Testing a generalized cooling procedure in the complex Langevin simulation of chiral Random Matrix Theory

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Outline

- Recent development of complex Langevin method (CLM) enables us to study some parameter-regimes of QCD.
  - heavy dense QCD [Seiler, Sexty, Stamatescu('12)]/full QCD at high temperature [Sexty ('14)]

- Observation of Silver Blaze phenomena (baryon number density is zero up to $m_N/3$) will be a milestone in the study of finite density lattice QCD.

- It is still difficult to apply CLM to small mass and low T
  - Dirac low-modes cause a problem [Mollgaard, Splittorff('13)]

- The purpose of this work is to develop a method to overcome this problem.

Problem in complex Langevin simulation for QCD at low temperature at small mass

Complex Langevin method(CLM) for sign problem

- Stochastic quantization with Langevin equation [Parisi-Wu('81)]

$$\frac{dx}{d\tau} = -\frac{\partial S}{\partial x} + \eta$$

- Its application to theories with complex action [Parisi('83), Klauder('83)]
  - originally real variables are extended to complex
  - this may cause ill-convergence problem

- CLM is justified if some conditions are satisfied [Aarts, et. al. PRD81, 054508('10), EPJC71,1756('11)].
  - fast fall-off of the probability distribution in the imaginary direction
  - regularity of drift terms [Nishimura, Shimasaki, PRD92 (2015) 1, 011501 arXiv:1504.08359 [hep-lat]] (Shimasaki's talk)

Excursion problem and "gauge cooling"

- Large probability in the imaginary direction leads to wrong convergence (excursion problem).

- Gauge cooling [Seiler, Sexty, Stamatescu('12), Sexty ('14)]
  - complexification of link variable SU(3) -> SL(3,C)
  - "unitarity norm" : measure of the distance from SU(3) matrices

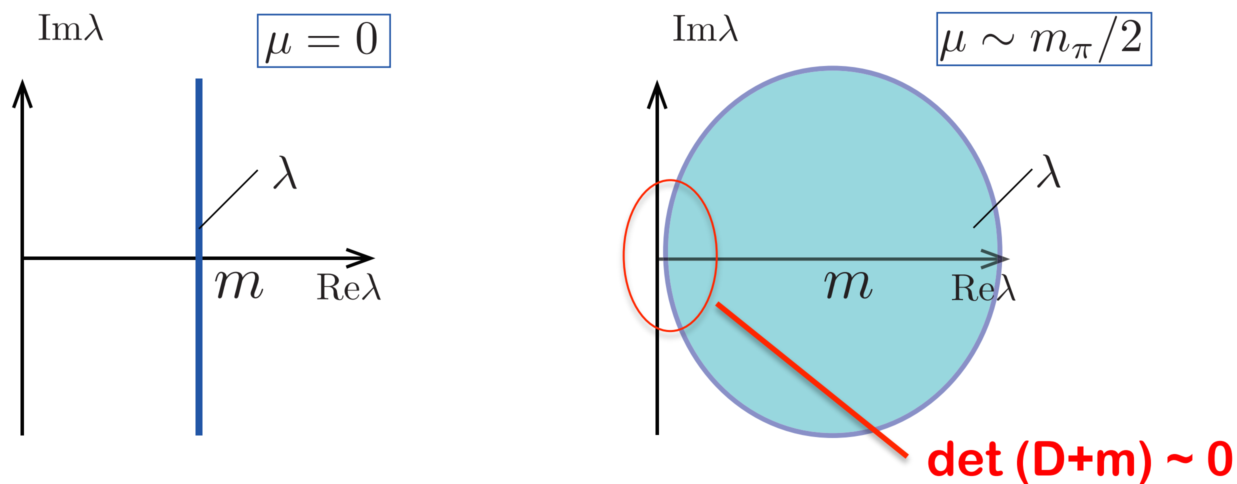
$$N = \sum_{x,\mu} \mathrm{tr}[U_{x,\mu}^{\dagger} U_{x,\mu} + (U_{x,\mu}^{\dagger})^{-1} U_{x,\mu}^{-1} - 2]$$

  - suppress this norm by using complexified gauge symmetry, i.e. SL(3,C).

