The microscopic Twisted Mass Dirac spectrum and the spectral determination of the LECs of Wilson χ-PT

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Lattice 2015
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Work in progress with Krzysztof Cichy (Universität Frankfurt), Elena Garcia-Ramos (DESY), Karl Jansen (DESY) and Kim Splittorff(NBI) .

Wilson Chiral Perturbation Theory for Twisted Mass fermions

Incorporating cutoff effects (UV) in the low-energy EFT describing the IR behavior?

Cutoff effects (UV) break chiral symmetry which determines the IR behavior

- combined extrapolation in $a \to 0$ and $m_\pi \to m_\pi^{phys}$
  - accounts for non-analyticities in a
    absent in a polynomial continuum extrapolation
    e.g. a^2 in chiral logs observed by MILC
  - relations between cutoff effects in different quantities
- non-perturbartive info regarding the phase structure of the theory

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- Wilson term breaks $\chi-$ symmetry explicitly
- Lattice spacing effects lead to new terms in $\chi - PT$

  Sharpe and Singleton (1998), Rupak and Shoresh (2002), Bär,Rupak and Shoresh (2004)
- $\epsilon-$ regime where in the thermodynamic, chiral and continuum limit $mV\Sigma$, $z_t V\Sigma$ and $a^2 V W_i$ kept fixed.
- At order a^2 it involves three Low Energy Constants (LECs)

$$Z_{N_f}(m, z; a) = \int_{\mathcal{M}} dU \; e^{-S[U]}$$

where the action is

$$S = -\frac{m}{2}\Sigma V \mathrm{tr}\,(U + U^\dagger) - \frac{z_t}{2}\Sigma V \mathrm{tr}\,(\tau_3 U - \tau_3 U^\dagger)$$
$$+ a^2 V W_6 [\mathrm{tr}\left(U + U^\dagger\right)]^2 + a^2 V W_7 [\mathrm{tr}\left(U - U^\dagger\right)]^2 + a^2 V W_8 \mathrm{tr}(U^2 + {U^\dagger}^2)$$

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Phase structure of the theory

- Aoki phase where $W_8 + 2W_6 > 0$
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Quoting Andreas Jüttner who quoted Heiri Leutwyler
"Please do not be content with reaching physical quark masses.
Extract the dependence on them, determine the LECs !"
Heiri Leutwyler at Chiral Dynamics 2015, Pisa

Determining the LECs of WchPT

- pion mass splittings

  Herdoiza et al (2013)

- lattice determination of the pion scattering lengths

  Aoki et al (2008) and Bernardoni et al (2011)

- mixed action setup

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- matching analytical predictions for the spectrum of D to lattice data for fixed ν in a finite volume

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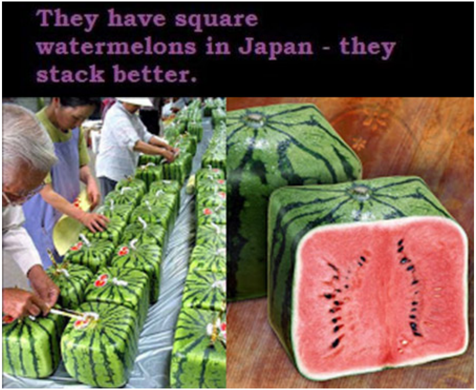
They have square
watermelons in Japan - they
stack better.

Lattice setup

- $N_{\rm f} = 2 + 1 + 1$ ETMC configurations
- Iwasaki gauge action
- 1 ensemble with $\beta = 1.95$, $a = 0.078$fm
  $a\mu = 0.0055$ with $m_\pi \approx 390$MeV
  with $32^3 \times 64$ volume with a physical extent of 2.5fm
  $m_\pi L \approx 5$
- very large statistics$\sim$ 5000 confs
  out of them ∼200 have $\nu = 0$, ∼400 have $|\nu| = 1, 2$ and
  ∼300 have $|\nu| = 3$
- field theoretical computation of the top. charge using the Gradient Flow Lüscher (2008)
  Many thanks to Andreas Athenodorou for providing his unpublished results

