Exotic Quantum Critical Points with Staggered Fermions

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Computational work done using the Open Science Grid and local Duke cluster.

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Introduction

- Symmetries can forbid fermion mass terms.
- Conventionally mass terms are introduced via Spontaneous Symmetry Breaking signalled through a non-zero fermion bilinear condensate.
- Can symmetry-preserving interactions generate fermion masses dynamically without a fermion bilinear condensate ?
- We have a lattice model with staggered fermions in which a four-fermion interaction makes this possible.

Our Lattice model

Staggered fermion action for two flavors ψ^1 and ψ^2

$$S = S_0 + S_I$$

- $S_0 = \sum_{x,y} \left\{ \overline{\psi^1_x} D_{x,y} \psi^1_y + \overline{\psi^2_x} D_{x,y} \psi^2_y \right\}$

  $$D_{x,y} = \frac{1}{2} \sum_{\hat{\alpha}} \left(\delta_{x,y+\hat{\alpha}} - \delta_{x,y-\hat{\alpha}} \right) \eta_{\alpha,x} \ .$$
- $S_I = -U \sum_x \overline{\psi^1_x} \psi^1_x \overline{\psi^2_x} \psi^2_x$

In addition to the usual discrete space-time symmetries[1], the action has a continuous $SU(4)$ symmetry.

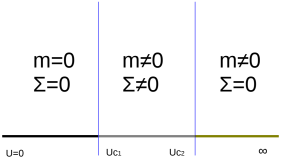
[1] M. Golterman and J Smit. Selfenergy and Flavor Interpretation of Staggered Fermions. Nucl.Phys., B245:61, 1984.

Phase Transition

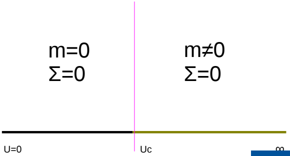
- Expectation from previous work →

  Mean field Calc by Lee,Shigemitsu, Shrock (1990),

  Hasenfratz, Neuhaus (1989)



- We seem to observe a direct 2nd order phase transition.
  Weak and strong coupling phases have the same symmetries without any spontaneous symmetry breaking [1]



[1] K. Slagle,Y.-Z. You, C. Xu, Phys. Rev. B 91, 115121 (2015)

The Fermion Bag approach [1,2]

$$Z = \int [d\overline{\psi} d\psi]\, e^{-S_0} e^{U \sum_x \overline{\psi}^1_x \psi^1_x \overline{\psi}^2_x \psi^2_x}$$

$$= \int [d\overline{\psi} d\psi]\, e^{-S_0} \prod_x e^{U \overline{\psi}^1_x \psi^1_x \overline{\psi}^2_x \psi^2_x}$$

$$= \int [d\overline{\psi} d\psi]\, e^{-S_0} \prod_x \left(1 + U \overline{\psi}^1_x \psi^1_x \overline{\psi}^2_x \psi^2_x\right)$$

$$= \sum_{\{m_x\}} \int [d\overline{\psi} d\psi]\, e^{-S_0} \prod_x \left(U \overline{\psi}^1_x \psi^1_x \overline{\psi}^2_x \psi^2_x\right)^{m_x}$$

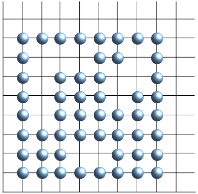


Figure : Fermion Bag configuration

Assigning $m_x = 0$ or 1 to each lattice site

- $m_x = 0 \equiv$ **free sites**
- $m_x = 1 \equiv$ **monomers**

Fermion Bag $\equiv$ Set of disconnected free sites

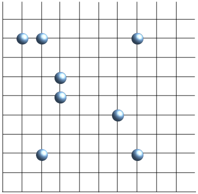
[1] S. Chandrasekharan - The Fermion bag approach to lattice field theories (2010)

[2] S. Chandrasekharan and A. Li - The generalized fermion bag approach (2011)

Weak coupling limit

$$Z = Det(D) \sum_{\{m_x\}} U^k Det(D_1^{-1}) Det(D_1^{-1})$$

where D^{-1} is a $k \times k$ matrix of propagators.

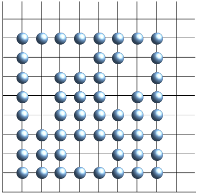


Strong coupling limit

$$Z = \sum_{\{m_x\}} U^k Det(W_1) Det(W_1)$$

where W_1 is a $(V - k) \times (V - k) matrix$.

