Mass spectrum of mesons containing charm quarks – continuum limit results from twisted mass fermions

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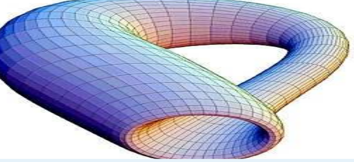


Presentation outline

1. Introduction
2. Lattice setup
3. Results
4. Conclusions and prospects

Charm mesons spectrum

- Several charm-containing mesons known experimentally:
  - ⋆ some of them well established and in good agreement with quark models,
  - ⋆ some of them known with large errors and with masses and/or widths not well predicted by quark models (e.g. D^*_{s0}, D_{s1} tetraquarks?).
- Hence, an *ab initio* investigation highly interesting – lattice.
- Charm physics on the lattice complicated due to the values of the lattice spacing – if too coarse, charm quark mass large in lattice units.
- Nevertheless, with current computing resources many questions can be addressed, including the spectrum of charmed mesons.
- Moreover, charm quarks can be treated as dynamical, so all systematic effects can be controlled with reasonable precision.
- Our aim: compute the spectrum of D (charm-light) mesons, D_s (charm-strange) mesons and charmonium (charm-charm) using fully dynamical twisted mass ensembles generated by ETMC.

Simulation setup – sea sector

- We use dynamical twisted mass configurations generated by ETMC with $N_f = 2+1+1$ dynamical quark flavours [R. Baron et al., 2010, 2011].
- Gauge action – Iwasaki action [Y. Iwasaki, 1985], i.e. $b_1 = -0.331$, $b_0 = 1 - 8b_1$,

$$S_G[U] = \frac{\beta}{3} \sum_x \Big(b_0 \sum_{\mu,\nu=1} \mathrm{Re\,Tr}\big(1 - P^{1\times 1}_{x;\mu,\nu}\big) + b_1 \sum_{\mu\neq\nu} \mathrm{Re\,Tr}\big(1 - P^{1\times 2}_{x;\mu,\nu}\big)\Big).$$

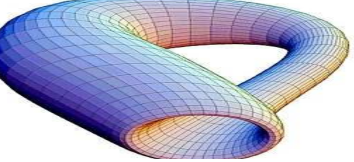
- Wilson twisted mass fermion action for the light sector
  [R. Frezzotti, P.A. Grassi, G.C. Rossi, S. Sint, P. Weisz, 2000-2004]

$$S_l[\psi,\bar\psi,U] = a^4 \sum_x \bar\chi_l(x)\big(D_W + m_{0,l} + i\mu_l\gamma_5\tau_3\big)\chi_l(x),$$

  $\chi_l = (\chi_u,\, \chi_d)$, $m_{0,l}$ and μ_l are the bare untwisted and twisted light quark masses.
- Twisted mass action for the heavy doublet [R. Frezzotti, G.C. Rossi, 2003, 2004]

$$S_h[\psi,\bar\psi,U] = a^4 \sum_x \bar\chi_h(x)\big(D_W + m_{0,h} + i\mu_\sigma\gamma_5\tau_1 + \mu_\delta\tau_3\big)\chi_h(x),$$

  $\chi_h = (\chi_c,\, \chi_s)$, $m_{0,h}$ – bare untwisted heavy quark mass, μ_σ – bare twisted mass with the twist along the τ_1 direction, μ_δ – mass splitting along the τ_3 direction that makes the strange and charm quark masses non-degenerate.
  Renormalized strange and charm quark masses: $m_R^{s,c} = Z_P^{-1}\left(\mu_\sigma \mp (Z_P/Z_S)\mu_\delta\right)$.



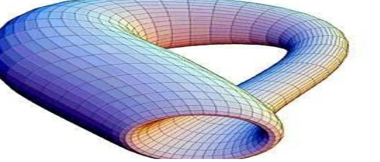
Simulation setup – valence sector

- Light quarks – the same action as in the sea.
- Strange and charm – introduce 2 strange (s, s') and 2 charm (c, c') quark flavours with the action for a single flavour f:

$$D_f = D_W + m_0 + i\mu_f\gamma_5.$$

- We take:
  - ⋆ either $\mu_{s/c} = -\mu_{s'/c'}$ – call this TM setup (however, it is still non-unitary)
  - ⋆ or $\mu_{s/c} = \mu_{s'/c'}$ – call this Osterwalder-Seiler (OS) setup.

  In this way, we avoid the mixing of strange and charm quarks, which would make the computations problematic.
- Formally, the lattice action includes a ghost action that exactly cancels the contributions of the additional valence quarks to the fermionic determinant.
- Such setup still guarantees **automatic** $\mathcal{O}(a)$ **improvement**.
- However, the non-unitarity has to be taken care of by appropriate matching.



Ensembles used

Ensemble	β	lattice	$a\mu_l$	$\mu_{l,R}$ [MeV]	κ_c	L [fm]	$m_\pi L$	a [fm]
A30.32	1.90	$32^3 \times 64$	0.0030	13	0.163272	2.8	3.5	0.0885
A40.32	1.90	$32^3 \times 64$	0.0040	17	0.163270	2.8	4.1	0.0885
A80.24	1.90	$24^3 \times 48$	0.0080	34	0.163260	2.1	4.3	0.0885
B25.32	1.95	$32^3 \times 64$	0.0025	12	0.161240	2.6	3.2	0.0815
B55.32	1.95	$32^3 \times 64$	0.0055	26	0.161236	2.6	4.6	0.0815
B85.24	1.95	$24^3 \times 48$	0.0085	40	0.161231	2.0	4.3	0.0815
D15.48	2.10	$48^3 \times 96$	0.0015	9	0.156361	3.0	3.2	0.0619
D20.48	2.10	$48^3 \times 96$	0.0020	12	0.156357	3.0	3.7	0.0619
D30.48	2.10	$48^3 \times 96$	0.0030	19	0.156355	3.0	4.5	0.0619

Values of the lattice spacings, $\mu_{l,R}$ ($\overline{\mathrm{MS}}$, 2 GeV) and $m_\pi L$ from:

N. Carrasco et al. (ETM Collaboration), *Up, down, strange and charm quark masses with* $N_f = 2+1+1$ *twisted mass lattice QCD*, Nucl. Phys. B887 (2014) 19-68, arXiv: 1403.4504 [hep-lat].

Our lattice meson creation operators are of the following from:

$$O^{\text{twisted}}_{\Gamma,\bar{\chi}^{(1)}\chi^{(2)}} \equiv \frac{1}{\sqrt{V/a^3}} \sum_{\mathbf{n}} \bar{\chi}^{(1)}(\mathbf{n}) \sum_{\Delta\mathbf{n}=\pm\mathbf{e}_x,\pm\mathbf{e}_y,\pm\mathbf{e}_z} U(\mathbf{n};\mathbf{n}+\Delta\mathbf{n})\Gamma(\Delta\mathbf{n})\chi^{(2)}(\mathbf{n}+\Delta\mathbf{n}),$$

where:

- $\sum_{\mathbf{n}}$ gives zero total momentum,
- $\sum_{\Delta\mathbf{n}=\pm\mathbf{e}_x,\pm\mathbf{e}_y,\pm\mathbf{e}_z}$ realizes spatial separation between quarks, such that the meson can have angular momentum,
- $\Gamma(\Delta\mathbf{n})$ is a suitable combination of spherical harmonics and γ matrices (determines total angular momentum, parity and charge conjugation properties (for charmonium)),
- $U(\mathbf{n};\mathbf{n}+\Delta\mathbf{n})$ is a gauge link,
- $\chi^{(1)}$, $\chi^{(2)}$ are twisted basis quark operators.