$$U_{x,\mu} \rightarrow V_x^{-1} U_{x,\mu} V_{x+\mu}$$

A difficulty in QCD at low-T and small mass

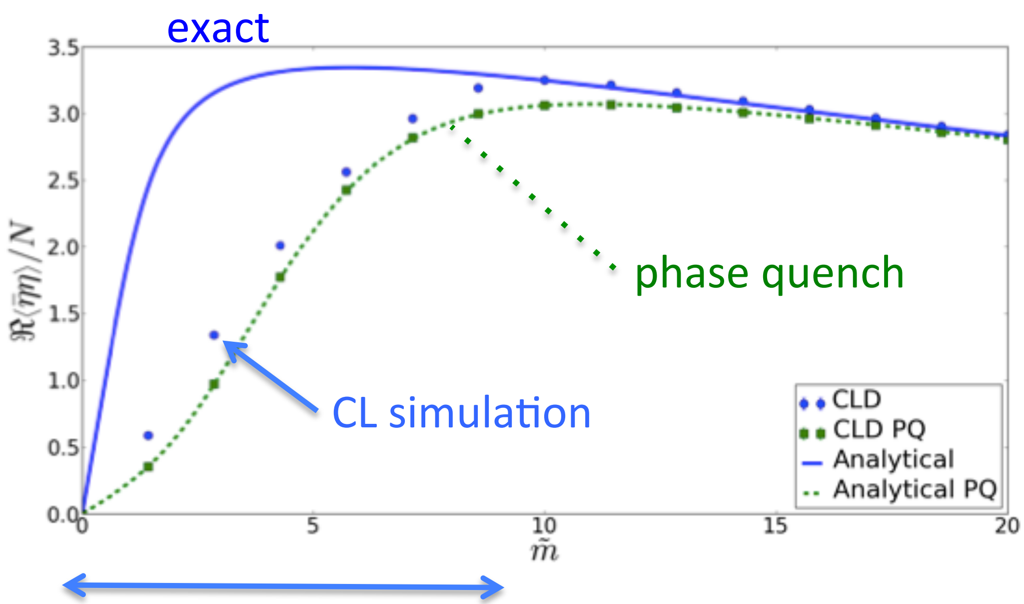
- Justification of CLM requires the regularity of drift terms
- Dirac low-modes cause
  - singularity in the fermion drift term : violation of the justification condition
  - closely related to Silver Blaze problem : unphysical increase of the baryon number density at $m_\pi/2$



Schematic figure of eigenvalues of *D+m*

Example in Random matrix theory

$$Z = \int d\Phi d\Psi [\det(D+m)]^{N_f} e^{-S_b},$$



Chiral condensate [Mollgaard, Splittorff, PRD88 (2013), 11,116007]
N_f=2, N = 30, μ=2/Sqrt(N), m~=m N

det(D+m) close to zero

This problem will happen in QCD at low T and small mass

A new method to overcome the problem caused by Dirac low-modes

Generalization of “gauge cooling”

- “Cooling” procedure can be generalized for
  - any theories with symmetries as large as degrees of freedom of the system
  - any norms suitable to control causes of wrong convergences.

- *We propose new types of norms to overcome problems caused by Dirac low-modes.*
  - *exploring suitable norms and its feasibility in RMT*
  - *application to QCD*

Random matrix theory

$$Z = \int d\Phi d\Psi [\det(D+m)]^{N_f} e^{-S_b} \qquad D = \begin{pmatrix} 0 & X \\ Y & 0 \end{pmatrix}$$

- Bosonic part

$$S_b = 2N \mathrm{Tr}[\Phi_1^2 + \Phi_2^2 + \Psi_1^2 + \Psi_2^2] \qquad \Phi_i = \Phi_i^\dagger, \Psi_i = \Psi_i^\dagger$$

- Fermionic part

$$X = e^{\mu}\Phi_+ + e^{-\mu}\Psi_+, \qquad \Phi_\pm = \Phi_1 \pm i\Phi_2,$$

$$Y = -e^{-\mu}\Phi_- - e^{\mu}\Psi_-, \qquad \Psi_\pm = \Psi_1 \pm i\Psi_2.$$

- Symmetry

$$\begin{cases} \Phi_+ \to \Phi'_+ = g\Phi_+ h^{-1} \\ \Phi_- \to \Phi'_- = h\Phi_- g^{-1} \end{cases} \begin{cases} \Psi_+ \to \Psi'_+ = g\Psi_+ h^{-1} \\ \Psi_- \to \Psi'_- = h\Psi_- g^{-1} \end{cases}$$

