

# Towards the continuum limit of the critical endline of finite temperature QCD

Yoshifumi Nakamura

RIKEN Advanced Institute for Computational Science

in collaboration with

X.-Y. Jin, Y. Kuramashi, S. Takeda & A. Ukawa

15 Jul. 2015,  
Lattice 2015 in Kobe

# Motivation

- Critical endpoint (CEP) obtained with staggered and Wilson type fermions is inconsistent. → **Results in the continuum limit is necessary**

$m_\pi$  at the endpoint at  $\mu = 0$  (bottom-left corner of Columbia plot)

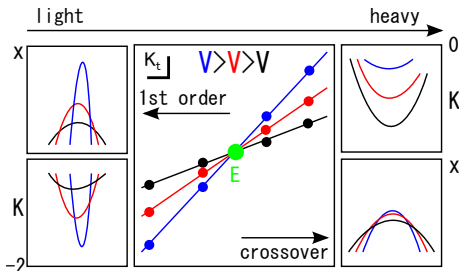
$N_t$	action	$m_\pi^E$ [MeV]	
4	unimproved staggered	260	de Forcrand, Philippsen '07
6	unimproved staggered	150	
4	p4-improved staggered	70	Karsch et al. '03
6	stout-improved staggered	$\lesssim 50$	Endrődi et al. '07
6	HISQ	$\lesssim 45$	Ding et al. '11
4	unimproved Wilson	$\sim 1100$	Iwasaki et al. '96

- $N_f = 3$  study is a stepping stone
  - curvature of critical surface → **talk by S. Takeda**
  - to the physical point → **we didn't include it because of highly preliminary**

We determine CEP on  $m_l = m_s$  line with clover fermions in the continuum limit and the critical endline around  $m_l = m_s$

## Method to determine CEP (kurtosis intersection)

- determine the transition point (peak position of susceptibility)
- determine kurtosis at transition point at each spatial lattice size
- find intersection point of kurtosis by fit,  $K_E + aN_l^{1/\nu}(\beta - \beta_E)$

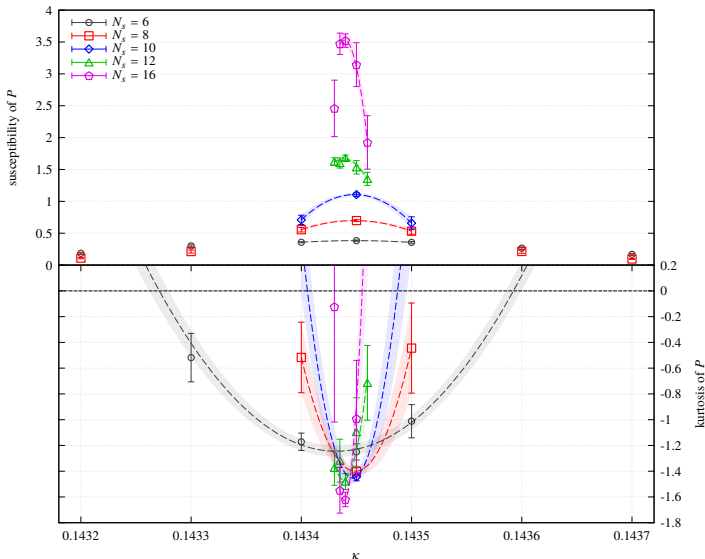


- interpolate/extrapolate  $\sqrt{t_0} m_{PS,t}$  measured at transition point to  $\beta_E$
- extrapolate  $\sqrt{t_0} m_{PS,E}$  to the continuum limit
- use scale determined from Wilson flow  $1/\sqrt{t_0} = 1.347(30)$  GeV [Borsanyi et al. '12]

