Chiral Magnetic Conductivity in an interacting lattice model of a parity-breaking Weyl semimetal

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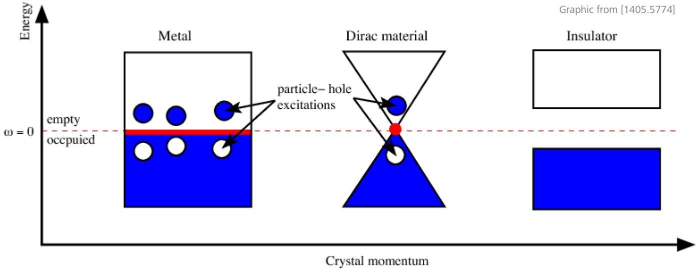
Introduction

What is a semimetal?

- Band structure

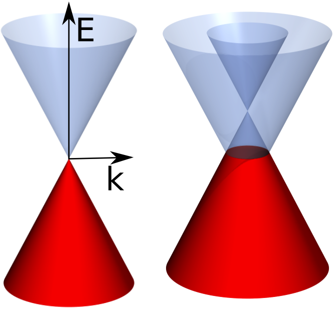


- Dirac semimetal: $\hat{H}_D(\mathbf{k}) = v_\mathsf{f} \begin{pmatrix} \sigma \cdot \mathbf{k} & 0 \\ 0 & -\sigma \cdot \mathbf{k} \end{pmatrix}$

Introduction

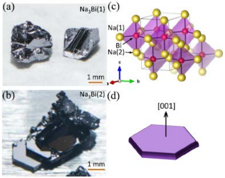
How to make a Weyl semimetal?

- Take a Dirac semimetal
- Break $\mathcal{P}$ (parity):
  e.g. chiral pumping: $\delta\hat{H} \sim \gamma_5 \mu_A$
- Dirac point splits $\Rightarrow$ Weyl points
  $\hat{H}_D(\mathbf{k}) \quad \Rightarrow \quad \hat{H}_W^{\pm} = \pm v_{\mathrm{f}}\, \sigma \cdot \mathbf{k}$
- Broken $\mathcal{P}$: energy shift



Graphic from [1303.5784] (modified)

Motivation



Graphic from [1503.08179]

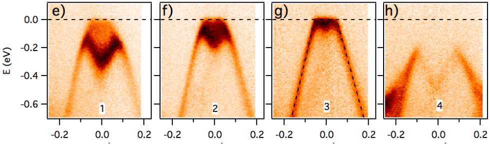
Why study Weyl semimetals?

- Anomalous transport
  - Energy shift of Weyl points:

Chiral Magnetic Effect $\rightarrow$ $\boxed{\vec{j} = \frac{e^2}{2\pi^2}\mu_A \cdot \vec{B}}$

- Realised in experiments[a]

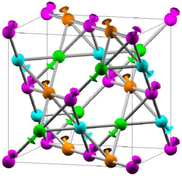
"Table-top" experiments to study anomalous transport!



Graphic from [1412.6543]

[a] See for example J. Xiong et al. [1503.08179], Q. Li et al. [1412.6543]

Modeling a Weyl semimetal


Graphic from [1007.0016]

The simplest model[b]:

- One flavour of massless Dirac fermions
- Electrons move with $v_\mathrm{f} \ll c$
  $\Rightarrow$ Magnetic interactions suppressed
- In practice:
  Only instantaneous Coulomb interactions important
- Effective coupling constant enhanced by factor $\frac{c}{v_\mathrm{f}}$
- $\alpha \cdot \frac{c}{v_\mathrm{f}} \sim 1 \Rightarrow$ Strongly coupled system

[b] *A. Sekine, K. Nomura* [1309.1079], *M. Vazifeh, M. Franz* [1303.5784], *P. Hosur, X. Qi* [1309.4464]