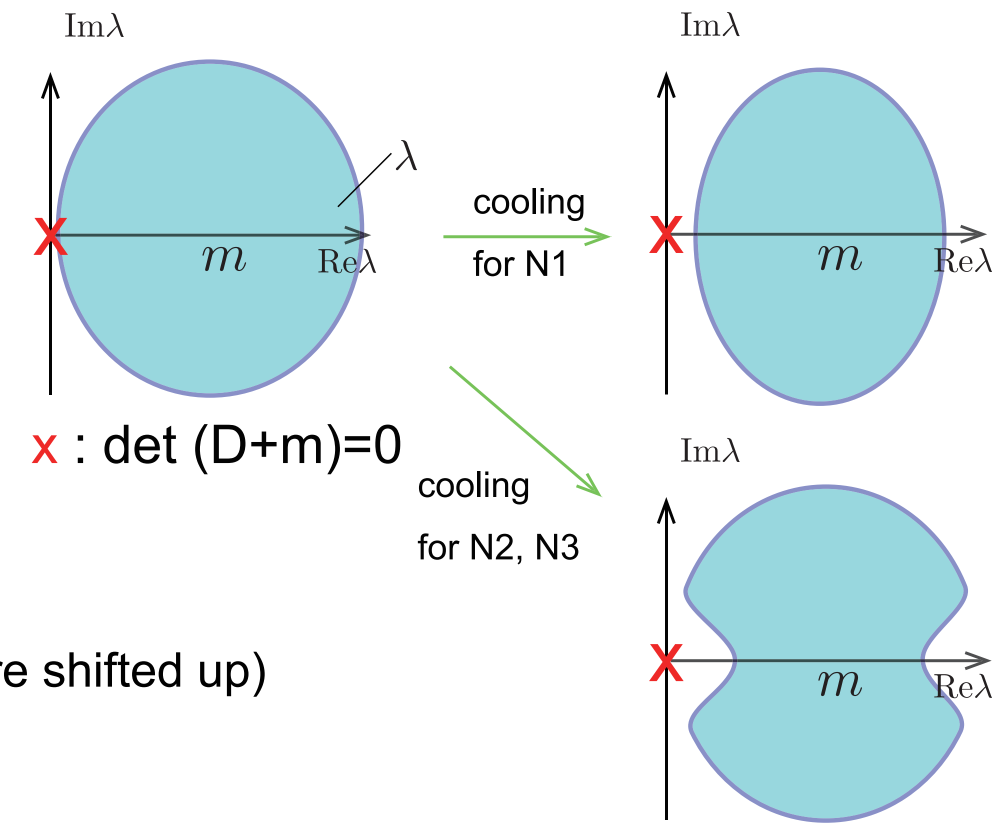
g and h originally belong to SU(N), then extended to SL(N,C)

This extended symmetry allows us to perform cooling procedure.

New norms to control logarithmic singularity

- Type 1 : anti-hermiticity

$$N_1 = \mathrm{tr}(D + D^\dagger)^2$$



- Type 2 : M^+M (M=D+m)

$$N_2 = \mathrm{tr} e^{-\xi \, \mathrm{tr} M^\dagger M}$$

(eigenvalues of M^+M smaller than $1/\xi$ are shifted up)

- Type 3 : use of some evs ($M^+M \, v = \alpha \, v$)

$$N_3 = \mathrm{tr} e^{-\xi \sum_i^n \alpha_i}$$

Norms to control logarithmic singularity

- New norms (M=D+m)

$$N_1 = \mathrm{tr}(D + D^\dagger)^2$$

$$N_2 = \mathrm{tr} e^{-\xi \, \mathrm{tr} M^\dagger M}$$

$$N_3 = \mathrm{tr} e^{-\xi \sum_i^n \alpha_i}$$

- Hermiticity norm : an analog of the unitarity norm in lattice QCD

$$N_H = \mathrm{tr}((\Phi_+ - \Phi_-^\dagger)(\Phi_+ - \Phi_-^\dagger)^\dagger + (\Phi \to \Psi))$$