Smearing of gauge links and quark fields

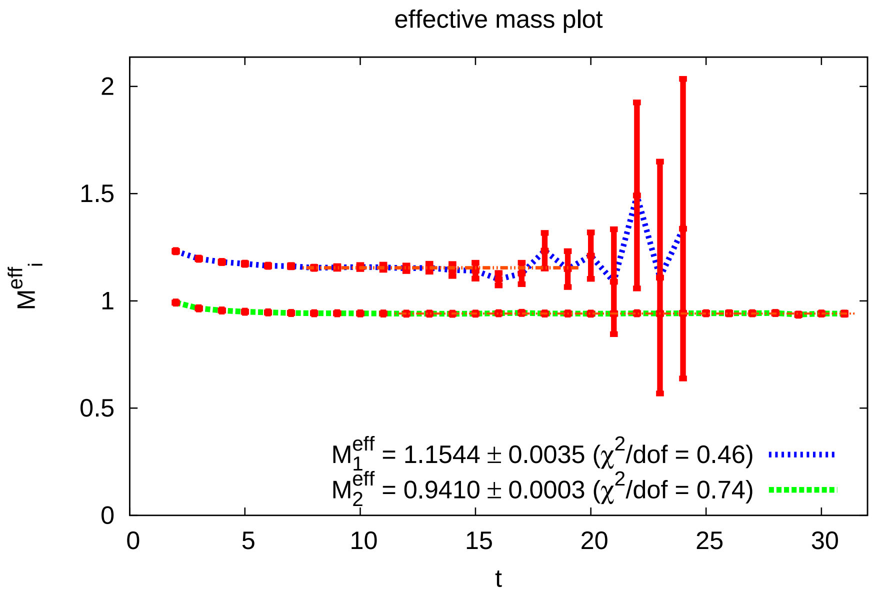
- We use standard smearing techniques to enhance the overlap between our trial states and low lying meson states:
  - ⋆ first, APE smearing of links, e.g. for A-ensembles: $N_{\text{APE}} = 10$, $\alpha_{\text{APE}} = 0.5$,
  - ⋆ second, Gaussian smearing of quark fields, e.g. for A-ensembles: $N_{\text{Gauss}} = 30$, $\kappa_{\text{Gauss}} = 0.5$.
- Smearing does not affect the irreducible representation of the cubic group and the total angular momentum O^J, parity $\mathcal{P}$ and charge conjugation $\mathcal{C}$ that are all determined by the meson creation operators.

Correlation matrices

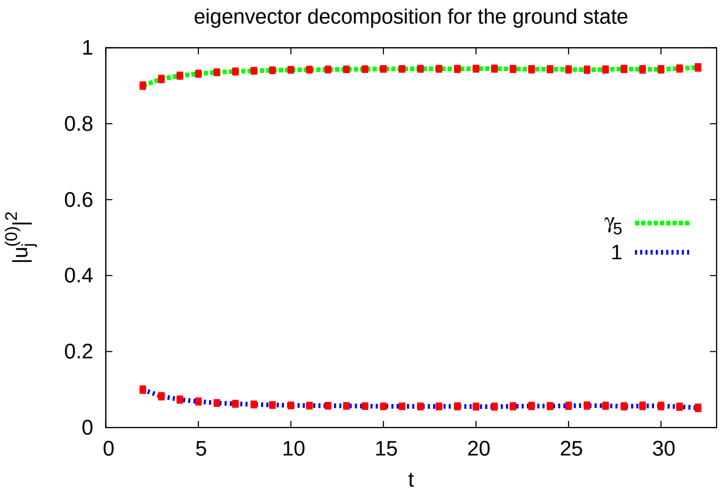
For each sector, i.e. the same

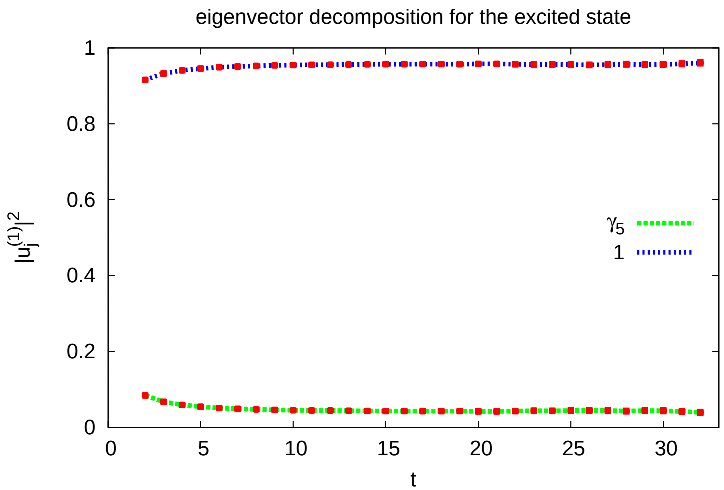
- flavours $\bar{\chi}^{(1)}\chi^{(2)}$,
- cubic representation O^J
- and $(\bar{c}c)$ $\mathcal{C}$ (OS) or $\mathcal{C} \circ \mathcal{P}^{(\mathrm{tm})}$ (TM),

we compute temporal correlation matrices of meson creation operators.
$\longrightarrow$ Correlators in a given correlation matrix have different $\mathcal{P}$ (parity broken by TM!) and spin (Γ structure).



D_s meson
$O^J = A_1$ $(J = 0)$
$\Gamma \in \{\gamma_5, 1\}$
Ensemble A30.32





Results

Matching and interpolating

For each ensemble, we compute:

- m_K at 2 different values of the strange quark mass μ_s,
- m_D at 2 different values of the charm quark mass μ_c.

