Heavy $Q\bar{Q}$ pair free energies and screening masses at the QCD physical points

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In collaboriation with
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Motivation

We are interested in deconfinement properties of QCD

- Quarkonia could give us a thermometer of the QGP, since the different quarkonia are expected to "dissociate" at different temperatures. Analogy: spectral analyis of stellar media
- WANTED: "dissociation" temperatures
- Problem: To calculate spectral functions, analytical continuation is needed, this is ill-posed
- Therefore, we consider a simpler, but related problem, static free energies
- Also, we are interested in the electric and magnetic screening lengths in the plasma

Static quark free energies

Excess free energy induced by a static test charge

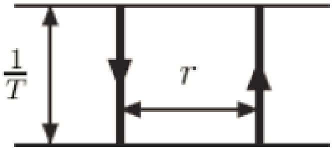
$$F_Q(T) = -T \log \langle \mathrm{Tr}\, L(\vec{x}) \rangle$$

In pure SU(N) gauge theory: order parameter

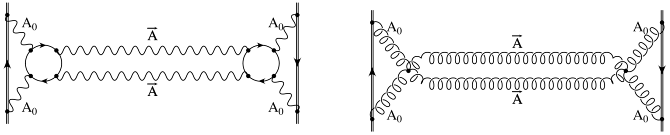
Excess free energy induced by a static $Q\bar{Q}$ pair

$$F_{Q\bar{Q}}(r, T) = -T \log \left\langle \mathrm{Tr}\, L(\vec{r}+\vec{x})\, \mathrm{Tr}\, L^{\dagger}(\vec{x}) \right\rangle$$

Given by the correlator of Polyakov loops.



The Polyakov-loop correlator



This couples directly only to electric gluon fields, since:

$$\mathrm{Tr}\, L = \mathrm{Tr}\, \mathcal{P} e^{ig \int_0^\beta d\tau A_4(\tau,\vec{x})}$$

but indirectly also to magnetic fields.

- In QED: $\langle PP \rangle$ falls algebraically, not exponentially, because of indirect coupling to magnetic fields, that are not screened.
- In QCD: magnetic fields also screened, but the screening mass is lower than the electric screening mass.

It would be nice to somehow disentangle the electric and magnetic sectors.

Screening masses - nonperturbative definition

Magnetic and electric screening masses

Use symmetry: Euclidean time reflection $\mathcal{R}$ = Real time $\mathcal{CT}$

Arnold, Yaffe '95

$$A_4(\tau,\mathbf{x}) \xrightarrow{\mathcal{R}} -A_4(-\tau,\mathbf{x}), \quad \mathrm{A_i}(\tau,\mathbf{x}) \xrightarrow{\mathcal{R}} \mathrm{A_i}(-\tau,\mathbf{x})$$

$$L \xrightarrow{\mathcal{R}} L^\dagger$$

$$L_M \equiv (L + L^\dagger)/2$$
$$L_E \equiv (L - L^\dagger)/2$$

Use also $\mathcal{C}$:

$$A_4 \xrightarrow{\mathcal{C}} A_4^* \quad \text{and} \quad \mathrm{L} \xrightarrow{\mathcal{C}} \mathrm{L}^*$$

$$L_{M\pm} = (L_M \pm L_M^*)/2$$
$$L_{E\pm} = (L_E \pm L_E^*)/2$$

Screening masses - nonperturbative definition

Magnetic and electric screening masses

$$L_{M\pm} = (L_M \pm L_M^*)/2 \qquad L_{E\pm} = (L_E \pm L_E^*)/2$$

Also $\mathrm{Tr}\, L_{E+} = 0 = \mathrm{Tr}\, L_{M-}$, so we have 2 correlators:

$$C_{M+}(r,T) \equiv \left\langle \sum_{\mathbf{x}} \mathrm{Tr} L_{M+}(\mathbf{x}) \mathrm{Tr} L_{M+}(\mathbf{x}+\mathbf{r}) \right\rangle - \left| \left\langle \sum_{\mathbf{x}} \mathrm{Tr} L(\mathbf{x}) \right\rangle \right|^2$$

$$C_{E-}(r,T) \equiv - \left\langle \sum_{\mathbf{x}} \mathrm{Tr} L_{E-}(\mathbf{x}) \mathrm{Tr} L_{E-}(\mathbf{x}+\mathbf{r}) \right\rangle$$

