Electromagnetic Effects on the Light Pseudoscalar Mesons & Determination of m_u/m_d

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(MILC Collaboration)

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Motivation

- Disentangling electromagnetic and isospin-violating effects in the pions and kaons is a long-standing issue.
- Crucial for determining light-quark masses.
  - Fundamental parameters in Standard Model; important for phenomenology.
  - Size of EM contributions is largest uncertainty in determination of m_u/m_d.

	m_u [MeV]	m_d [MeV]	m_u/m_d
value	1.9	4.6	0.42
statistical error	0	0	0
lat. syst.error	0.1	0.2	0.01
perturbative error	0.1	0.2	--
EM error	0.1	0.1	0.04

MILC, arXiv:0903.3598; Rev.Mod.Phys. 82:1349, (2010)

  - Reduce error by calculating EM effects on the lattice.

Background

- ✦ EM error in m_u/m_d dominated by error in $(M^2_{K^+} - M^2_{K^0})^\gamma$, where γ indicates the EM contribution (*i.e.*, is it not an exponent).
- ✦ Dashen (1960) showed that at leading order EM splittings are mass independent: $(M^2_{K^+} - M^2_{K^0})^\gamma = (M^2_{\pi^+} - M^2_{\pi^0})^\gamma$
- ✦ Parameterize higher order effects ("corrections to Dashen's theorem") by $(M^2_{K^+} - M^2_{K^0})^\gamma = (1+\epsilon)(M^2_{\pi^+} - M^2_{\pi^0})^{\rm expt}$
  - Note: ε as defined by FLAG (Colangelo et al., arXiv:1310.8555) is based on experimental pion splittings. But EM splitting should be ≈ experimental splitting, since isospin violations for pions are small. Using our calculated electromagnetic splitting gives an alternative result, which enters systematic error estimate. We neglect disconnected diagrams in neutral pion, which should be a small effect.

Ensembles

Ensembles

✦ Table of ensembles used in the analysis:

$\approx a$[fm]	Volume	β	m_l/m_s	# configs.	L (fm)	$m_\pi L$
0.12	$12^3 \times 64$	6.76	0.01/0.05	1000	1.4	2.7
	$16^3 \times 64$	6.76	0.01/0.05	1303	1.8	3.6
	$20^3 \times 64$	6.76	0.01/0.05	2254	2.3	4.5
	$28^3 \times 64$	6.76	0.01/0.05	274	3.2	6.3
	$40^3 \times 64$	6.76	0.01/0.05	115	4.6	9.0
	$48^3 \times 64$	6.76	0.01/0.05	132+52	5.5	10.8
	$20^3 \times 64$	6.76	0.007/0.05	1261	2.3	3.8
	$24^3 \times 64$	6.76	0.005/0.05	2099	2.7	3.8
0.09	$28^3 \times 96$	7.09	0.0062/0.031	1930	2.3	4.1
	$40^3 \times 96$	7.08	0.0031/0.031	1015	3.3	4.2
0.06	$48^3 \times 144$	7.47	0.0036/0.018	670	2.8	4.5
0.06	$56^3 \times 144$	7.47	0.0025/0.018	798	3.3	4.4
0.06	$64^3 \times 144$	7.47	0.0018/0.018	826	3.8	4.3
0.045	$64^3 \times 192$	7.47	0.0028/0.014	801	2.8	4.6

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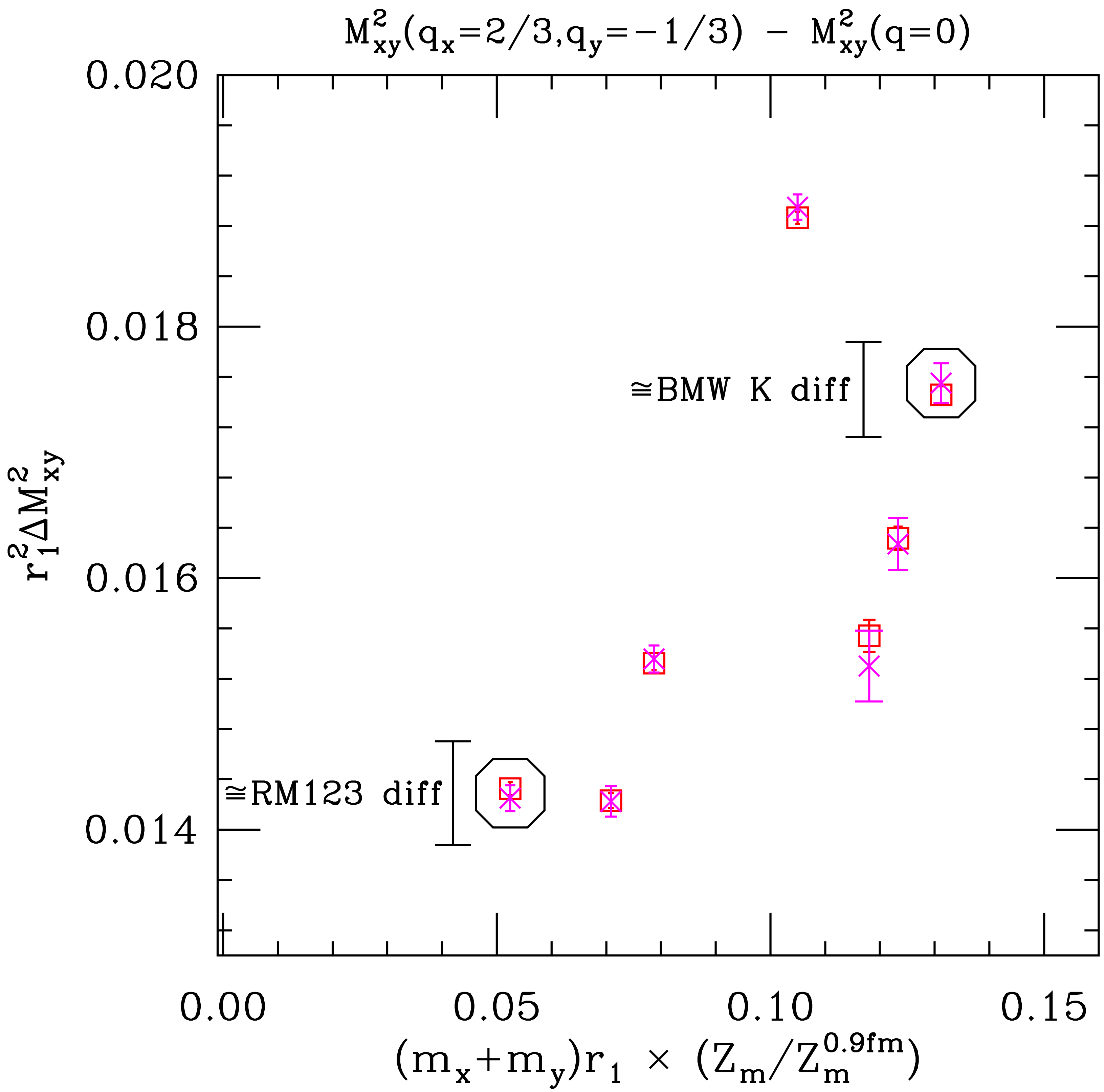
- New since last year.
- Can now drop *a* =0.12 fm ensembles in final chiral extrapolation --- fits much better behaved.
  - But *a* = 0.12 fm ensembles are used for finite volume study.

