Friedrich-Schiller-Universität Jena

Critical Flavour Number of the Thirring Model in Three Dimensions

Daniel Schmidt Björn Wellegehausen Andreas Wipf

Kobe, July 18, 2015

Research Training Group
Quantum and Gravitational Fields

Research Training Group (1523/2)
Quantum and Gravitational Fields

seit 1558

Outline

What is the Thirring model?

QFT with N_f flavours of massless fermions with current interaction
$\rightarrow$ special case of a four-fermi theory

Euclidean Lagrangian

$$\mathcal{L} = \bar{\psi}_j i\partial\!\!\!/ \psi_j - \frac{g^2}{2N_f}\left(\bar{\psi}_j \gamma^\mu \psi_j\right)^2 \qquad j = 1, \ldots, N_f$$

History

- Introduced by Walter Thirring in 1958 as soluble QFT in 2D.
- Renewed interest in 3D model due to similarity to QED_3 and possible applications in superconductors, graphene, ...

What model exactly?

Massless, reducible Thirring model in 3D

- 3-dimensional euclidean spacetime
- 4-dimensional reducible representation of Clifford algebra (4-component spinors)

What model exactly?

Massless, reducible Thirring model in 3D

- 3-dimensional euclidean spacetime
- 4-dimensional reducible representation of Clifford algebra (4-component spinors)

Hubbard-Stratonovich transformation

Introduction of auxiliary vector field V_μ to eliminate quartic term.

$$\mathcal{L} = \bar{\psi}_j i\gamma^\mu \left(\partial_\mu - iV_\mu\right)\psi_j + \lambda_{\mathrm{Th}} V^\mu V_\mu$$

with $\lambda_{\mathrm{Th}} = \frac{N_f}{2g^2}$. Integration $\int \mathcal{D}V_\mu \; e^{-S}$ leads back to original action.

Symmetries

- chiral symmetry, generated by $\{\mathbb{1}, \gamma_4, \gamma_5, i\gamma_4\gamma_5\}$
- flavour symmetry

together: $U(N_f, N_f)$, can be spontaneously broken to $U(N_f) \otimes U(N_f)$

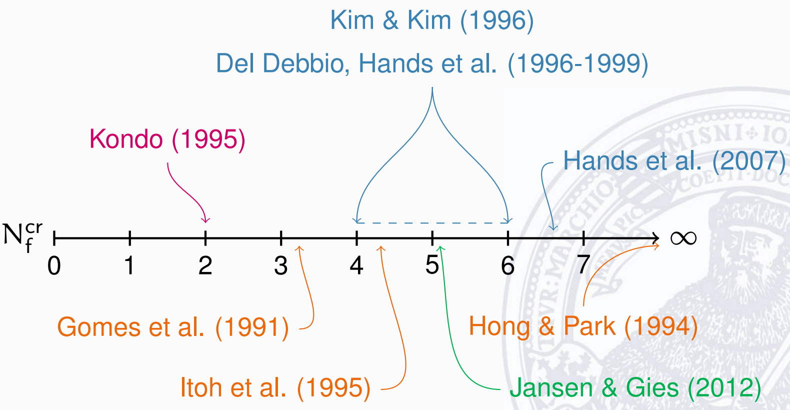
Chiral symmetry breaking

"$N_f = 0.5$" irreducible representation for $N_f = 1$ corresponds to Gross-Neveu model with chiral symmetry breaking

$N_f \to \infty$ no chiral symmetry breaking

$\Rightarrow$ there is a N_f^{cr} where chiral behaviour changes

Results for N_f^{cr} from Schwinger-Dyson equations, $\frac{1}{N_f}$-expansion, functional renormalization group, Lattice simulation with staggered fermions:



Previous Lattice Simulations

First result from Kim & Kim (1996), further publications from Del Debbio, Hands et al. from 1996-1999 and 2007.

fermions	staggered with $m > 0$ (no full chiral symmetry!)
algorithms	Hybrid Monte Carlo ($N_f = 2, 4, 6$)
	Hybrid Molecular Dynamics (N_f real valued)
measurements	fits to "equation of state" relating $\langle \bar{\psi}\psi \rangle$ to m