Can show that each determinant can be expressed as a square of smaller determinants.



Observables

- Average monomer density :

$$\rho = \frac{1}{V} \sum_x \langle \overline{\psi_x^1} \psi_x^1 \overline{\psi_x^1} \psi_x^1 \rangle$$

- Bosonic correlators :

$$C_1 = \langle \overline{\psi_0^1} \psi_0^1 \overline{\psi_{\frac{L}{2}}^1} \psi_{\frac{L}{2}}^1 \rangle$$

$$C_2 = \langle \overline{\psi_0^1} \psi_0^1 \overline{\psi_{\frac{L}{2}}^2} \psi_{\frac{L}{2}}^2 \rangle$$

- Fermionic correlator :

$$F(x, y) = \langle \overline{\psi_x^1} \psi_y^1 \rangle$$

Extracting the condensate

- Σ is usually defined as: $\Sigma = \lim_{m\to 0} \lim_{V\to\infty} \langle \overline{\psi}\psi \rangle$
- With massless fermions, we can instead compute the bosonic correlator:

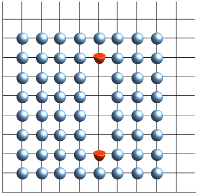
$$C = \langle \overline{\psi^1_0} \psi^1_0 \overline{\psi^1_{\frac{L}{2}}} \psi^1_{\frac{L}{2}} \rangle$$

- We expect :

$$\lim_{V\to\infty} C \sim \Sigma^2$$

Fermion Mass

- Small $U \rightarrow$ Irrelevant coupling $\implies$ massless fermions
- Very large U, $F(x,y) \sim e^{-(y-x)\ln U} \sim e^{-m(y-x)} \implies$ massive fermions
- Exponential decay of all correlators indicates a zero condensate at very large U.



Computational approach

Configuration weights and observables are functions of determinants of propagators between monomers.

Previous approach[1] :

- Configuration weights are computed at each step.
- Re-tracement results in recalculation of weights.
- Waste of computer time if acceptance is low.

[1] Ayyar, Chandrasekharan, Phys. Rev. D 91,065035, March 2014.

New approach :

- We compute all possible configuration weights of perturbations at the start and store them.
- Weights of subsequent configurations can be computed from these.
- Once perturbations are large, repeat the setup.
- Advantages: Faster updates and no re-tracement costs.
- Drawbacks : Memory requirement rises, so we need to update the lattice in sub-blocks.

Preliminary results

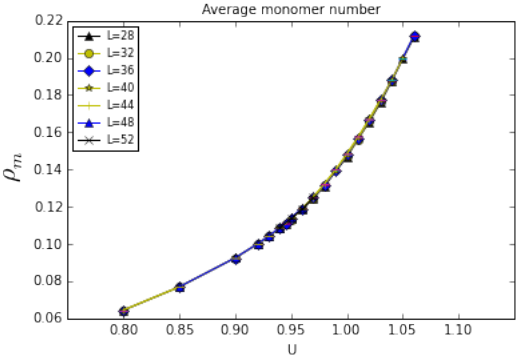


Figure : Average monomer density rises sharply without any discontinuity

Have used Open Science Grid (OSG) for computations.

- Average monomer density rises sharply without any discontinuity
- Hints of a second order phase transition

Bosonic Correlator vs U

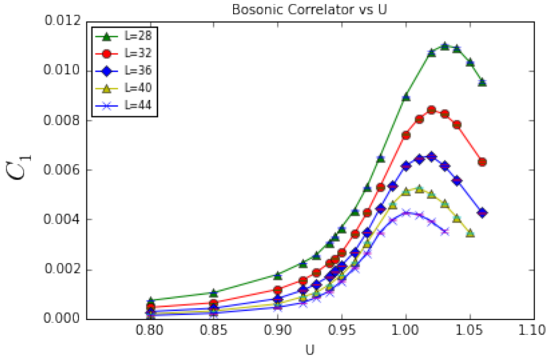


Figure : Correlators peak for a certain value of U and then decay.

- Calculations on lattices upto size 60^3
- Correlator increases with coupling U.
- It reaches a maximum for intermediate U and then decreases.

