

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Outline

The color confinement problem

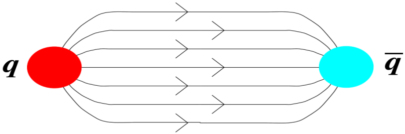

FIGURE : $q\bar{q}$ pair at distance R in the QCD vacuum

Deconfined phase

$$E_0(R) \stackrel{R\to\infty}{\longrightarrow} 2m$$

Confined phase

$$E(R) \longrightarrow \sigma R, \quad \sqrt{\sigma} = 420 \text{ MeV}$$

At the scale of color confinement non perturbative methods are needed

Dual superconductivity

Dual superconductor picture of confinement in QCD by Mandelstam and 't Hooft.
[G. 't Hooft, in High Energy Physics, EPS International Conference, (1975)]
[S. Mandelstam, Phys. Rep. 23, (1976)]

QCD vacuum as a dual superconductor

- Color confinement due to the dual Meissner effect by condensation of chromomagnetic monopoles
- Chromoelectric field connecting a $q\bar{q}$ static pair squeezed inside a tube structure: Abrikosov vortex

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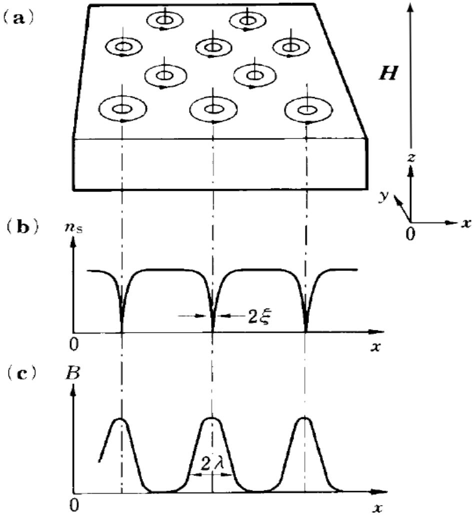
Relevance of nonperturbative study of chromoelectric flux tubes at $T \neq 0$ to clarify the formation of $c\bar{c}$ and $b\bar{b}$ bound states in heavy ion collisions.

Coherence length and London penetration depth



[A. C. Rose-Innes and E. H. Rhoderick, Introduction to Superconductivity (Pergamon Press, Second edition, 1978)]

- λ London penetration depth: characteristic length of the exponential decrease of $\vec{B}$ in a superconductor
- ξ Coherence length: length scale on which the density of Cooper pairs can change appreciably

Fitting functions for $E_l(x_t)$

Here enters the dual superconductor model

- Superconductivity: magnetic field as function of the distance from a vortex line in the mixed state
- Fit functions by dual analogy, from either London or Ginzburg-Landau theory

1. Vortex as a line singularity

$$E_l(x_t) = \frac{\phi}{2\pi}\mu^2 K_0(\mu x_t), \qquad x_t > 0, \qquad \lambda \gg \xi \;\leftrightarrow\; \kappa \gg 1$$

[P. Cea and L. Cosmai, Phys.Rev. D52 (1995)]

2. Cylindrical vortex

$$E_l(x_t) = \frac{\phi}{2\pi}\frac{1}{\lambda\xi_v}\frac{K_0(R/\lambda)}{K_1(\xi_v/\lambda)} ,$$

[J. R. Clem, J. Low Temp. Phys. 18, 427 (1975)]

[P. Cea, L. Cosmai, and A. Papa, Phys. Rev. D 86, (2012)]

Fitting function in our work

$$E_l(x_t) = \frac{\phi}{2\pi}\frac{\mu^2}{\alpha}\frac{K_0[(\mu^2 x_t^2 + \alpha^2)^{1/2}]}{K_1[\alpha]} \qquad x_t \geq 0,$$

$$R = \sqrt{x_t^2 + \xi_v^2}, \qquad \mu = \frac{1}{\lambda}, \qquad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}, \qquad \kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha}\left[1 - K_0^2(\alpha)/K_1^2(\alpha)\right]^{1/2}.$$

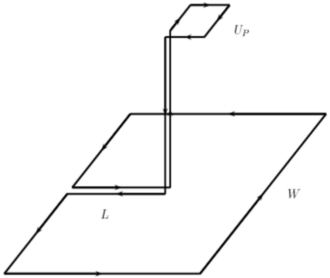
1. ϕ external flux
2. $\mu = 1/\lambda$ London penetration depth inverse
3. $1/\alpha = \lambda/\xi_v$ with ξ_v variational core-radius parameter
4. $\kappa = \lambda/\xi$ Ginzburg-Landau parameter

