Chiral behavior of light meson form factors in 2+1 flavor QCD with exact chiral symmetry

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introduction

- **kaon semileptonic form factors (FFs) :**

$$\langle \pi(p') | V_\mu | K(p) \rangle = (p+p')_\mu \, f_+(t) + (p-p')_\mu \, f_-(t) \quad \left(t = (p-p')^2 \right)$$

$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

- **normalization :** $f_+(0) = f_0(0)$
  - determination of CKM ME $|V_{us}|$ through $K \to \pi l \nu$ decay rate Γ
  - non-lattice inputs (Γ, I, $\delta_{SU(2),EM}$) 0.2% $\Rightarrow$ need $\lesssim$1% accuracy
  - direct calculation @ t = 0 + physical m_{ud} $\Rightarrow$ FNAL/MILC, RBC/UKQCD,$\cdots$

- **shape :** $d f_{+,0} / d t, \ldots$
  - comparison w/ exp't and ChPT $\Rightarrow$ reliability of precision calc.

a different approach exploiting exact chiral symmetry

this talk

study FF normalization and shape simultaneously

by chiral extrapolation based on NNLO ChPT

light meson (π^+, K^+, K^0) EM form factors : share LECs

$\Rightarrow$ chiral behavior of these light meson FFs

1. simulation set up
2. FFs in ChPT
3. EM FFs
4. kaon semileptonic FFs

simulation set up

JLQCD's simulations using overlap quarks

- $N_f = 2+1$ QCD w/ exact chiral symmetry
- $a = 0.1120(5)(3)$ fm $\Rightarrow$ $O((a\Lambda)^2) \sim 8\%$ error
- 4 m_{ud}'s $\Rightarrow$ $M_\pi = 290 - 540$ MeV
- $m_s = 0.060,\ 0.080$ $\Leftrightarrow$ $m_{s,\mathrm{phys}} = 0.081$
- $16^3 \times 48$ or $24^3 \times 48$ $\Rightarrow$ $M_\pi L \gtrsim 4$ $\Rightarrow$ suppress finite V correction
- extra-Wilson fermions $\det[H_W^2] / \det[H_W^2+\mu^2]$ $\Rightarrow$ speed-up + fixed Q
  - finite V effects $\propto 1/V$ (Aoki et al., 2007); $\lesssim$ stat. error for $F_V^{\pi^+}(t)$ (JLQCD, 2009)
- 2500 HMC trajectories @ each simulation point
- all-to-all propagator, reweighting, twisted boundary conditions

chiral expansion of FFs

in NNLO SU(3) ChPT

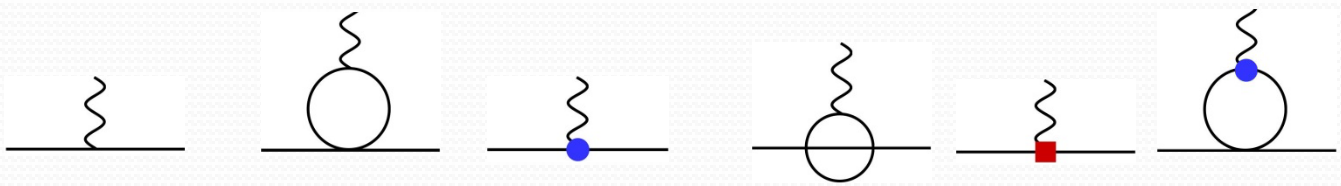
LO = 1 (CVC)

NLO 1-loop $f_{+,2,B}$, NNLO 2-loop $f_{+,4,B}$

- parameter-free w/ ξ-expansion

$$m_q / (4\pi F_0)^2 \rightarrow \xi = M_P^2 / (4\pi F_\pi)^2$$

$$f_+ = f_{+,0} + f_{+,2,B} + f_{+,2,L} + f_{+,4,B} + f_{+,4,C} + f_{+,4,L} + f_{+,6}$$



???

NLO, NNLO analytic, NNLO one-loop

- $f_{+,2,L}$, $f_{+,4,L}$: "$\mathcal{O}(p^4)$" couplings L_i in $\mathcal{O}(p^4)$ chiral Lagrangian $\mathcal{L}_4$
- $f_{+,4,C}$: $\mathcal{O}(p^6)$ couplings C_i in $\mathcal{L}_6$

chiral extrapolation = fix unknown LECs

$O(p^4)$ couplings L_i

- have been studied phenomenologically

"fit 10", Amoros-Bijnens-Talavera, 2001 : $M_{\{\pi,K,\eta\}}$, $F_{\{\pi,K\}}$, K_{e4} FFs

"new fit", Bijnens-Jemos, 2012 : $M_{\{\pi,K,\eta\}}$, $F_{\{\pi,K\}}$, K_{e4} FFs, $\pi\pi$, $K\pi$, $\cdots$

"BE14", Bijnens-Ecker, 2014 : (updated) $M_{\{\pi,K,\eta\}}$, $F_{\{\pi,K\}}$, K_{e4} FFs, $\pi\pi$, $K\pi$, $\cdots$

▶ in our analysis of f_+

- L_9 @ NLO : shared w/ charged meson EM FFs

  $\Rightarrow$ can be determined from EM FFs

- $L_{1\text{-}8}$ @ NNLO $\Rightarrow$ reasonably small NNLO corrections

  $\Rightarrow$ input "BE14" values

$O(p^6)$ couplings C_i

- poorly known $\Rightarrow$ have to be determined on the lattice

for $f_+(t)$

- NNLO analytic term $\Rightarrow$ 8 C_i's in 4 coefficients (*Bijnens-Talavera, 2003*)

$$F_\pi^4 f_{+,4,C} = c_{+,\pi K}\left(M_K^2 - M_\pi^2\right)^2 + c_{+,\pi t} M_\pi^2 t + c_{+,Kt} M_K^2 t + c_t t^2$$

$$c_{+,\pi K} = -8\left(C_{12} + C_{34}\right),$$

$$c_{+,\pi t} = -4\left(2C_{12} + 4C_{13} + C_{64} + C_{65} + C_{90}\right),$$

$$c_{+,Kt} = -4\left(2C_{12} + 8C_{13} + 2C_{63} + 2C_{64} - C_{90}\right),$$

$$c_{+,t} = -4\left(C_{88} - C_{90}\right)$$

$\Rightarrow$ 4 fitting parameters ; too many to obtain a stable fit?

$O(p^6)$ couplings C_i

- 3 of them : determined from π^+, K^+, K^0 EM FFs

$$F_\pi^4 F_{4,C}^{\pi^+} = c_{\pi t}^{\pi^+} M_\pi^2 t + c_{Kt}^{\pi^+} M_K^2 t + c_t^{\pi^+} t^2$$

$$F_\pi^4 F_{4,C}^{K^+} = c_{\pi t}^{K^+} M_\pi^2 t + c_{Kt}^{K^+} M_K^2 t + c_t^{K^+} t^2,$$

$$F_\pi^4 F_{4,C}^{K^0} = c^{K^0} \left(M_K^2 - M_\pi^2\right) t$$

$$c_{+,\pi t} = -2\left(c_{\pi t}^{\pi^+} + c_{Kt}^{\pi^+} - c^{K^0}\right),$$

$$c_{+,Kt} = -2\left(c_{\pi t}^{\pi^+} + 3c_{Kt}^{\pi^+} + c^{K^0}\right)$$

$$c_{+,t} = c_t^{\pi^+} = c_t^{K^+}$$

$c_{+,\pi K}$: SU(3) breaking @ $t = 0$ $\Rightarrow$ to be determined from $K \to \pi$ FFs

only 1 free parameter $c_{+,\pi K}$ @ NNLO + possible $\mathrm{N}^{3(+)}$LO $\Rightarrow$ a stable fit