Bosonic Correlator vs L

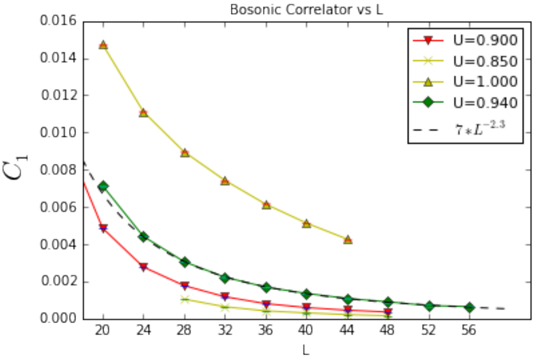
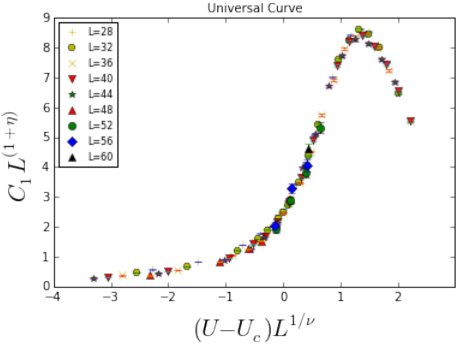


Figure : Correlators decay exponentially indicating a zero condensate

- Correlators decay $\Longrightarrow$ *condensate* $= 0$
- Near $U_c = 0.94$, C decays like a power law

Evidence that U_c is a 2^{nd} order critical point

For a second order transition, we expect: $C = L^{1+\eta} f\left[(U - U_c)L^{\frac{1}{\nu}}\right]$



- Fitting to find the critical exponents seems quite difficult due to the absence of the condensate.
- Preliminary calc. of Critical exponents:

$$\eta = 1.0(1), \ \nu = 1.2(1), \\ U_c = 0.946(5)$$

4 D results

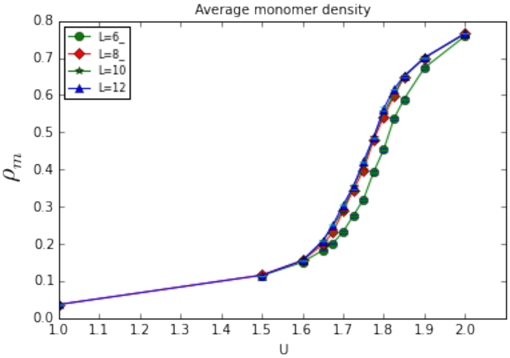


Figure : Average monomer density rises sharply without any discontinuity

- Results qualitatively similar to 3D. Correlator shows a peak around U=1.75
- Average monomer density rises sharply around U=1.75 without any discontinuity.
- Hints of a second-order phase transition.

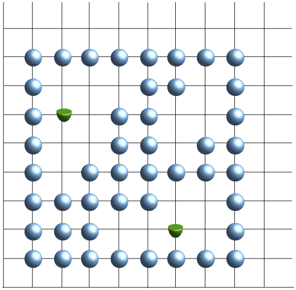
Conclusions

- Our lattice model suggests that fermions can acquire a mass, but without a fermion bilinear condensate(no Spontaneous Symmetry Breaking).
- In 3D, the transition from massless to massive phase is second order. Hence, we could have an interesting 3D continuum field theory.
- In 4D, preliminary results point to the possibility of a similar second order transition. This could be interesting for particle physics.

THANK YOU

Back-up slides

$$\langle \overline{\psi_x^1}\psi_x^1\overline{\psi_y^1}\psi_y^1\rangle = \frac{1}{Z}\det(W_1)\det(W_2)$$



For $\det \neq 0, N_{even} = N_{odd}$

Can show that,

$\det(W_1) \neq 0 \implies \det(W_2) = 0;$

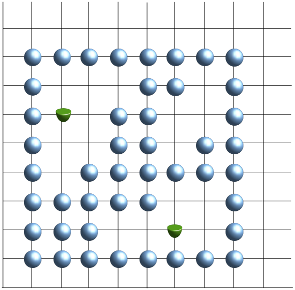
$\det(W_2) \neq 0 \implies \det(W_1) = 0$

Fcorr is zero

Fermionic correlator

$$\langle \overline{\psi_x^1} \psi_y^1 \rangle = \frac{1}{Z} \det(W) {W^{-1}}_{x,y}$$

- W can be written in terms of bags in block diagonal form.
- If x and y belong to the different bags, the correlator is zero.



Similarily, for bosonic correlator $\langle \overline{\psi_x^1} \psi_x^1 \overline{\psi_y^1} \psi_y^1 \rangle$: Can also show that we need a path of connecting free sites.