Previous Lattice Simulations

First result from Kim & Kim (1996), further publications from Del Debbio, Hands et al. from 1996-1999 and 2007.

fermions	staggered with $m > 0$ (no full chiral symmetry!)
algorithms	Hybrid Monte Carlo ($N_f = 2, 4, 6$)
	Hybrid Molecular Dynamics (N_f real valued)
measurements	fits to "equation of state" relating $\langle \bar{\psi}\psi \rangle$ to m

Coupling constant renormalization in $\frac{1}{N_f}$-expansion

renormalizability in $\frac{1}{N_f}$-expansion → transversity of vacuum polarization → renormalization with $g_R^2 = \frac{g^2}{1 - g^2 J(m)}$

Previous Lattice Simulations

First result from Kim & Kim (1996), further publications from Del Debbio, Hands et al. from 1996-1999 and 2007.

fermions	staggered with $m > 0$ (no full chiral symmetry!)
algorithms	Hybrid Monte Carlo ($N_f = 2, 4, 6$)
	Hybrid Molecular Dynamics (N_f real valued)
measurements	fits to “equation of state” relating $\langle \bar{\psi}\psi \rangle$ to m

Coupling constant renormalization in $\frac{1}{N_f}$-expansion

renormalizability in $\frac{1}{N_f}$-expansion → transversity of vacuum polarization → renormalization with $g_R^2 = \frac{g^2}{1 - g^2 J(m)}$

unphysical phase: $g_R^2 < 0$ for bare coupling $g_{\text{lim}}^{-2} < J(m) \xrightarrow[m \to 0]{} \frac{2}{3}$

Outline

Current approach

fermions use derivative with exact chiral symmetry

algorithm rational Hybrid Monte Carlo $\rightarrow$ non-integer N_f possible

SLAC derivative

$$\bar{\psi}\partial\!\!\!/\psi \rightarrow \sum_{x,y=1}^{L} \bar{\psi}_x \partial\!\!\!/_{xy}^{\mathrm{SLAC}} \psi_y \quad \text{with}$$

$$\partial_{xy}^{\mathrm{SLAC}} = \begin{cases} 0 & x = y \\ \frac{\pi}{L}(-1)^{x-y} \frac{1}{\sin\left(\frac{\pi}{L}(x-y)\right)} & x \neq y \end{cases}$$

- not local: needs all grid points in a line
- in momentum space: multiplication by $i\gamma^{\mu}p_{\mu}$

Technical problem

Chiral condensate is always zero due to exact chiral symmetry and integration over fermions.

1. Include new action preserving $U(1)$ symmetry

$$1 = \lim_{g_g \to 0} e^{-S_g} = \lim_{g_g \to 0} e^{\frac{g_g}{2N_f}\left(\Sigma^2 - \Pi^2\right)}$$

with $\Sigma = \frac{1}{V}\sum_x \bar{\psi}_x \psi_x$ and $\Pi = \frac{1}{V}\sum_x \bar{\psi}_x \gamma_5 \psi_x$.

2. Introduce global fields $\sigma, \hat{\pi}$ via Hubbard-Stratonovich transformation:

$$e^{-S_g} = \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\hat{\pi} \, \frac{N_f}{2\pi g_g} e^{-\frac{N_f}{2g_g}\left(\sigma^2 + \hat{\pi}^2\right) + \Sigma\sigma + i\Pi\hat{\pi}}$$

Coupling to a Global Field

3. Rescale fields by $\sqrt{V}$, define $\lambda_g := \frac{VN_f}{2g_g}$. Resulting action:

$$e^{-S_g} = \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\hat{\pi} \frac{\lambda_g}{\pi} e^{-\lambda_g\left(\sigma^2+\hat{\pi}^2\right)+\sqrt{V}(\Sigma\sigma+i\Pi\hat{\pi})}$$

4. Include global fields into Dirac operator:

$$\bar{\psi}D\psi = \bar{\psi}\left(i\partial\!\!\!/ + V\!\!\!/ - \frac{1}{\sqrt{V}}(\sigma + i\gamma_5\pi)\right)\psi$$

