D meson semileptonic form factors at zero momentum transfer in 2+1+1 flavor lattice QCD

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Motivation

- Standard Model predicts unitary CKM matrix.
- Determination of $|V_{cd}|$ and $|V_{cs}|$ requires both lattice and experiment.
- Determination from leptonic $D_{(s)}$ decays limited by experimental error:

$$|V_{cd}| = 0.217(1)_{LQCD}(5)_{expt}(1)_{EM}$$

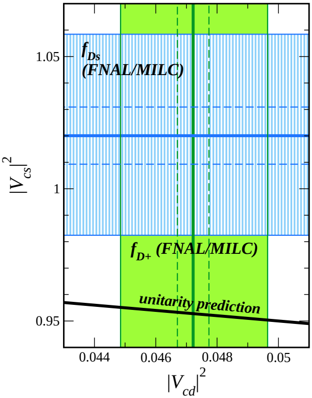
$$|V_{cs}| = 1.010(5)_{LQCD}(18)_{expt}(6)_{EM}$$

- Still room for lattice QCD to improve determination from semileptonic $D \to K(\pi)\ell\nu$ decays[1]:.

$$|V_{cd}| = 0.214(9)_{LQCD}(3)_{expt}$$

$$|V_{cs}| = 0.977(14)_{LQCD}(7)_{expt}$$

- Our goal is to at least match the experimental error in $f_+(0) \times |V|$.



Fermilab/MILC Phys.Rev.D90, 074509, (2014) [arXiv:1407.3772v2 [hep-lat]], $N_f = 2 + 1 + 1$.

[1] HPQCD, $N_f = 2 + 1$, [arXiv:1109.1501 [hep-lat]], [arXiv:1305.1462v1 [hep-lat]], HFAG [arXiv:1412.7515v1 [hep-ex]]

Calculation Method

- Vector current $V^\mu = \bar{q}\gamma^\mu c$,

$$\langle K(\pi)|V^\mu|D\rangle = f_+^{D\to K(\pi)}(q^2)\left[p_D^\mu + p_{K(\pi)}^\mu - \frac{M_D^2 - M_{K(\pi)}^2}{q^2}q^\mu\right] + f_0^{D\to K(\pi)}(q^2)\frac{M_D^2 - M_{K(\pi)}^2}{q^2}q^\mu. \quad (1)$$

- Scalar current $S = \bar{q}c$,

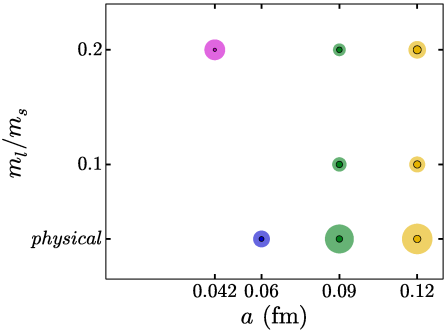
$$\langle K(\pi)|S|D\rangle = \frac{M_D^2 - M_{K(\pi)}^2}{m_c - m_{s(d)}} f_0^{D\to K(\pi)}(q^2), \quad (2)$$

- With staggered quarks the local scalar current yields an absolutely normalized f_0.
- Kinematic constraint requires that $f_+(0) = f_0(0)$.
- We follow this scalar current approach, which was introduced by HPQCD[1].

[1] HPQCD Phys.Rev.D82, 114506, (2010) [arXiv:1008.4562v2 [hep-lat]]

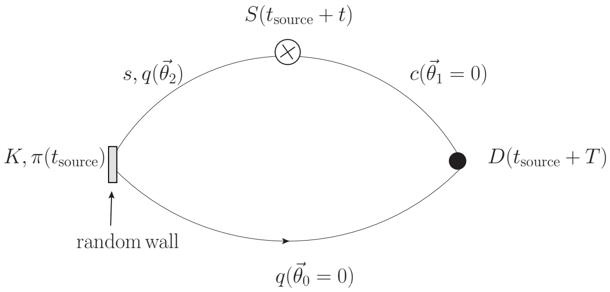
Simulation Details

- MILC $2+1+1$ flavor HISQ ensembles.
- Light, strange and charm valence quarks also use the HISQ action.
- Inner symbols radius indicates N_{conf}.
- Outer symbol radius indicates $N_{conf} \times N_{tsrc}$ which is at least 3000 for each ensemble.
- $M_\pi L > 3.5$ for all ensembles.