Outline

Connected correlator from previous studies

$$\rho_W^{\text{conn}} = \frac{\left\langle \text{tr}\left(W L U_P L^\dagger\right)\right\rangle}{\langle \text{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \text{tr}(U_P)\text{tr}(W)\rangle}{\langle \text{tr}(W)\rangle}$$

[Di Giacomo, Maggiore, Olejnik, Nucl.Phys. B347 (1990)]
[Cea, Cosmai, Phys.Rev. D52 (1995)]



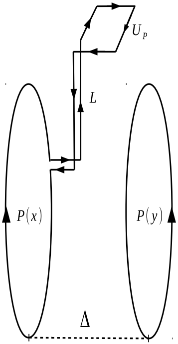
- W Wilson loop
- L Schwinger line
- U_p Plaquette

Connected correlator with Polyakov loops

$$\rho_P^{\text{conn}} = \frac{\left\langle \mathrm{tr}\left(P(x)\, L U_P L^\dagger\right) \mathrm{tr} P(y)\right\rangle}{\left\langle \mathrm{tr}\left(P(x)\right) \mathrm{tr}\left(P(y)\right)\right\rangle} - \frac{1}{3}\frac{\left\langle \mathrm{tr}\left(P(x)\right) \mathrm{tr}\left(P(y)\right) \mathrm{tr}\left(U_P\right)\right\rangle}{\left\langle \mathrm{tr}\left(P(x)\right) \mathrm{tr}\left(P(y)\right)\right\rangle}$$

- ρ_P^{conn} suited for the $T \neq 0$ case

[Di Giacomo, Maggiore, Olejnik, Nucl.Phys. B347 (1990)]



- $P(x)$, $P(y)$ Polyakov lines separated by a distance Δ
- L Schwinger line
- U_p Plaquette

Numerical evidence for $\rho_P^{\rm conn}$ measuring fields

$\rho_P^{\rm conn}$ changes sign under the transformation $U_P \to U_P^\dagger$

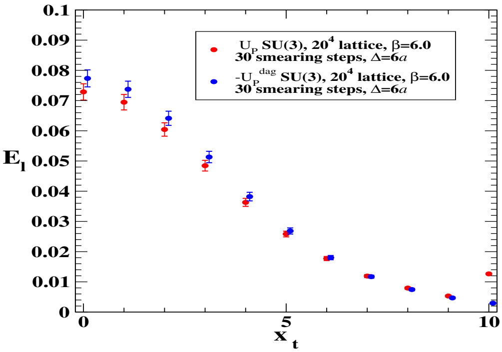


Figure : Longitudinal chromoelectric field E_l versus x_t, in lattice units for $\Delta = 6a$, at $\beta = 6.0$ and after 30 smearing steps

Our investigation in few steps

For different $N_s \times N_t$ and β

- Smearing over thermalized configuration
- Measurement of $E_l(x_t)$ through ρ_P^{conn}
- Fit of $E_l(x_t)$ to extract ϕ, μ, λ/ξ_v, κ
- Analysis of $E_l(x_t)$ versus smearing
- λ, ξ and κ in physical units from a scaling analysis

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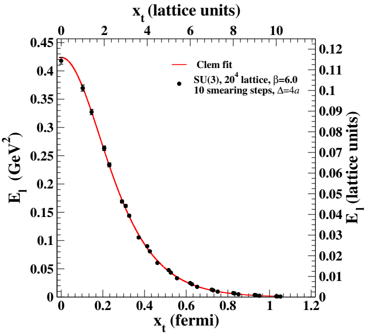

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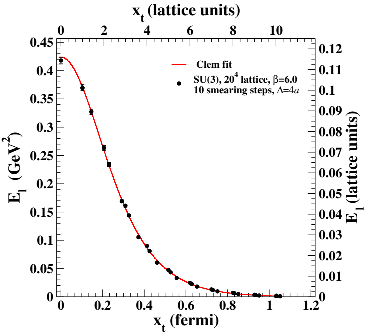

