Towards the heavy dense QCD phase diagram using Complex Langevin simulations

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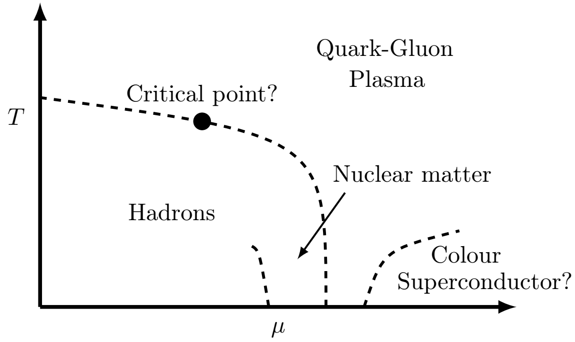
Goal: QCD phase diagram



- Still largely unknown
- Full thermodynamical study of QCD (phase transitions, etc)
- Applications in cosmology (e.g. neutron stars, early universe)
- Possible guide for heavy-ion collision experiments

CLE: Motivation

- *Sign problem*: Inclusion of a chemical potential to an Euclidean path integral makes the action complex
- Average values for observables then rely on precise cancellations of oscillating quantities
- In QCD this is manifested in the fermion determinant

$$[\det M(U,\mu)]^* = \det M(U,-\mu^*)$$

  which is, for real chemical potential μ, complex
- Results from Hybrid Monte-Carlo simulations become unreliable if $\mu/T \gg 1$, where the sign problem is severe

CLE: Stochastic quantization

- Add fictitious time dimension θ to gauge links
- Evolve them according to a Langevin equation

$$U_{x\mu}(\theta+\varepsilon) = R_{x\mu} U_{x\mu}(\theta)\,, \quad R_{x\mu} = \exp\left[i\lambda^a(\varepsilon D^a_{x\mu} S + \sqrt{\varepsilon}\eta^a_{x\mu})\right]\,,$$

where λ^a are the Gell-Mann matrices, ε is the stepsize, $\eta^a_{x\mu}$ are white noise fields satisfying

$$\langle \eta^a_{x\mu} \rangle = 0\,, \quad \langle \eta^a_{x\mu} \eta^b_{y\nu} \rangle = 2\delta^{ab}\delta_{xy}\delta_{\mu\nu}\,,$$

S is the QCD action and $D^a_{x\mu}$ is defined as

$$D^a_{x\mu} f(U) = \frac{\partial}{\partial\alpha} f(e^{i\alpha\lambda^a} U_{x\mu})\Big|_{\alpha=0}$$

- Quantum expectation values are computed as averages over the Langevin time θ after the system reaches equilibrium at $\theta = T_{therm}$

$$\langle O \rangle = \lim_{T\to\infty} \frac{1}{T - T_{therm}} \int_{T_{therm}}^{T} O(\theta)\, d\theta$$

CLE: Complexification

- Allow gauge fields to be complex, i.e., $\mathbb{R} \ni A_\mu^a(x) \to A_\mu^a(x) \in \mathbb{C}$
- On the lattice this means $\mathrm{SU}(3) \ni U_{x\mu} \to U_{x\mu} \in \mathrm{SL}(3, \mathbb{C})$
- Use $U_{x\mu}^{-1}$ instead of $U_{x\mu}^\dagger$ as
  - it keeps the action holomorphic;
  - they coincide on $\mathrm{SU}(3)$ but on $\mathrm{SL}(3, \mathbb{C})$ it is U^{-1} that represents the backwards-pointing link.
- That means the plaquette is now

$$U_{x,\mu\nu} = U_{x\mu} \, U_{x+\mu,\nu} \, U_{x+\nu,\mu}^{-1} \, U_{x,\nu}^{-1} \, ,$$

and the Wilson action reads

$$S[U] = -\frac{\beta}{3} \sum_x \sum_{\mu<\nu} \mathbf{Tr} \left[\frac{1}{2} \left(U_{x,\mu\nu} + U_{x,\mu\nu}^{-1} \right) - \mathbb{1} \right]$$

CLE: Complexification - Gauge cooling

- During simulations monitor the distance from the unitary manifold with

$$d = \mathbf{Tr} \sum_{x,\mu} \left[U_{x\mu} U_{x\mu}^\dagger - \mathbb{1} \right] \geq 0$$