- Total norm

$$N = (1-r)N_i + rN_H \qquad 0 \le r \le 1 \quad \text{tunable parameter}$$

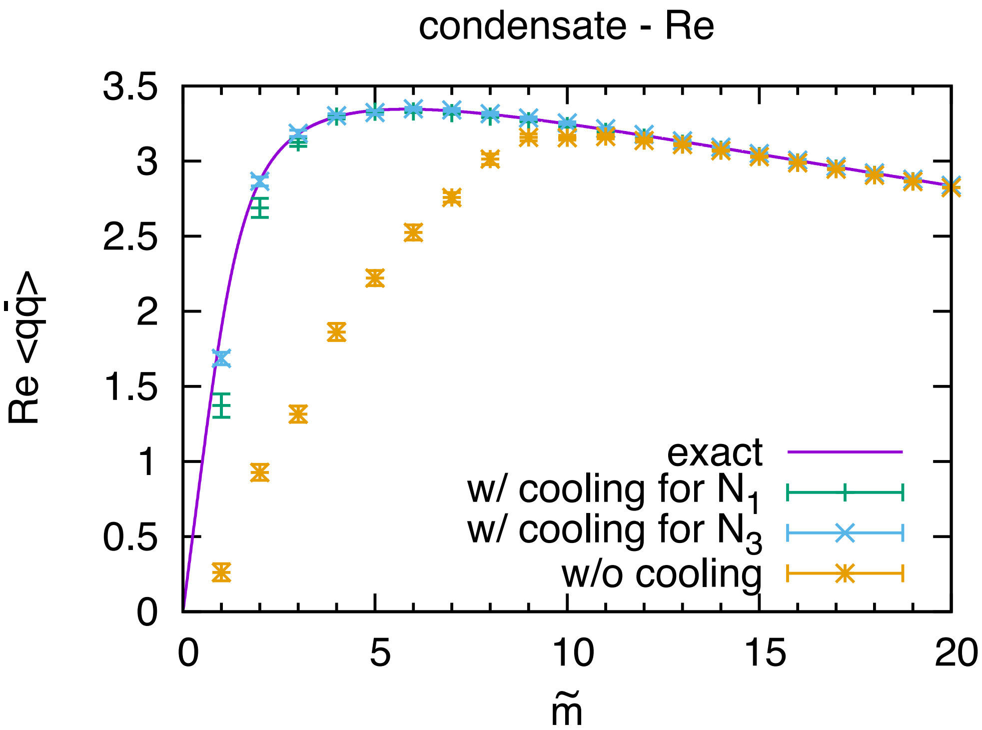
Setup for Random matrix theory

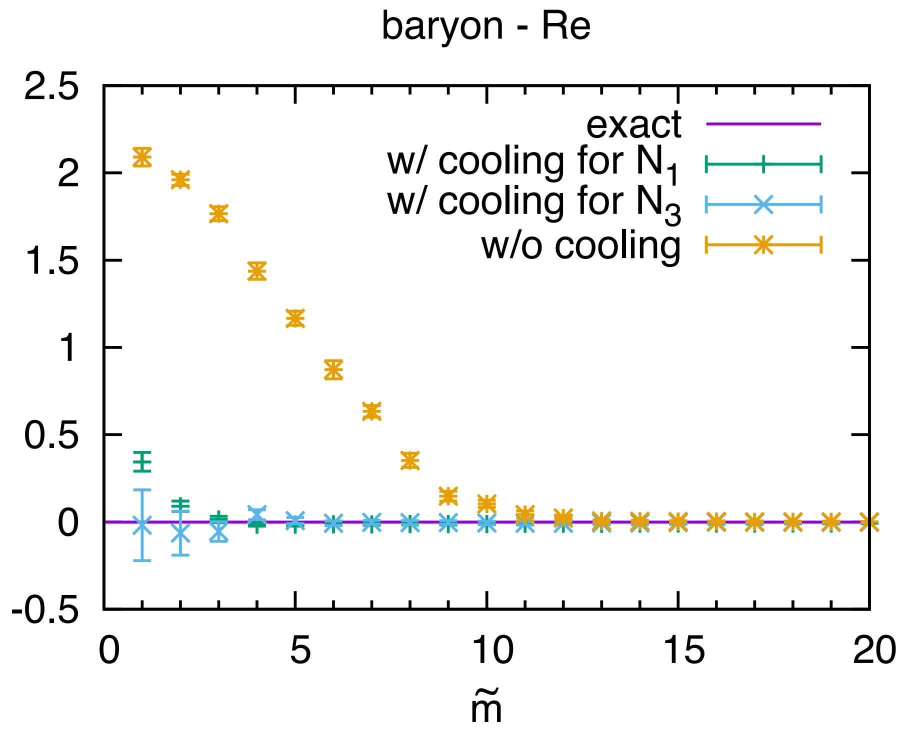
- Physical setup (same as in Mollgaard & Splitorff(2013))
  - N = 30, Nf = 2, μ = 2/Sqrt(N)

- Setup for Langevin step
  - dtau = 1.d-4, # of steps = 50 000
  - total Langevin time = 5.0
  - measurement for each 100 Langevin steps.