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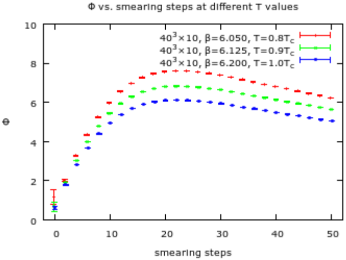

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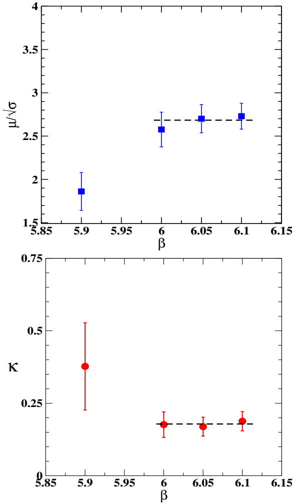


From lattice to physical units

Scaling of the plateau values of $a\mu$ with the string tension through the parametrization.

$$\begin{aligned}\sqrt{\sigma}(g) &= f_{\mathrm{SU(3)}}(g^2)[1 + 0.2731\,\hat{a}^2(g) \\ &- 0.01545\,\hat{a}^4(g) + 0.01975\,\hat{a}^6(g)]/0.01364\end{aligned}$$

$$\hat{a}(g) = \frac{f_{\mathrm{SU(3)}}(g^2)}{f_{\mathrm{SU(3)}}(g^2(\beta=6))}, \quad \beta = \frac{6}{g^2}, \quad 5.6 \leq \beta \leq 6.5$$

$$f_{\mathrm{SU(3)}}(g^2) = \left(b_0 g^2\right)^{\frac{-b_1}{2b_0^2}} \exp\left(\frac{-1}{2b_0 g^2}\right), \quad b_0 = \frac{11}{(4\pi)^2}, \quad b_1 = \frac{102}{(4\pi)^4}$$

[R. G. Edwards, U. M. Heller, and T. R. Klassen, Nucl. Phys. B 517, (1998)]

Summary of our previous results

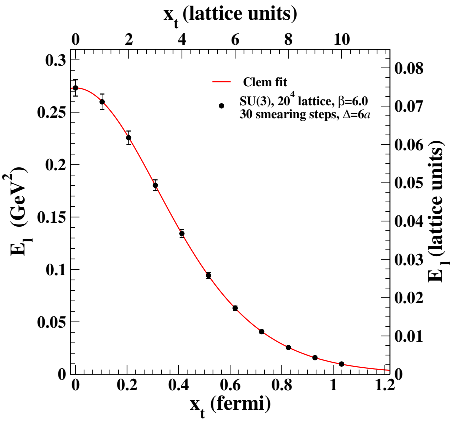


FIGURE : E_l vs x_t, lattice and physical units

- Connected correlator with Polyakov loops and smearing:
  - $\lambda = 1/\mu = 0.1750(63)$ fm
  - $\xi = 0.983(121)$ fm
  - $\kappa = 0.178(21)$
- λ, ξ, κ in agreement with $T = 0$ studies using Wilson-plaquette connected correlator and cooling
- SU(3) vacuum as a type-I dual superconductor

[P. Cea, L. Cosmai, F. C. and A. Papa, Phys. Rev. D 89, (2014)]

Current investigations $T \neq 0$

- Polyakov connected correlator with spatial APE and temporal HYP smearing
- Wilson connected correlator in the spatial sublattice with spatial APE smearing

Model & Algorithms

- Numerical computations within the MILC collaboration's public lattice gauge theory code (`http://physics.utah.edu/~detar/milc.html`)
- $SU(3)$ pure gauge LGT:

$$S = \beta \sum_{x,\mu>\nu} [1 - \frac{1}{3}\mathrm{Re}\mathrm{Tr} U_{\mu\nu}(x)]$$

- Updating:
  - Cabibbo-Marinari algorithm combined with overrelaxation
- Smoothing:
  - Single HYP smearing step on links in temporal direction
  - Many APE smearing steps on links in spatial directions

Outline

Polyakov connected correlator

Motivations

- Naive extension of the $T = 0$ study.
- Check of dual superconductor scenario by measuring changes in parameters of flux tubes across the deconfining transition.

