Direct calculation of hadronic light-by-light scattering

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Outline

1. Introduction
2. Lattice four-point function
3. Light-by-light scattering amplitude
4. Strategy for $g - 2$
5. Summary and outlook

Some of these results were posted in arXiv:1507.01577

The muon $g-2$

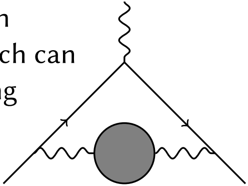
One of the most precise tests of the Standard Model

$$a_\mu \equiv \left(\frac{g-2}{2}\right)_\mu = \begin{cases} 116592080(63) \times 10^{-11} & \text{experiment} \\ 116591790(65) \times 10^{-11} & \text{theory} \end{cases}$$

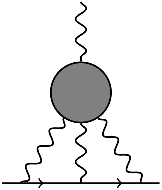
$\delta a_\mu = (290 \pm 90) \times 10^{-11}$, a 3σ deviation

- Fermilab 989 has goal to reduce experimental error by factor of 4
- Leading theory errors come from:

Hadronic vacuum polarization, which can be improved using $e^+e^- \to$ hadrons experiments

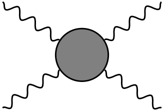


Hadronic light-by-light (HLbL) scattering, which is not easily obtained from experiments



Light-by-light scattering

Before computing a_μ^{HLbL}, start by studying light-by-light scattering by itself.



This has much more information than just a_μ^{HLbL}. We can:

- Compare against phenomenology.
- Test models used to compute a_μ^{HLbL}.

Lattice four-point function

Directly compute four-point function of vector currents

- Use one local current $Z_V J^l_\mu$ at the source point.
- Use three conserved currents J^c_μ.

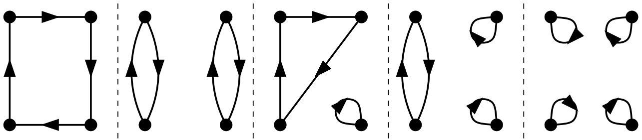
In position space:

$$\begin{aligned}\Pi^{\text{pos}}_{\mu_1\mu_2\mu_3\mu_4}(x_1,x_2,0,x_4) = \Big\langle Z_V J^l_{\mu_3}(0)\big[&J^c_{\mu_1}(x_1)J^c_{\mu_2}(x_2)J^c_{\mu_4}(x_4)\\ &+\delta_{\mu_1\mu_2}\delta_{x_1x_2}T_{\mu_1}(x_1)J^c_{\mu_4}(x_4)\\ &+\delta_{\mu_1\mu_4}\delta_{x_1x_4}T_{\mu_4}(x_4)J^c_{\mu_2}(x_2)\\ &+\delta_{\mu_2\mu_4}\delta_{x_2x_4}T_{\mu_4}(x_4)J^c_{\mu_1}(x_1)\\ &+\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_4}\delta_{x_1x_4}\delta_{x_2x_4}J^c_{\mu_4}(x_4)\big]\Big\rangle,\end{aligned}$$

where $T_\mu(x)$ is a “tadpole” contact operator. This satisfies the conserved-current relations,

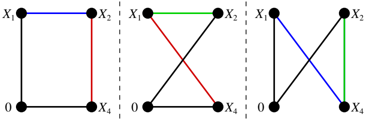
$$\Delta^{x_1}_{\mu_1}\Pi^{\text{pos}}_{\mu_1\mu_2\mu_3\mu_4} = \Delta^{x_2}_{\mu_2}\Pi^{\text{pos}}_{\mu_1\mu_2\mu_3\mu_4} = \Delta^{x_4}_{\mu_4}\Pi^{\text{pos}}_{\mu_1\mu_2\mu_3\mu_4} = 0.$$

Quark contractions



Compute only the *fully-connected* contractions, with fixed kernels summed over x_1 and x_2:

$$\Pi^{\text{pos}'}_{\mu_1\mu_2\mu_3\mu_4}(x_4; f_1, f_2) = \sum_{x_1,x_2} f_1(x_1) f_2(x_2) \Pi^{\text{pos}}_{\mu_1\mu_2\mu_3\mu_4}(x_1, x_2, 0, x_4)$$



Generically, need the following propagators:

- 1 point-source propagator from $x_3 = 0$
- 8 sequential propagators through x_1, for each μ_1 and f_1 or f_1^*
- 8 sequential propagators through x_2
- 32 double-sequential propagators through x_1 and x_2, for each (μ_1, μ_2) and (f_1, f_2) or (f_1^*, f_2^*)

Kinematical setup

Obtain momentum-space Euclidean four-point function using plane waves:

$$\Pi^{E}_{\mu_1\mu_2\mu_3\mu_4}(p_4; p_1, p_2) = \sum_{x_4} e^{-ip_4\cdot x_4} \Pi^{\text{pos}'}_{\mu_1\mu_2\mu_3\mu_4}(x_4; f_1, f_2)\Big|_{f_a(x)=e^{-ip_a\cdot x}}.$$

Thus, we can efficiently fix $p_{1,2}$ and choose arbitrary p_4.