Model Hamiltonian

$$\hat{H} = \underbrace{\sum_{x,y} \hat{\psi}^\dagger_x h^{(0)}_{x,y} \hat{\psi}_y}_{\hat{H}_0} + \underbrace{U \sum_x \left(\hat{\psi}^\dagger_x \hat{\psi}_x - 2 \right)^2}_{\hat{H}_I}$$

- $\hat{\psi}_x = \left\{ \hat{\psi}_{\uparrow R,x} \, , \hat{\psi}_{\downarrow R,x} \, , \hat{\psi}_{\uparrow L,x} \, , \hat{\psi}_{\downarrow L,x} \right\}$
- charge operator: $\left(\hat{\psi}^\dagger_x \hat{\psi}_x - 2 \right)$
- on-site repulsive interaction: $U > 0$
- single-particle Hamiltonian: $h^{(0)}_{x,y} = \sum_k e^{ik(x-y)} h^{(0)}(k)$
- continuum Dirac Hamiltonian (P. V. Buividovich, [1408.4573])

Model Hamiltonian

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- In this talk: Wilson-Dirac Hamiltonian
  (P. V. Buividovich, MP, S. N. Valgushev [1505.04582])

Wilson-Dirac Hamiltonian with chiral chemical potential:

$$h^{(0)}(k) = \sum_{i=1}^{3} \big(v_{\mathrm{f}}\alpha_i \sin(k_i) + 2\gamma_0 \sin^2(k_i/2)\big) + \mu_A^{(0)}\gamma_5 + m^{(0)}\gamma_0,$$

with bare chiral chemical potential $\mu_A^{(0)}$ and bare mass $m^{(0)}$

Advantage:

- More realistic description of solid state systems, where

$$\varepsilon_k \neq v_{\mathrm{f}} k \quad \text{for} \quad |k| \gg 0$$

- No regularisation

Mean-field calculation

- Compute partition function

$$\mathcal{Z} = \mathrm{tr}\left(e^{-\beta \hat{H}}\right)$$

- $\hat{H}$ contains four fermion interactions $\left(\hat{\psi}_x^\dagger \hat{\psi}_x - 2\right)^2$
- Apply Suzuki-Trotter decomposition ...

$$e^{-\beta(\hat{H}_0 + \hat{H}_I)} = e^{-\Delta\tau \hat{H}_0} e^{-\Delta\tau \hat{H}_I} e^{-\Delta\tau \hat{H}_0} e^{-\Delta\tau \hat{H}_I} \ldots$$

- ... and Hubbard-Stratonovich (HS) transformation

$$e^{-U\Delta\tau \hat{\psi}_\alpha^\dagger \hat{\psi}_\alpha \hat{\psi}_\beta^\dagger \hat{\psi}_\beta} = \int D\Phi_{\alpha\beta} \exp\left(-\frac{\Delta\tau}{4U} \Phi_{\alpha\beta}\Phi_{\beta\alpha} - \Delta\tau \Phi_{\alpha\beta} \hat{\psi}_\alpha^\dagger \hat{\psi}_\beta\right)$$

Mean-field calculation

- In MF approximation: Type of HS-field important!
- Choose general ansatz

$$\Phi_x = \sum_{A=1}^{15} \Gamma_A \Phi_{x,A}, \qquad \text{i.e.} \quad \Phi \sim \langle \psi^\dagger \Gamma_A \psi \rangle$$

- Basis set of 15 traceless Hermitian matrices

$$\Gamma_A \in \{\gamma_5\alpha_k, \gamma_0, \gamma_0\gamma_5\alpha_k, -i\gamma_5\gamma_0, i\gamma_0\alpha_k, \gamma_5, \alpha_k\}$$

- Assumptions: Φ_x static, rotation/translation symmetry

Mean-field calculation

Saddle point HS-field

$$\varPhi = (m_r - m^{(0)})\gamma_0 + m_i\gamma_0\gamma_5 + (\mu_A - \mu_A^{(0)})\gamma_5$$

- m_r/μ_A renormalised mass/chemical potential
- m_i $\mathcal{CP}$-breaking mass term