We decomposed the full Polyakov-loop correlator into 2 pieces:

$$C(r,T) - C(r \to \infty, T) = C_{M+}(r,T) + C_{E-}(r,T),$$

Two correlation lengths: m_M and m_E

Simulation details

Lattice details

- Tree-level Symanzik improved gauge action
- Stout-improved staggered fermion action with 2+1 dynamical quarks
- Physical quark masses, obtained by reproducing the physical ratios m_π/f_K and m_K/f_K
- $N_s^3 \times N_t = 24^3 \times 6, 24^3 \times 8, 32^3 \times 10, 32^3 \times 12$, $48^3 \times 16$
- Temperature range: 150MeV...450MeV
- Same action as in: JHEP 0601 (2006), 089
  The Equation of state in lattice QCD: With physical quark masses towards the continuum limit

Free energy - renormalization

Prescription

$$F^{ren}_{\bar{Q}Q}(r,\beta,T) = F_{\bar{Q}Q}(r,\beta,T) - F_{\bar{Q}Q}(r\to\infty,\beta,T_0)$$

Single quark free energy

Since the divergence in the $Q\bar{Q}$ free energy is independent of r, it is enough to remove it at $r\to\infty$ i.e. it is enough to renormalize the single heavy quark free energy. Prescription:

$$F^{ren}_{Q}(\beta,T;T_0) = F_{Q}(\beta,T) - F_{Q}(\beta,T_0)$$

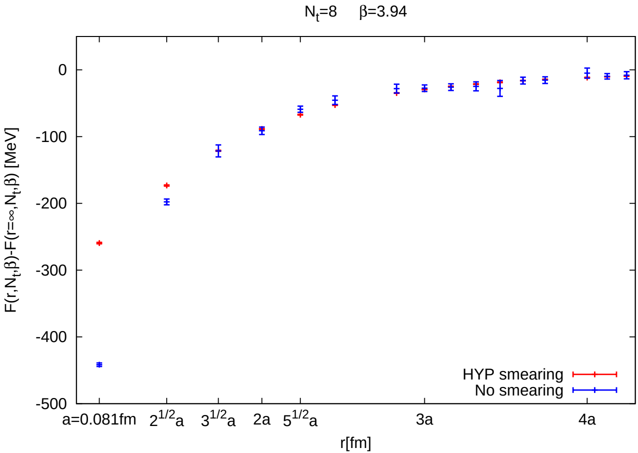
$Q\bar{Q}$ free energy

$$\tilde{F}_{\bar{Q}Q}(r,\beta,T) = F_{\bar{Q}Q}(r,\beta,T) - F_{\bar{Q}Q}(r\to\infty,\beta,T)$$

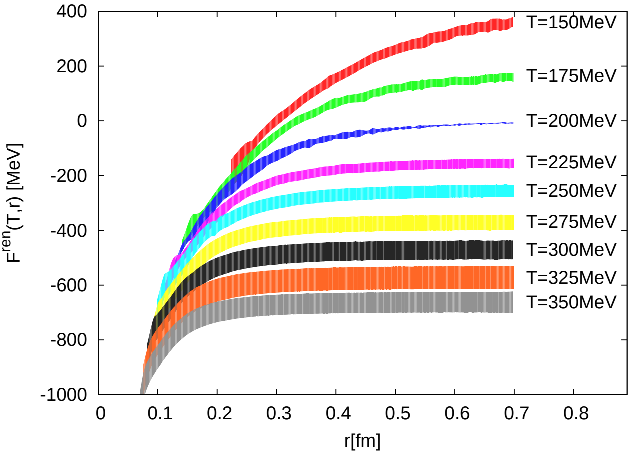
$$F_{\bar{Q}Q}(r\to\infty,\beta,T) = 2F_Q(T)$$

$$F^{ren}_{\bar{Q}Q}(r,\beta,T;T_0) = \tilde{F}_{\bar{Q}Q}(r,\beta,T) + 2F^{ren}_{Q}(\beta,T;T_0)$$

Free energy - effect of gauge field smearing (HYP)