This allows to extrapolate to the physical μ_s and μ_c quark masses by requiring that:

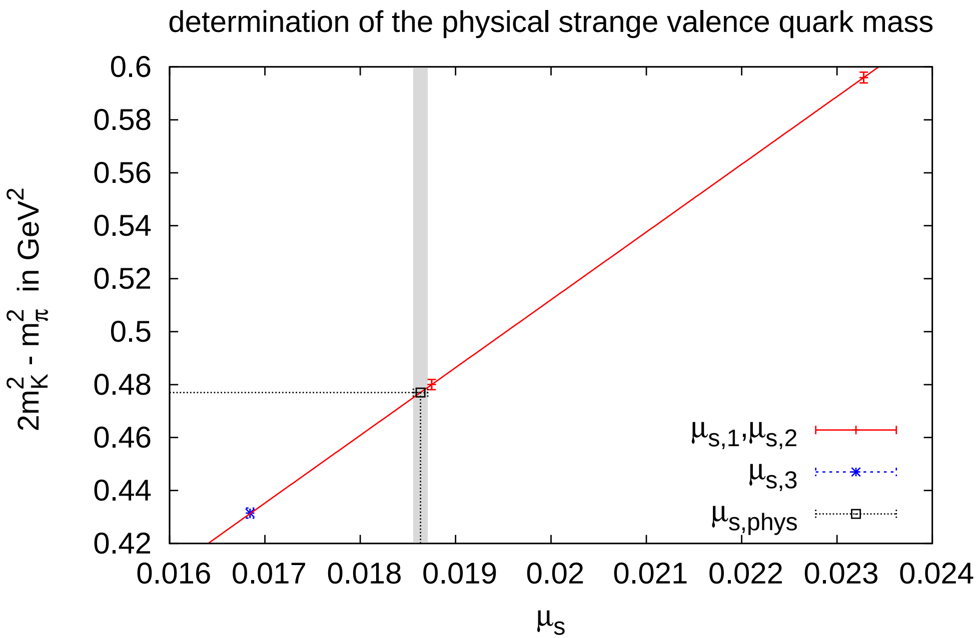
- $2m_K^2 - m_\pi^2$ takes its physical value of $0.477\,\text{GeV}^2$,
- m_D takes its physical value of 1.865 GeV

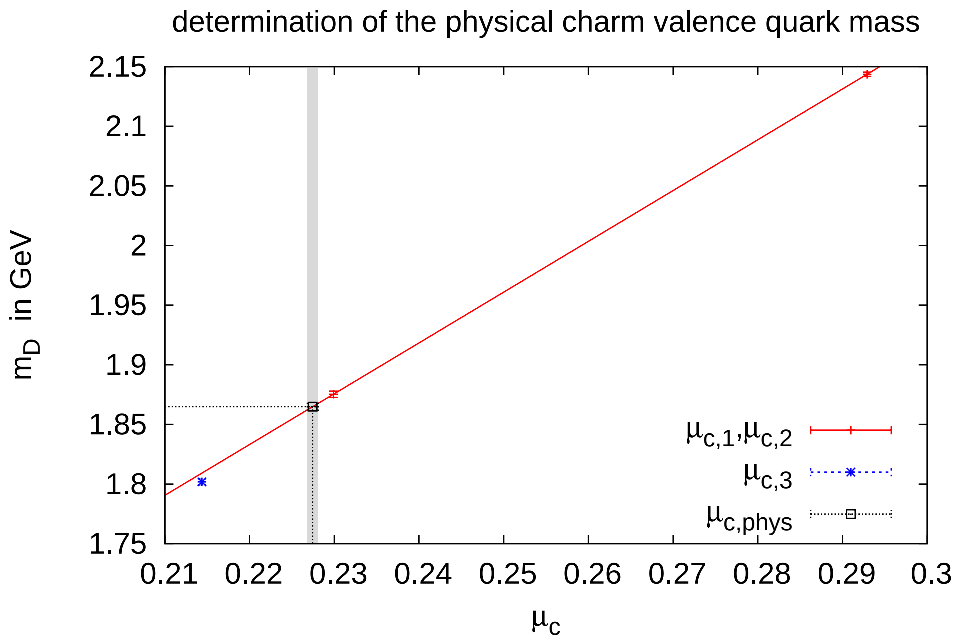
for each ensemble in the TM non-unitary setup ($\mu_{s,c} = -\mu_{s',c'}$).

After this procedure, the OS non-unitary setup ($\mu_{s,c} = \mu_{s',c'}$) should give the same masses $2m_K^2 - m_\pi^2$ and m_D, but only in the continuum limit.

Other meson masses should also be the physical ones after extrapolating to the physical pion mass and to the continuum limit.

Matching to physical meson masses





Our procedure

To extract the physical meson masses, we use the following procedure:

1. We compute the relevant TM/OS correlation functions for:
   - 3 lattice spacings,
   - 3 light quark masses for each lattice spacing,
   - 2 strange quark masses per light quark mass,
   - 2 charm quark masses per strange quark mass (i.e. 4 pairs $(\mu_{s,1}, \mu_{c,1})$, $(\mu_{s,1}, \mu_{c,2})$, $(\mu_{s,2}, \mu_{c,1})$, $(\mu_{s,2}, \mu_{c,2})$ for each light quark mass μ_l).
2. We perform extra-/interpolations in strange/charm quark masses to obtain the correlators at the physical strange and charm quark masses (use jackknife with binning to account for autocorrelations and propagate the error from this tuning).
3. This gives us a set of 18 points per correlator (3 lattice spacings $\times$ 3 quark masses $\times$ 2 discretizations).

Chiral and continuum extrapolations

Having this set of 18 points per correlator, we perform a combined **chiral and continuum extrapolation**, using the following fitting ansatz:

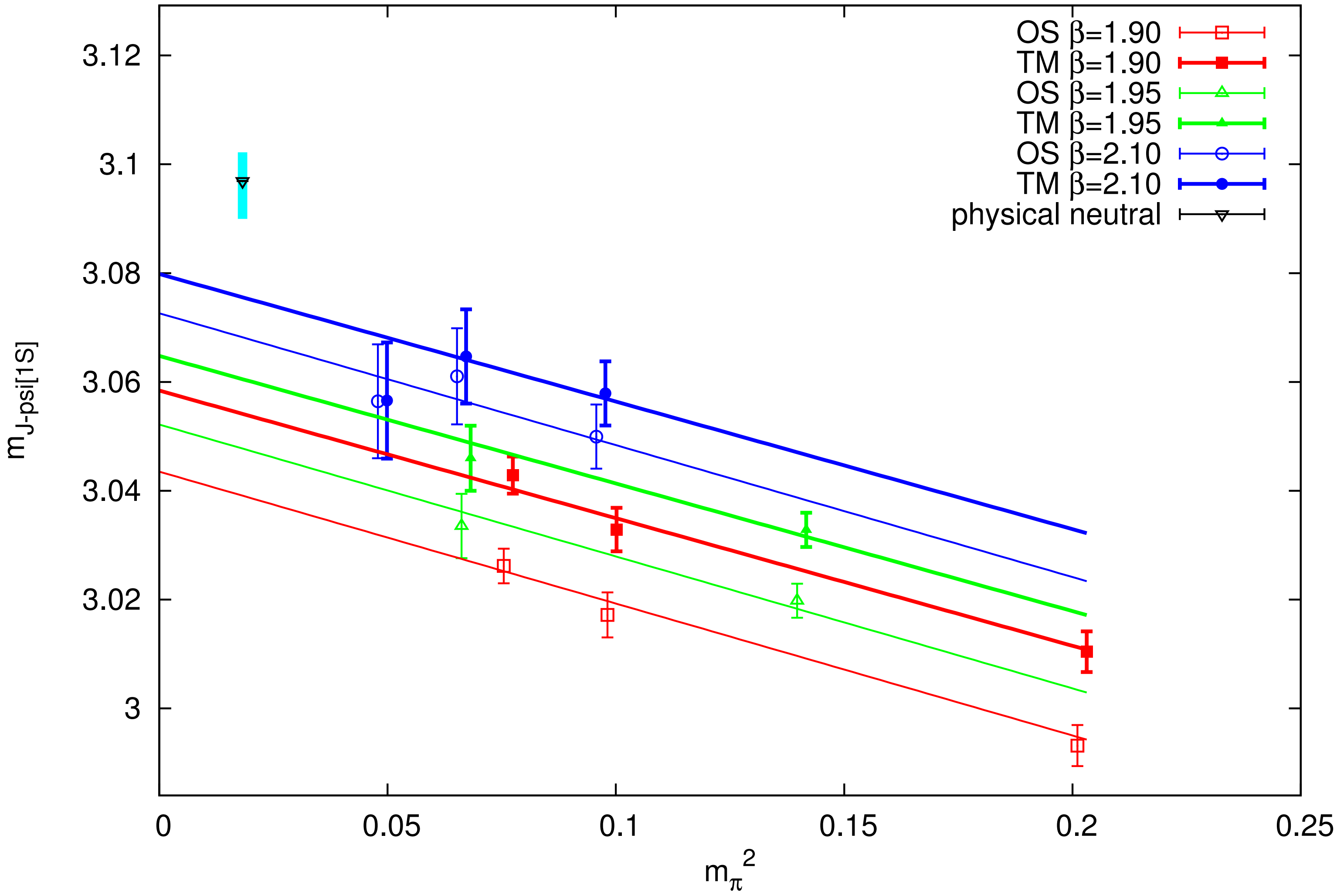
$$M^{TM}(a, m_\pi) = M + c^{TM} a^2 + \alpha^{TM}(m_\pi^2 - m_{\pi,phys}^2)$$