Lattice and correlator features

- Polyakov connected correlator with loops at distance $\Delta = 6a$
- $N_t = 6, 8, 10, 12$ with aspect ratio fixed to 4
- $\beta = 6.050, 6.170$ corresponding to $0.67 T_c \leq T \leq 1.6 T_c$

Field in lattice and physical units at $\beta = 6.05$

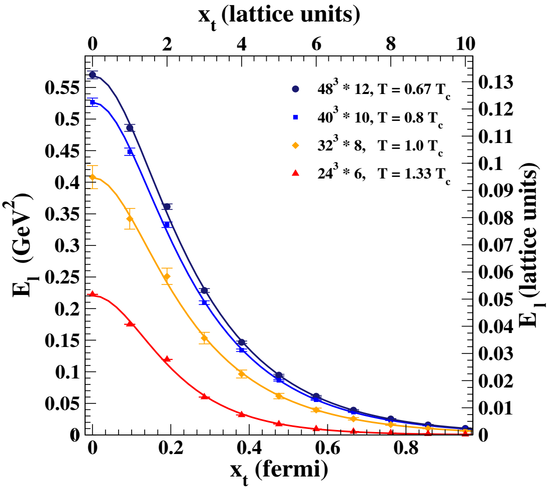


Figure : Longitudinal chromoelectric field E_l versus x_t, in lattice units and in physical units, at $\beta = 6.05$, for $\Delta = 6a$ and after 80 spatial smearing steps

Field in lattice and physical units at $\beta = 6.17$

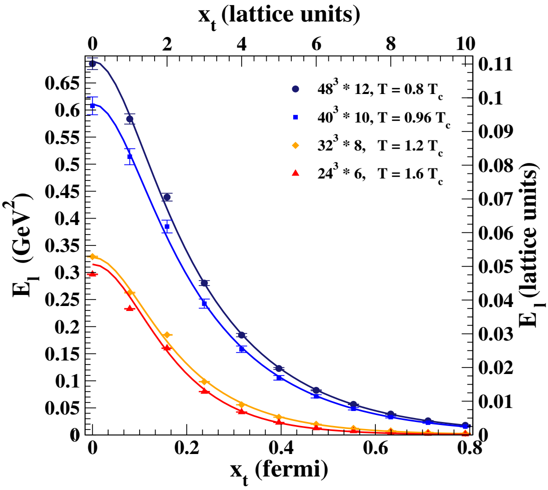
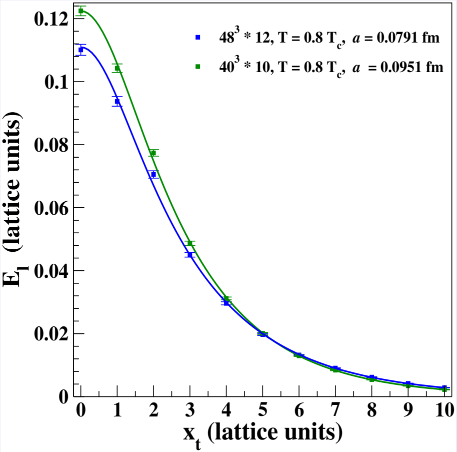


Figure : Longitudinal chromoelectric field E_l versus x_t, in lattice units and in physical units, at $\beta = 6.17$, for $\Delta = 6a$ and after 80 spatial smearing steps

Parameters scaling at $T = 0.8\,T_c$, $\Delta = 6a$



$\beta = 6.050,\ a = 0.0951$ fm

$$\mu/\sqrt{\sigma} = 1.935 \pm 0.023$$

$$\kappa = 1.448 \pm 0.058$$

$$\lambda = 1/\mu = 0.243 \pm 0.003 \text{ fm}$$

$$\xi = 0.168 \pm 0.007 \text{ fm}$$

$\beta = 6.170,\ a = 0.0791$ fm

$$\mu/\sqrt{\sigma} = 2.077 \pm 0.029$$

$$\kappa = 1.830 \pm 0.085$$

$$\lambda = 1/\mu = 0.226 \pm 0.003 \text{ fm}$$

$$\xi = 0.124 \pm 0.006 \text{ fm}$$

Plateau values vs T/T_c at fixed β

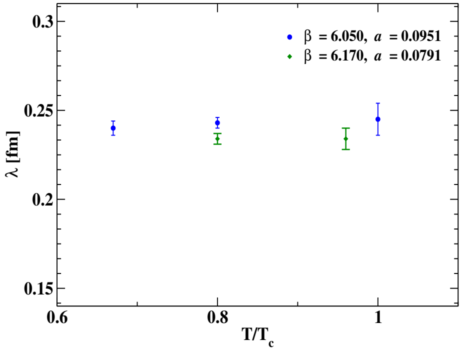


Figure : Plateau values for λ vs T/T_c. $T = 0$ value was $\lambda_{T=0} = 0.1750(63)$ fm