Finite-Volume Effects



- Difference between 20^3 (□) and 28^3 (×) ensembles at a≃0.12 fm is small compared to what we expect from BMW [arXiv:1201.2787], and RM123 [arXiv:1303.4896] results.
- We are not currently able to resolve the differences (consistent with zero).
  - Sign of the difference actually varies fairly randomly as quark masses change.
- Our recent work has been focused on understanding the (surprisingly small) FV effects in our data.

Finite-Volume Effects in ChPT

✦ Hayakawa and Uno [arXiv:0804.2044] calculated the EM finite-volume effects in ChPT.

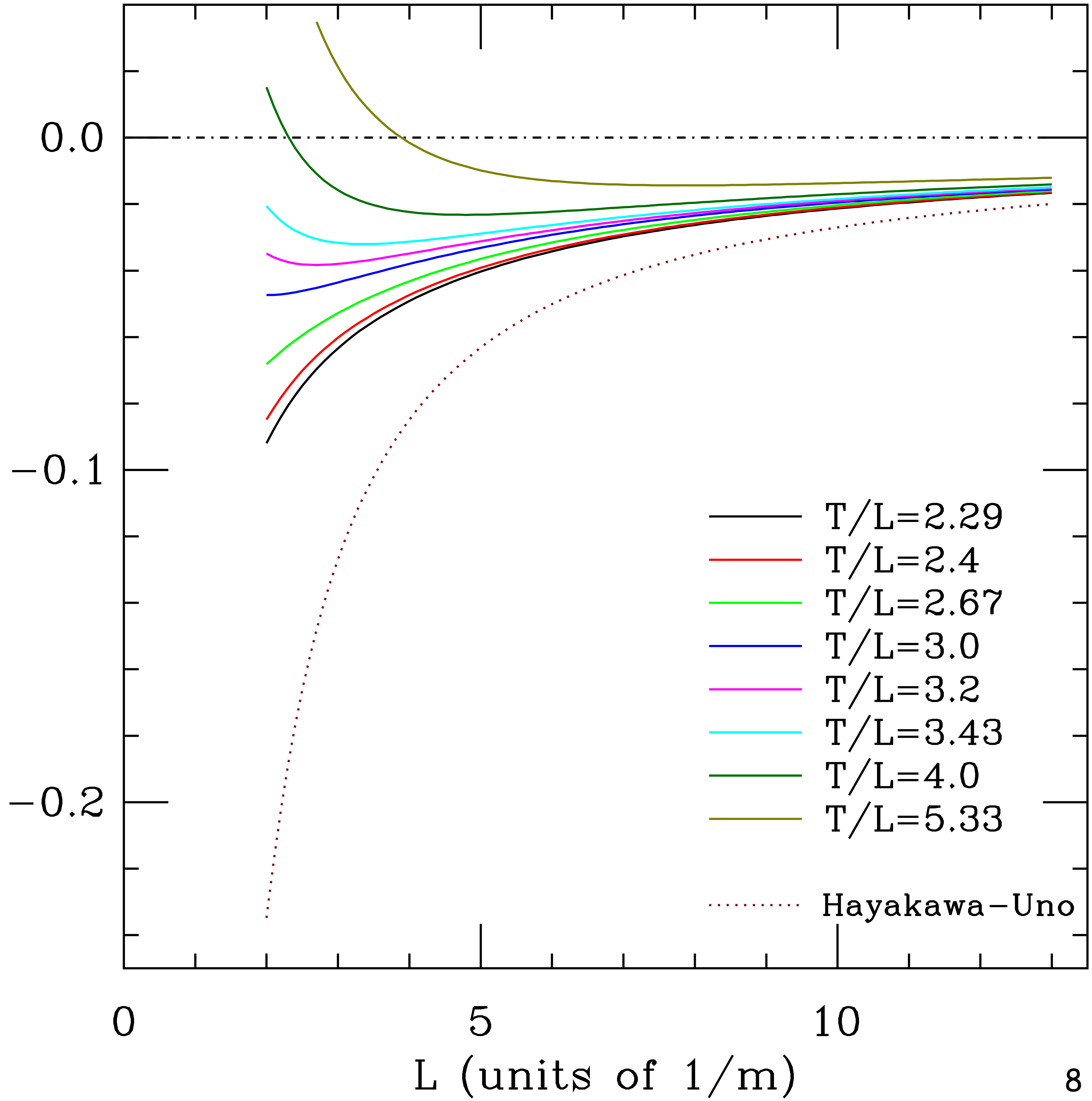
- Use noncompact realization of QED on the lattice, as we do.
- Found rather large effects.
- But noncompact QED in finite-volume is not uniquely defined:
  - It is necessary to drop some zero modes, but dropping others appears to be optional.
  - In Coulomb gauge, action for A_0 is: $\frac{1}{2}\int(\partial_i A_0)^2$.
    - For path integral to be convergent, need to drop A_0 modes for 3-momentum $\vec{k}$=0, any k_0.
  - Action for A_i is: $\frac{1}{2}\int\left[(\partial_0 A_i)^2 + (\partial_j A_i)^2\right]$.
    - Here, only required to drop mode with 4-momentum k_μ=0.
    - Hayakawa & Uno drop all A_i modes with $\vec{k}$=0.
    - MILC keeps modes with $\vec{k}$=0, $k_0\neq0$.