5. Full action:

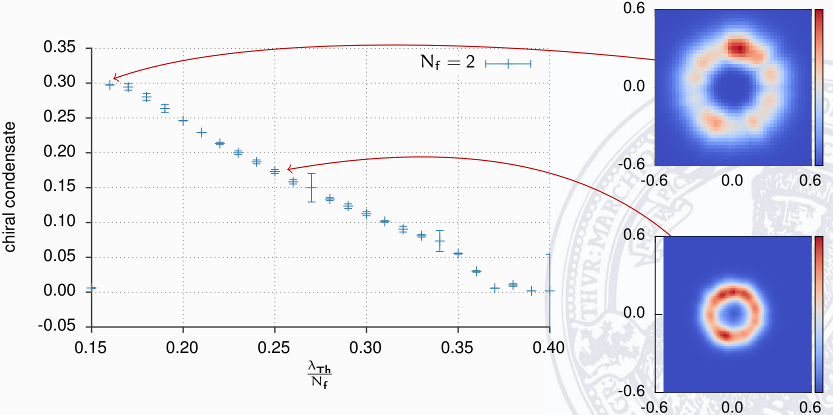
$$S = \sum_x \bar{\psi}D\psi + \lambda_{\mathrm{Th}} \sum_x V_\mu V^\mu + \lambda_g\left(\sigma^2 + \hat{\pi}^2\right)$$

$$Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}V_\mu d\sigma d\hat{\pi}\, e^{-S}$$

Outline

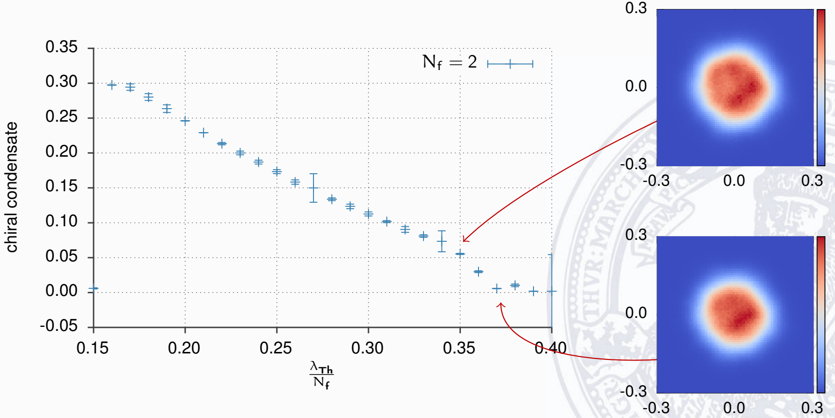
Fixed Global Coupling

Local histograms of chiral condensate for lattice size 12x11x11, $\lambda_g = 0.5$.

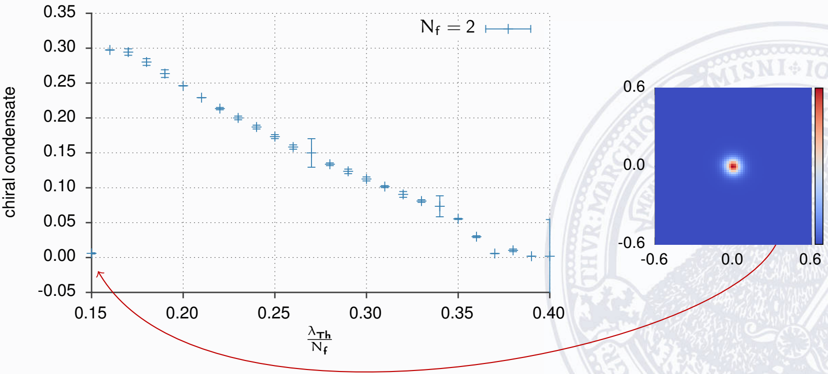


Fixed Global Coupling

Local histograms of chiral condensate for lattice size 12x11x11, $\lambda_g = 0.5$.