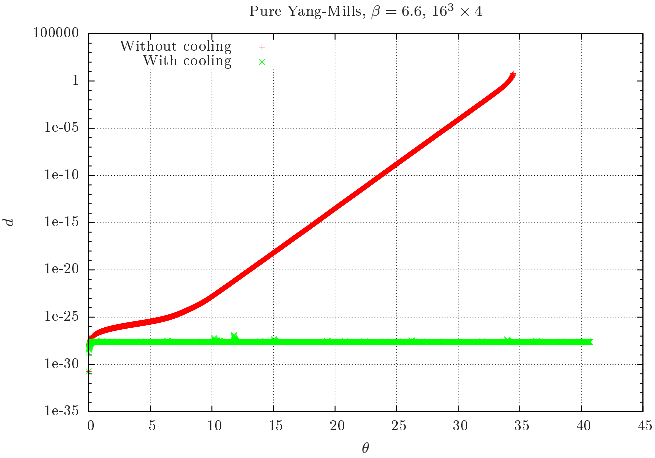
- *Gauge cooling*: Use gauge transformations to keep the system as close as possible to SU(3), i.e., minimise the imaginary part of $A_\mu^a(x)$

$$U_{x\mu} \to \Lambda_x U_{x\mu} \Lambda_{x+\mu}^{-1}, \quad \Lambda_x = \exp\left[-\varepsilon\alpha\lambda^a f_x^a\right]$$

where

$$f_x^a = 2\mathbf{Tr} \left[\lambda^a \left(U_{x\mu} U_{x\mu}^\dagger - U_{x-\mu,\mu}^\dagger U_{x-\mu,\mu} \right) \right]$$

Gauge cooling example



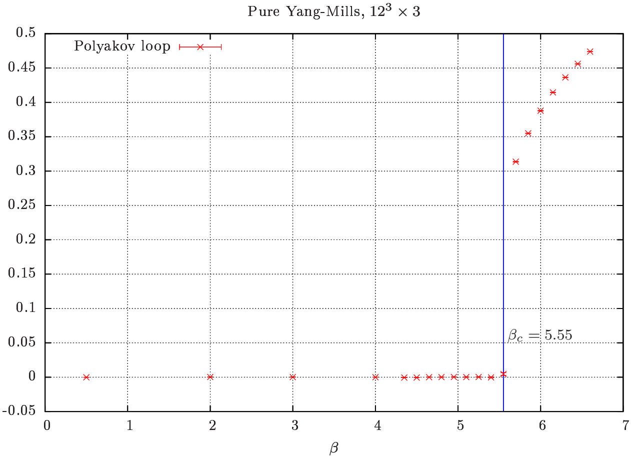
In $SL(3, \mathbb{C})$ the system has more freedom to "wander" around. Gauge cooling helps preventing convergence to wrong results.

Deconfinement transition

Goals:

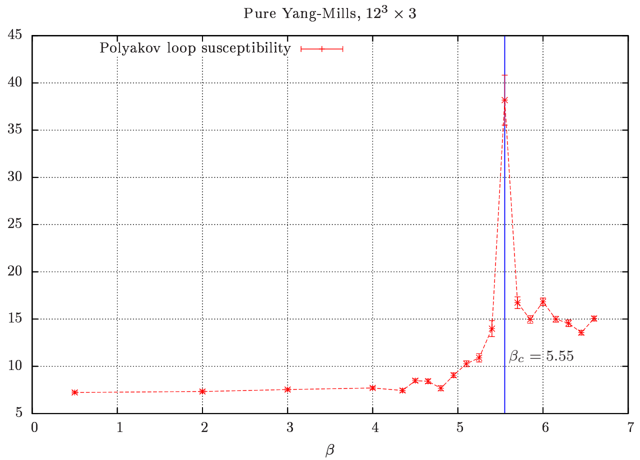
- Show that CLE works for real theories as well
- Test if gauge cooling can replace re-unitarisation in those cases
- Reproduce known values for the deconfinement transition

Deconfinement transition



CLE reproduces the transition in agreement with the literature (hep-lat/9405018).