Chiral Perturbation Theory

- ✦ Claude Bernard has calculated and carefully compared with results of Hayakawa and Uno.
  - C.Bernard & E. Freeland, arXiv:1011.3994, provides the basic staggered-quark chiral perturbation theory.
  - Additional details may be found in Bernard's talk at Lattice 2014, arXiv:1409.7139.
    - Detailed formulae in slides available via Lattice '14 indico site
  - See arXiv:1406.4088 for finite volume study by BMW collaboration with similar results.
- ✦ We skip to graphic display of result.

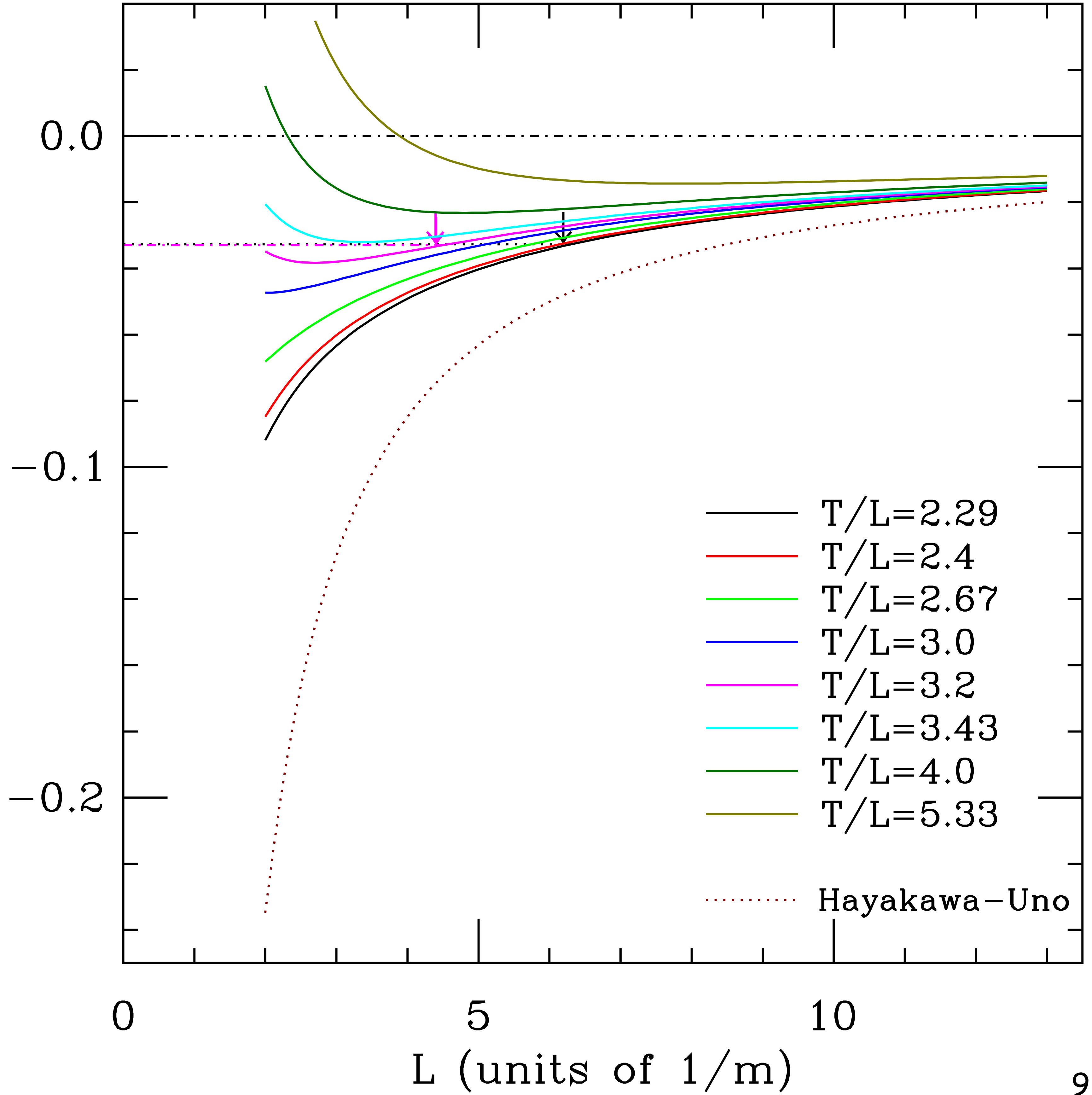
Finite-Volume Corrections

- Comparison of MILC and H-U FV corrections.
  - An overall factor of $e^2 m^2$, (where e and m are charge & mass of the meson) has been taken out.
- *T/L* values are the ones of our lattices.
  - *T/L* = 4.0, 5.33 are the small lattices (~1.4 fm, ~1.8 fm) used only for investigating FV effects.
- H-U results are insensitive to *T* in this range. (In their paper, they calculate in the *T*= ∞ limit only.)
- Our FV corrections are a factor of 2-3 less in most of the relevant range!