Mean-field free energy $(T=0)$

$$\frac{\mathcal{F}}{L^3} = -\frac{1}{L^3}\sum_{\epsilon_i<0}\epsilon_i + \frac{\varPhi_{\alpha\beta}\varPhi_{\beta\alpha}}{4U}$$

- ϵ_i: Energy levels of effective Hamiltonian $h(k) = h^{(0)}(k) + \varPhi$

$$\epsilon_{\pm,s}(\mathbf{k}) = \pm\sqrt{(\sqrt{\sum_i \sin^2(k_i)} - s\mu_A)^2 + m_i^2 + (m_r + \sum_i \sin^2(k_i/2))^2}, \quad s = \pm 1$$

Mean-field phase diagram

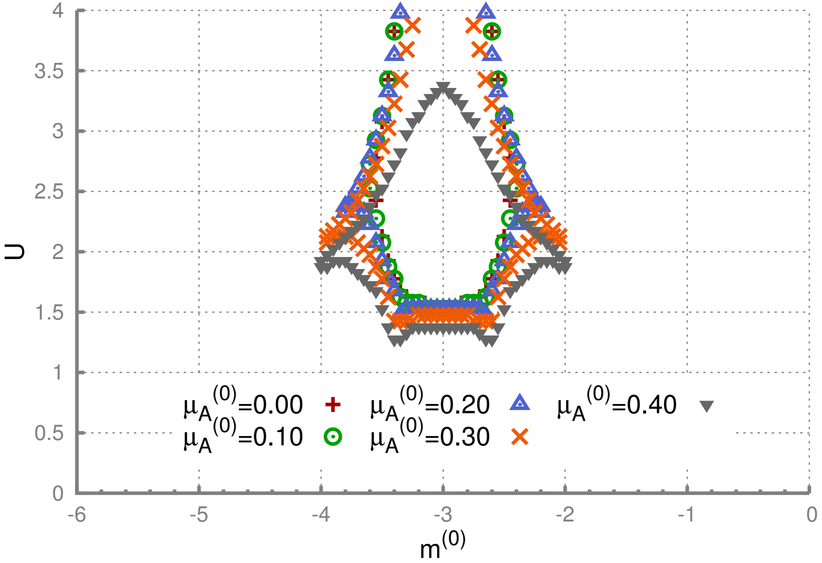
- Input parameters: $U, \mu_A^{(0)}$ and $m^{(0)}$
- m_r, m_i and μ_A: numerical minimisation of free energy $\mathcal{F}$
- Wilson term breaks $U(1)_A$ to Z_2: $m_i \leftrightarrow -m_i$
- "Aoki phase": pion condensation/$\mathcal{CP}$ broken

$$m_i \propto \langle \hat{\psi}^\dagger \gamma_0 \gamma_5 \hat{\psi} \rangle \neq 0$$

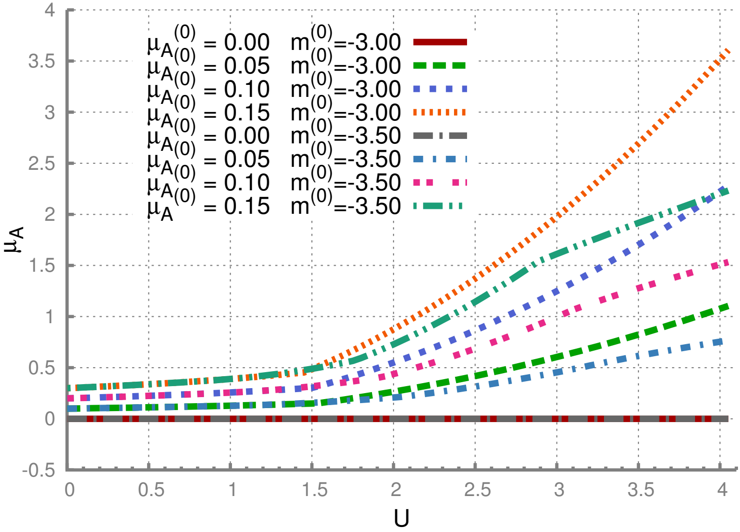
- Massless excitations only for $-6 \leq m^{(0)} \leq 0$
- N_f depends on m_r (Doublers)