$O(p^6)$ couplings C_i

for $f_0(t)$

- many additional C_i's through $f_-(t)$ (*Bijnens-Talavera, 2003*) $\Rightarrow$ ~~naïve fit~~
- use a quantity inspired by Callan-Treiman th. (*Bijenns-Talavera, 2003*)

$$\tilde{f}_0(t) = f_0(t) + \frac{t}{M_K^2 - M_\pi^2}\left(1 - \frac{F_K}{F_\pi}\right)$$

- no LECs @ NLO : L_5 does NOT appear even @ $t \neq 0$
- huge cancellation b/w NNLO analytic terms

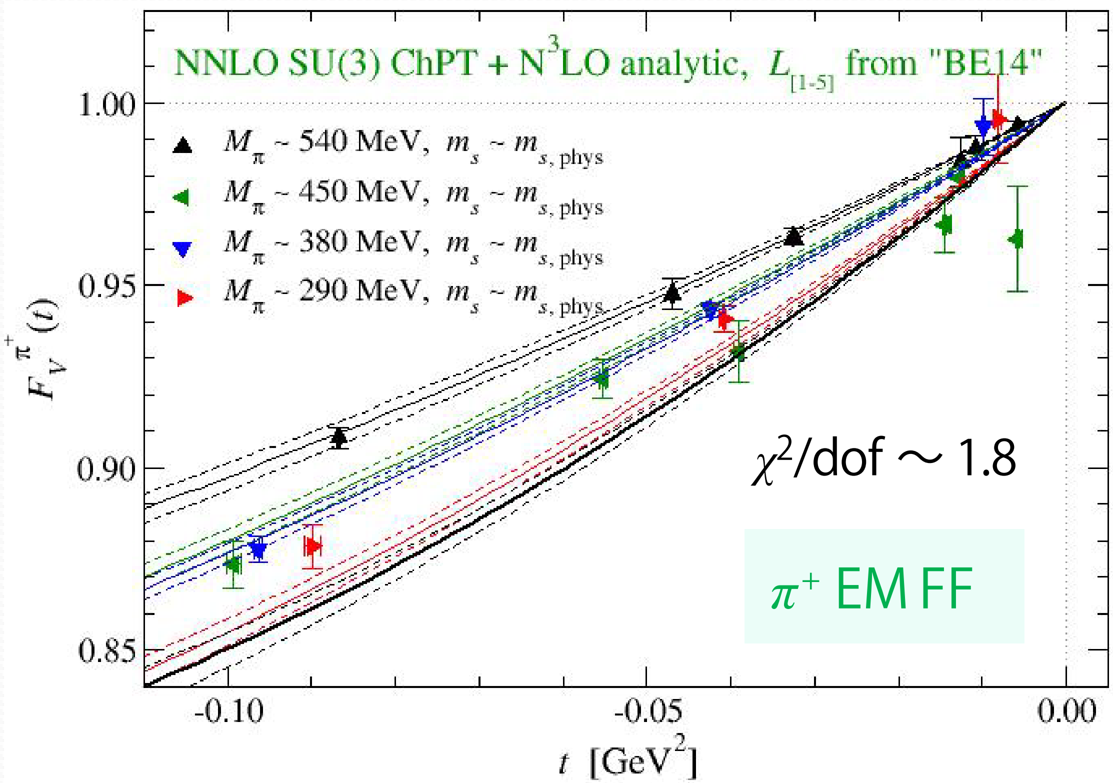
$$F_\pi^4\, \tilde{f}_{0,4,C}(t) = c_{+,\pi K}\left(M_K^2 - M_\pi^2\right)^2 + \left(8C_{12} - c_{+,\pi K}\right)\left(M_K^2 + M_\pi^2\right)t - 8C_{12}t^2$$

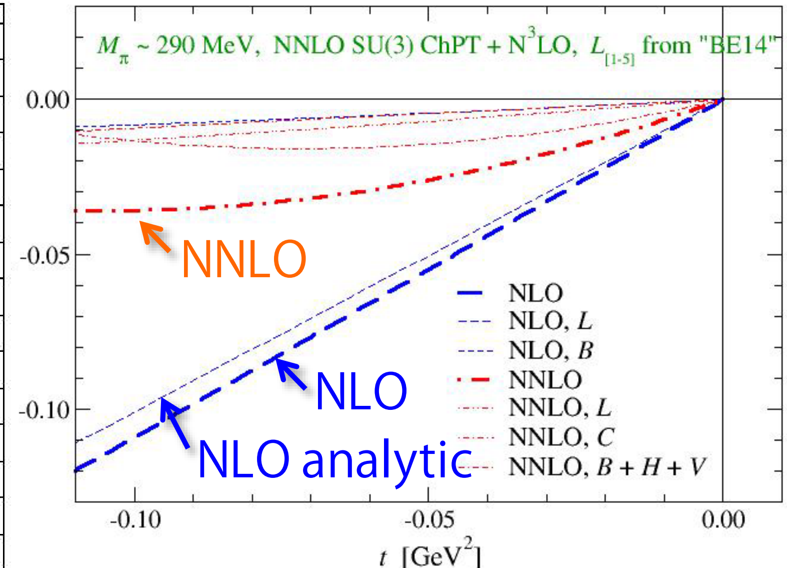
$\Rightarrow$ only 1 additional parameter C_{12} @ NNLO