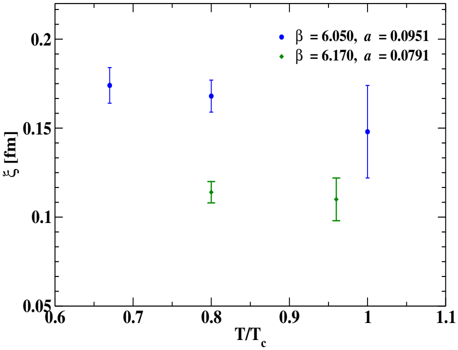


Figure : Plateau values for ξ vs T/T_c. $T = 0$ value was $\xi_{T=0} = 0.983(121)$ fm

Plateau values vs T/T_c at fixed β

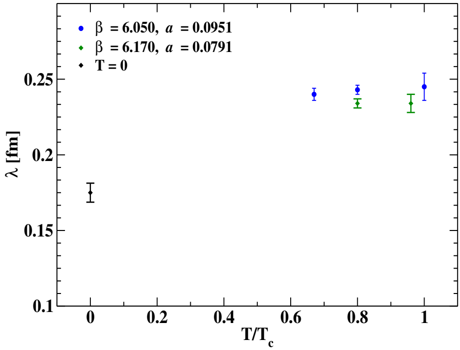


FIGURE : Plateau values for λ vs T/T_c. $T = 0$ value was $\lambda_{T=0} = 0.1750(63)$ fm

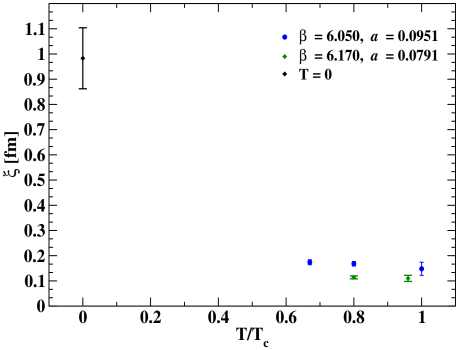


FIGURE : Plateau values for ξ vs T/T_c. $T = 0$ value was $\xi_{T=0} = 0.983(121)$ fm

Summary

Expectations

In ordinary superconductivity:

- T-dependent λ and ξ

$$\lambda, \xi \propto \frac{1}{\sqrt{T_c - T}}$$

- T-independent $\kappa = \frac{\lambda(T)}{\xi(T)}$

Results

- Φ decreases with T
- λ (ξ) bigger (much smaller) than its $T = 0$ value, but:
  - Weak T-dependence for $T \lesssim T_c$
  - No signal of divergence for $T \to T_c$
- κ tells SU(3) vacuum is a type-II dual superconductor: disagreement with $T = 0$ results
- Unreliable fits for $T > T_c$
- Transverse components of the chromoelectric field negligibly small $\forall$ T

Outline

Wilson connected correlator: chromomagnetic sector

Motivations

- Dual superconductivity not obviously at work in this sector
- No need for any kind of smoothing in the temporal direction

Lattice and correlator features

- Wilson connected correlator in the spatial sublattice (loop size $\Delta = 6a$)
- $N_t = 8, 10$ with aspect ratio fixed to 4

Consistency check: measurement of the spatial string tension $\sqrt{\sigma_s}$ [F. Karsch, E. Laermann, M. Lutgemeier, Phys. Lett. B346 (1995)]

- $N_s = 32$, $N_t = 2, 4$ and $\beta = 6.0$
- Spatial Wilson loops of sizes up to 12×12

Field in lattice and physical units

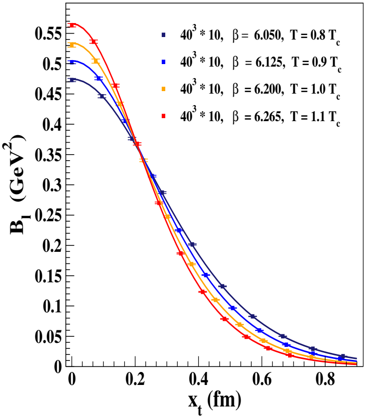


Figure : Longitudinal chromomagnetic field B_l versus x_t, in physical units, for $\Delta = 6a$ and after 12 smearing steps