$$M^{OS}(a, m_\pi) = M + c^{OS} a^2 + \alpha^{OS}(m_\pi^2 - m_{\pi,phys}^2)$$

with 5 fitting parameters: M, c^{TM}, c^{OS}, α^{TM}, α^{OS}.

Note that we **enforce** a common continuum and physical pion mass limit for both discretizations.

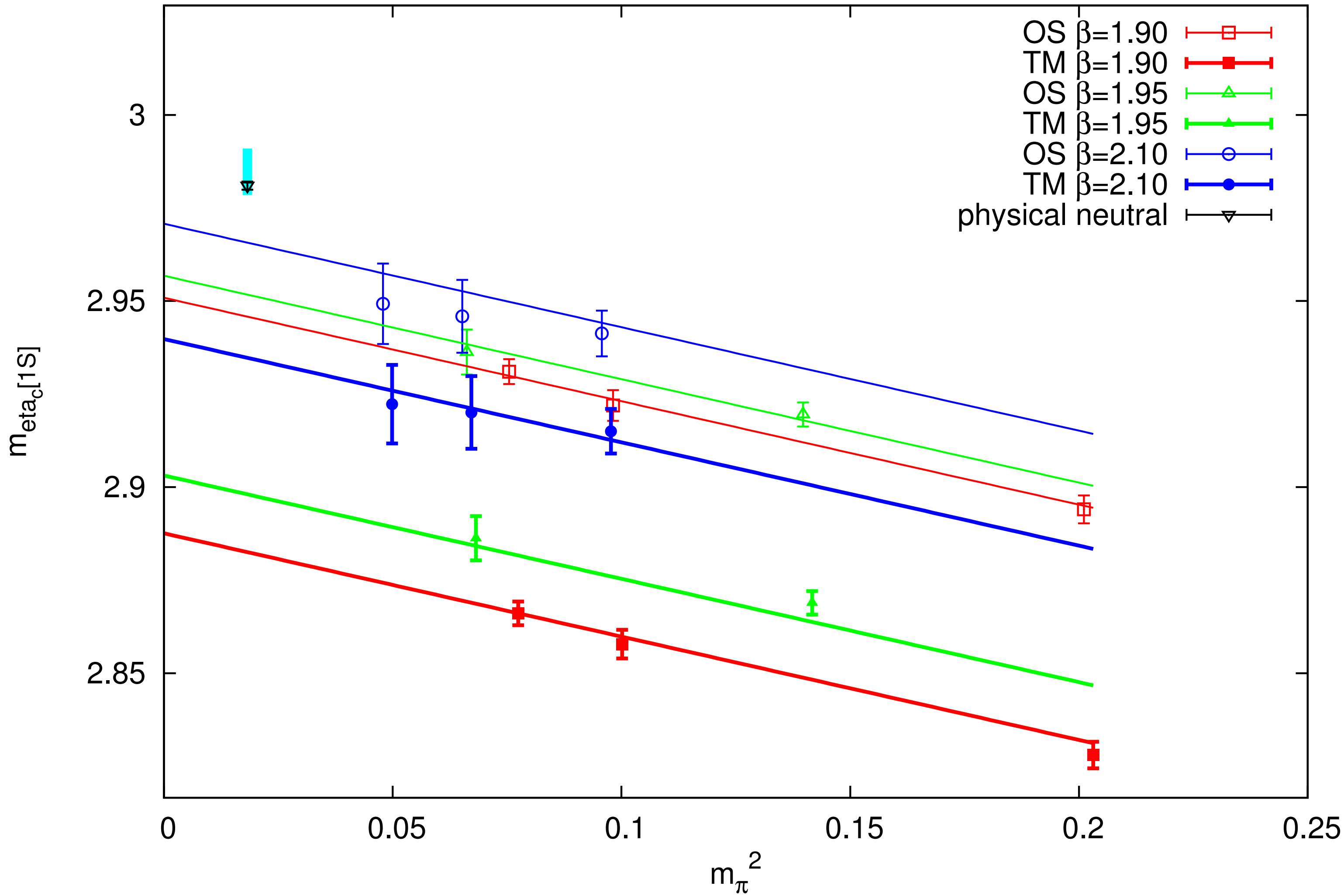
Example extrapolations: J/ψ $(J^{\mathcal{PC}} = 1^{--})$