- Setup for gauge cooling
  - 10 times cooling after each Langevin step
  - ξ=300
  - r=0 for N_1 and r=0.01 for N_3

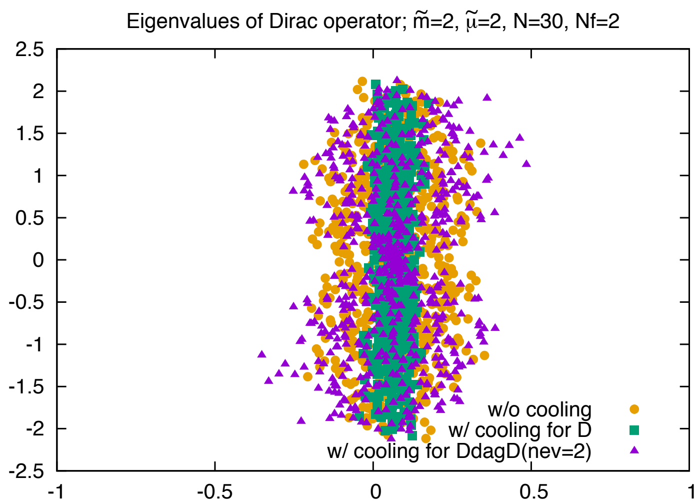
Result - chiral condensate and baryon number density



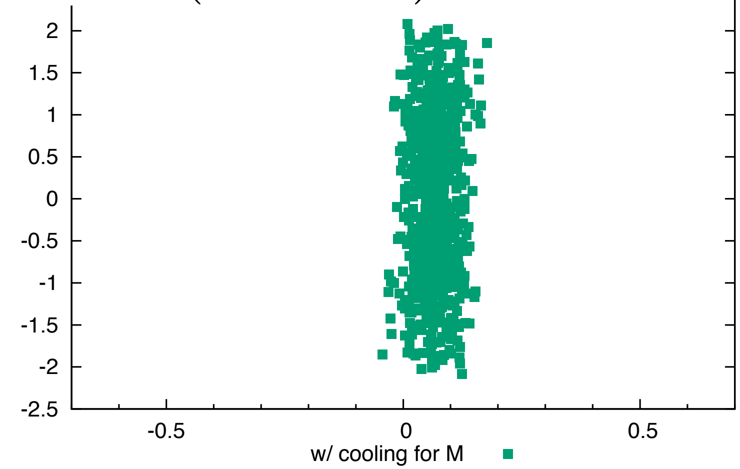


New types of cooling works well even for quite small mass.

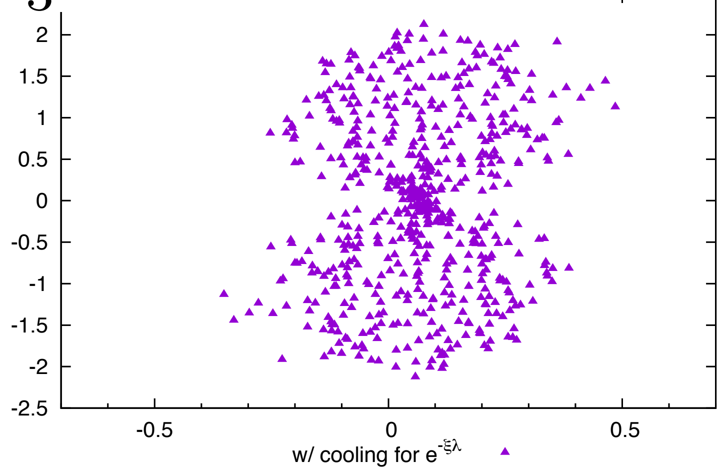
Result – eigenvalues of *D*+*m*



$$N_1 = \mathrm{tr}(D + D^\dagger)^2$$



$$N_3 = \mathrm{tr} e^{-\xi \sum_i^n \alpha_i}$$



New types of cooling successfully removes problematic eigenvalues in a gauge invariant manner.