$$N_f = 1 \quad m_r \in \{0, -6\}\ ,\ N_f = 3 \quad m_r \in \{-2, -4\}$$

Mean-field phase diagram



Renormalisation of chiral chemical potential



Chiral magnetic conductivity

- Chiral magnetic conductivity σ_{CME}

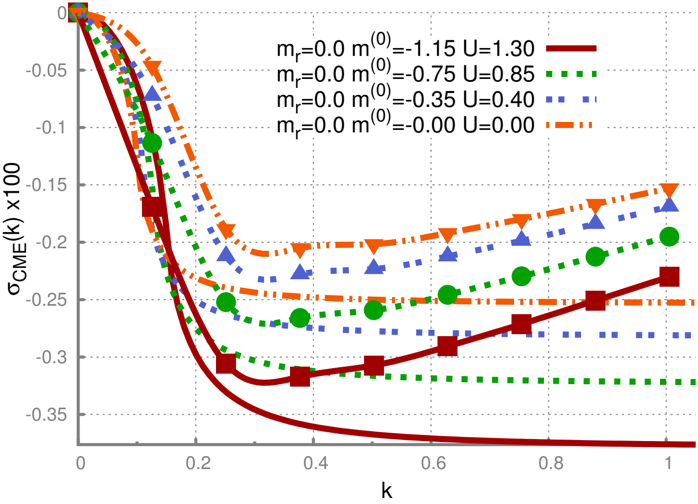
$$\vec{j} = \sigma_{\text{CME}}\ \vec{B}$$

- Static external $\vec{B} = \vec{\nabla} \times \vec{A}$ modulated with wave vector $\vec{k}$
- From linear response theory $(\vec{A} \parallel \hat{x}_2\ ,\ \vec{k} \parallel \hat{x}_3)$

$$\sigma_{\text{CME}}(k_3) = -\frac{i}{k_3}\frac{1}{L^3}\sum_{x,y} e^{ik_3(x_3-y_3)}\frac{\delta^2 \mathcal{F}[A_{x,k}]}{\delta A_{x,1}\delta A_{y,2}}\Bigg|_{A_{x,i}=0}$$

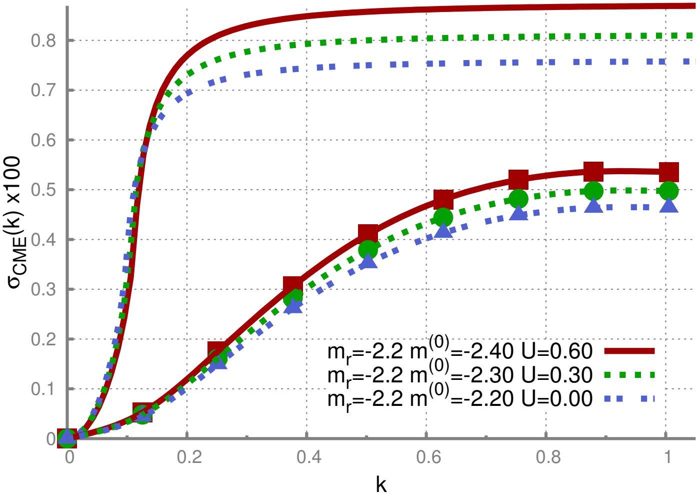
- $\frac{\delta^2 \mathcal{F}[A_{x,k}]}{\delta A_{x,1}\delta A_{y,2}}$ from perturbation theory

Chiral magnetic conductivity



Lines without symbols: Results for free Dirac fermions (P. V. Buividovich [1312.1843])