PDG value of the mass: 3096.920(10) GeV

Our value of the mass: 3096(6) GeV $\qquad \chi^2$/d.o.f. of our fit: 0.36

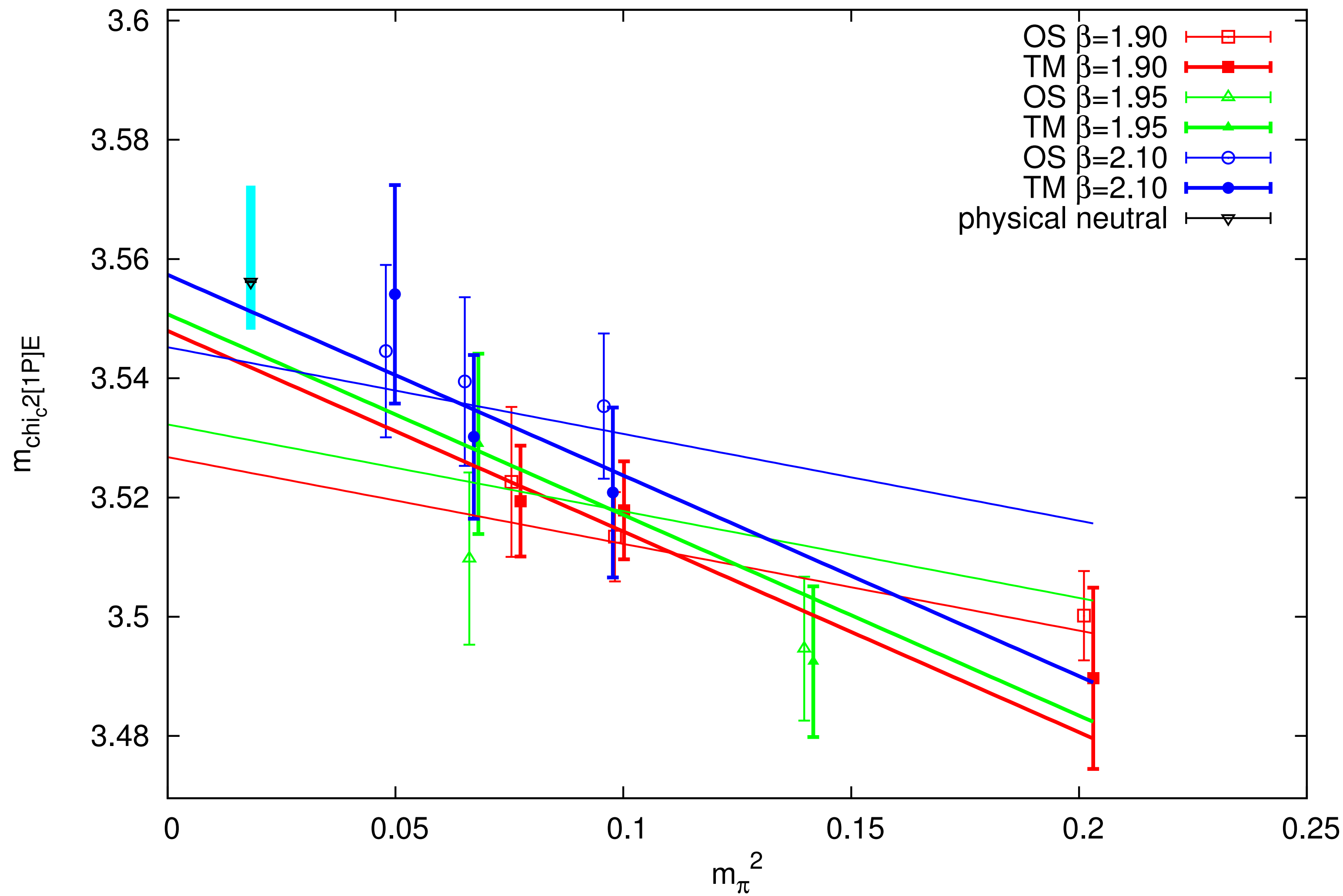
Example extrapolations: η_c $(J^{\mathcal{PC}} = 0^{-+})$