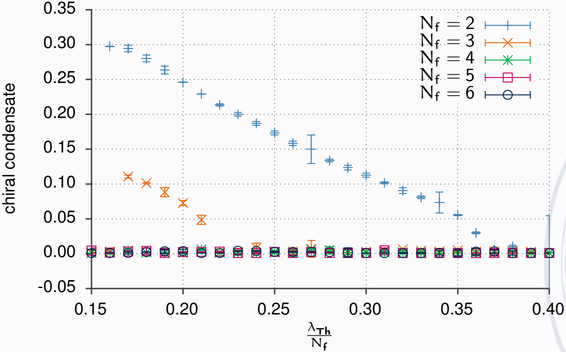


Local histograms of chiral condensate for lattice size 12x11x11, $\lambda_g = 0.5$.



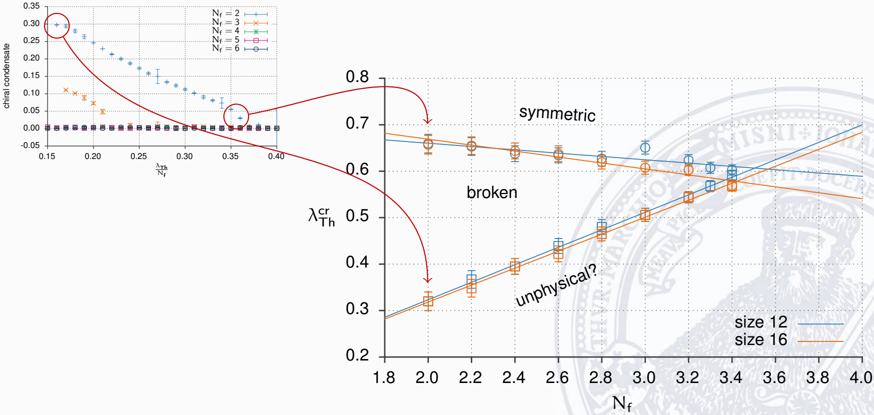
Fixed Global Coupling

Chiral condensate dependence on N_f for lattice size 12x11x11, $\lambda_g = 0.5$.

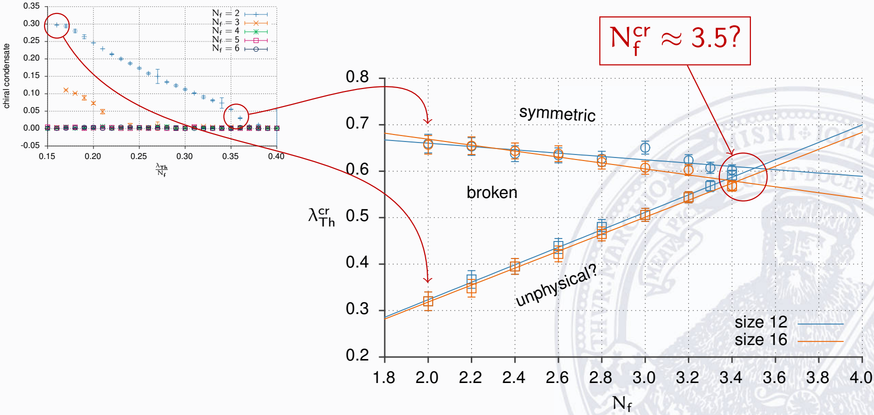


Fixed Global Coupling

Critical couplings for both transitions depending on N_f, for $\lambda_g = 0.5$.



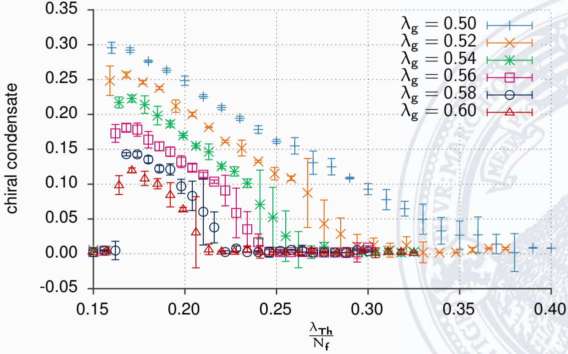
Critical couplings for both transitions depending on N_f, for $\lambda_g = 0.5$.