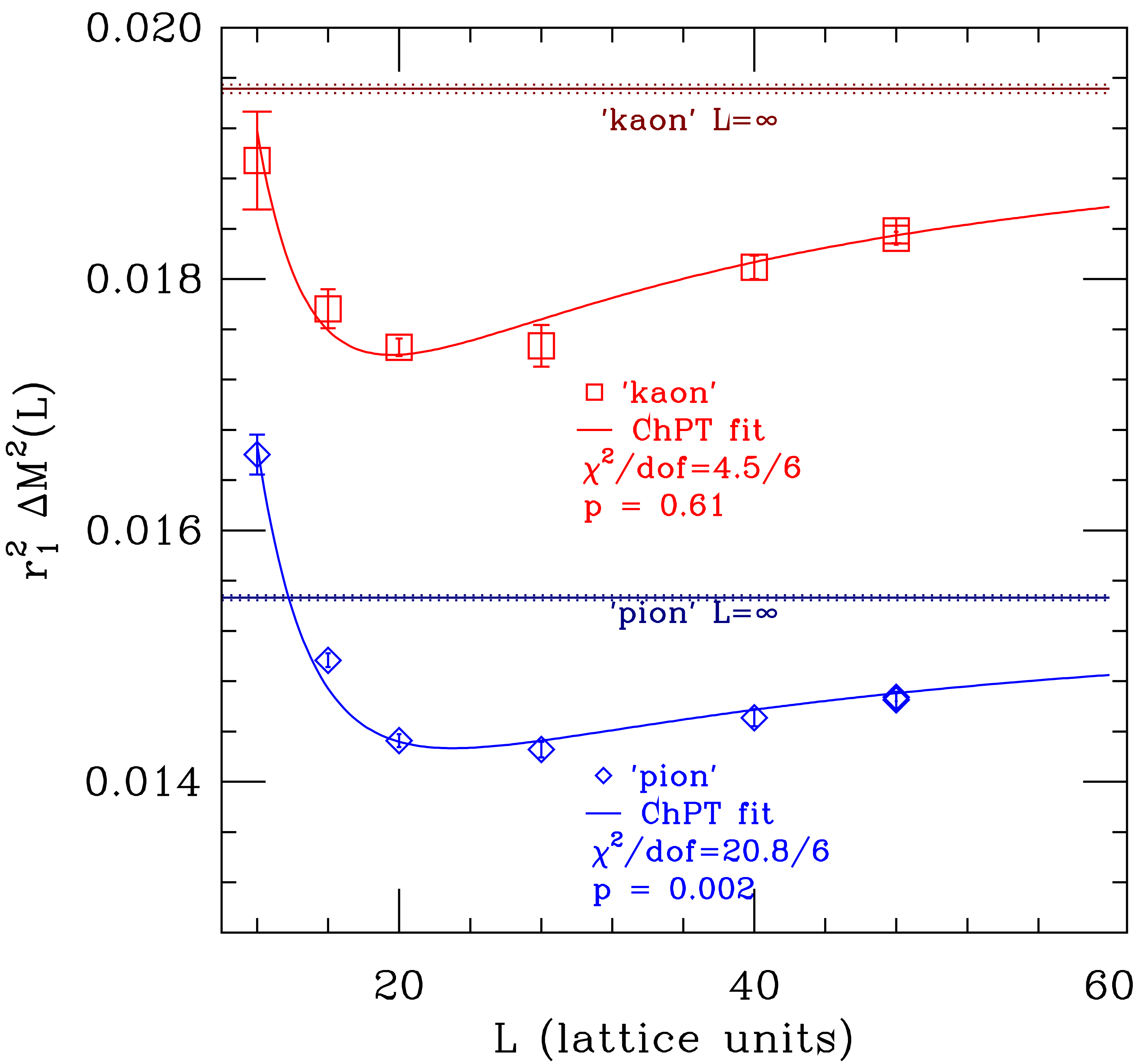
FV Corrections: Comparison

- Accidental very small FV difference between $20^3 \times 64$ (magenta) and $28^3 \times 64$ (black) lattices at "pion" point.