Free energy - continuum results



Screening masses

Recap

In the beginning, we decomposed the full Polyakov-loop correlator into 2 pieces:

$$C(r,T) - C(r \to \infty, T) = C_{M+}(r,T) + C_{E-}(r,T),$$

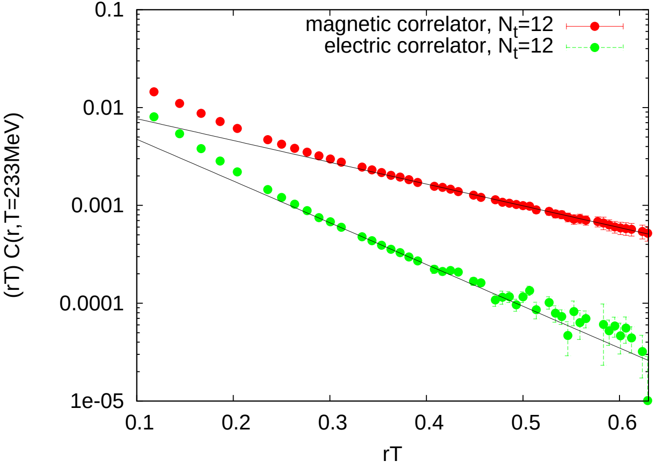
The two pieces define two correlation lengths:

$$C_{M+}(r,T) \xrightarrow{r\to\infty} K_M(T)\frac{\mathrm{e}^{-m_M(T)r}}{r},$$
$$C_{E-}(r,T) \xrightarrow{r\to\infty} K_E(T)\frac{\mathrm{e}^{-m_E(T)r}}{r},$$

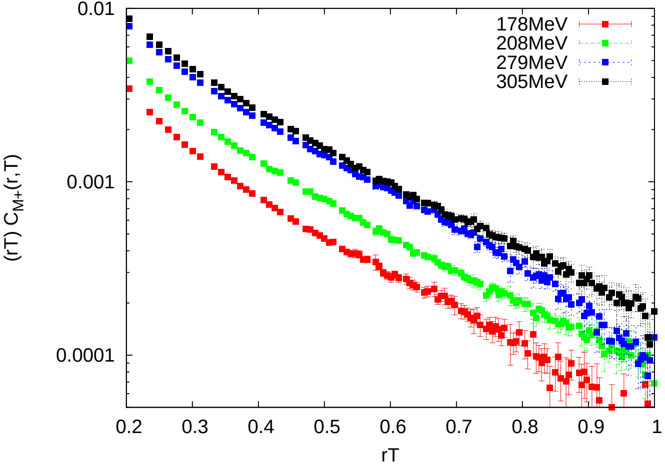
Outline

- Some simple properties already seen from the raw data
- Detailed analysis

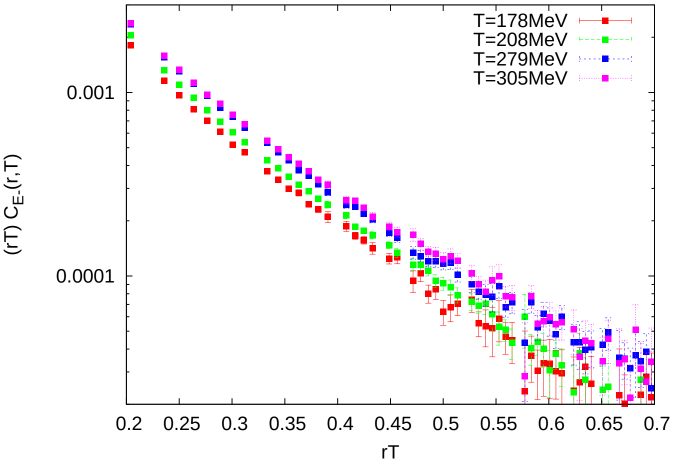
Screening masses - illustrating $m_E > m_M$



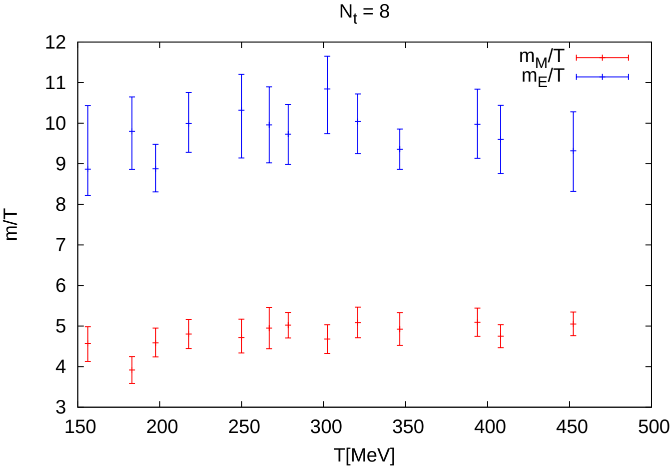
Screening masses - illustrating $m_M \propto T$