Deconfinement transition



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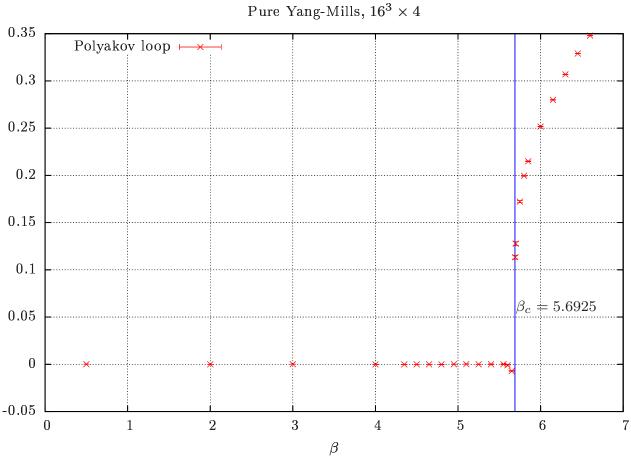
Deconfinement transition



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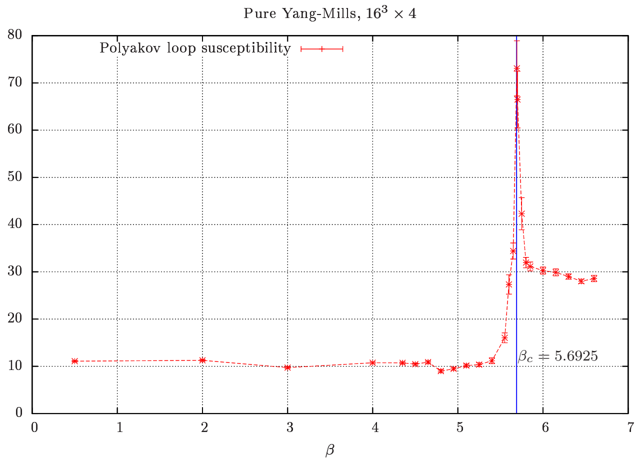
Deconfinement transition



CLE reproduces the transition in agreement with the literature (hep-lat/9405018).

Heavy-dense QCD

Heavy-dense approximation

- Heavy quarks → spatial part of fermion determinant does not contribute, but temporal part is exact ($\kappa \to 0, \mu \to \infty\,, \kappa e^{\mu} = const$):

$$\det M(U,\mu) = \prod_{\vec{x}} \left\{ \det \left[1 + \left(2\kappa e^{\mu}\right)^{N_\tau} \mathcal{P}_{\vec{x}} \right]^2 \times \det \left[1 + \left(2\kappa e^{-\mu}\right)^{N_\tau} \mathcal{P}_{\vec{x}}^{-1} \right]^2 \right\}$$

- Polyakov loop

$$\mathcal{P}_{\vec{x}} = \prod_{\tau} U_4(\vec{x},\tau)$$

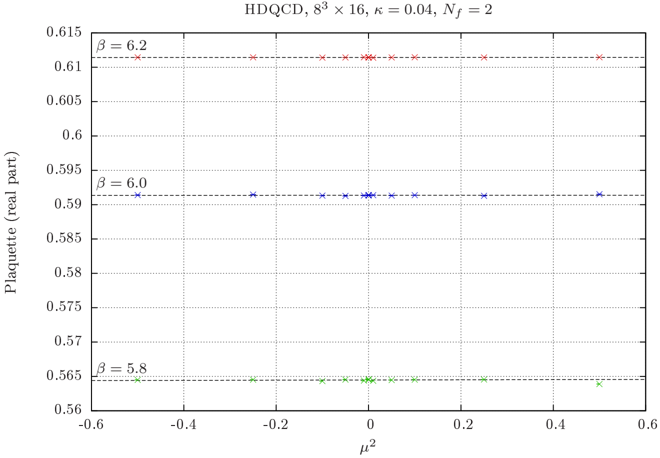
- Exhibits the sign problem: $[\det M(U,\mu)]^* = \det M(U,-\mu^*)$

Heavy-dense QCD

Goals

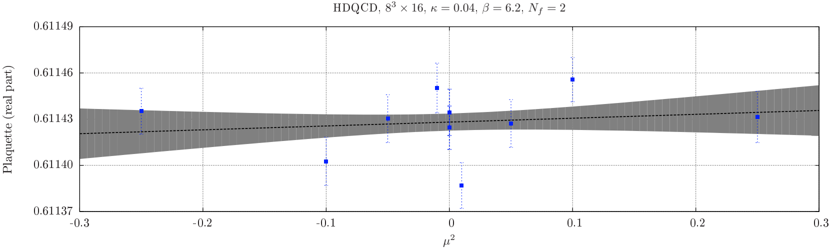
- Assert the plaquette's continuity from imaginary to real chemical potential
- Check for limitations of CLE in a complex theory