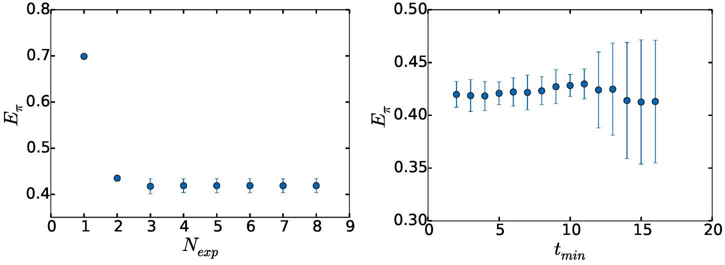
Correlators

- Perform calculation directly at $q^2 = 0$ using twisted boundary conditions.
- A twist of θ in each spatial direction gives $\vec{p} = \theta \frac{\pi}{L}(1, 1, 1)$.
- Required twists for $D \to K$ and $D \to \pi$ are in the range of $\theta = 2$ to 5, resulting in large momenta.
- 5 different external source times (T) for each three-point correlator.
- Also calculate two-point kaons and pions with and without twisted boundaries and D mesons with no twist.



Correlator Fitting procedure

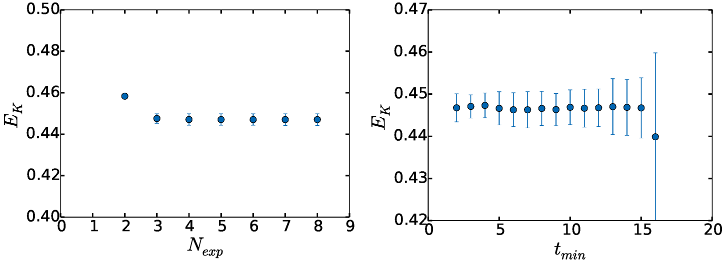
- Two-point fits with $N_{exp} + N_{exp}$ states, odd and even parity, increasing N_{exp} until fit is stable.
- t_{min} chosen from earliest option giving good p-value and consistent fit result.
- t_{max} for noisy (non-zero momentum) correlators chosen as last time slice with error $< 30\%$.
- Bayesian priors with broad widths used to help fit stability only.



Pion energy values from the 0.09 fm physical quark mass ensemble as a function of the number of states of each parity (left) and as a function of t_{min} (right). p-value is ≈ 1 for every fit shown except $N_{exp} = 1$.

Correlator Fitting procedure

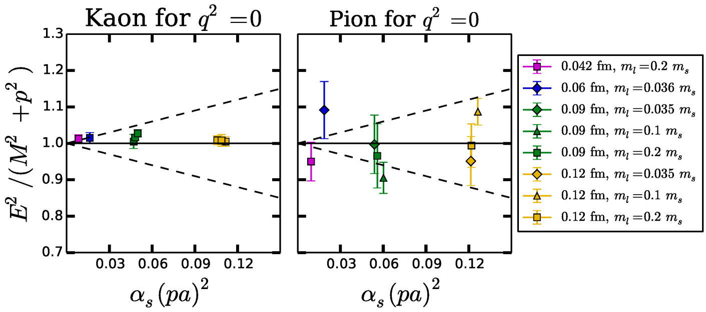
- Non-zero momentum Kaon correlators have significantly smaller statistical errors and better stability than their pion counterparts.
- This is despite requiring only a marginally smaller momentum to achieve $q^2 = 0$.
- Statistical errors in $q^2 = 0$ form factor results are dominated by non-zero momentum Kaons/pions and three-point correlators that include them.
- Zero momentum D meson, Kaon and pion fits are much more precise and stable and make negligible contributions to the f_0 uncertainty.



Kaon energy values from the 0.09 fm physical quark mass ensemble as a function of the number of states of each parity (left) and as a function of t_{min} (right). p-value is ≈ 1 for every fit shown except $N_{exp} = 1$.