PDG value of the mass: 2981.1(1.1) GeV

Our value of the mass: 2985(6) GeV $\qquad$ χ^2/d.o.f. of our fit: 0.54

Example extrapolations: χ_{c2} $(J^{\mathcal{PC}} = 2^{++})$

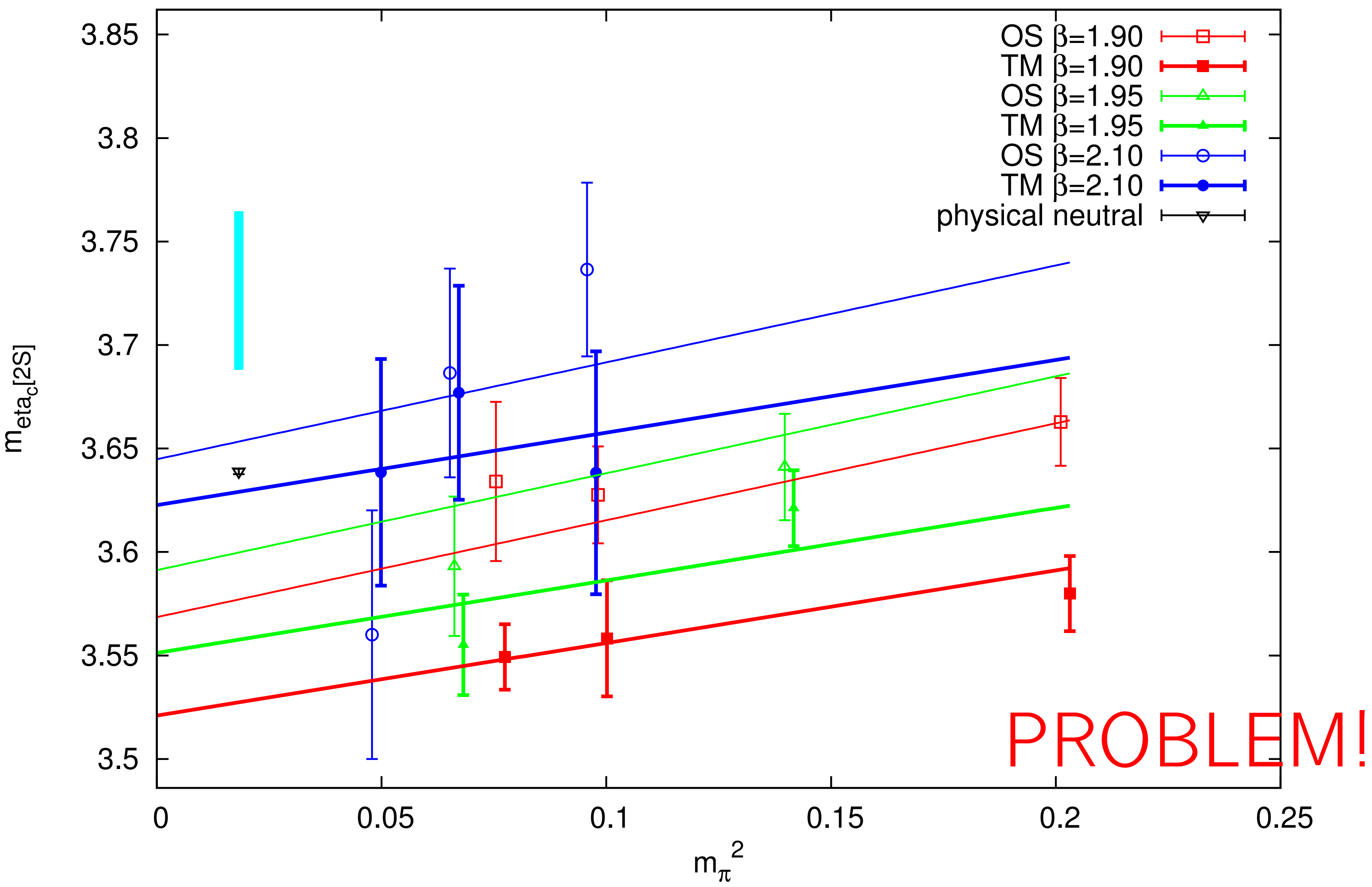


PDG value of the mass: 3556.20(9) GeV

Our value of the mass: 3560(12) GeV

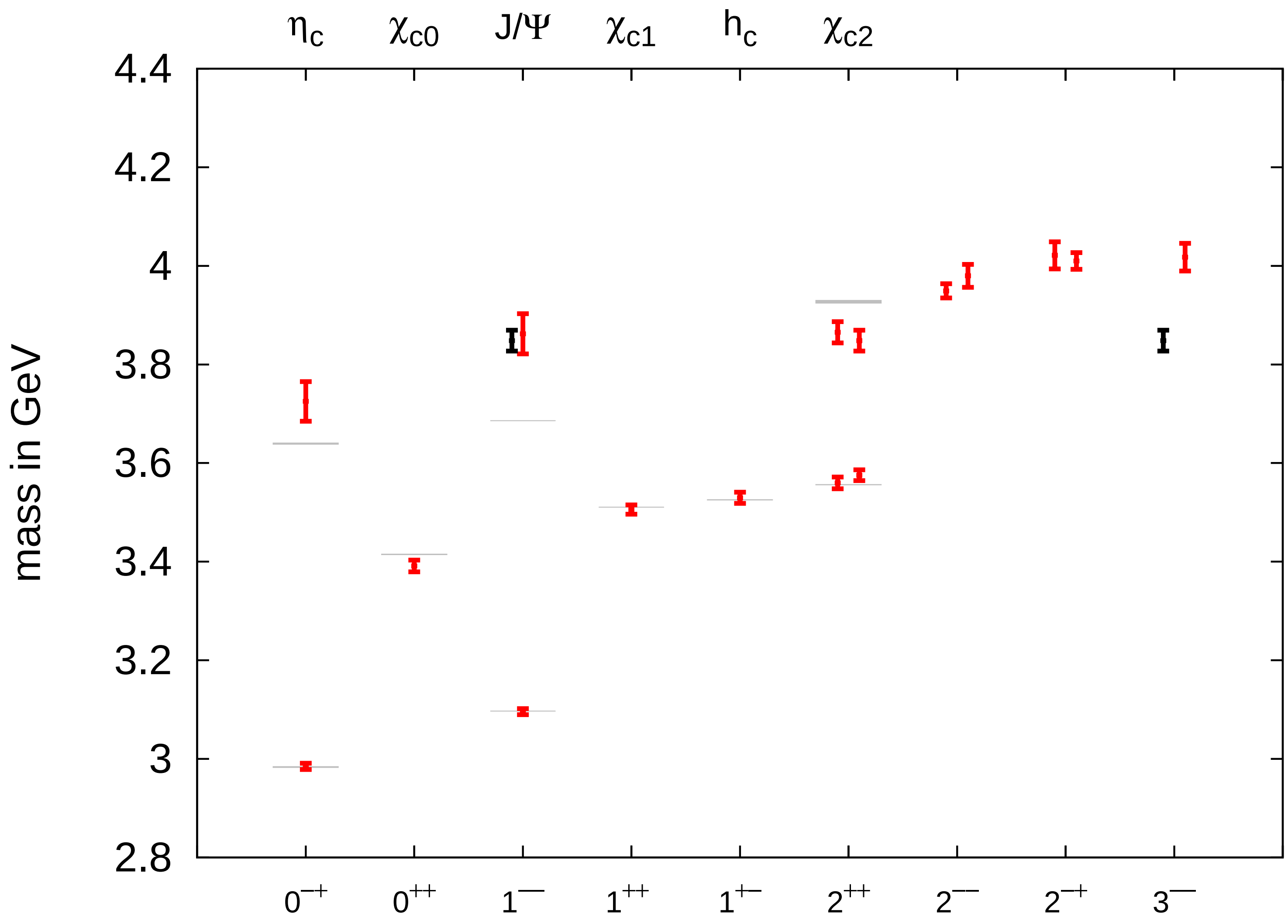
χ^2/d.o.f. of our fit: 0.53

Example extrapolations: $\eta_c[2S]$ ($J^{\mathcal{PC}} = 0^{-+}$)