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Analytical derivation of $\rho_5^\nu(\lambda^5, z_t; a)$

$$Z_{3|1}^\nu(\mathcal{Z}; a) = \int dU_{Gl(3|1)} \ \mathrm{Sdet}(iU)^\nu e^{\frac{i}{2}\mathrm{Str}\,(\mathcal{Z}[U+U^{-1}])+a^2\mathrm{Str}\,(U^2+U^{-2})}$$

with $\mathcal{Z} \equiv \mathrm{diag}(iz_t, -iz_t, z, \tilde{z})$

Splittorff and Verbaarschot (2012)

- $G_{3|1}^\nu(z, z_t; a) = \lim_{\tilde{z}\to z} \frac{d}{dz} \mathcal{Z}_{3|1}^\nu(iz_t, -iz_t, z, \tilde{z}; a)$
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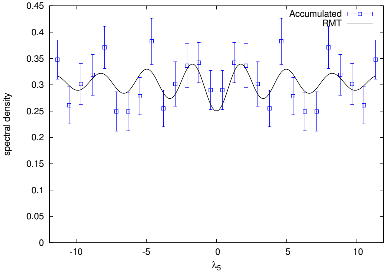
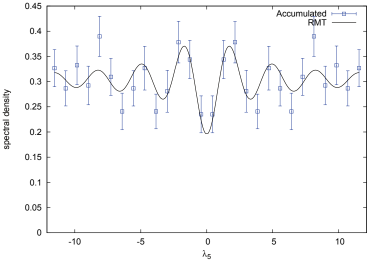
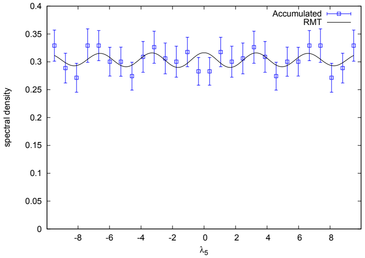
Comparison RMT vs lattice for $\nu = 0$

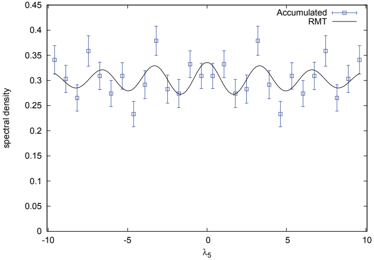




LHS $\hat{z}_t = 38.5$ $\hat{a} = 0.7$ (full sample) RHS $\hat{z}_t = 39.188$ $\hat{a} = 0.454$ (bootstrapped sample)

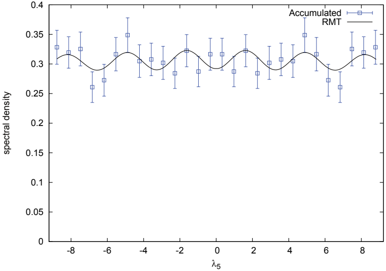
Comparison RMT vs lattice for $\nu = 1$

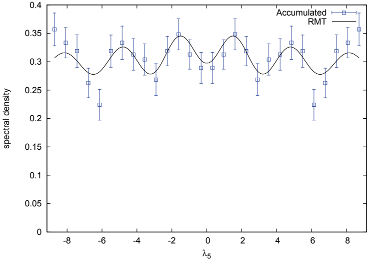




LHS $\hat{z}_t = 32.25$ $\hat{a} = 1.25$ (full sample) RHS $\hat{z}_t = 32.593$ $\hat{a} = 0.806$ (bootstrapped sample)

Comparison RMT vs lattice for $\nu = 2$





LHS $\hat{z}_t = 29.82$ $\hat{a} = 1.12$ (full sample) RHS $\hat{z}_t = 29.61$ $\hat{a} = 0.807$ (bootstrapped sample)