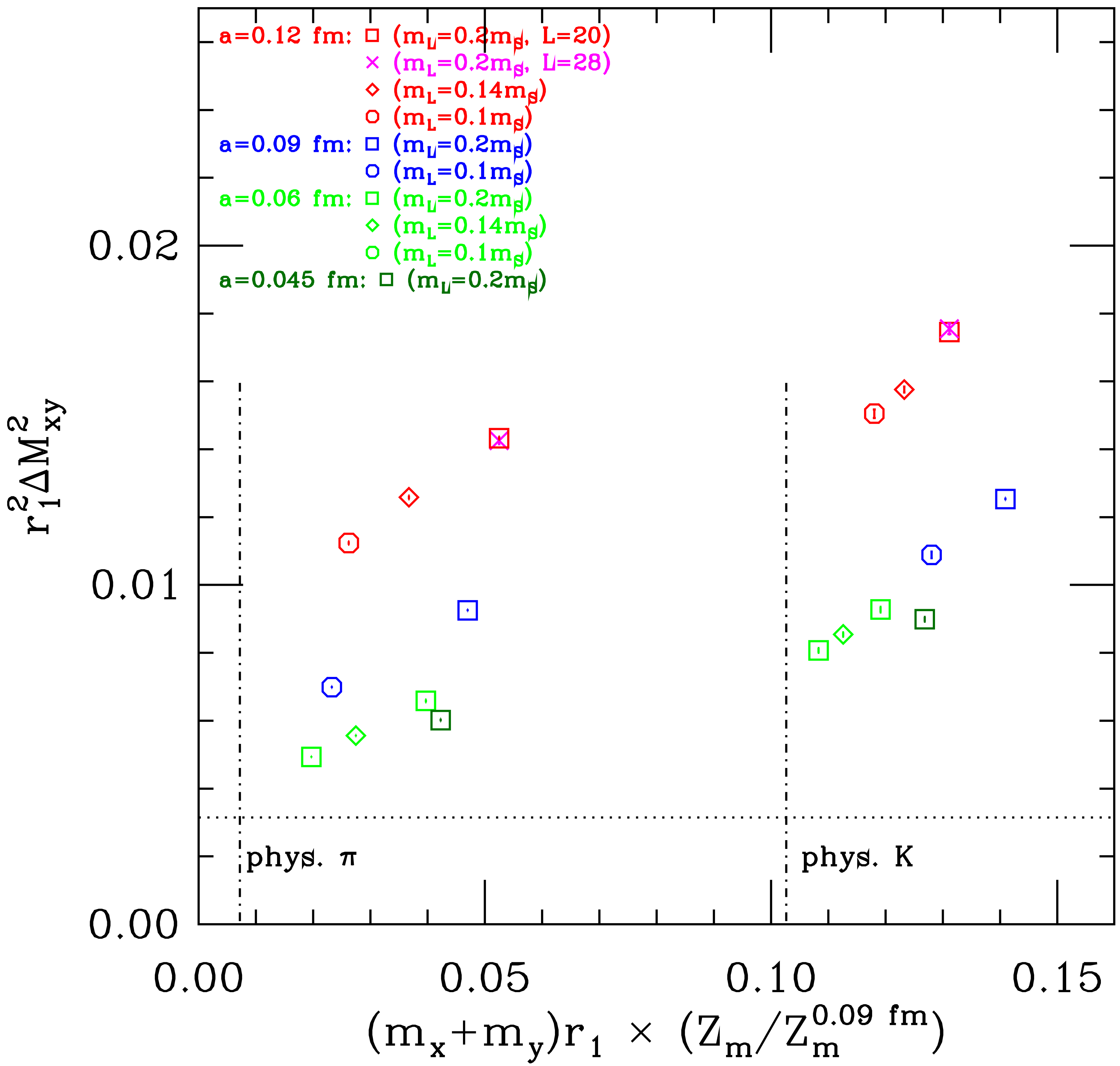
FV Corrections: Comparison with Data

- ‘kaon’ and ‘pion’ points are the (am_x, am_y) = (0.01, 0.04) and (0.01, 0.01) respectively.
- Each fit has 1 free parameter (overall height); shape is completely determined by ChPT at NLO.
- ChPT gives reasonable description of FV effects.
- Note that FV effect actually changes sign in ‘pion’ case.
- Can see why it is difficult to observe difference between results on L=20 and L=28 ensembles.
- L=16 point high for ‘pion’, need to check fit ranges



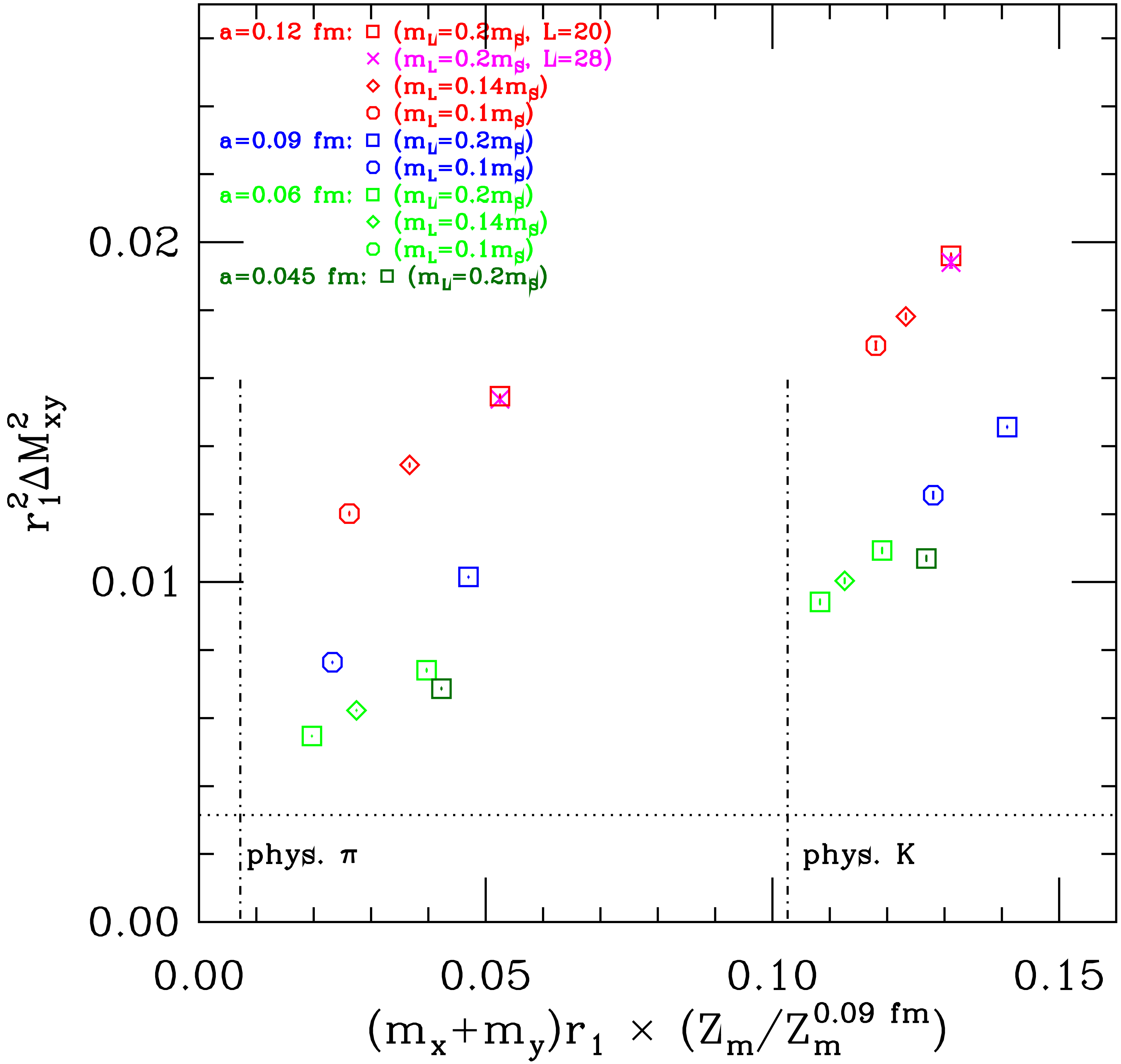
$a \simeq 0.12$ fm, $m_l/m_s = 0.01/0.05$