$\Rightarrow$ simultaneous fit to $f_+(t)$ and $\bar{f}_0(t)$: a viable option

EM FFs

simultaneous fit to π^+, K^+, K^0 EM FFs





$\pi^+, K^+, K \to \pi$

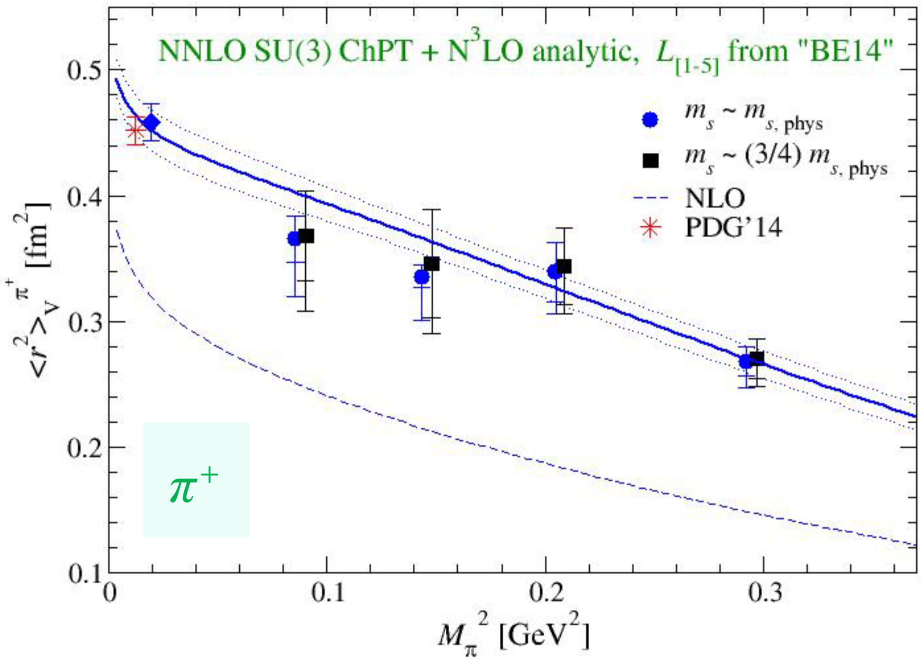
- reasonable convergence

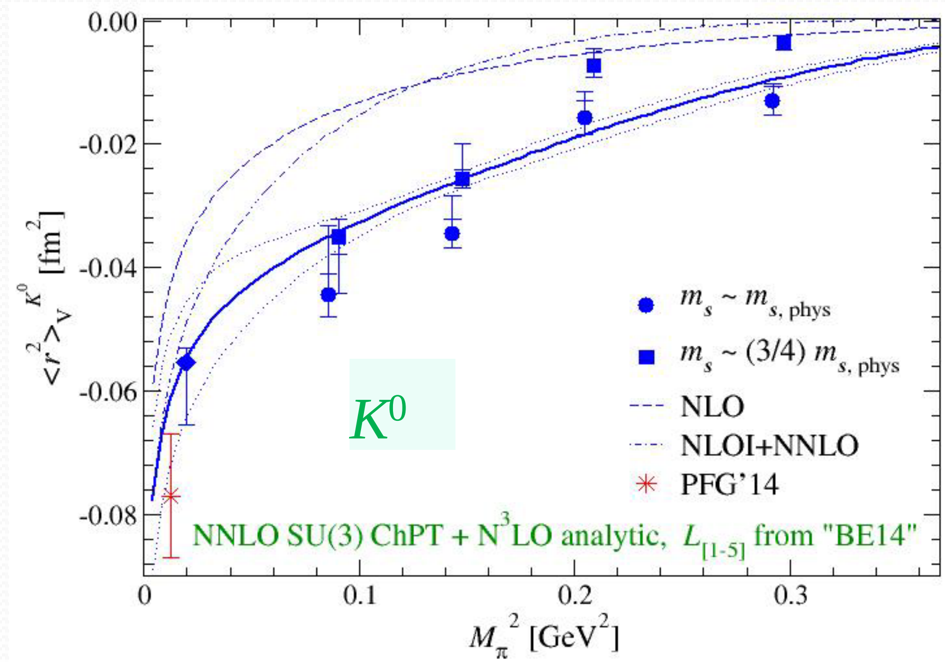
= dominated by NLO $2L_9 t/F^2$

- $L_9(M_\rho) =$ 4.6(1.1)(0.5) x 10^{-3} ⇔ 5.9(0.4) x 10^{-3} ($F_V^{\pi^+}$, Bijnens-Talavera, 2002)
- $C_t = C_{88} - C_{90} =$ - 6.4(1.1)(0.1) x 10^{-5} ⇔ - 5.5(0.5) x 10^{-5} (BT, 2002)
- $|C^{\pi^+}{}_{\pi t}|$, $|C^{\pi^+}{}_{K\pi t}|$, $|C^{K0}|$ ~ 1 - 6 x 10^{-5} ⇔ poorly known, $C_i \sim (4\pi)^{-4} = 4$ x 10^{-5}

EM FFs

charge radii





- circles/squares : value @ simulation pts. from $F(t) = 1/(1- t/M_V^2) + \ldots$
- radii are consistent with experiment

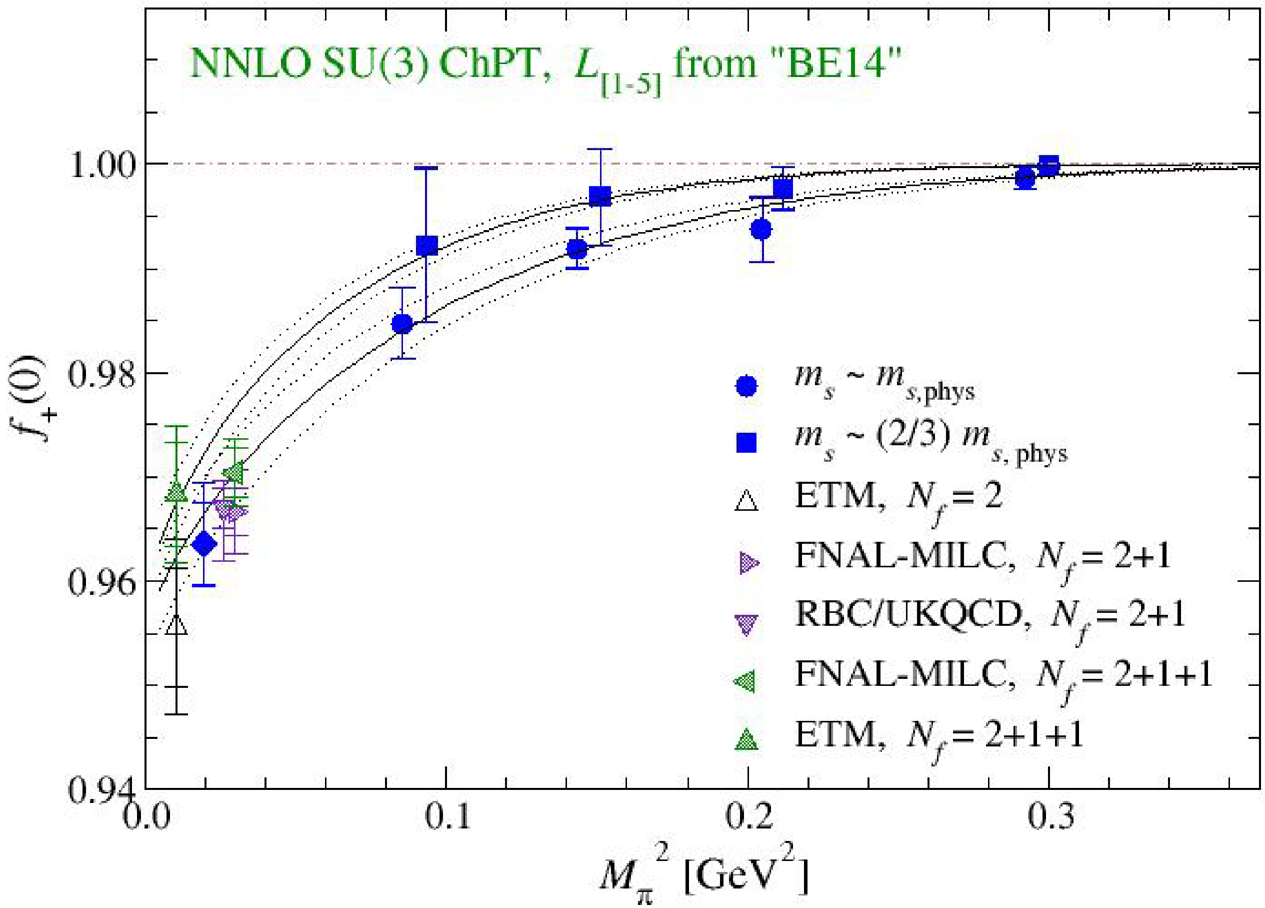
$$\langle r^2 \rangle_V^{\pi^+} = 0.458(15)(38)\,\text{fm}^2,\ \langle r^2 \rangle_V^{K^+} = 0.380(12)(32)\,\text{fm}^2,\ \langle r^2 \rangle_V^{K^0} = \text{-}0.055(10)(45)\,\text{fm}^2$$

$\Leftrightarrow$ PDG'14 : $\langle r^2 \rangle_V^{\pi^+} = 0.452(11)\,\text{fm}^2,\ \langle r^2 \rangle_V^{K^+} = 0.314(35)\,\text{fm}^2,\ \langle r^2 \rangle_V^{K^0} = \text{-}0.077(10)\,\text{fm}^2$