QCD at low temperature with light quarks

- Physical setup
  - Nx = Ny = Nz = 2, Nt=20, beta = 5.7
  - staggered fermion with Nf = 4

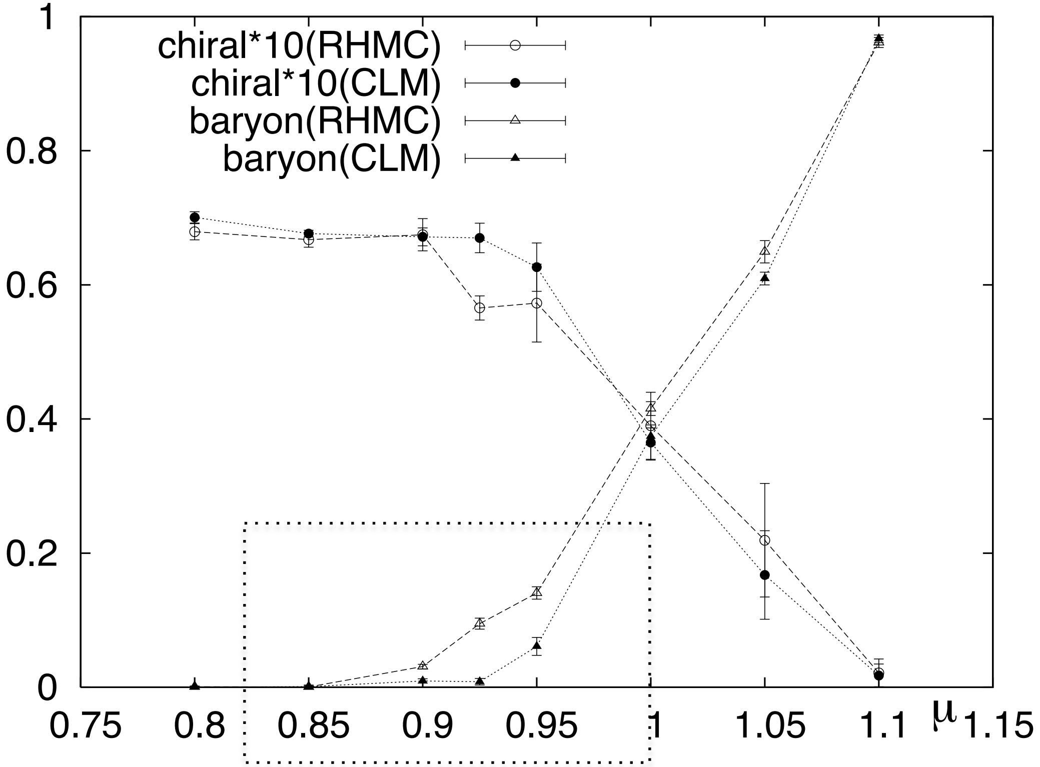
- Setup for Langevin step
  - dtau = 5.d-5, # of steps = 10 000
  - measurement for each 10 Langevin steps.

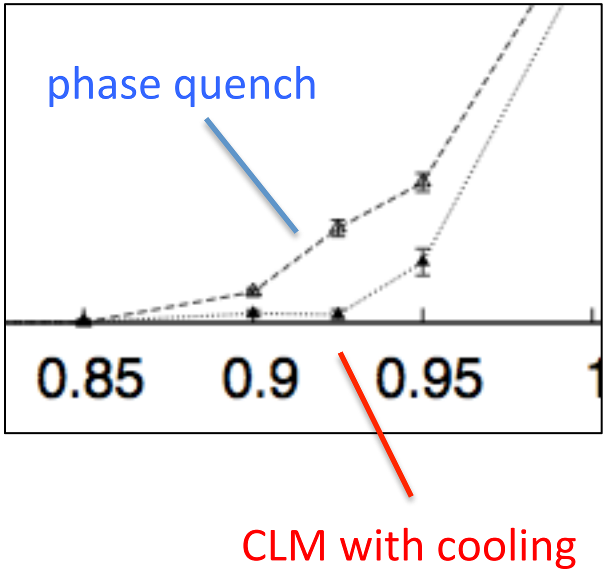
- Setup for gauge cooling
  - 10 times cooling after each Langevin step
  - ξ=100
  - r=0.99 (r=0.999 for some parameters) for N_2