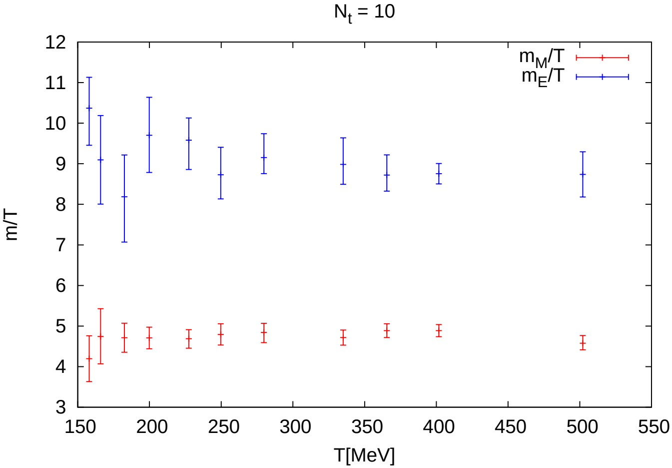
Screening masses - illustrating $m_E \propto T$



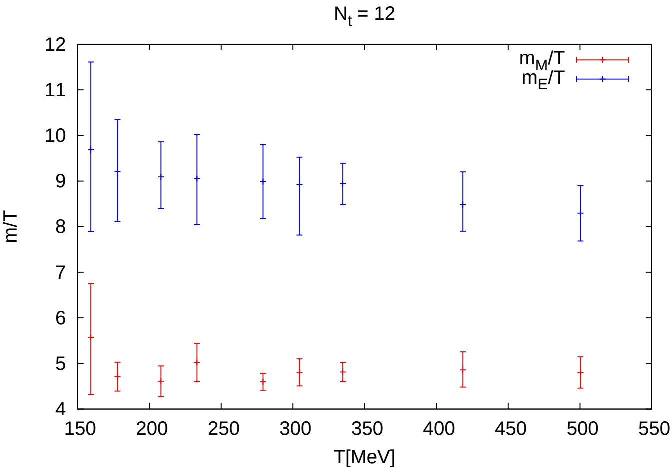
Screening masses - results at finite lattice spacing



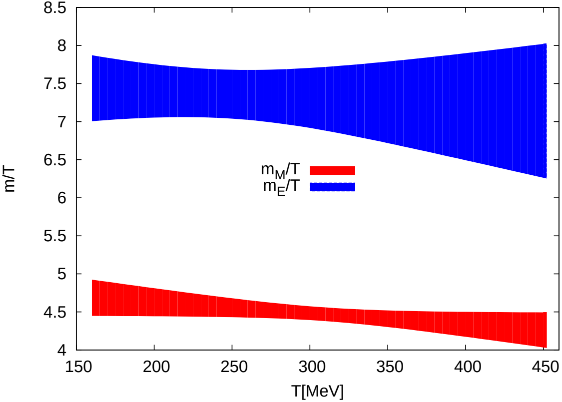
Screening masses - results at finite lattice spacing



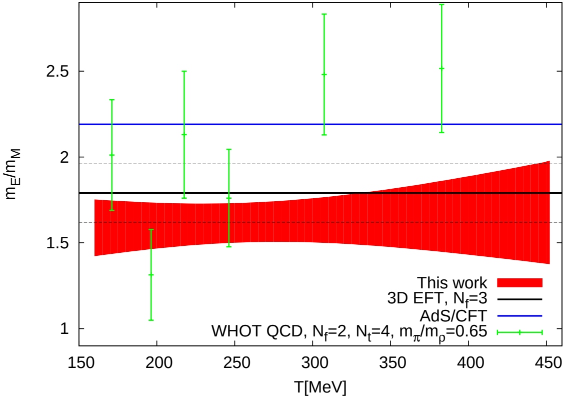
Screening masses - results at finite lattice spacing



Screening masses - continuum results



Screening masses - continuum results and comparisons



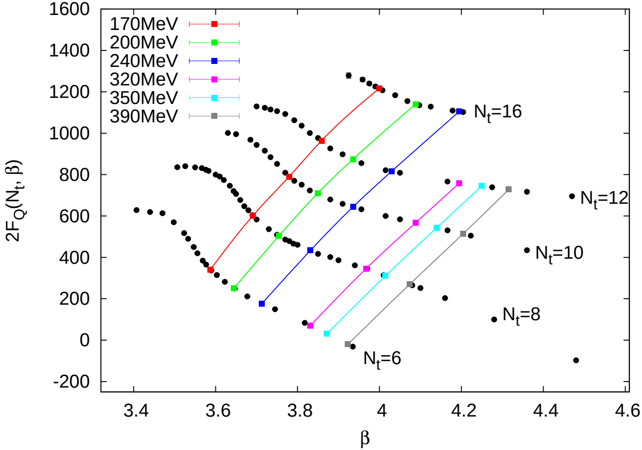
Summary

We studied Polyakov loop correlators in finite temperature QCD with physical quark masses.