Thirring Model Limit $\lambda_g \to \infty$

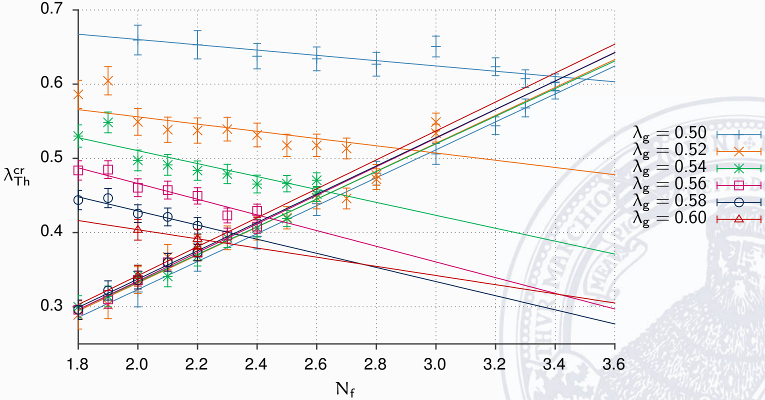
Increasing global coupling for $N_f = 2$

- Artefact transition has $\frac{\lambda_{Th}}{N_f}$ roughly constant.
- Physical transition moves to smaller coupling.



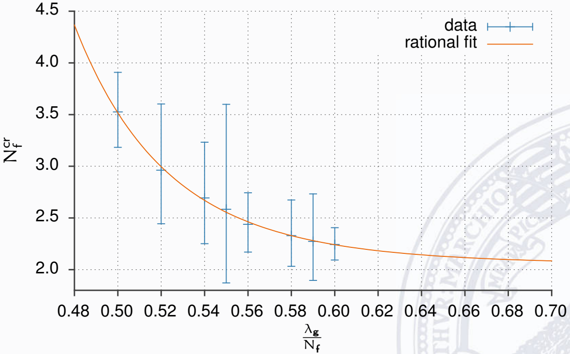
Thirring Model Limit $\lambda_g \to \infty$

Phase transitions get closer for increasing $\lambda_g \Rightarrow N_f^{cr}$ gets smaller.



Thirring Model Limit $\lambda_g \to \infty$

Intersections of the lines of physical and artefact transition, depending on λ_g:

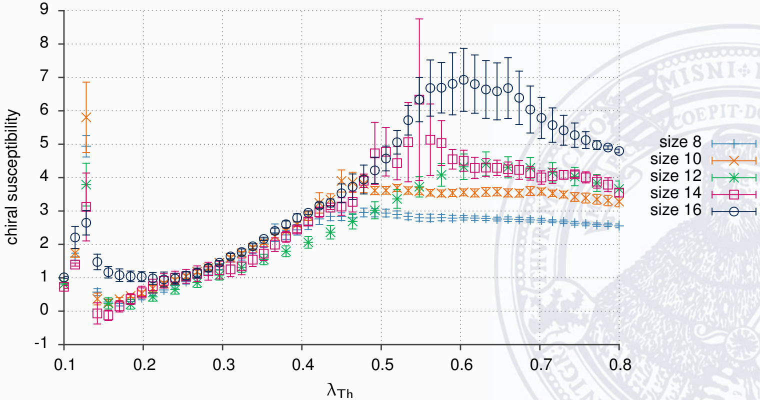


Fit to data with $ax^{-b} + c$ gives $N_f^{cr}(\lambda_g \to \infty) = 2.05 \pm 0.05$.

Can we define a continuum limit? At which transition?

- Peaks in susceptibility agree with transitions in chiral condensate.
- Current work: scaling of susceptibility with the lattice volume.

Example for $N_f = 1$, $\lambda_g = 0.5$:



Summary

- The Thirring model shows chiral symmetry breaking for $N_f < N_f^{cr}$, for which there are many different predictions.
- Lattice formulation with SLAC fermions has the same chiral symmetry as the continuum model.
- Coupling to a global model allows direct measurement of chiral condensate while preserving a $U(1)$-symmetry.
- There is a second phase transition, where the model might get unphysical.
- When recovering the Thirring model, the chiral symmetry breaking transition merges with the second one for $N_f > 2$ and probably not for $N_f \leqslant 2$.

Thank you for your attention!