Neutral B meson mixings and B meson decay constants in the infinite b quark mass limit with domain-wall light quarks

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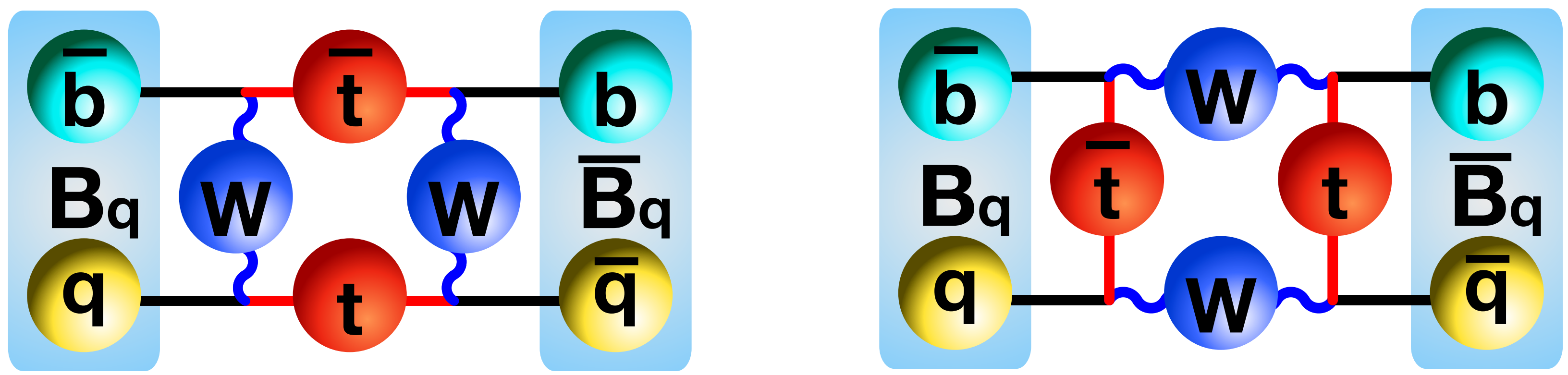
LATTICE 2015 KOBE

$B^0 - \bar{B}^0$ mixing: constrains on CKM

- $B^0 - \bar{B}^0$ mixing



$q = \{d, s\}$

- Neutral mesons are not eigenstates of the weak interactions.
- New Physics comes through loop diagrams.
- Mass difference between physical eigenstates:

$$\Delta m_q = \frac{G_F^2 m_W^2}{16\pi^2 m_{B_q}} |V_{tq}^* V_{tb}|^2 S_0 \left(\frac{m_t^2}{m_W^2}\right) \eta_B \mathcal{M}_{B_q}$$

$\longrightarrow$ constraints to V_{td}, V_{ts}

- $\Delta B = 2$ mixing matrix elements (non-perturbative hadronic)

$$\mathcal{M}_{B_q} = \langle \overline{B}_q^0 | [\bar{b}\gamma_\mu P_L q][\bar{b}\gamma_\mu P_L q] | B_q^0 \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}$$

$B^0 - \bar{B}^0$ mixing: constrains on CKM

SU(3) breaking ratio ξ

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi\sqrt{\frac{\Delta m_d}{\Delta m_s}\frac{m_{B_s}}{m_{B_d}}} \qquad \xi = \frac{m_{B_d}}{m_{B_s}}\sqrt{\frac{\mathcal{M}_{B_s}}{\mathcal{M}_{B_d}}}$$

- The most attractive quantity in the mixing phenomena.
- Many of the uncertainties are canceled in the ratio.
- In the simulation, fluctuations are largely canceled in the ratio.

Other quantities in this work

- B meson decay constants

$$f_{B_d}, f_{B_s}$$

- B-parameters

$$B_q = \frac{3}{8}\frac{\mathcal{M}_{B_q}}{m_{B_q}^2 f_{B_q}^2}$$

Static limit

- Static approximation (leading order of HQET)
  - Easy to implement (Static quark propagator is almost free.)
  - Symmetries (HQ spin symmetry + chiral symmetry)

    reduced operator mixing
  - Continuum limit exists even in the perturbative renormalization.
  - But, we always have the error coming from static approx.