Chiral magnetic conductivity



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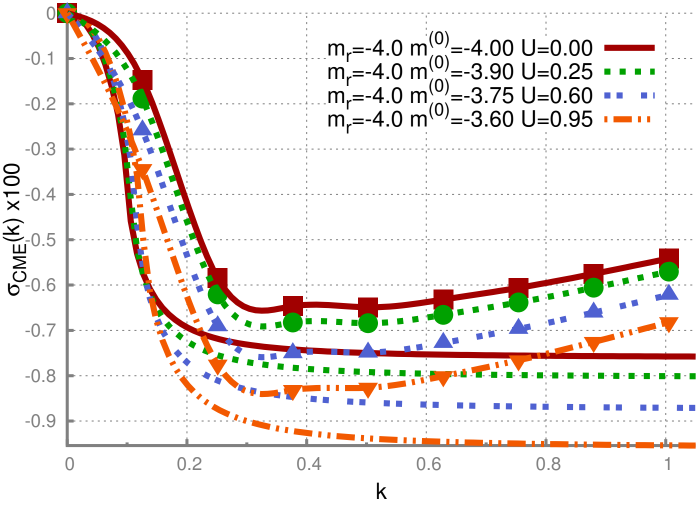


Lines without symbols: Results for free Dirac fermions (P. V. Buividovich [1312.1843])

Chiral magnetic conductivity

- Limit of static and homogeneous $\vec{B}$
- "Literature value" for free fermions

$$\sigma_{\mathsf{CME}} = \frac{N_f \mu_A}{2\pi^2}$$

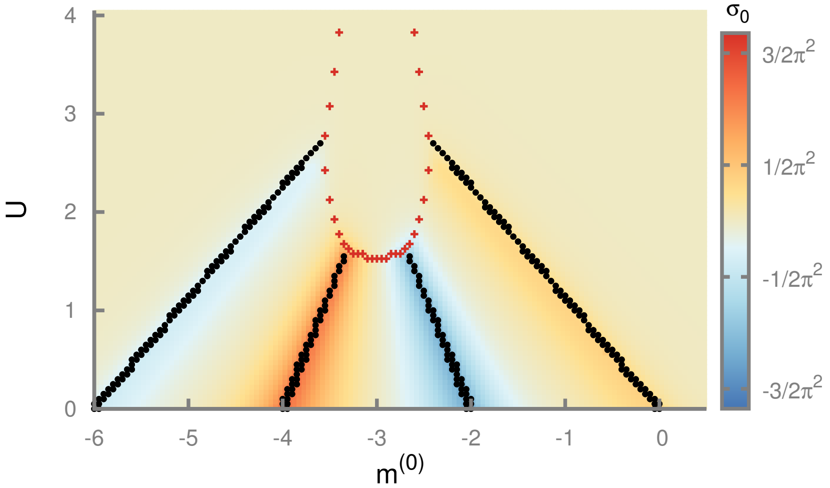
- Connection to our results
  - First expand around $\mu_A \ll 1$ for fixed k_3

$$\sigma_{\mathsf{CME}}(k_3, \mu_A) = \sigma(k_3)\mu_A + \mathcal{O}(\mu_A^2)$$

  - Then take the limit $k_3 \to 0$

$$\sigma_0 := \lim_{k_3 \to 0} \sigma(k_3)$$

Chiral magnetic conductivity



Summary

- MF for Weyl semimetal model with Wilson-Dirac fermions
- Phase diagram
  - Aoki phase shrinks with increasing $\mu_A^{(0)}$
  - Onset moves to smaller values of U
- Strong multiplicative renormalisation of μ_A in all phases
- Static chiral magnetic conductivity
  - Free fermion result along $m_r \in \{0, -2, -4, -6\}$
  - Vanishes in Aoki phase
  - No enhancement due to interactions
  - In stark contrast to results for continuum Dirac Hamiltonian

Outlook

- Allow for non-static HS-field

$$\Phi = \Phi_{\alpha\beta}(x)$$

- Explore possibility of non-homogeneous ground state
- Challenges
  - No analytic solution for ϵ_i
  - To find ground state
    $\rightarrow$ numerical minimisation with $\mathcal{O}(L^3)$ parameters

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Thank you!