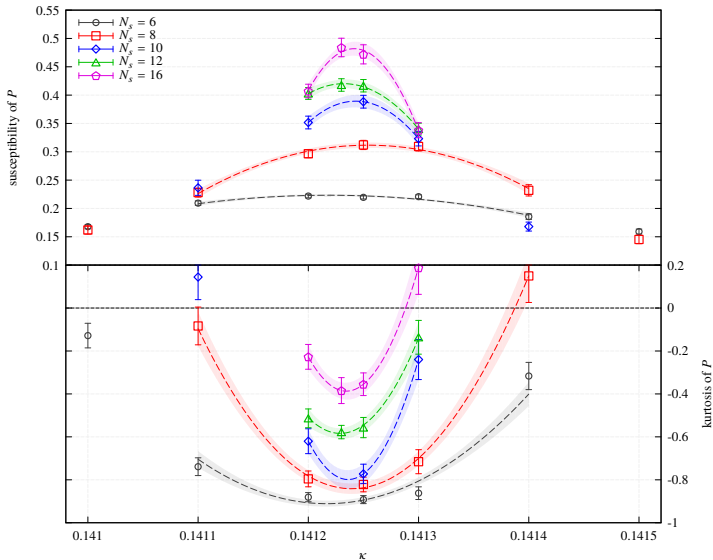
# Simulations

- action: Iwasaki gluon +  $N_f = 3$  clover (non perturbative  $c_{SW}$ , degenerate)
- observables
  - gauge action density,  $G$
  - plaquette,  $P$
  - Polyakov loop,  $L$
  - chiral condensate,  $\Sigma$
  - and their higher moments
- temporal lattice size  $N_t = 4, 6, 8$ 
  - statistics: O(100K) traj
- preliminary  $N_t = 10$ 
  - statistics: O(1K) traj

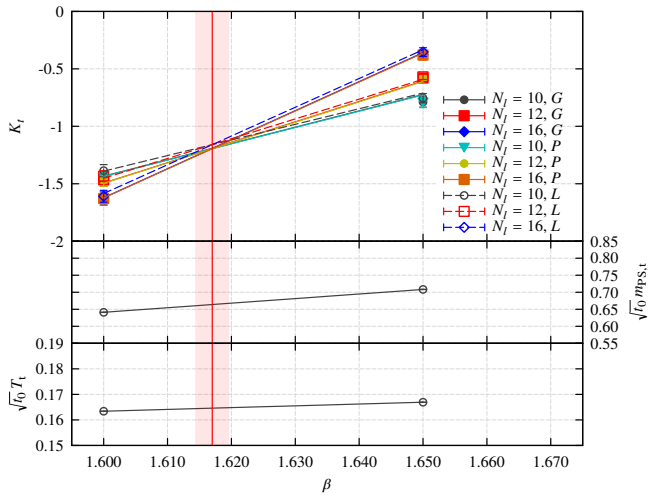
# plaquette at $\beta = 1.60, N_t = 4$



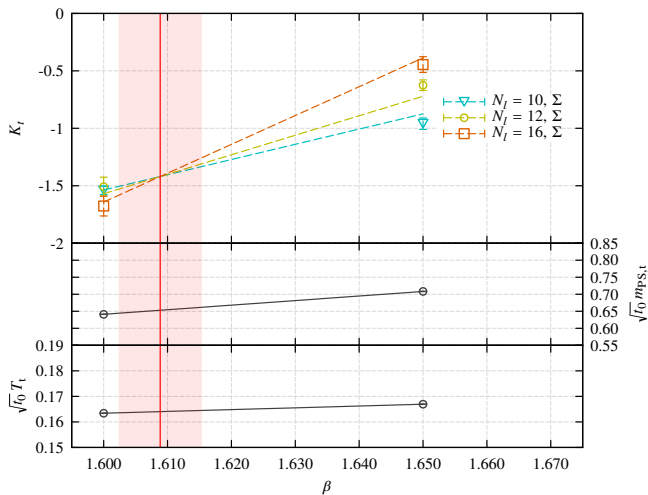
# plaquette at $\beta = 1.65$ , $N_t = 4$