1. We performed a continuum limit extrapolation of the single quark and $Q\bar{Q}$ pair free energies, calculable from the Polyakov loop.
2. Using a decomposition of the Polyakov loop correlator to an Euclidean time reflection odd and even part, we determined electric and magnetic screening masses in the QGP.

Backup - renormalization

Single quark free energy



Backup - renormalization

Single quark free energy

- Problem: limited temperature range for the continuum limit
- E.g. for $T_0 = 200\mathrm{MeV}$ if I want to do c.l. from the $N_t = 8, 10, 12$ lattices, the temperature range will be $(6/8)200\mathrm{MeV} = 150\mathrm{MeV}$ to $(16/12)200\mathrm{MeV} = 266.7\mathrm{MeV}$.
- Simple trick: use more values of T_0 and shift them together in the continuum:

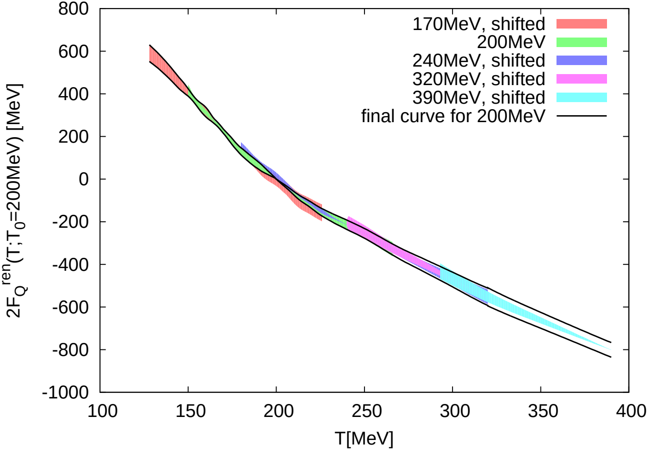
$$F_Q^{\mathrm{ren}}(T;T_0) - F_Q^{\mathrm{ren}}(T;T_1) = F_Q^{\mathrm{ren}}(T_1;T_0)$$

Backup - Free energy - error estimation

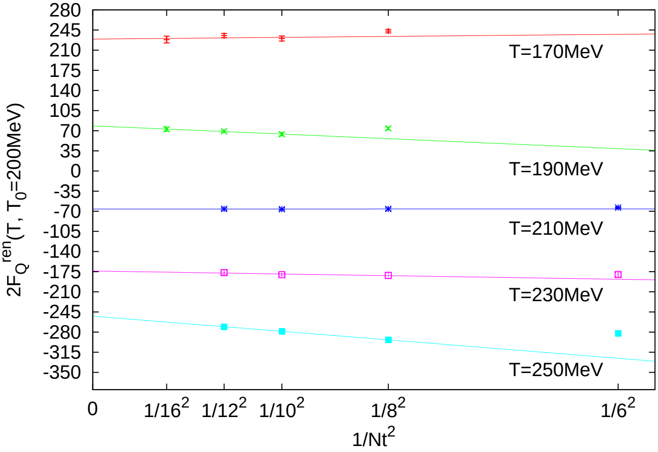
Error sources

- Statistical error (jacknife)
- Method of the different interpolations
- Different point for continuum extrapolation ($N_t = 8, 10, 12$ vs $N_t = 8, 10, 12, 16$)
- Histogram method, AIC weights

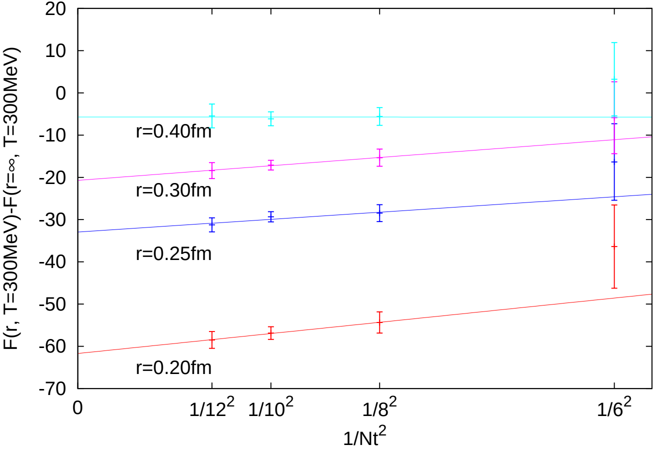
Backup - single quark free energy continuum results