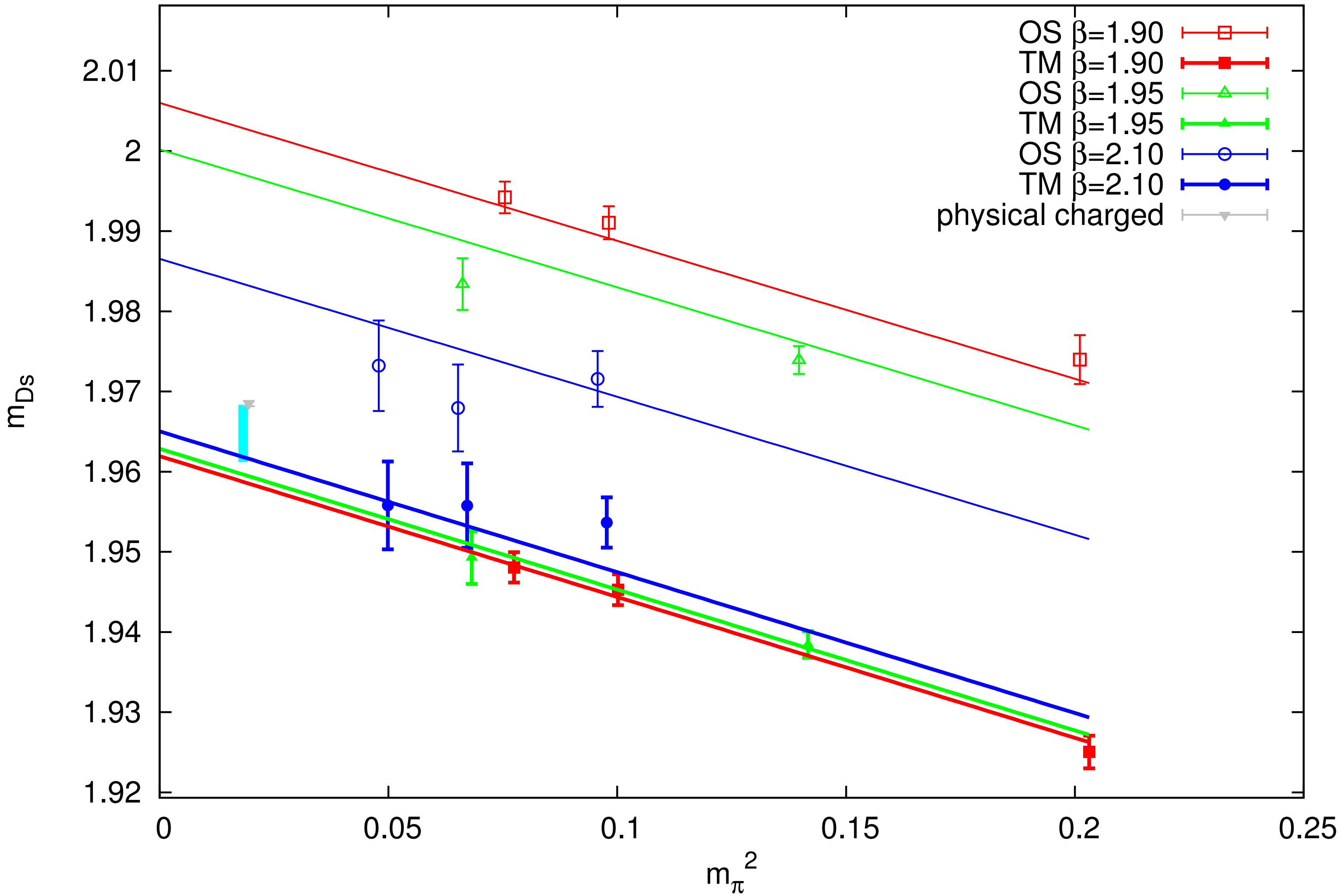


PDG value of the mass: 3638.9(1.3) GeV
Our value of the mass: 3726(38) GeV χ^2/d.o.f. of our fit: 0.85

Summary – charmonium spectrum



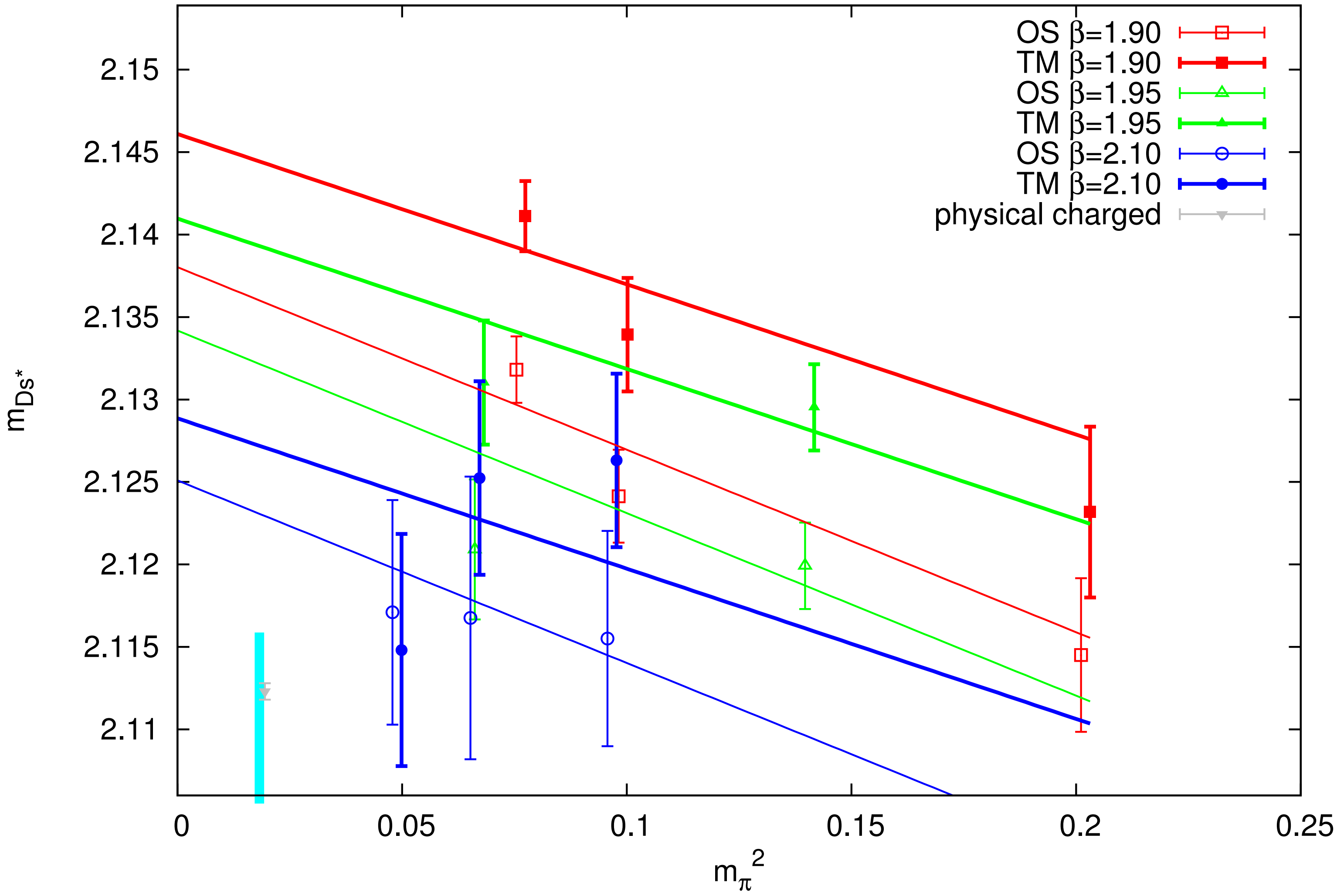
Example extrapolations: D_s ($J^{\mathcal{P}} = 0^-$)