    $O(\Lambda_{\rm QCD}/m_b) \sim 10\%$

- Ratio quantities (ξ , f_{B_s}/f_{B_d}) in the static limit
  - Error coming from static approximation is reduced to:

$$O\left(\frac{m_s - m_d}{\Lambda_{\rm QCD}} \times \frac{\Lambda_{\rm QCD}}{m_b}\right) \sim 2\%$$

Static limit

Static limit as a valuable anchor point

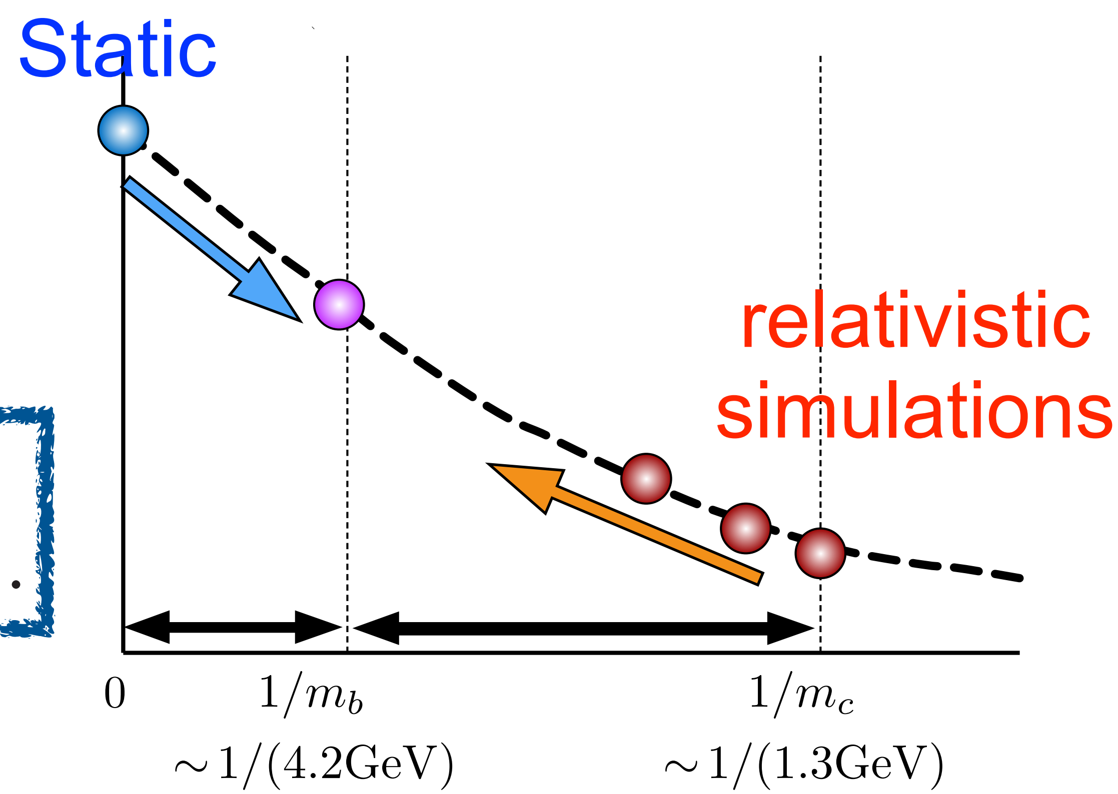
- HQ expansion:

$$\Phi_{\rm hl}(1/m_Q) = \Phi_{\rm hl}(1/m_{Q_A}) \exp\left[\sum_{p=1}^{\infty} \gamma_p \left\{ \left(\frac{\Lambda_{\rm QCD}}{m_Q}\right)^p - \left(\frac{\Lambda_{\rm QCD}}{m_{Q_A}}\right)^p \right\}\right].$$

m_{Q_A} : anchor point

- Once γ_p is determined, what we need is the overall factor at some anchor point.