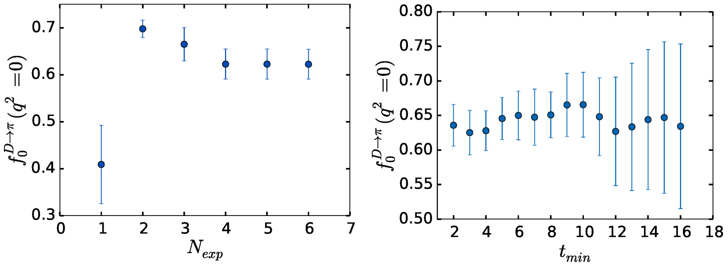
Dispersion relation

- $q^2 = M^2_{K(\pi)} + M^2_D - 2M_D E_{K(\pi)}$
- Kaon and pion momenta needed to get $q^2 = 0$ determined via the dispersion relation.
- Large D-K and D-π mass differences mean a large twist/momenta which leads to large statistical errors.
- We find dispersion relation violations to be within expectations and therefore chose to use $\sqrt{M^2 + p^2}$ in place of the fit energy in our chiral fits.



Three-point fits

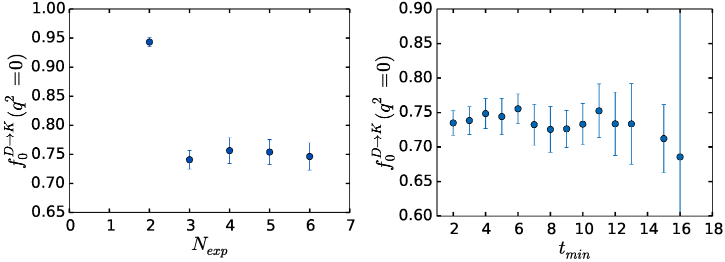
- Tried both simultaneous and sequential fits, results were consistent but the latter gave slightly better stability.
- $t^{3pt}_{min} = t^{K(\pi)}_{min}$ and $t^{3pt}_{max} = T_{ext} - t^{D}_{min}$.
- 3 out of 5 T_{ext} included in each fit, more than this shows no improvement in errors or stability and can make the fit more difficult.



$D \to \pi$ form factor fit values from the 0.09 fm physical quark mass ensemble as a function of the number of states of each parity (left) and as a function of t_{min} (right). p-value is ≈ 1 for every fit shown except $N_{exp} < 4$.

Three-point fits

- $D \to K$ three-point fits have smaller statistical errors than the $D \to \pi$ ones, but not to the same extent as the difference between Kaon and pion two-point correlator fits.



$D \to K$ form factor fit values from the 0.09 fm physical quark mass ensemble as a function of the number of states of each parity (left) and as a function of t_{min} (right). p-value is ≈ 1 for every fit shown except $N_{exp} < 4$.

Chiral perturbation theory

- We apply continuum NLO Heavy Meson χPT expressions calculated by Bećirević, *et al.*[1], in the hard pion/kaon[2] limit.

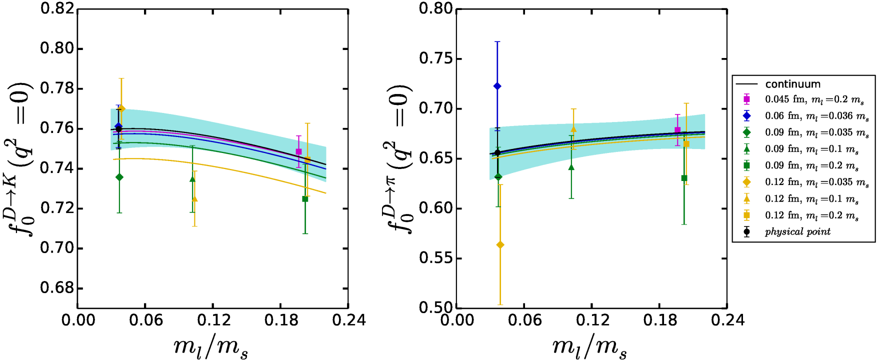
$$f_0(q^2) \quad = \quad c_0(1 + df_{logs}) + c_1\chi_\ell + c_2\chi_{a^2} \tag{3}$$

- Terms highlighted in red are the fit parameters.
- g_π is also a fit parameter with a prior of 0.52 ± 0.07.
- Use dimensionless parameters χ_i such that fit parameters are expected to be of order one and use priors $c_i = 0 \pm 2$.
- Do not include strange quark mass, Kaon/pion energy or sea quark mass terms in our central fit; first two are approximately constant across our ensembles and sea masses are very close or equal to the valence masses.
- Consider these and other chiral-continuum fit variations in systematic error analysis.
- The fits are stable under inclusion of such terms.