PDG value of the mass: 1968.49(32) GeV

Our value of the mass: 1964.8(3.6) GeV

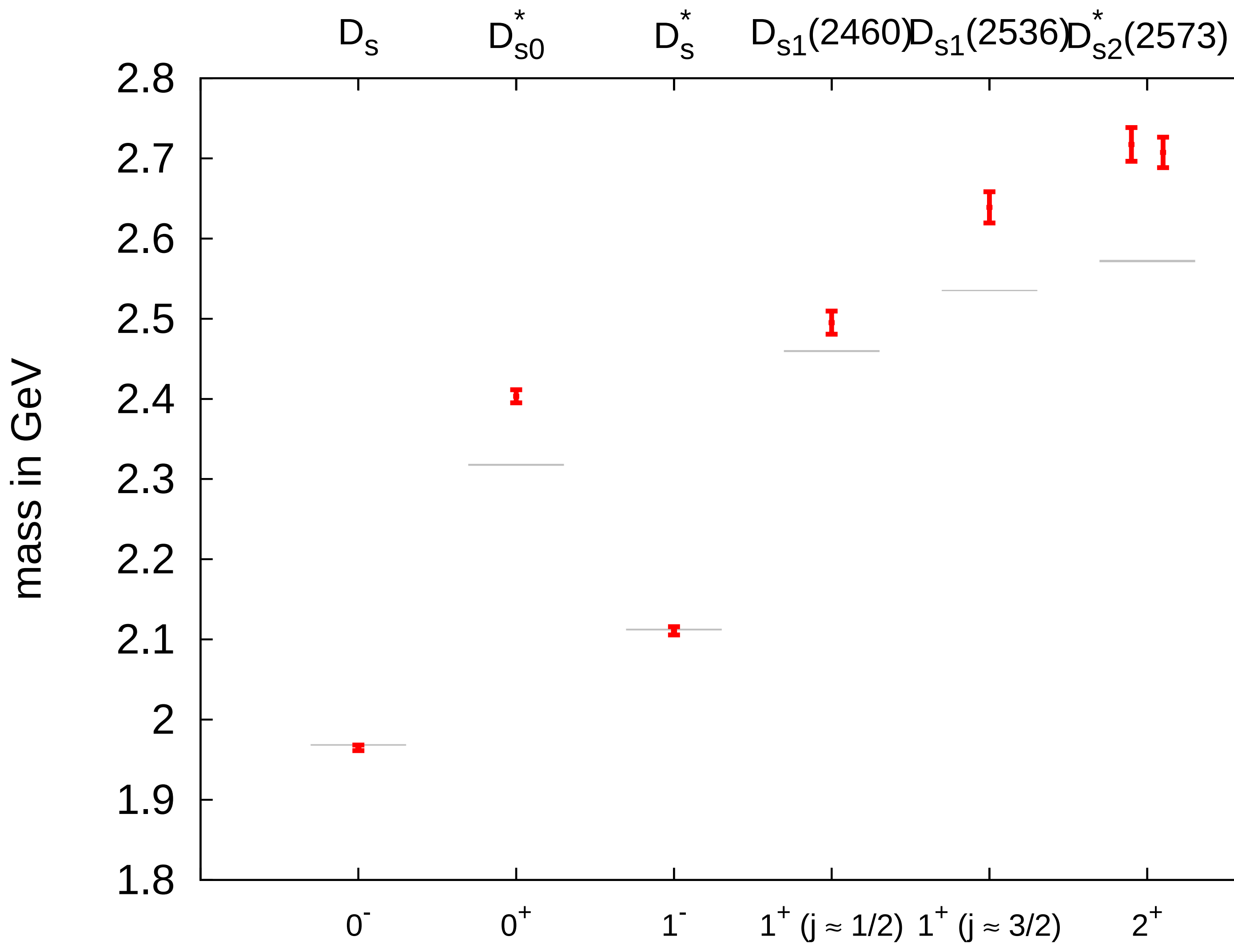
χ^2/d.o.f. of our fit: 1.24

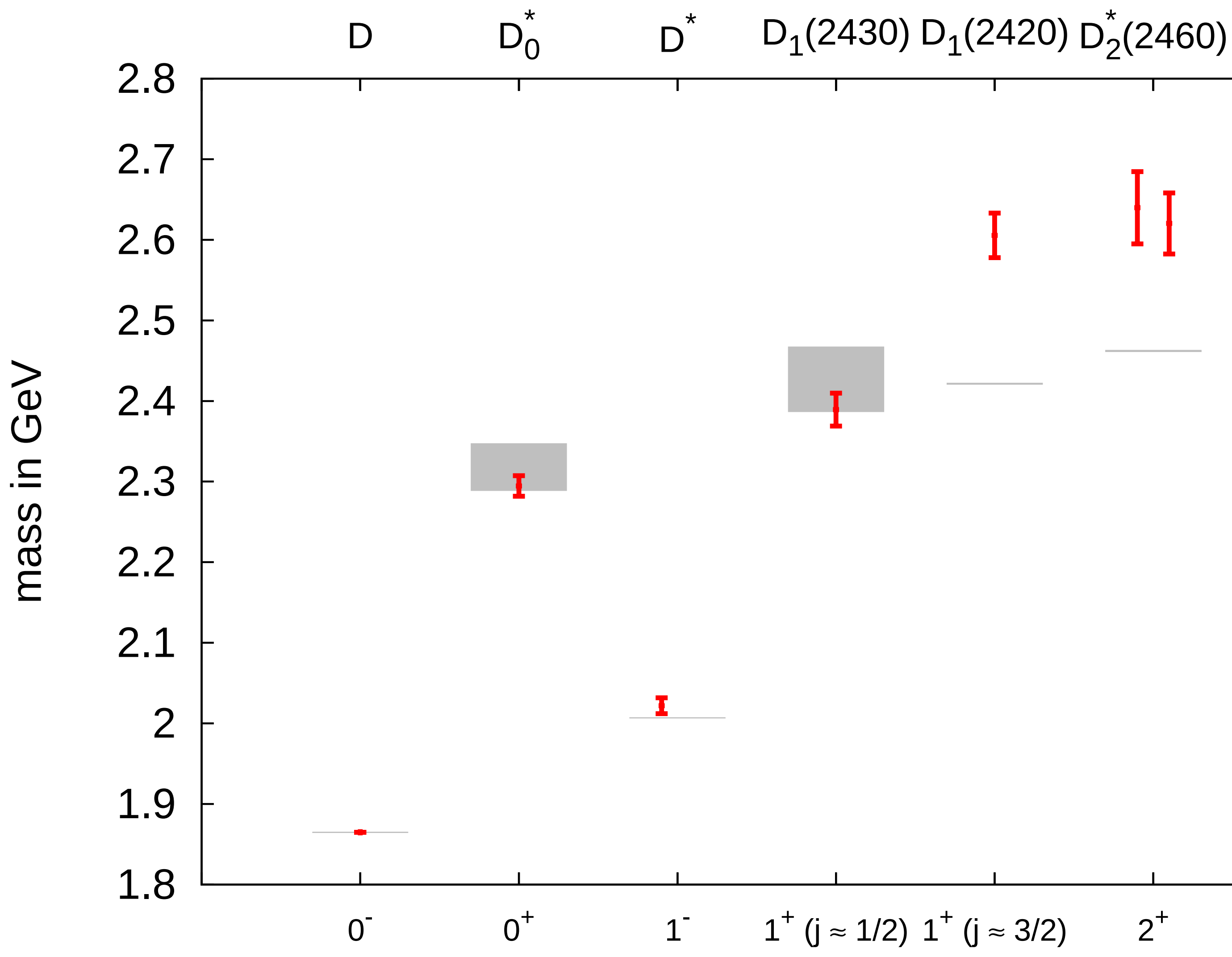


PDG value of the mass: 2112.3(5) GeV
Our value of the mass: 2110.7(5.2) GeV

χ^2/d.o.f. of our fit: 1.08

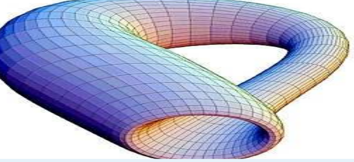
Summary – D_s spectrum




D
D*0
D*
D1(2430)
D1(2420)
D*2(2460)
mass in GeV
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2
1.9
1.8
0-
0+
1-
1+ (j ≈ 1/2)
1+ (j ≈ 3/2)
2+

Conclusions and prospects

- We have shown a computation of the spectrum of D mesons, D_s mesons and charmonium, using maximally twisted mass sea quarks and 2 different valence quark discretizations.
- We have rather good control over the light quark mass dependence and cut-off effects.
- Problems with plateaus in certain cases – needs a systematic analysis.
- Our plans:
  - ⋆ different fitting ansätze for chiral/continuum extrapolation,
  - ⋆ systematic analysis of plateaus (assign systematic error from plateau choice),
  - ⋆ comparison of TM/OS not enforcing a common continuum value,
  - ⋆ 3rd light quark mass missing at one of the lattice spacings.



Thank you for your attention!