Backup - discretization errors in the free energy



Backup - discretization errors in the free energy



Screening masses - outline of analysis

Backup - Screening Masses - Steps of analysis

1. fix N_t, β, $\rightarrow$ fits for m_M, m_E
2. fix N_t, interpolation in T (polynomial fit)
3. Continuum limit in $1/N_t^2 \propto a^2$ from $N_t = 8, 10, 12$
4. Comment: screening mass can be extracted without renormalizing the correlator

The first step is a little tricky, if you don't do it carefully you can be off by lots of σs ($\rightarrow$ Kolmogorov-Smirnov test).

Backup - Screening masses - correlated fitting

Procedure

- Correlated fitting
- Careful choice of fit interval: Kolmogorov-Smirnov test for the uniform distribution of Q
- Systematic error: choice of fit interval (within the limits where the previous test is OK) AIC or Q weights
- Stat. error: jackknife

Choice of fit interval

- $m \sim T$ in both cases $\rightarrow$ choose the fit intervals in units of rT
- If χ^2 calc. from data has χ^2 distribution $\rightarrow$ Q has uniform distribution
- we fix the range of all the fits in rT and perform a KS test

Backup - Screening masses - Kolmogorov-Smirnov test

Correlator type	$(rT)_{\mathrm{min}}$	$(rT)_{\mathrm{max}}$	$Pr\,(\mathrm{KS, uniform})$
Magnetic	0.43	0.9	0.007
Magnetic	0.45	0.9	0.016
Magnetic	0.465	0.9	0.30
Magnetic	0.5	0.9	0.38
Magnetic	0.61	0.9	0.96
Electric	0.3	0.65	$3 \cdot 10^{-7}$
Electric	0.32	0.65	0.018
Electric	0.35	0.65	0.31
Electric	0.43	0.65	0.94

Backup - screening mass comparisons

- This work:

$$m_E/T = 7.31(25) \quad m_M/T = 4.48(9)$$
$$m_E/m_M = 1.63(8)$$

- $N_f = 2$ lQCD, $m_\pi/m_\rho = 0.65$, $N_t = 4$ Maezawa, Aoki et al. 2010

$$m_E/T = 13.0(11) \quad m_M/T = 5.8(2)$$
$$m_E/m_M = 2.3(3)$$

- $\mathcal{N} = 4$ SYM, large N_c limit, AdS/CFT Bak, Karch, Yaffe 2007

$$m_E/T = 16.05 \quad m_M/T = 7.34$$
$$m_E/m_M = 2.19$$

- dimensionally reduced 3D effective theory, $N_f = 3$ massless quarks Hart, Laine, Philipsen 2000

$$m_E/T = 7.9(4) \quad m_M/T = 4.5(2)$$
$$m_E/m_M = 1.76(17)$$

Backup - the choice of the ansatz

Kolmogorov-Smirnov probabilities

Típus	$(rT)_{\min}$	$Pr\,(\mathrm{KS}, \mathrm{exp.})$	$Pr\,(\mathrm{KS}, \mathrm{Yuk.})$
M	0.43	$1 \cdot 10^{-16}$	0.007
M	0.45	$2 \cdot 10^{-14}$	0.016
M	0.465	$3 \cdot 10^{-8}$	0.30
M	0.5	$4 \cdot 10^{-5}$	0.38
M	0.61	0.86	0.96
M	0.7	0.63	0.96
E	0.3	$1 \cdot 10^{-17}$	$3 \cdot 10^{-7}$
E	0.32	$4 \cdot 10^{-6}$	0.018
E	0.35	$5 \cdot 10^{-3}$	0.31
E	0.43	0.99	0.94
E	0.45	0.99	0.99

Exp. ansatz gives masses approx. $20 - 30\%$ higher

Backup - the choice of the ansatz

EFT approach

QCD = 4D YM + quarks, $\omega_n \sim 2\pi T$

$\downarrow$ PT

EQCD = 3D YM + A_0, $m_{el} \sim gT$

$\downarrow$ PT

MQCD = 3D YM, $g_3^2 \sim g^2 T$

Non-perturbative