# Kurtosis intersection at $N_t = 4$

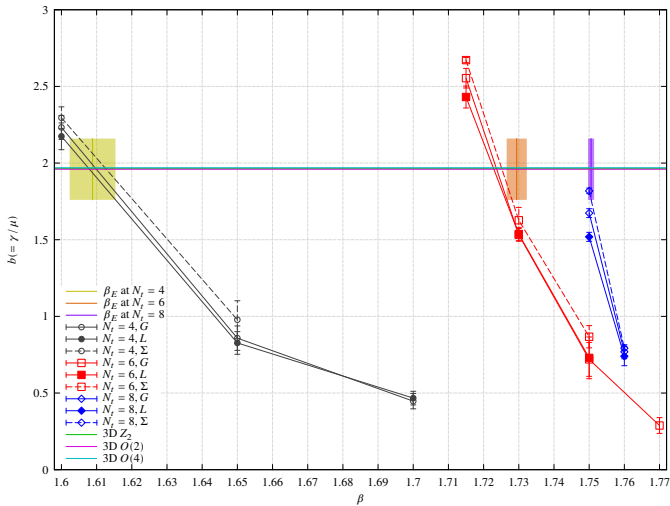


# Kurtosis intersection at $N_t = 4$



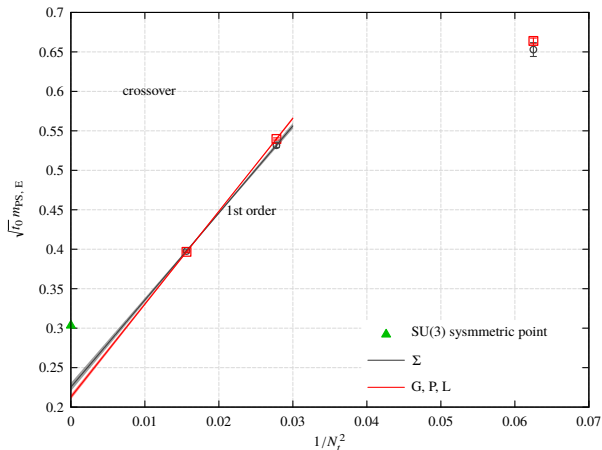


# $\gamma/\nu$ v.s. $\beta$



$$\chi_{\max} = a N_l^{\gamma/\nu}$$

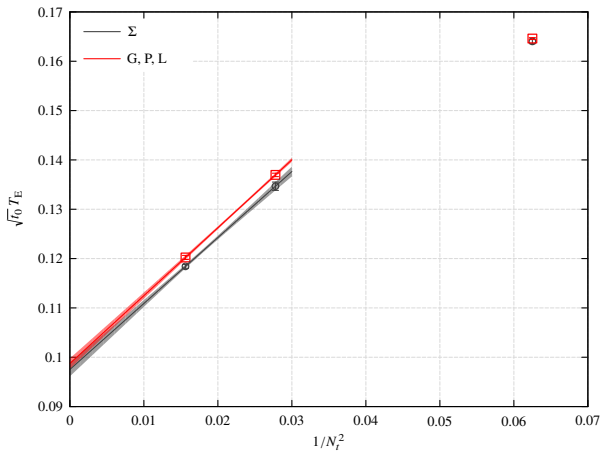
# continuum extrapolation for $\sqrt{t_0}m_{\text{PS,E}}$



$$\blacktriangle : \sqrt{t_0}m_{\text{PS}}^{\text{phy;sym}} = \sqrt{t_0} \sqrt{(m_\pi^2 + 2m_K^2)/3} \sim 0.305$$