K→π FFs

conventional analysis : $f_+(0)$ vs $M_{\{\pi,K\}}{}^2$

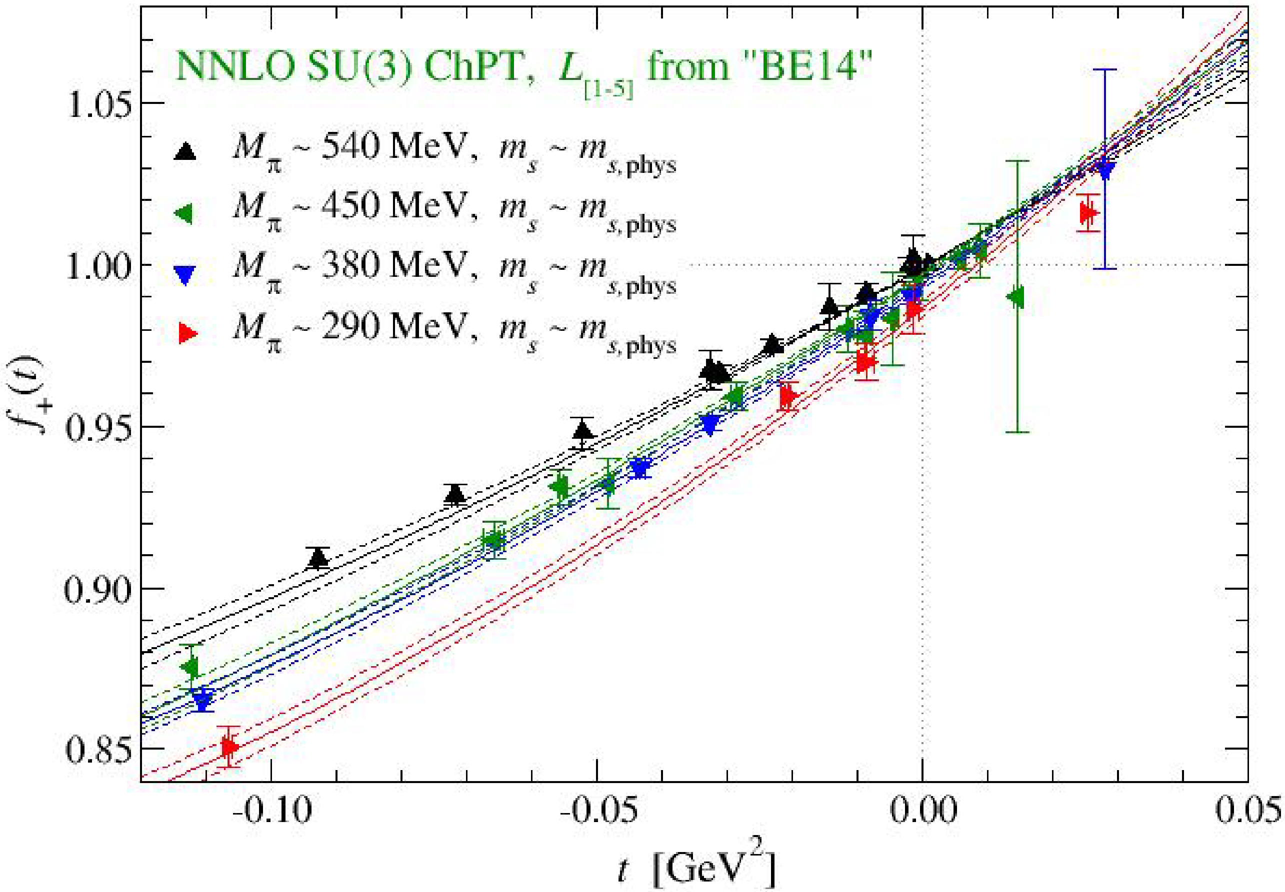
- estimate $f_+(0)$ @ each $(M_\pi{}^2, M_K{}^2)$ by $f_{+,0}(t) = f_+(0) / (1 - t / M_{\rm pole}{}^2) + \ldots$
- extrapolate to physical point based on (NNLO) ChPT



- reasonable fit
  χ^2 / dof ≈ 0.2
- $C_{+,\pi K}$ x 10^5
  $= 0.52(7)_{\rm stat}(6)_{Li}$
  ⇔ $C_i \sim (4\pi)^{-4}$
  ⇔ 0.46(4)(9)
  (FNAL/MILC, Nf=3, 2012)
- $f_+(0) = 0.964(4)(+4)_{Li}$

K→π FFs

a global fit of $f_+(t)$ vs $M_{\{\pi,K\}}{}^2$ and t



- reasonable fit

  χ^2 / dof ≈ 0.2

- $C_{+,\pi K}$ x 10^5

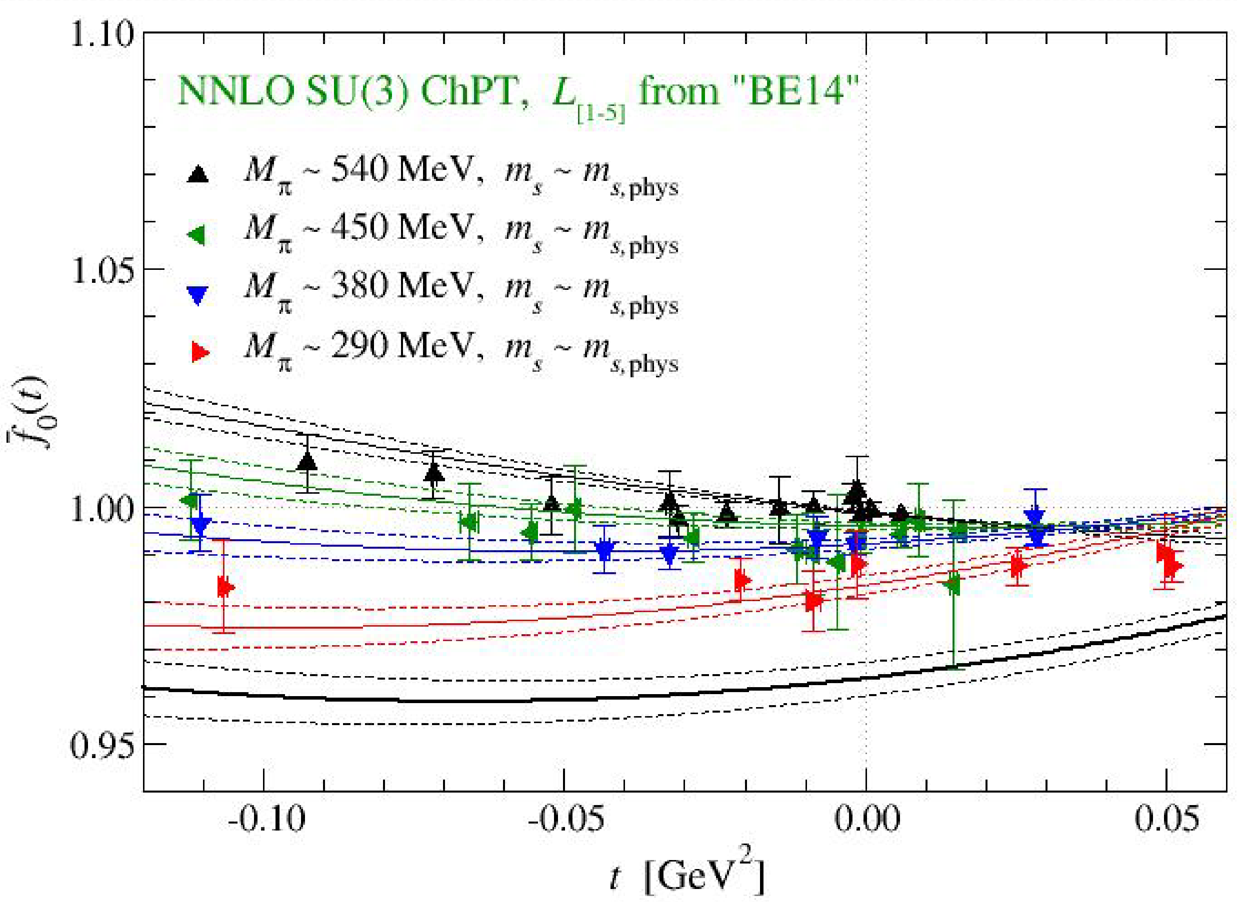
  $= 0.43(7)_{\mathrm{stat}}(7)_{Li}$

- $f_+(0) = 0.969(4)(+5)_{Li}$

- consistent w/ conventional analysis

K→π FF

simultaneous fit to $f_+(t)$ and $\bar{f}_0(t)$ vs $M_{\{\pi,K\}}^2, t$



- reasonable fit

  χ^2 / dof ≈ 0.7

- $C_{+,\pi K}$ x 10^6

  $= 0.52(6)(-7)_{Li}$

  C_{12} x $10^6 = -2.3(0.7)$

  C_{34} x $10^6 = +7.6(1.1)$

- $f_+(0) = 0.964(4)(+5)_{Li}$

- no NLO analytic ⇒ mild dependence on t
- mainly use this fit to study FF normalization and shape