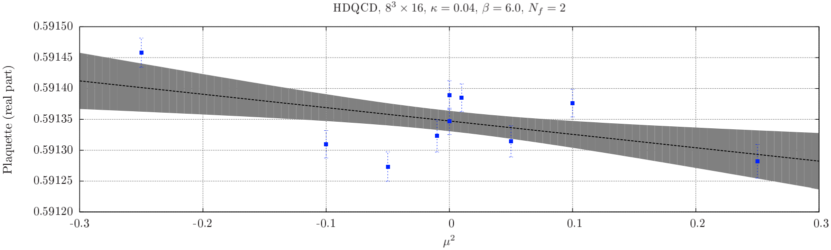
Plaquette as a function of μ^2 in HDQCD



Continuity between real theory at $\mu^2 < 0$ and complex one at $\mu^2 > 0$.
Dashed lines are linear fits.

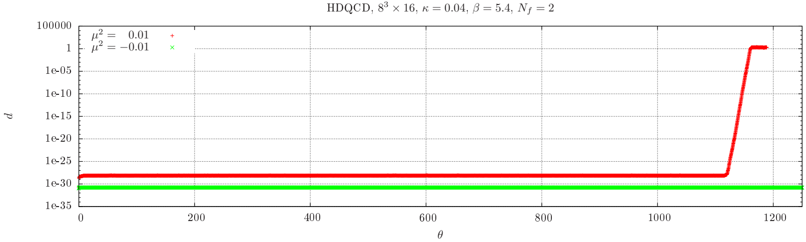
Plaquette in HDQCD

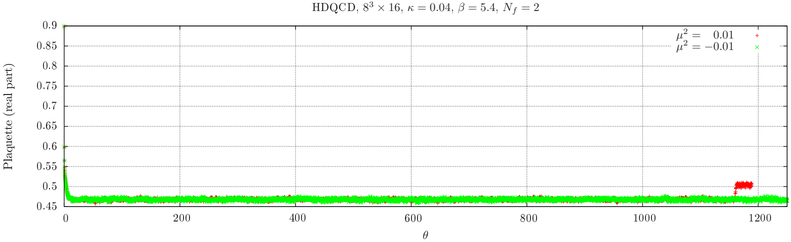




Statistical fluctuations are compatible with linear fit for $\beta = 6.2$ and 6.0.

HDQCD at $\beta = 5.4$





When the distance from SU(3) gets too large the simulation breaks down. The imaginary part is no longer a "small deformation" to the QCD action.

Summary

Complex Langevin equation

- Reproduces known results for theories without the sign problem
- Allows simulations of theories that exhibit the sign problem
- Gives consistent results when transitioning from real to complex theories
- Opens the door to probing the entire phase diagram of QCD
- Stay tuned for Ben Jäger's talk on the HDQCD phase diagram

Gauge cooling

Gauge cooling analogy

- In a real theory gauge fixing minimises

$$W \sim \sum_{x,\mu} \mathbf{Tr}\left[U_{x\mu} + U^{\dagger}_{x\mu}\right] \sim a^2 \sum_{x,\mu} \mathbf{Tr}\left[A^2_{\mu}(x)\right] + \mathcal{O}(a^3)$$

with a "force"

$$f^a_x = \mathbf{Tr}\left[\lambda^a \left(U_{x\mu} - U_{x-\mu,\mu}\right) - \text{h.c.}\right] = a^2 \partial_{\mu} A^a_{\mu}(x) + \mathcal{O}(a^3)$$

- Analogously, gauge cooling minimises

$$d \sim \sum_{x,\mu} \mathbf{Tr}\left[U_{x\mu} U^{\dagger}_{x\mu}\right] \sim a^2 \sum_{x,\mu} \mathbf{Tr}\left[B^2_{\mu}(x)\right] + \mathcal{O}(a^3)\,, \quad B_{\mu}(x) = \mathbf{Im}\left[A_{\mu}(x)\right]$$

with a "force"

$$f^a_x = 2\mathbf{Tr}\left[\lambda^a \left(U_{x\mu} U^{\dagger}_{x\mu} - U^{\dagger}_{x-\mu,\mu} U_{x-\mu,\mu}\right)\right] = a^2 \partial_{\mu} B^a_{\mu}(x) + \mathcal{O}(a^3)$$