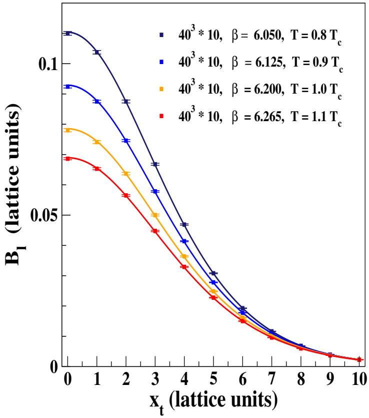


Figure : Longitudinal chromomagnetic field B_l versus x_t, in lattice units, for $\Delta = 6a$ and after 12 smearing steps

Plateau values vs T/T_c at fixed β

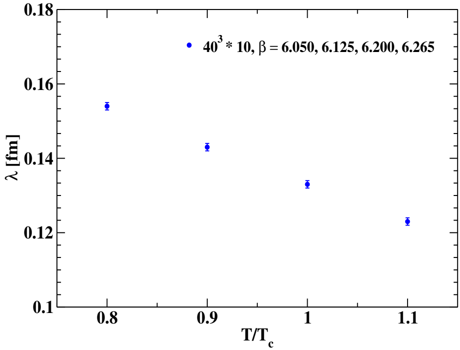


Figure : Plateau values for λ vs T/T_c.

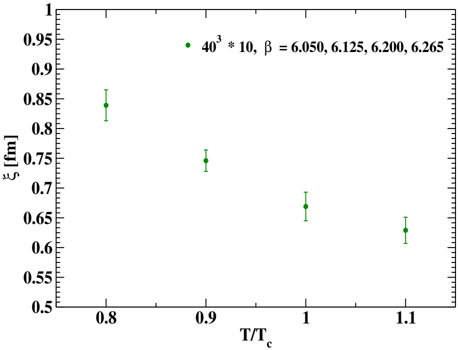


Figure : Plateau values for ξ vs T/T_c.

Summary

Expectations

- Area law for Wilson loops expected also for $T \geq T_c$
- Spatial string tension almost linearly growing with T above T_c

Results

- $B_l(x_t)$ well-fitted by Clem function
- chromomagnetic flux-tube structures surviving the deconfinement transition
- $\sqrt{\sigma_s}$, measured within the same code, in agreement with literature

Outline

Conclusions

Open questions

- The thickness of flux tubes does not diverge towards T_c, while our fit results become unreliable for $T \geq T_c$
- The observed dynamics of flux tubes with T does not match dual superconductivity expectations
- Clem function well fits measurements of fields in the chromomagnetic sector

Outlook

- Introduction of dynamical quarks d.o.f.

Remarks on our lattice observable

Field strength and the Gauss Law

$$\frac{\left\langle \mathrm{tr}\left\{\left[ga^2\nabla\cdot E\left(x_t\right)\right]^{x_t} P^{x_t}\left(x\right)\right\}\mathrm{tr}P^*\left(y\right)\right\rangle}{\left\langle \mathrm{tr}P\left(x\right)\mathrm{tr}P^*\left(y\right)\right\rangle} = i\frac{4}{3}g^2\delta_{x_t,x}$$

$$\left[\nabla\cdot E\left(x_t\right)\right]^{x_t} = \sum_{i=1}^{3}\left[E_i^{x_t}\left(x_t\right) - E_i^{x_t}\left(x_t - \hat{i}\right)\right]$$

$$\left\langle E_i^x\left(x_t\right)\right\rangle_{q\bar{q}} = \frac{\left\langle \mathrm{tr}\left[L\left(\mathcal{C}_x^{x_t}\right)E_i^{x_t}\left(x_t\right)L^\dagger\left(\mathcal{C}_x^{x_t}\right)P\left(x\right)\right]\mathrm{tr}P^*\left(y\right)\right\rangle}{\left\langle \mathrm{tr}P\left(x\right)\mathrm{tr}P^*\left(y\right)\right\rangle}$$

[P. Skala, M. Faber, and M. Zach, Nucl. Phys. B494 (1997)]