- Full 4-point tensor is very complicated: it can be decomposed into 41 scalar functions of 6 kinematic invariants.
- Forward case is simpler:

$$Q_1 \equiv p_2 = -p_1, \quad Q_2 \equiv p_4.$$

  Then there are 8 scalar functions that depend on 3 kinematic invariants.

Lattice ensembles

Use CLS ensembles: $N_f = 2$ $O(a)$-improved Wilson, with $a = 0.063$ fm.

1. $m_\pi = 451$ MeV, 64×32^3
2. $m_\pi = 324$ MeV, 96×48^3
3. $m_\pi = 277$ MeV, 96×48^3

Keep only u and d quarks in the electromagnetic current, i.e.,

$$J^l_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d.$$

Study forward case with a few different Q_1 and also more general kinematics.

Forward LbL amplitude

Take the amplitude for forward scattering of transversely polarized virtual photons,

$$\mathcal{M}_{TT}(-Q_1^2, -Q_2^2, \nu) = \frac{e^4}{4} R_{\mu_1\mu_2} R_{\mu_3\mu_4} \Pi^E_{\mu_1\mu_2\mu_3\mu_4}(-Q_2; -Q_1, Q_1),$$

where $\nu = -Q_1 \cdot Q_2$ and $R_{\mu\nu}$ projects onto the plane orthogonal to Q_1, Q_2.

A subtracted dispersion relation at fixed spacelike Q_1^2, Q_2^2 relates this to the $\gamma^*\gamma^* \to$ hadrons cross sections $\sigma_{0,2}$:

$$\mathcal{M}_{TT}(q_1^2, q_2^2, \nu) - \mathcal{M}_{TT}(q_1^2, q_2^2, 0) = \frac{2\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\sqrt{\nu'^2 - q_1^2 q_2^2}}{\nu'(\nu'^2 - \nu^2 - i\epsilon)} \left[\sigma_0(\nu') + \sigma_2(\nu')\right]$$

This is model-independent and will allow for systematically improvable comparisons between lattice and experiment.

Model for $\sigma(\gamma^*\gamma^* \to \text{hadrons})$

(V. Pascalutsa, V. Pauk, M. Vanderhaeghen, Phys. Rev. D **85** (2012) 116001)

Include single mesons and $\pi^+\pi^-$ final states:

$$\sigma_0 + \sigma_2 = \sum_M \sigma(\gamma^*\gamma^* \to M) + \sigma(\gamma^*\gamma^* \to \pi^+\pi^-)$$

Mesons:

- pseudoscalar (π^0, η')
- scalar (a_0, f_0)
- axial vector (f_1)
- tensor (a_2, f_2)

$\sigma(\gamma^*\gamma^* \to M)$ depends on the meson's:

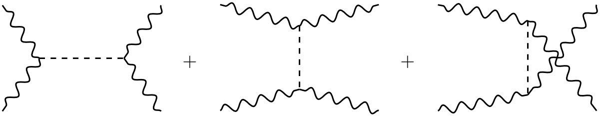
- mass m and width Γ
- two-photon decay width $\Gamma_{\gamma\gamma}$
- two-photon transition form factor $F(q_1^2, q_2^2)$

assume $F(q_1^2, q_2^2) = F(q_1^2, 0)F(0, q_2^2)/F(0,0)$

Use scalar QED dressed with form factors for $\sigma(\gamma^*\gamma^* \to \pi^+\pi^-)$.

Aside: π^0 contribution

Leading HLbL contributions to muon $g - 2$ are expected to come from π^0 exchange diagrams, which dominate at long distances.