Chiral Fit and Extrapolation



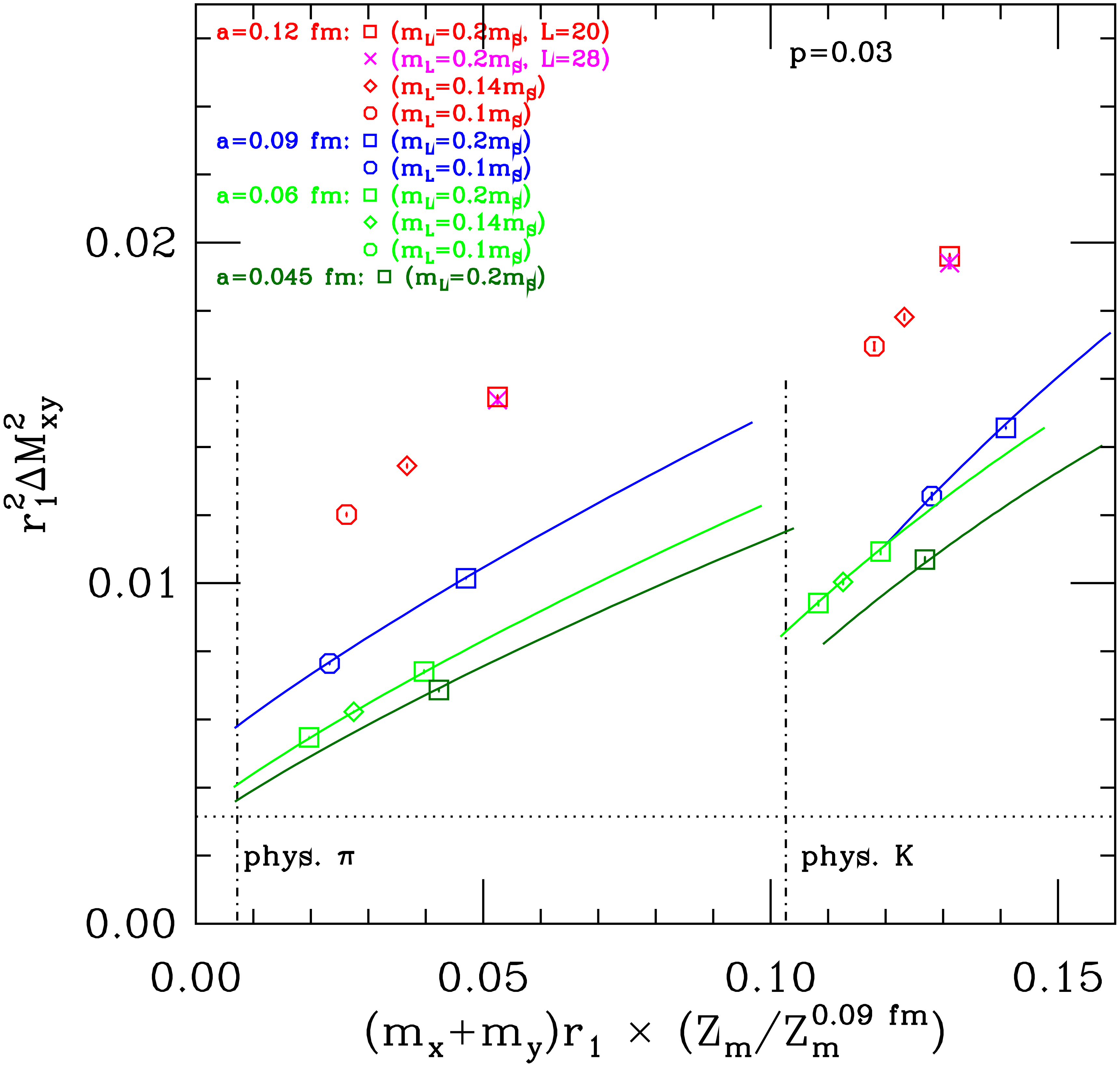
- Mass-square difference between charge +1 mesons (π^+ & K^+) and ones made from uncharged valence quarks
- Shows unitary points only.
- We have many partially quenched points, for charged and neutral mesons, as well as points with 2 × physical charges.
  - ~200-450 pts. in typical fit
- A big part the difference between results from different lattice spacings is from mis-tuned m_s, not discretization effects.

Chiral Fit and Extrapolation



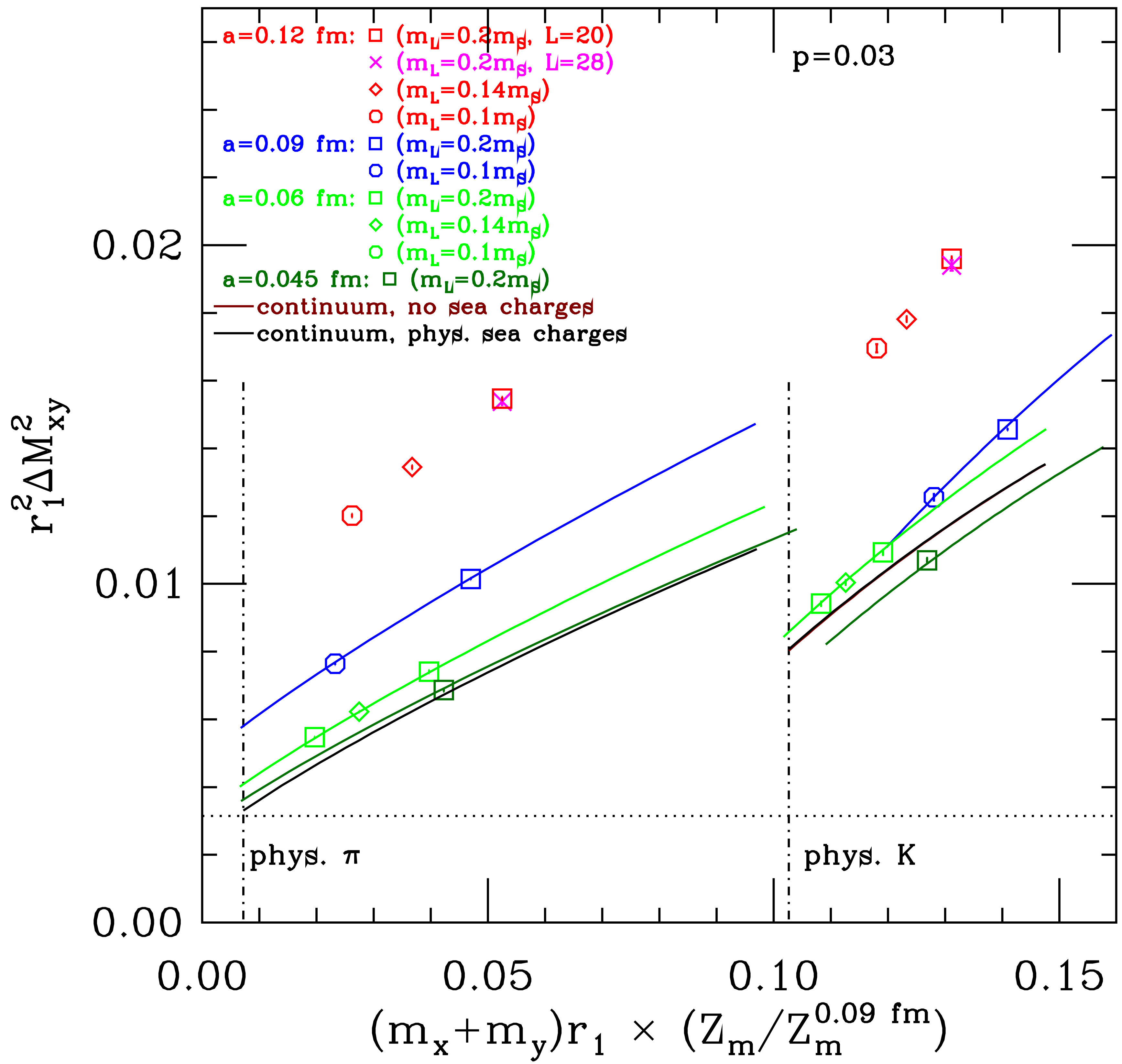
- Points after correction for finite-volume effects.
- Correction is ~7--10% (pions) and ~10--18% (kaons).
  - *Bigger* correction at higher mass because of overall factor of m^2 in 1-loop diagrams, but not at LO (Dashen's theorem).
- Note that $a \simeq 0.12$ fm, $m_l \simeq 0.2 m_s$ points for L=20 (□) and L=28 (×) are consistent.