FF normalization

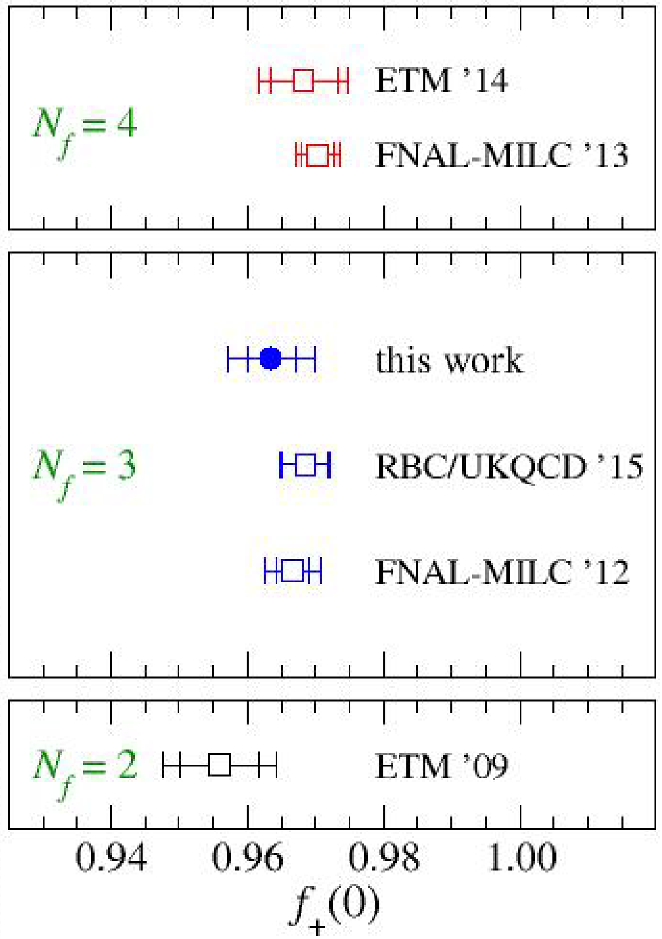
$$f_+(0) = 0.9636(36)_{\rm stat}\left({}^{+0}_{-45}\right)_{\rm N3LO}\left({}^{+41}_{-3}\right)_{Li}(29)_{a\neq 0} = 0.9636\left({}^{+62}_{-65}\right)$$

- systematics
  - $N^{3(+)}LO$: fits incl. various N^3LO terms
  - $\{L_1,\ldots,L_8\}$ input : fits w/ different $\{L_i\}$
  - $a \neq 0$: order counting $O((a\Lambda)^2)$
  - finite V : $\lesssim 0.2\%$ (neglected)

$f_+(0) - 1 =$ NLO + NNLO + $\cdots$ ($\lesssim$10%)

$\exp[-M_\pi L] \cong 2\%$ effects to these corrections

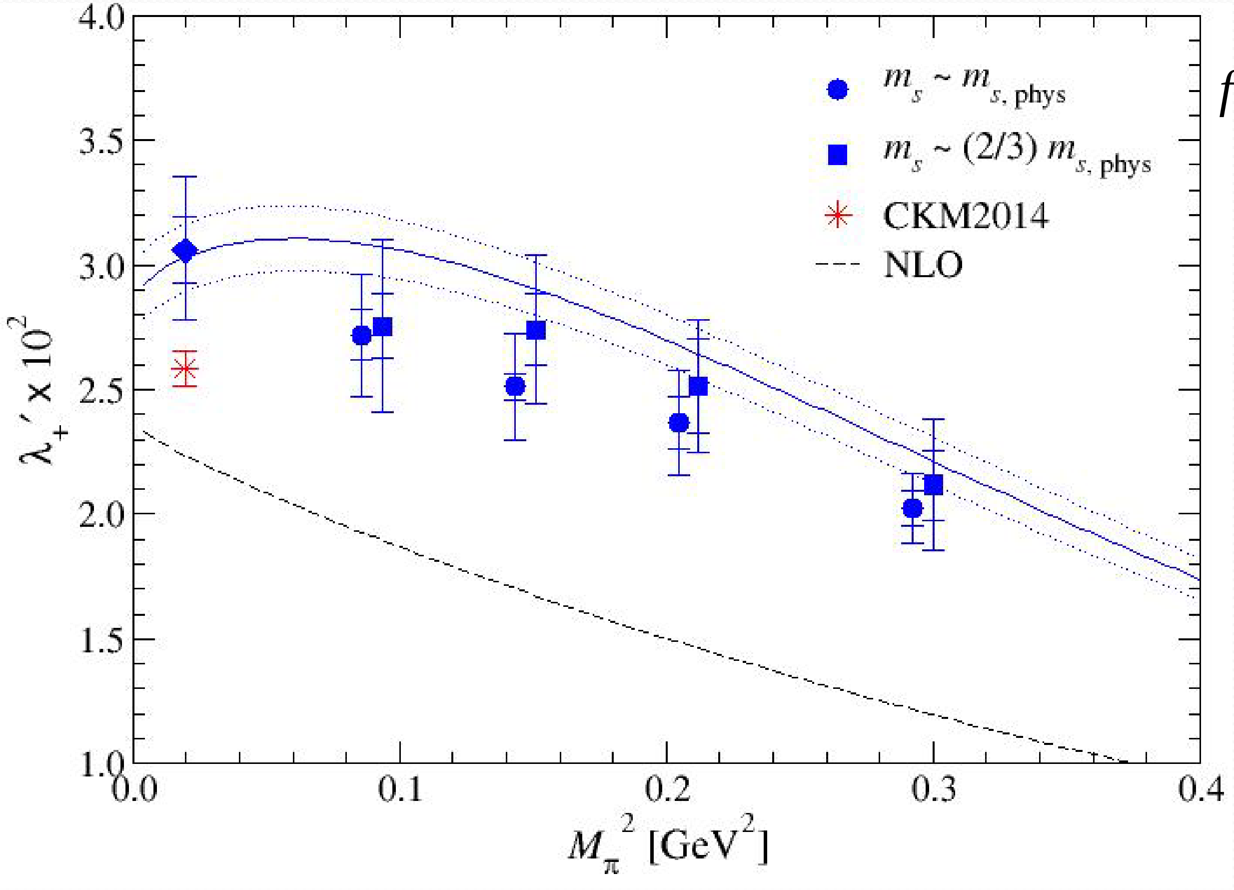
stat., chiral fit, $a \neq 0$ give equally large errors



- independent calculation of $f_+(0)$ w/ sub-% accuracy

FF shape

slope of $f_+(t)$



$$f_+(t) = f_+(0)\left\{1 + \frac{\lambda_+{}'}{M_\pi^2}t + O(t^2)\right\}$$

$$\lambda_+{}' = \frac{M_{\pi,\rm phys}^2}{f_+(0)} \left.\frac{df_+(t)}{dt}\right|_{t=0}$$

- NNLO is significant
- $\lambda_+{}' \times 10^2 =$ 3.08(14)(31)

  consistent wit exp't

  2.58(7) (CKM 2014)

- consistency also for $\lambda_0{}' \times 10^2 = 1.98(15)(44)$ ⇔ 1.37(9) (CKM 2014)
- largest uncertainty = discretization (8%, $\lambda_+{}'$), $N^{3(+)}LO$ (21%, $\lambda_0{}'$)

summary

JLQCD's study of light meson FFs

- exact chiral symmetry + chiral fits based on NNLO ChPT
  - normalization and shape of $K \to \pi$ FFs from a unique fit
  - small model dependence through small $N^{3(+)}$LO corrections