Extracted values for Σ and W_8

$\|\nu\|$	0	1	2	combined
$\Sigma^{1/3}$ [MeV]	288.9(2.7)	272.3(4.1)	265.3(3.7)	270.8(2.4)
W_8 $[r_0^6 W_0^2]$	0.0020(12)	0.0055(19)	0.0051(17)	0.0064(12)

Comparison with other (ETMC) results
continuum value $\Sigma^{1/3} = 290 \pm 11$ MeV
computed by the method of spectral projectors Cichy et al (2012)
our results are very close to this continuum value (hint for small cutoff effects)
$W_8[r_0^6 W_0^2] = 0.0064(2)(24)$ from the mixed action setup Cichy et al (2012)
$W_8[r_0^6 W_0^2] = 0.0138(22)$ from the pion mass splittings Herdoiza et al (2012)

Improving and Cost

- diagonalization of 1300 confs IDRIS-CNRS BGQ in Paris, cost of 5Mh
- larger volumes? smaller pion mass? smaller lattice spacing?
- there is another long ETMC ensemble with $m_\pi \sim 200\text{MeV}$ and $V = 48^3 \times 96$
- compared to the used one $m_\pi \sim 390\text{MeV}$ and $V = 32^3 \times 64$
- would bring the total cost up to ∼75Mh

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Conclusions and Outlook

- Determined the LECs (Σ and W_8) of WchPT
- Compared the analytical RMT expression for ρ^5 with lattice data
- Detailed analysis of the systematic uncertainties
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for upcoming results . . .

Thank you for your attention!

Many thanks to Andreas Athenodorou and Jac Verbaarschot

Extraction through the pion mass splittings

Münster (2004), Scorzato (2004), Sharpe et al (2004), Herdoiza et al (2013)

$$\begin{aligned} M^2_{\pi^\pm} &= 2B_0\mu_\ell \\ M^2_{\pi^0} &= 2B_0\mu_\ell - 8a^2\,(2w'_6 + w'_8) \\ M^2_{\pi^{(0,\mathrm{c})}} &= 2B_0\mu_\ell - 8a^2\,w'_8 \end{aligned}$$

w'_k defined through the Wilson LEC W'_k by

$$w'_k = \frac{16W_0^2\,W'_k}{f^2} \qquad (k = 6, 8)$$

Let's consider the following mass-splittings

$$\begin{aligned} M^2_{\pi^\pm} - M^2_{\pi^{(0,\mathrm{c})}} &= 8a^2\,w'_8 \\ \frac{1}{2}\left(M^2_{\pi^{(0,\mathrm{c})}} - M^2_{\pi^0}\right) &= 8a^2\,w'_6 \end{aligned}$$

$$c_2 = -\frac{32W_0^2}{f^2}(2W'_6 + W'_8)$$

Pion scattering lengths in WchPT

Aoki et al (2008) and Bernardoni et al (2011)

two-pion scattering process

$$\pi^\alpha(p) + \pi^\beta(k) \longrightarrow \pi^\gamma(p') + \pi^\delta(k')\,.$$

$$A(s,t,u) = \frac{1}{f^2}(s - M_0^2 - 2c_2 a^2)\,.$$

$a_0^0 = \frac{7}{32\pi f^2}\left(M_0^2 - \frac{5}{7}\,2c_2 a^2\right)$
$a_0^2 = \frac{1}{16\pi f^2}\left(M_0^2 + 2c_2 a^2\right)$

Mixed action setup

Furchner (2010), Cichy et al (2012)

$$
\begin{aligned}
M^2_{\mathrm{SS},\pm} &= 2B_0\mu \\
M^2_{\mathrm{SS,0,conn}} &= 2B_0\mu - \hat{a}^2\,\frac{32}{f^2}\,W_8' \\
M^2_{\mathrm{VV}} &= 2B_0 m_{\mathrm{ov}}\,, \\
M^2_{\mathrm{VS}} &= B_0(m_{\mathrm{ov}}+\mu) + \hat{a}^2\,\frac{4}{f^2}\,W_M - \hat{a}^2\,\frac{8}{f^2}\,W_8'
\end{aligned}
$$