Chiral Fit and Extrapolation



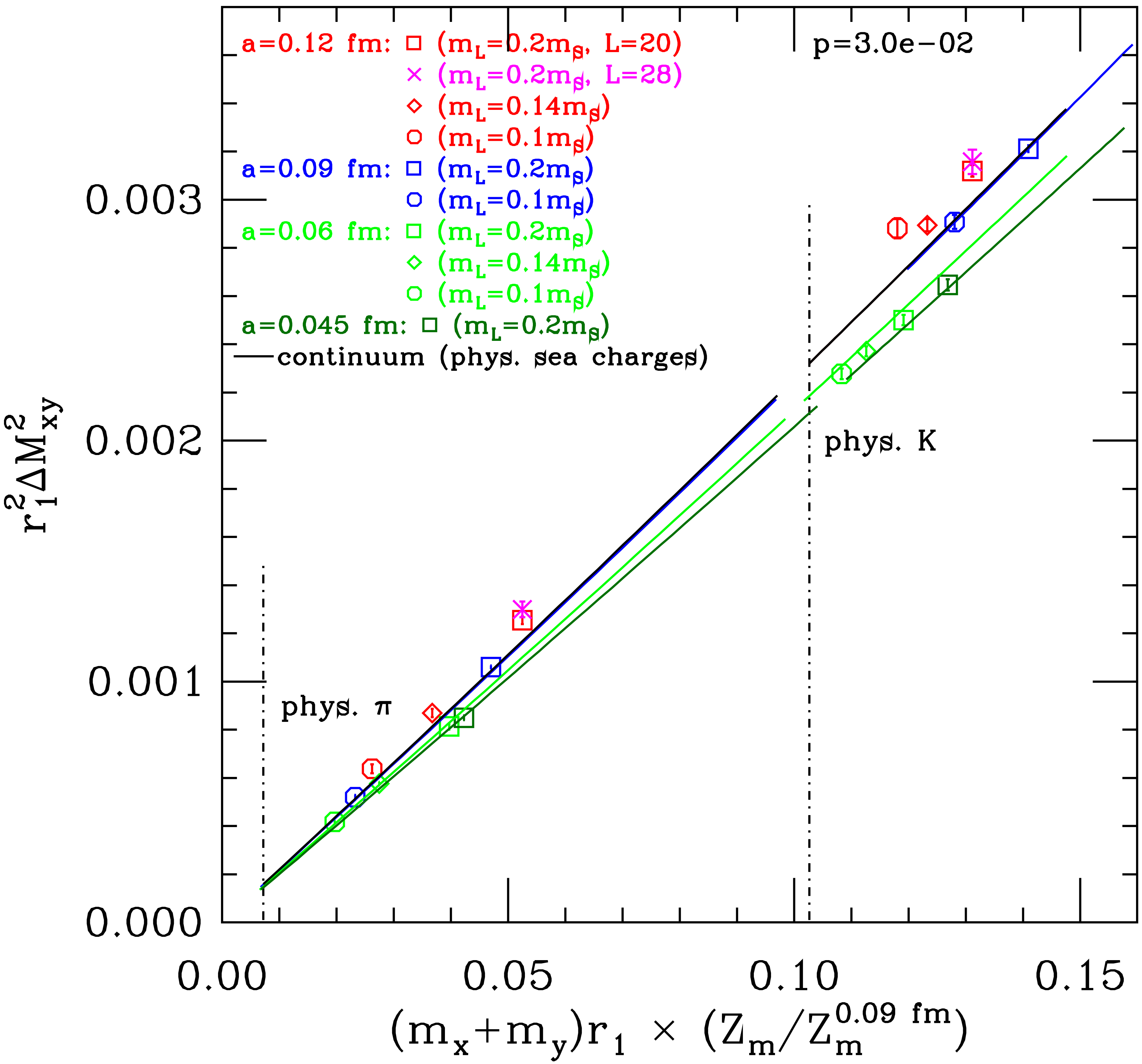
- Chiral fit to infinite-volume (corrected) points.
- Data has very high correlations for different valence masses or charges on the same ensembles: covariance matrix nearly singular.
- For that reason, and because errors are tiny (0.4--0.8%), it is difficult to get decent correlated fits.
- Here we apply SVD cut; has 444 data points, 37 parameters, χ^2/dof=486/397, p=0.03.
- Fits are generally significantly better than earlier ones without FV corrections.