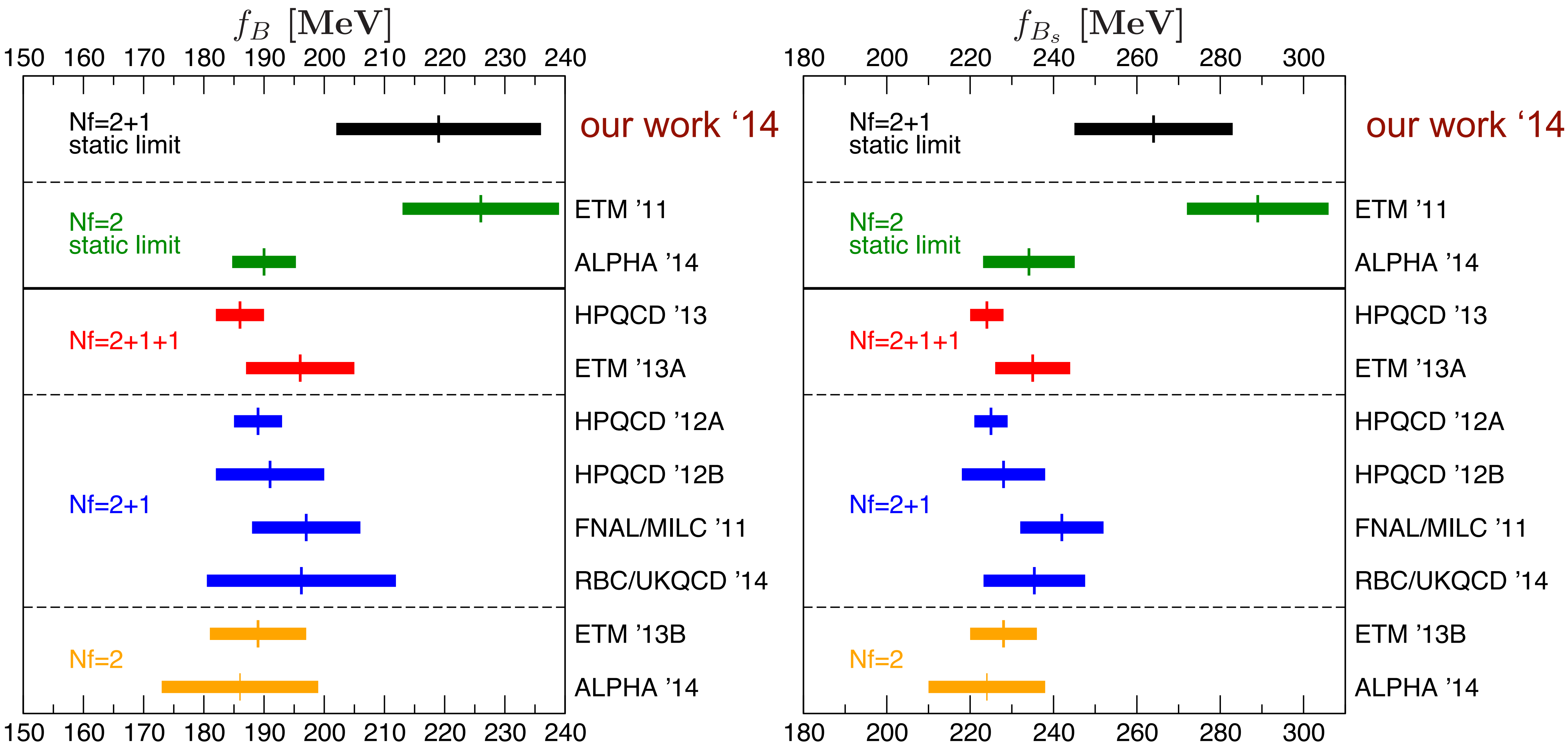
- Static limit $m_Q \to \infty$ is close to target point m_b in terms of $1/m_Q$.



Summary of our previous work

Comparison (decay constants)

[PRD91 (2015) 11, 114505]