$$m_{\text{PS,E}} = 304(7)(14)(7) \text{ MeV}$$

# continuum extrapolation for $\sqrt{t_0}T_E$



$$T_E = 131(2)(1)(3) \text{ MeV}$$

## Summary at $N_t = 4, 6, 8$

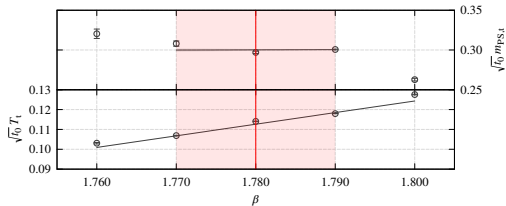
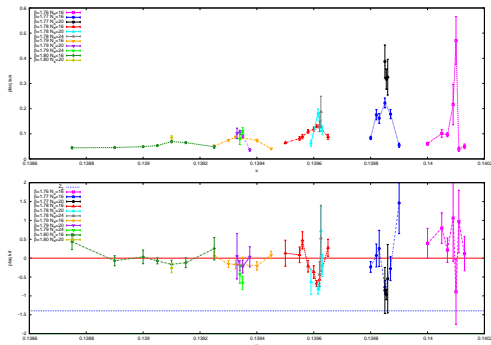
- kurtosis intersection analysis is consistent with  $\chi_{\max}$  analysis
- results at  $N_t = 4$  is out of scaling region
- $\sqrt{t_0}m_{\text{PS,E}}$  in the continuum limit is smaller than the SU(3) symmetric point,

$$m_{\text{PS,E}}/m_{\text{PS}}^{\text{phys,sym}} = 0.739(17)(34)(17)$$

- further studies at larger temporal sizes to obtain conclusive results are needed

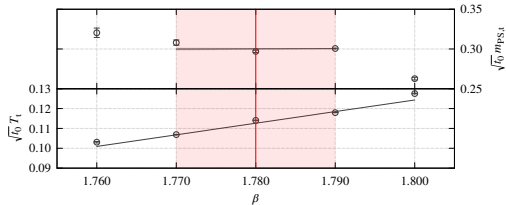
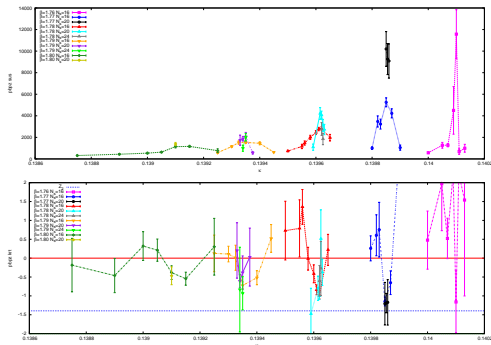
Phys. Rev. D 91, 014508 (2015)

# $P$ at $N_t = 10$ (preliminary)



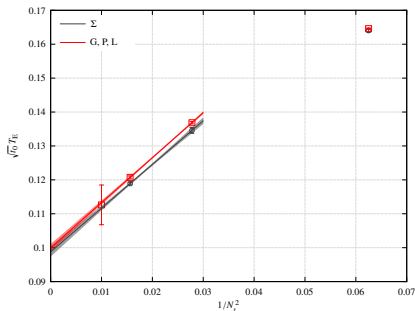
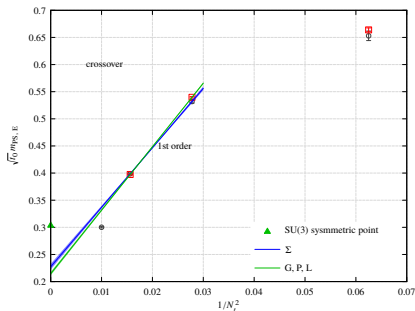
assuming  $\beta_E = 1.78(1)$

# $\Sigma$ at $N_t = 10$ (preliminary)



assuming  $\beta_E = 1.78(1)$

# continuum extrapolation and results at $N_t = 10$ (preliminary)



- assuming  $\beta_E = 1.78(1)$  at  $N_t = 10$
- excluding results at  $N_t = 10$  from continuum extrapolation
- $T_E$  would not change very much
- $m_{PS,E}$  may become smaller than results at smaller  $N_t$

# Summary

We have investigated the critical endpoint of QCD with clover fermions and determined the critical endpoint by using the intersection points of the Binder cumulants and extrapolated to the continuum limit

- $T_E$  in the continuum limit would not change very much

$$T_E \approx 130 \text{ MeV}$$

- $m_{\text{PS},E}$  in the continuum limit may become smaller than results at smaller  $N_t$

$$m_{\text{PS},E} < 304(7)(14)(7) \text{ MeV?}$$

$$m_{\text{PS},E}/m_{\text{PS}}^{\text{phys,sym}} < 0.739(17)(34)(17)?$$

- we are doing further studies with high statistics at larger temporal sizes to obtain conclusive results