Baryon number density

- m = 0.1

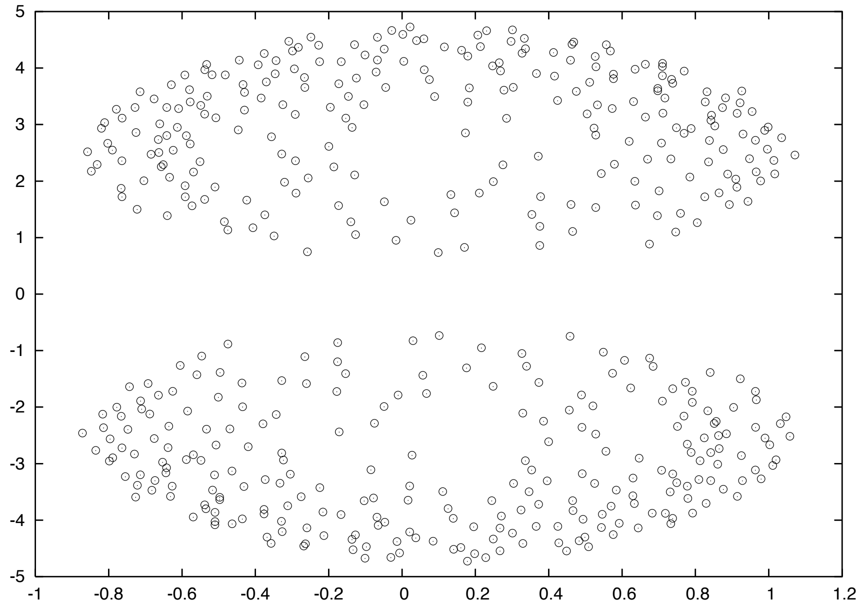
preliminary





- Silver Blaze phenomena seems to be observed.
  - This can be achieved only with the cooling for the unitarity norm.

Snapshot of eigenvalues



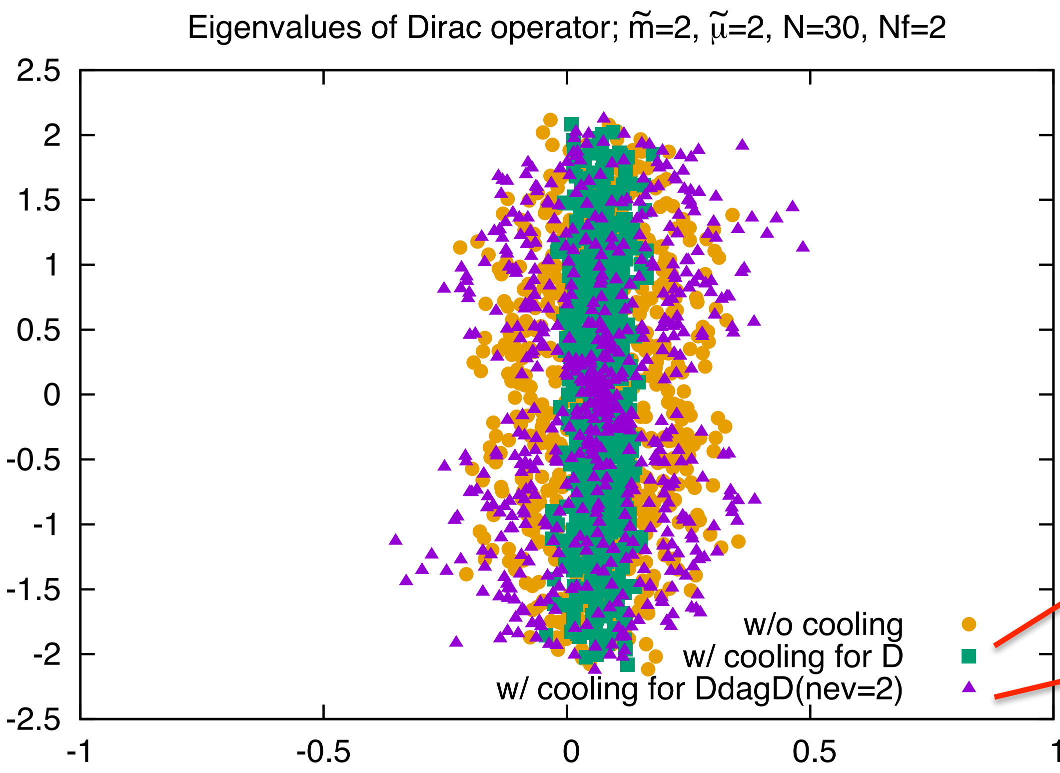
- In the present setup, there is a void near the origin.
- The new types of cooling will be needed when eigenvalues are distributed near the origin (work in progress).