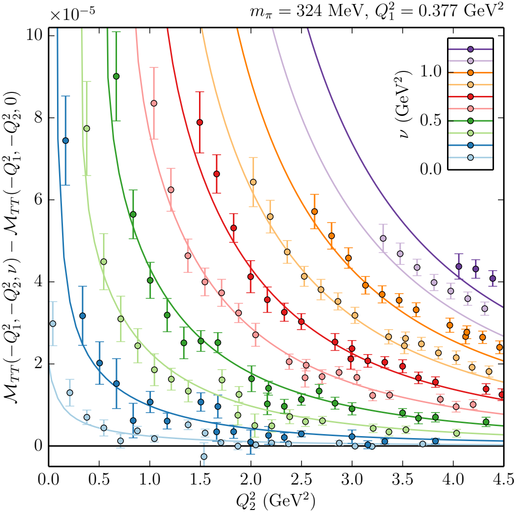
Their contribution to the four-point function:

$$\Pi^{E,\pi^0}_{\mu_1\mu_2\mu_3\mu_4}(p_4; p_1, p_2)$$
$$= -p_{1\alpha}p_{2\beta}p_{3\sigma}p_{4\tau}\left(\frac{\mathcal{F}_{12}\epsilon_{\mu_1\mu_2\alpha\beta}\mathcal{F}_{34}\epsilon_{\mu_3\mu_4\sigma\tau}}{(p_1+p_2)^2+m_\pi^2} + \frac{\mathcal{F}_{13}\epsilon_{\mu_1\mu_3\alpha\sigma}\mathcal{F}_{24}\epsilon_{\mu_2\mu_4\beta\tau}}{(p_1+p_3)^2+m_\pi^2} + \frac{\mathcal{F}_{14}\epsilon_{\mu_1\mu_4\alpha\tau}\mathcal{F}_{23}\epsilon_{\mu_2\mu_3\beta\sigma}}{(p_2+p_3)^2+m_\pi^2}\right),$$

where $p_3 = -(p_1 + p_2 + p_4)$ and $\mathcal{F}_{ij} = \mathcal{F}(p_i^2, p_j^2)$.

This is consistent with the dispersion relation using $\sigma(\gamma^*\gamma^* \to \pi^0)$.

$\mathcal{M}_{TT}$: dependence on ν and Q_2^2



For scalar, tensor mesons there is no data from expt; we use

$$F(q^2, 0) = F(0, q^2) = \frac{1}{1 - q^2/\Lambda^2}$$

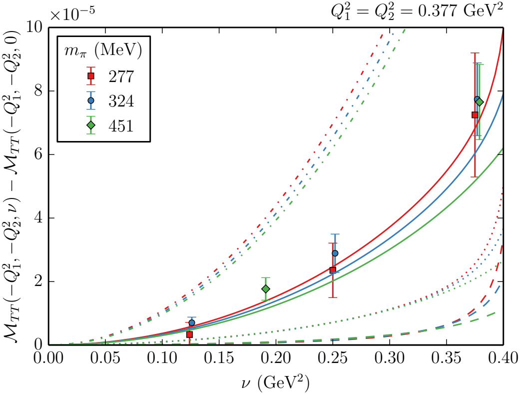
with Λ set by hand to 1.6 GeV

Changing Λ by ± 0.4 GeV adjusts curves by up to $\pm 50\%$.

Points: lattice data.

Curves: dispersion relation + model for cross section.

$\mathcal{M}_{TT}$: dependence on ν and m_π



Points: lattice data.

Curves: dispersion relation + model for cross section.

In increasing order:

- π^0
- $\pi^0 + \eta'$
- full model
- full model + high-energy $\sigma(\gamma\gamma \to \text{hadrons})$ at physical m_π

General kinematics case

To study off-forward kinematics, we fix $p_1^2 = p_2^2 = (p_1 + p_2)^2 = 0.33\ \text{GeV}^2$ and consider contractions of $\Pi^E_{\mu_1\mu_2\mu_3\mu_4}(p_4; p_1, p_2)$ with two different tensors:

1. $\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}$ yields π^0 contribution

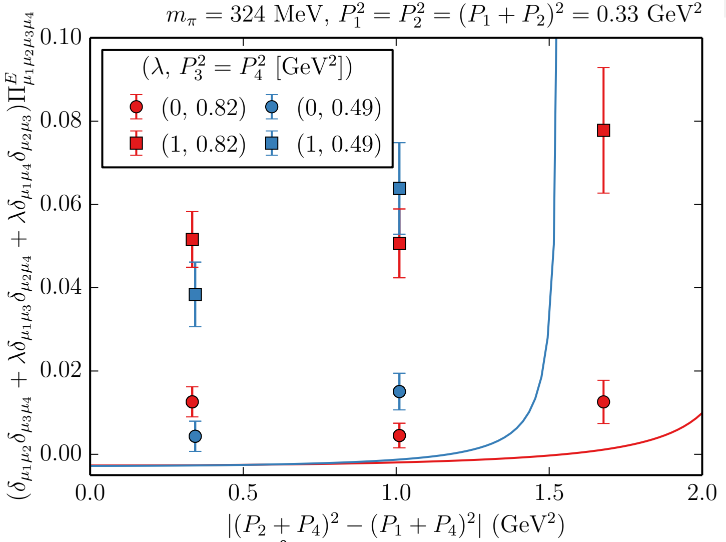
$$-2\left(\frac{(p_1\cdot p_2)(p_3\cdot p_4)-(p_1\cdot p_4)(p_2\cdot p_3)}{(p_1+p_3)^2+m_\pi^2}\mathcal{F}(p_1^2,p_3^2)\mathcal{F}(p_2^2,p_4^2)\right.$$
$$\left.+\frac{(p_1\cdot p_2)(p_3\cdot p_4)-(p_1\cdot p_3)(p_2\cdot p_4)}{(p_2+p_3)^2+m_\pi^2}\mathcal{F}(p_1^2,p_4^2)\mathcal{F}(p_2^2,p_3^2)\right),$$