[1] Phys.Rev.D68, 074003,(2003) [arXiv:hep-lat/0305001v3]

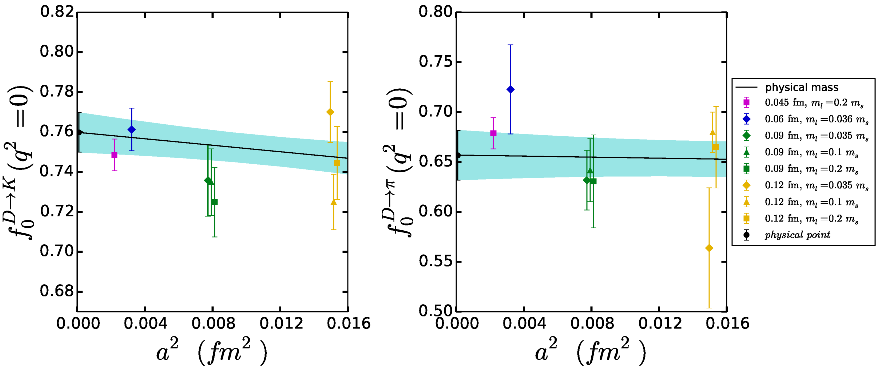
[2] Nucl.Phys.B840, 54-66, (2010) [arXiv:1006.1197v2 [hep-ph]]

Chiral and continuum extrapolation



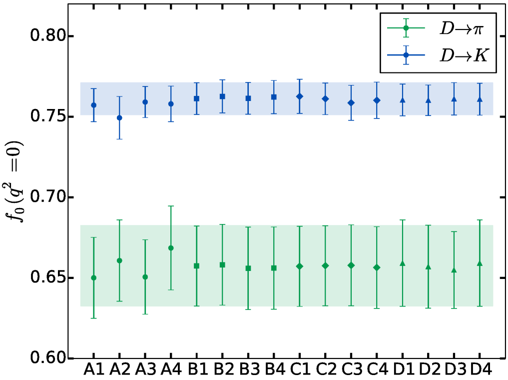
Chiral/continuum fits of $f_0^{D\to K}(0)$ (left) and $f_0^{D\to\pi}(0)$ (right) as a function of m_l/m_s.

Chiral and continuum extrapolation



Chiral/continuum fits of $f_0^{D\to K}(0)$ (left) and $f_0^{D\to\pi}(0)$ (right) as a function of a^2.

Chiral and continuum extrapolation



- *A1-4*. Different fit window approaches in correlator fits.
- *B1-4*. Include analytic NLO terms in m_h, m_{sea} and $E_{K(\pi)}$.
- *C1-4*. Add analytic NNLO terms in combinations m_ℓ and a^2.
- *D1-4*. Different ways of parameterizing the a^2 dependence.

Very Preliminary Error Budget

Source of uncertainty	% Error $f_+^{D\to\pi}(0)$	$f_+^{D\to K}(0)$
Chiral fit	4.5	1.5
(Statistics)		
(Truncation of chiral model)		
(discretization errors)		
$m_s^{val} \neq m_s^{sea}$	0.04	0.15
Finite volume (*est.*)	(0.2)	(0.2)
Scale a	0.02	0.3
Total	4.5	1.6

- Sea quark mistuning effects estimated by the difference between using valence or sea quark masses in the chiral fit equation.
- Scale setting uncertainty effects determined by rerunning the chiral fit with each a varied by $\pm\sigma$, stated uncertainty is the largest change.
- Finite volume errors will be resolved by additional calculations at multiple volumes.
- These errors are comparable to those from HPQCD who used 2+1 flavor *asqtad* ensembles with HISQ valence quarks.

Conclusion

- In this work we are calculating $D \to K$ and $D \to \pi$ semileptonic form factors at $q^2 = 0$.
  - χPT extrapolation to the physical point and continuum limit.
  - Anticipate total errors of $\sim 5\%$ ($\sim 2\%$) for $f_+^{D\to\pi}(0)$ ($f_+^{D\to K}(0)$).
  - Before this analysis is completed we will add an additional 0.06 fm ensemble at an unphysical quark mass, use additional spatial volumes to determine finite volume errors, and expand our error analysis.
- Future work:
  - Calculations including scalar and vector currents at a variety of q^2 values, employing a z-expansion, to get the normalization and shape of the form factors.
  - Combine this with experiment to improve the result for the CKM matrix elements.
- Both of these projects use MILC 2+1+1 flavor HISQ ensembles.