Chiral Fit and Extrapolation



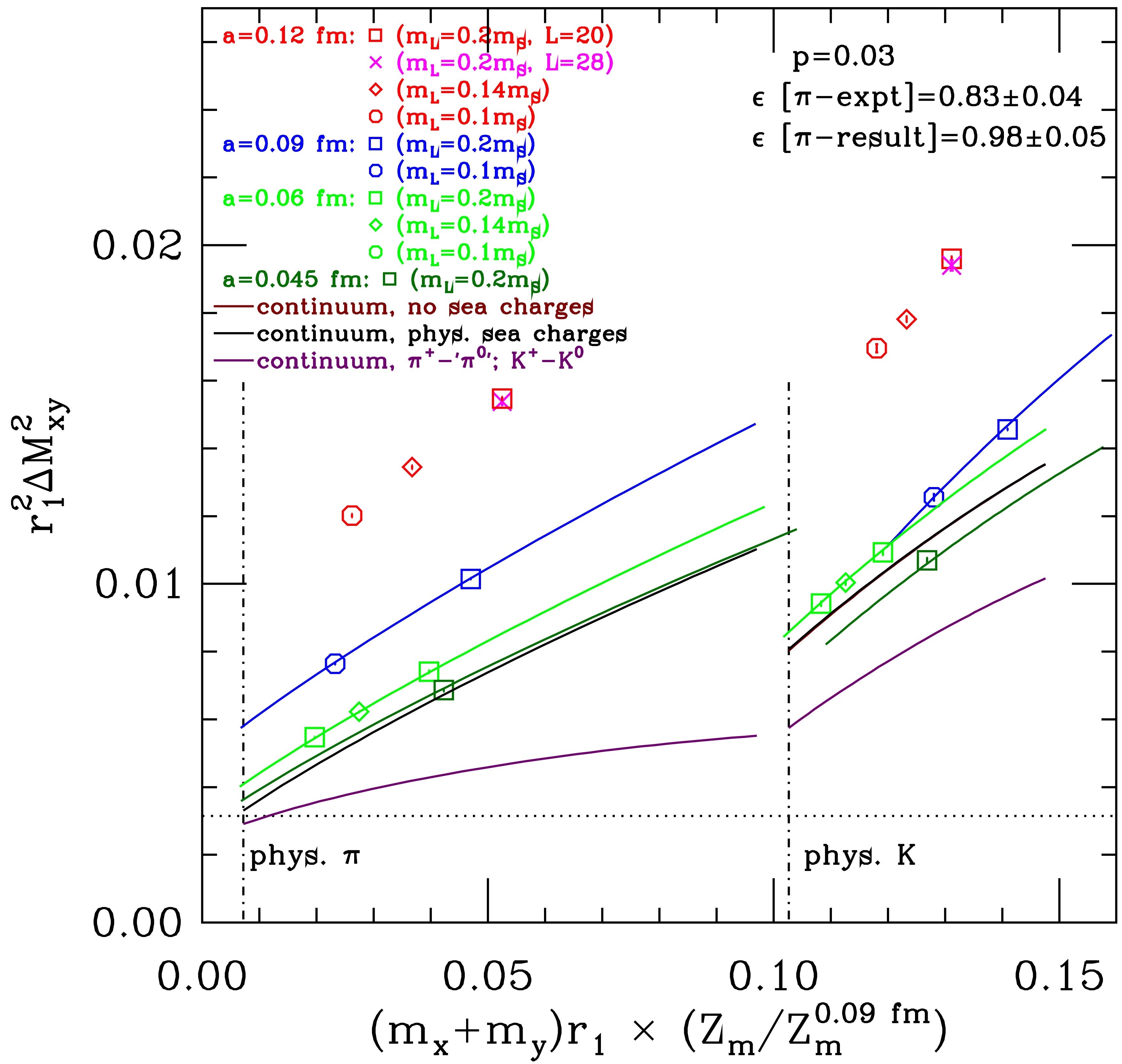
- First with sea quarks uncharged:
  - Extrapolate to continuum, and set valence, sea masses equal.
  - Adjust m_s to physical value.
- Then:
  - Set sea quark charges to their physical values, using NLO chiral logs.
- Difference with previous case is very small for kaon; vanishes identically for pion.

Chiral Fit and Extrapolation



- Neutral $d\bar{d}$-like mesons ($q_x = q_y = 1/3$) for same fit.
- Note difference in scale from charged meson plot.
- Function of (m_x+m_y) only (π and K line up).
- Nearly linear: chiral logs vanish for neutrals.

Chiral Fit and Extrapolation



- Now subtract neutral masses from charged masses to give purple lines.
- We are not including disconnected EM graphs for π^0, which is why we call it 'π^0'.
- Horizontal dotted line shows experimental value of π splitting; difference between it and intercept of purple line with vertical, dashed-dotted physical π line is a measure of systematic errors.
- Can now read off ratio of K and π splittings:

$$\epsilon = 0.83(4)$$

Current Result

- Get (preliminary):

$$\epsilon = 0.83(4)_{\text{stat}}(15)_{a^2}(5)_{\text{FV}}$$

or:

$$\epsilon = 0.83(16)$$

- Using this number with the current HISQ light meson analysis gives (preliminary):

$$m_u/m_d = 0.4510(41)_{\text{stat}}(^{+10}_{-81})_{a^2}(1)_{\text{FV}_{\text{QCD}}}(124)_{\text{EM}}$$

  - where here "EM" denotes all errors from ε, while "FV_{QCD}" refers to finite-volume effects in the pure QCD calculation on the HISQ ensembles.
  - Electromagnetic error reduced by more than a factor of three from RMP paper.

Future Plans

- ✦ EM effects in baryons also being studied.
- ✦ Extension to MILC HISQ ensembles is straightforward, and should reduce errors significantly:
  - Smaller discretization effects.
  - Nearly absent chiral extrapolation errors, since ensembles with physical masses are included.
  - Smaller FV effects, since our HISQ lattices are generally larger than the older asqtad ones. Max size ~5.5 fm.
- ✦ Extension to fully dynamical SU(3)×U(1) will make possible controlled calculations of many additional quantities.
  - Dynamic (unquenched) QED code has been written, and has passed some basic tests.

- Thank you for your attention

- PS Don’t forget to wish Carleton a Happy Birthday!

Topological Charge

- Time history of topological charge of asqtad ensembles with m_l/m_s=0.2, for (top to bottom) a=0.12, 0.09, 0.06, 0.045 fm.
  - PRD 81 (2010) 114501, arXiv: 1003.5695