   where $\mathcal{F}(0,0) = -1/(4\pi^2 F_\pi)$ (Wess-Zumino-Witten) and we use vector meson dominance for dependence on p^2.

2. $\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4} + \delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4} + \delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3}$, which is totally symmetric and thus has no π^0 contribution.

We also fix $p_3^2 = p_4^2$ to two different values and plot versus the one remaining kinematic variable.

Off-forward kinematics

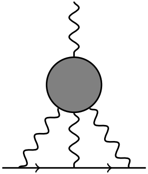


Squares: contraction without π^0 contribution.

Circles: contraction containing π^0 contribution.

Curves: π^0 contribution assuming model for $\mathcal{F}(p_1^2, p_2^2)$.

Strategy for muon $g-2$: kernel



In Euclidean space, give muon momentum $p = im\hat{\epsilon}$, $\hat{\epsilon}^2 = 1$.
Apply QED Feynman rules and isolate $F_2(0)$; obtain

$$a_\mu^{\mathrm{HLbL}} = \int d^4x\, d^4y\, \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(\hat{\epsilon},x,y)\, i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y),$$

where $$\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x_1,x_2) = \int d^4x_4\, (ix_4)_\rho \left\langle J_\mu(x_1)J_\nu(x_2)J_\lambda(0)J_\sigma(x_4)\right\rangle.$$

The integrand for a_μ is a scalar function of 5 invariants: x^2, y^2, $x\cdot y$, $x\cdot\epsilon$, and $y\cdot\epsilon$, so 3 of the 8 dimensions in the integral are trivial.
Five dimensions is still too many. Result is independent of $\hat{\epsilon}$, so we can eliminate it by averaging in the integrand:

$$\mathcal{L}(\hat{\epsilon},x,y) \to \bar{\mathcal{L}}(x,y) \equiv \langle \mathcal{L}(\hat{\epsilon},x,y)\rangle_{\hat{\epsilon}}$$

Then the integrand depends only on x^2, y^2, and $x\cdot y$.

Strategy for muon $g-2$: lattice

$$a_\mu^{\text{HLbL}} = \int d^4x \int d^4y\, d^4z\, \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)(-z)_\rho \left\langle J_\mu(x)J_\nu(y)J_\lambda(0)J_\sigma(z)\right\rangle$$
$$= 2\pi^2 \int_0^\infty x^3 dx \int d^4y\, d^4z\, \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)(-z)_\rho \left\langle J_\mu(x)J_\nu(y)J_\lambda(0)J_\sigma(z)\right\rangle .$$

Evaluate the y and z integrals in the following way:

1. Fix local currents at the origin and x, and compute point-source propagators.
2. Evaluate the integral over z using sequential propagators.
3. Contract with $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ and sum over y.

The above has similar cost to evaluating scattering amplitudes at fixed p_1,p_2. Do this several times to perform the one-dimensional integral over $|x|$.

Summary and outlook

- The contribution from fully-connected four-point function to the light-by-light scattering amplitude can be efficiently evaluated if two of the three momenta are fixed.
- Forward-scattering case is related to $\sigma(\gamma^*\gamma^* \to \text{hadrons})$; lattice is consistent with phenomenology, within the latter's large uncertainty.
- For typical Euclidean kinematics the π^0 contribution is not dominant.
- A strategy is in place for computing the leading-order HLbL contribution to the muon $g-2$.
  Work is ongoing to evaluate the kernel $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$.
- Phenomenology indicates the π^0 contribution is dominant for $g-2$; reaching this regime (physical m_π, large volumes) may be challenging on the lattice.
- Results on the HLbL scattering amplitude were posted in arXiv:1507.01577.