- precision calculation of $f_+(0)$
  - independent determination with sub-% accuracy
- reliability check
  - different fits ($f_+(0)$, $f_+(t)$ and $\overline{f}_0(t)$) yield consistent results for $f_+(0)$
  - charge radii, λ_+', λ_0' : in reasonable agreement with experiment

- future directions
  - heavy flavor physics $\Rightarrow$ *T. Suzuki, Thu 16, D semileptonic decays*

backup slides

NNLO analytic： EM FF

$$F_V^P = F_0^P + F_{2,L}^P + F_{2,B}^P + F_{4,B}^P + F_{4,C}^P + F_{4,L}^P + F_6^P$$

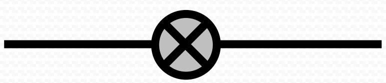
- NNLO analytic terms
  - symmetries $\Rightarrow$ 11 parameter dependences of $F_V^{\pi^+}$, $F_V^{K^+}$, $F_V^{K^0}$, f_+

$$F_\pi^4 F_{4,C}^{\pi^+} = c_{\pi t}^{\pi^+} M_\pi^2 t + c_{Kt}^{\pi^+} M_K^2 t + c_{t^2}^{\pi^+} t^2$$

$$F_\pi^4 F_{4,C}^{K^+} = c_{\pi t}^{K^+} M_\pi^2 t + c_{Kt}^{K^+} M_K^2 t + c_{t^2}^{K^+} t^2,$$

$$F_\pi^4 F_{4,C}^{K^0} = c^{K^0} \left(M_K^2 - M_\pi^2\right) t,$$

$$F_\pi^4 f_{4,C} = c_{K\pi}^+ \left(M_K^2 - M_\pi^2\right)^2 + c_{\pi t}^+ M_\pi^2 t + c_{Kt}^+ M_K^2 t + c_{t^2}^+ t^2$$



$\mathcal{L}_6$ vertex

  - 11 coefficients involve 8 LECs C_{12}, C_{13}, C_{34}, C_{63}, C_{64}, C_{65}, C_{88}, C_{90}

e.g. $c_{\pi t}^{\pi^+} = 4C_{12} + 4C_{13} + 2C_{63} + C_{64} + C_{65} + 2C_{90}$

NNLO analytic : EM FF

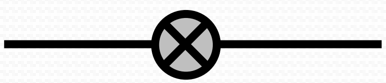
$$F_V^P = F_0^P + F_{2,L}^P + F_{2,B}^P + F_{4,B}^P + F_{4,C}^P + F_{4,L}^P + F_6^P$$

- NNLO analytic terms
  - can be written by using 5 independent linear combinations

$$C_{\pi t}^{\pi^+} = 4C_{12} + 4C_{13} + 2C_{63} + C_{64} + C_{65} + 2C_{90}$$

$$C_{\pi t}^{\pi^+} = 4C_{13} + C_{64}, \qquad C_{t^2} = C_{88} - C_{90},$$

$$C^{K^0} = 2C_{63} - C_{65}, \qquad C_{+} = C_{12} + C_{34}$$



$\mathcal{L}_6$ vertex

$$e.g. \quad c_{\pi t}^{\pi^+} = C_{\pi t}^{\pi^+}, \quad C_{\pi t}^{K^+} = C_{\pi t}^{\pi^+} + C^{K^0}/3, \quad c_{t^2}^{\pi^+} = c_{t^2}^{K^+} = c_{t^2}^{+} = C_{t^2}$$

  - these combinations are poorly known in general

$\Rightarrow$ treated as fit parameters

$\Rightarrow$ fix by fitting lattice data to ChPT formulae

higher order corrections

$$F_V^P = F_0^P + F_{2,L}^P + F_{2,B}^P + F_{4,B}^P + F_{4,C}^P + F_{4,L}^P + F_6^P$$

$$F_\pi^6 F_6^{\pi^+}, \quad F_\pi^6 F_6^{K^+} = 0$$

$$F_\pi^6 F_6^{K^0} = d^{K^0} M_\pi^2 \left(M_K^2 - M_\pi^2 \right) t$$

$$F_\pi^6 f_{+,6} = d_{+,\pi t} M_\pi^4 t, \quad d'_{+,\pi t} M_\pi^2 t^2, \quad d_{+,\pi K t} M_\pi^2 M_K^2 t, \quad d_{+,\pi K} M_\pi^2 \left(M_K^2 - M_\pi^2 \right)^2$$

$$F_\pi^6 \tilde{f}_{0,6} = d_{0,\pi t} M_\pi^4 t, \quad d'_{0,\pi t} M_\pi^2 t^2, \quad d_{0,\pi K t} M_\pi^2 M_K^2 t, \quad d_{0,\pi K} M_\pi^2 \left(M_K^2 - M_\pi^2 \right)^2$$