Summary

- We developed new types of cooling procedure to overcome the problem caused by Dirac low modes in the fermion drift.
- They reproduced exacts results even for quite small quark mass in RMT
- They are now being tested for QCD.
  - Silver Blaze phenomena seems to be observed !
- The new technique extends the range of applicability of the complex Langevin method to finite density QCD at low temperatures with light quarks.

Buck up slides

Result - Dirac eigenvalues



$$N_1 = \mathrm{tr}(D + D^\dagger)$$

$$N_3 = \mathrm{tr} e^{-\xi \sum_i^n \lambda_i}$$

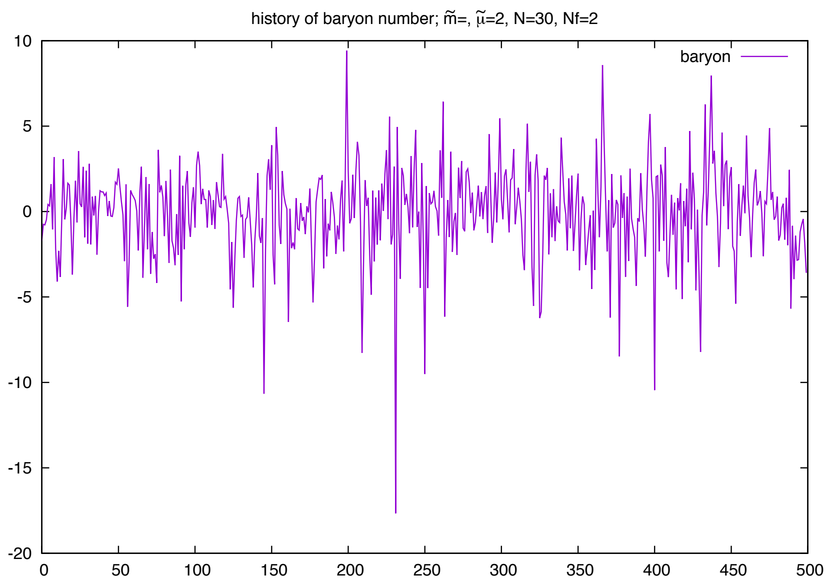
New types of cooling successfully removes problematic eigenvalues in a gauge invariant manner.

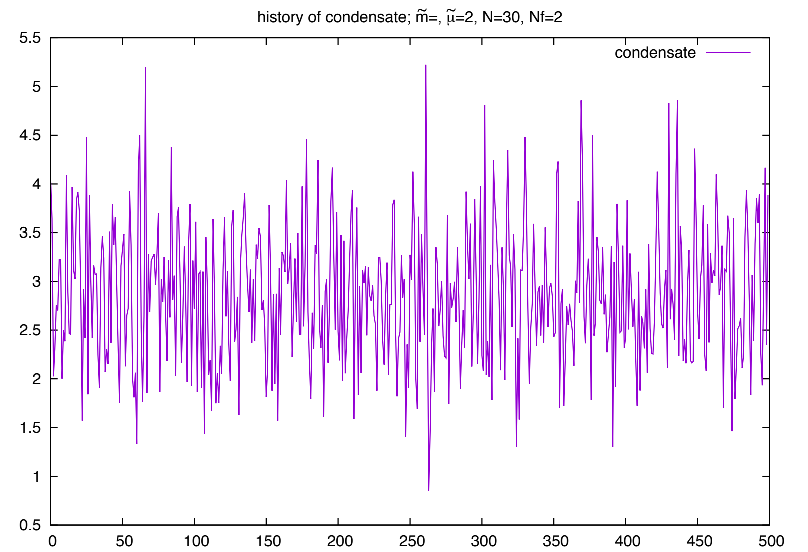
Note on new cooling

- *Eigenvalue distribution is invariant under SL(3,C) transformation.*
- *But, imposing the cooling for each Langevin step, the configurations are changed to reduce the norms.*
- *Note that the gauge cooling does not change expectation values of SL(3,C) invariant(originally gauge invariant) quantities.*

Langevin time history (RMT)

- Left : Baryon Number, Right : chiral condensate





History

- solid : CLM with cooling
- dashed : RHMC for phase quenched model