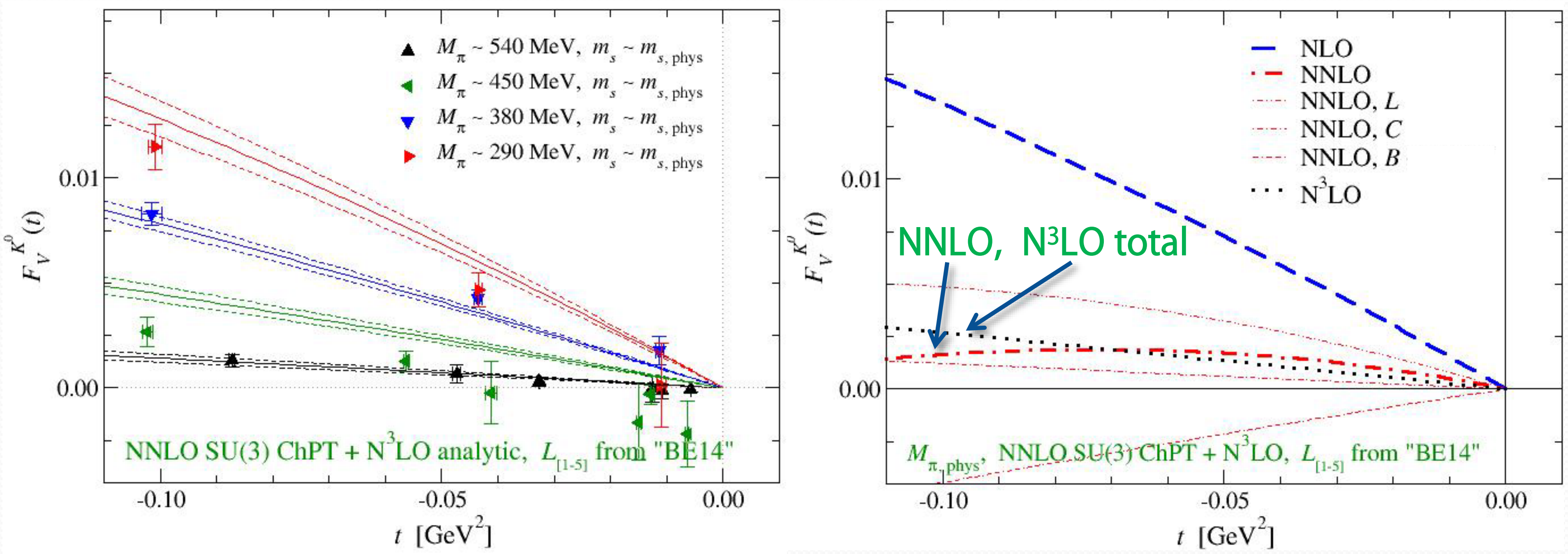
genuine 2-loop correction

only a part of $F_{V,4,B}{}^{\pi}\cdots$

$$\left(\frac{5}{2}M_\pi^4-\frac{7}{3}M_\pi^2t\right)V_{1,1}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(M_\pi^4-\frac{2}{3}M_\pi^2t+\frac{1}{12}t^2\right)V_{1,1}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\frac{1}{18}M_\pi^4V_{1,1}(M_\pi^2,M_\pi^2,M_\eta^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$+\frac{1}{18}\left(\frac{3}{2}M_\pi^4-\frac{17}{12}M_\pi^2t+\frac{1}{6}t^2\right)V_{1,1}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(\frac{2}{3}M_\pi^4-\frac{2}{3}M_\pi^2t+\frac{1}{8}t^2\right)V_{1,1}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-6M_\pi^2+t\right)V_{2,1}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-2M_\pi^2+\frac{2}{3}t\right)V_{2,1}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-4M_\pi^2+\frac{4}{3}t\right)V_{2,1}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-2M_\pi^2+t\right)V_{2,1}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-6M_\pi^4+\frac{10}{3}M_\pi^2t\right)V_{2,2}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-2M_\pi^4+\frac{4}{3}M_\pi^2t-\frac{1}{6}t^2\right)V_{2,2}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-4M_\pi^4+\frac{11}{4}M_\pi^2t-\frac{1}{3}t^2\right)V_{2,2}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-2M_\pi^4+\frac{5}{3}M_\pi^2t-\frac{1}{4}t^2\right)V_{2,2}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(\frac{5}{3}M_\pi^2t+\frac{1}{2}t^2\right)V_{2,4}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\frac{1}{3}M_\pi^2tV_{2,4}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)+\frac{5}{6}M_\pi^2tV_{2,4}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\frac{1}{3}M_\pi^2tV_{2,4}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-4M_\pi^2+\frac{4}{3}t\right)V_{2,5}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-2M_\pi^2+\frac{1}{2}t\right)V_{2,5}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-3M_\pi^2+\frac{17}{12}t\right)V_{2,5}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-2M_\pi^2+t\right)V_{2,5}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-4M_\pi^4+\frac{7}{3}M_\pi^2t+\frac{1}{3}t^2\right)V_{2,6}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(-2M_\pi^4+M_\pi^2t\right)V_{2,6}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(6M_\pi^2-2t\right)V_{3,1}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(3M_\pi^2-t\right)V_{3,1}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(6M_\pi^2-2t\right)V_{3,1}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(\frac{9}{2}M_\pi^2-\frac{3}{2}t\right)V_{3,1}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$-\frac{1}{3}tV_{3,2}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)-\frac{2}{3}tV_{3,2}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$-\frac{1}{2}tV_{3,2}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(2M_\pi^4-M_\pi^2t+\frac{1}{3}t^2\right)V_{3,3}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(M_\pi^4-\frac{2}{3}M_\pi^2t+\frac{1}{12}t^2\right)V_{3,3}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(2M_\pi^4-\frac{4}{3}M_\pi^2t+\frac{1}{6}t^2\right)V_{3,3}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(\frac{3}{2}M_\pi^4-M_\pi^2t+\frac{1}{8}t^2\right)V_{3,3}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$-\frac{1}{2}t^2V_{3,5}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)-\frac{1}{3}M_\pi^2tV_{3,5}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$-\frac{2}{3}M_\pi^2tV_{3,5}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)-\frac{1}{2}M_\pi^2tV_{3,5}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$-\frac{1}{2}t^2V_{3,6}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)-\frac{1}{12}t^2V_{3,6}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$-\frac{1}{6}t^2V_{3,6}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)-\frac{1}{8}M_\pi^2tV_{3,6}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$+4M_\pi^2V_{3,7}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(2M_\pi^2-\frac{1}{2}t\right)V_{3,7}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(4M_\pi^2-t\right)V_{3,7}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(3M_\pi^2-\frac{3}{4}t\right)V_{3,7}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$
$$+2tV_{3,8}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(8M_\pi^2-4t\right)V_{3,9}(M_\pi^2,M_\pi^2,M_\pi^2,M_\pi^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(4M_\pi^2-\frac{3}{2}t\right)V_{3,9}(M_\pi^2,M_\pi^2,M_K^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(8M_\pi^2-3t\right)V_{3,9}(M_K^2,M_K^2,M_\pi^2,M_K^2;M_\pi^2,t,M_\pi^2)$$
$$+\left(6M_\pi^2-\frac{9}{4}t\right)V_{3,9}(M_K^2,M_K^2,M_K^2,M_\eta^2;M_\pi^2,t,M_\pi^2)$$

EM FFs

neutral kaon



- $F_V^{K^0} \propto \langle \bar{s}\gamma s - \bar{d}\gamma d \rangle$: $F_0^{K^0}$, $F_{2,L}^{K^0}$ vanish $\Rightarrow$ poor convergence

simulated M_π : NNLO $\leq$ NLO, N^{3}LO

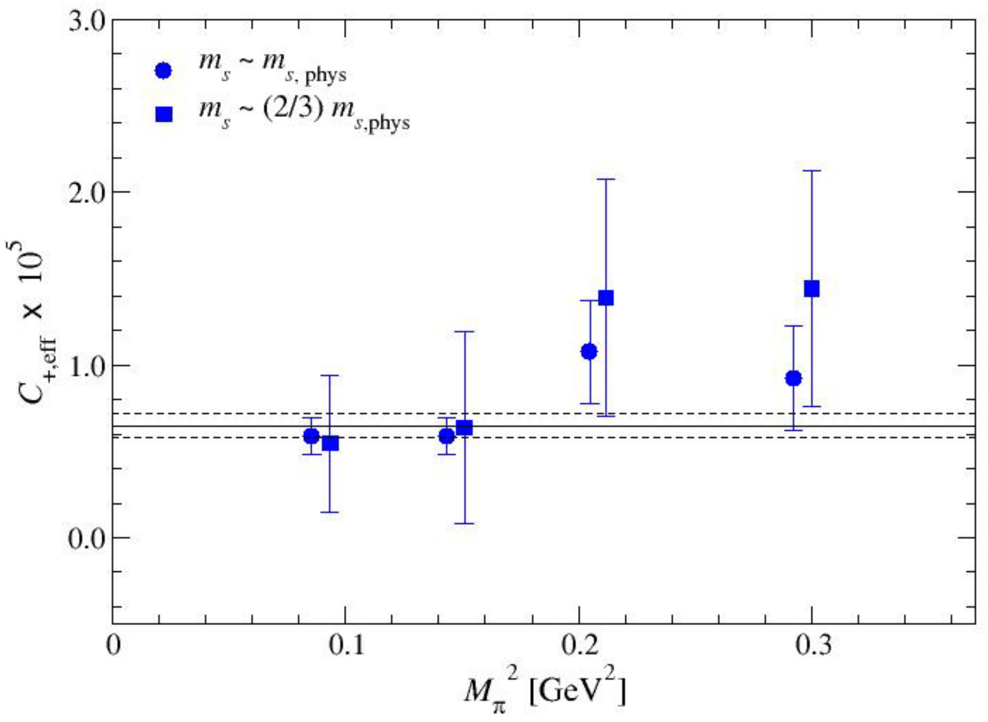
physical M_π : NLO $\gg$ NNLO $\sim$ N^{3}LO

K→π FF

N³LO is significant?

$$\Delta f_+ \equiv f_{+,4,C} + f_{+,6} = f_+ - (1 + f_{2,B} + f_{4,L} + f_{4,B})$$

$$\Delta f_+ = -8C_+(M_K^2 - M_\pi^2)^2 + f_{+,6}$$
$$= -8\, C_{+,\mathrm{eff}}\, (M_K^2 - M_\pi^2)^2$$



- if N³LO is small

  $\Rightarrow$ $c_{+,\mathrm{eff}}$ is constant

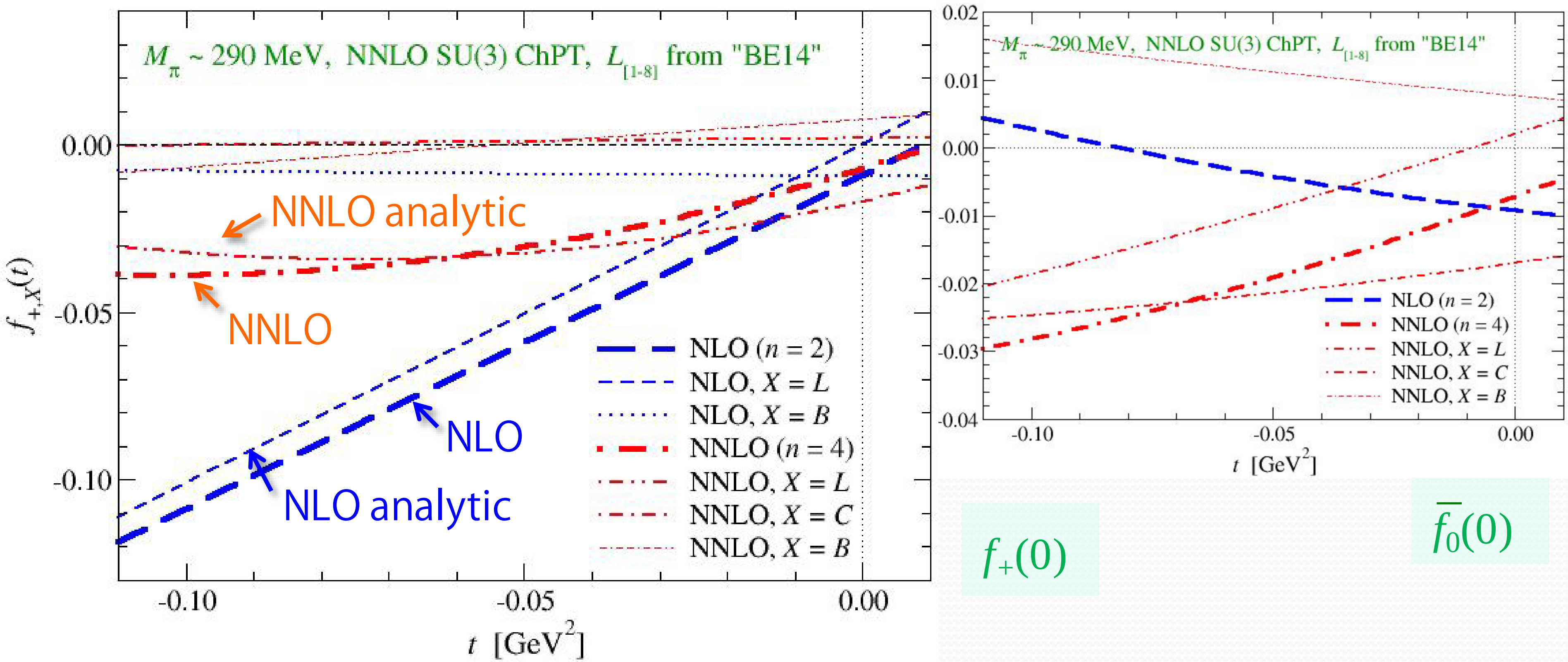
- $M^2_{\{\pi,K\}}$ dependence

  $\Rightarrow$ N³LO is not small

- our data : small dependence on $M^2_{\{\pi,K\}}$ $\Rightarrow$ N³LO can be neglected

K→π FFs

convergence of NNLO fit



- reasonable convergence

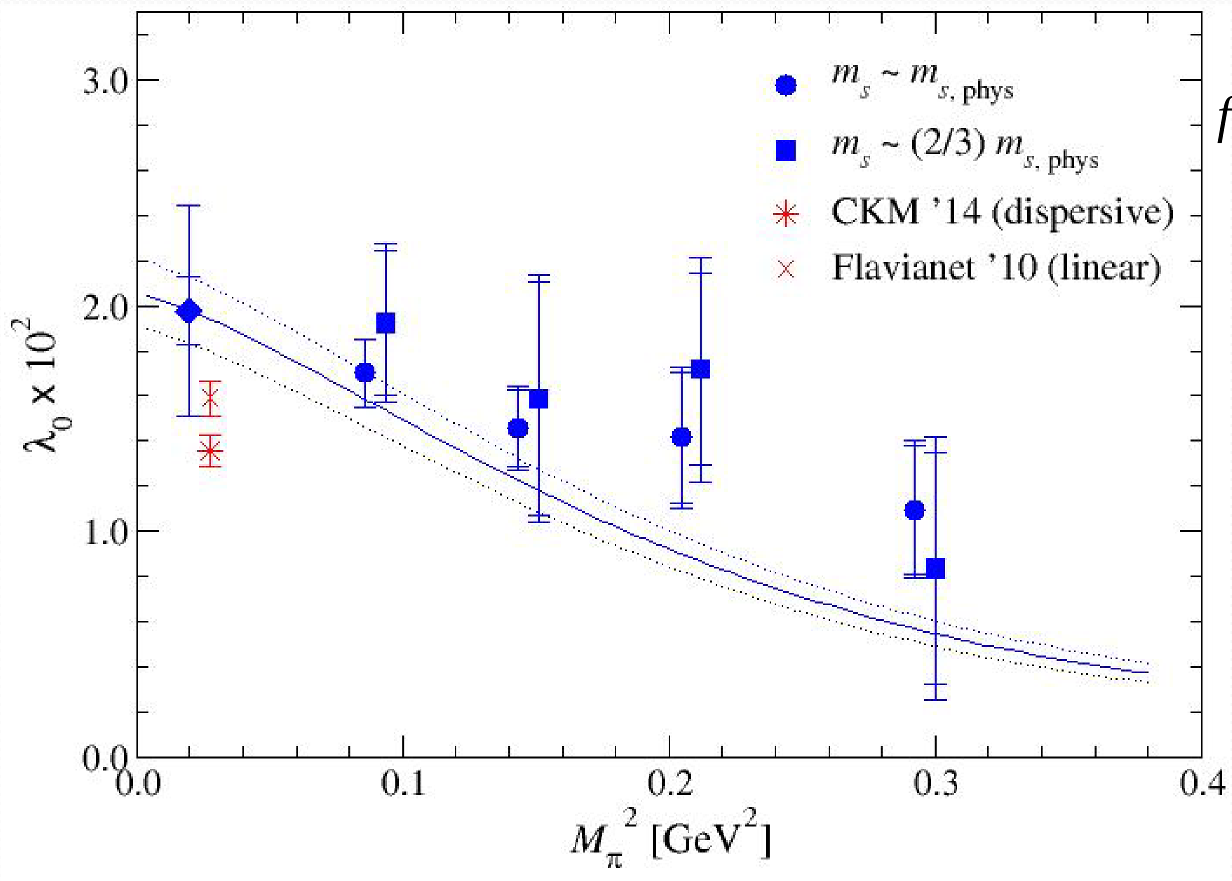
  total nontrivial correction = dominated by NLO $2L_9 t / F^2$

- analytic terms dominate NLO and NNLO contributions

  cancellations b/w loop corrections ?

FF shape

slope of $f_0(t)$



$$f_0(t) = f_+(0)\left\{1 + \frac{\lambda_0'}{M_\pi^2}t + O\left(t^2\right)\right\}$$

$$\lambda_0' = \frac{M_{\pi,\text{phys}}^2}{f_+(0)} \frac{df_0(t)}{dt}\bigg|_{t=0}$$

- $\lambda_0' \times 10^2 = 1.98(15)(44)$

  consistent wit exp't

  1.37(9) (CKM 2014)

- uncertainty larger than λ_+' : L_i (↑), NN$^{3(+)}$LO (↓)