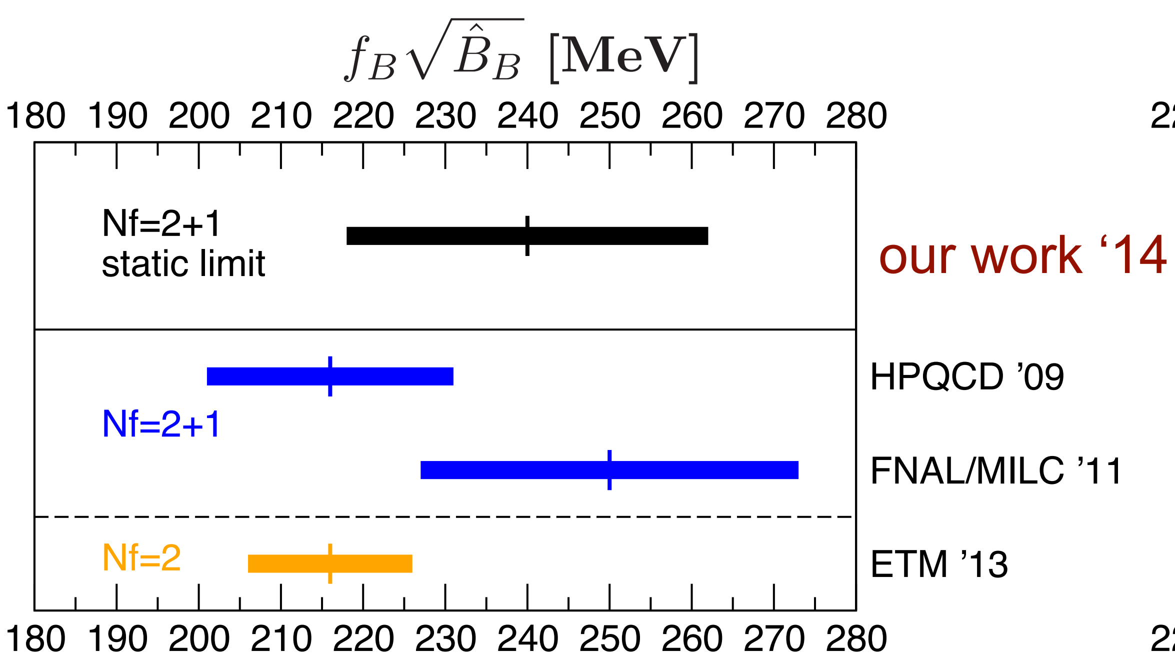


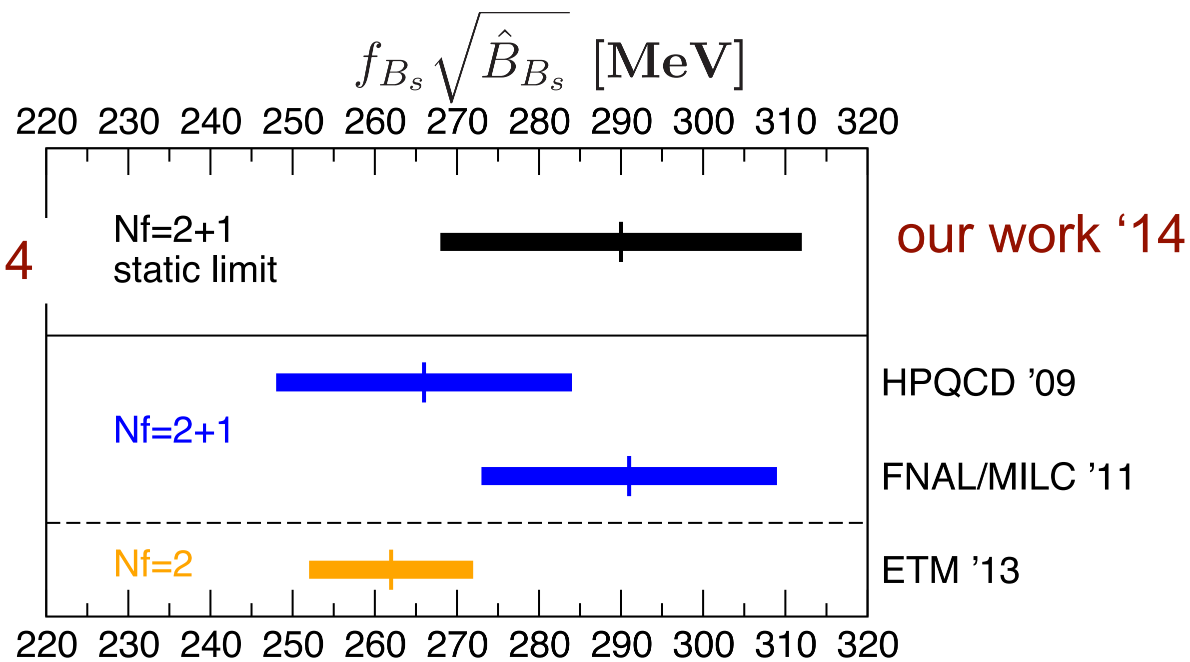
Decay constants have ~10% deviation from physical b results.

Summary of our previous work

- Comparison (mixing matrix elements)

[PRD91 (2015) 11, 114505]



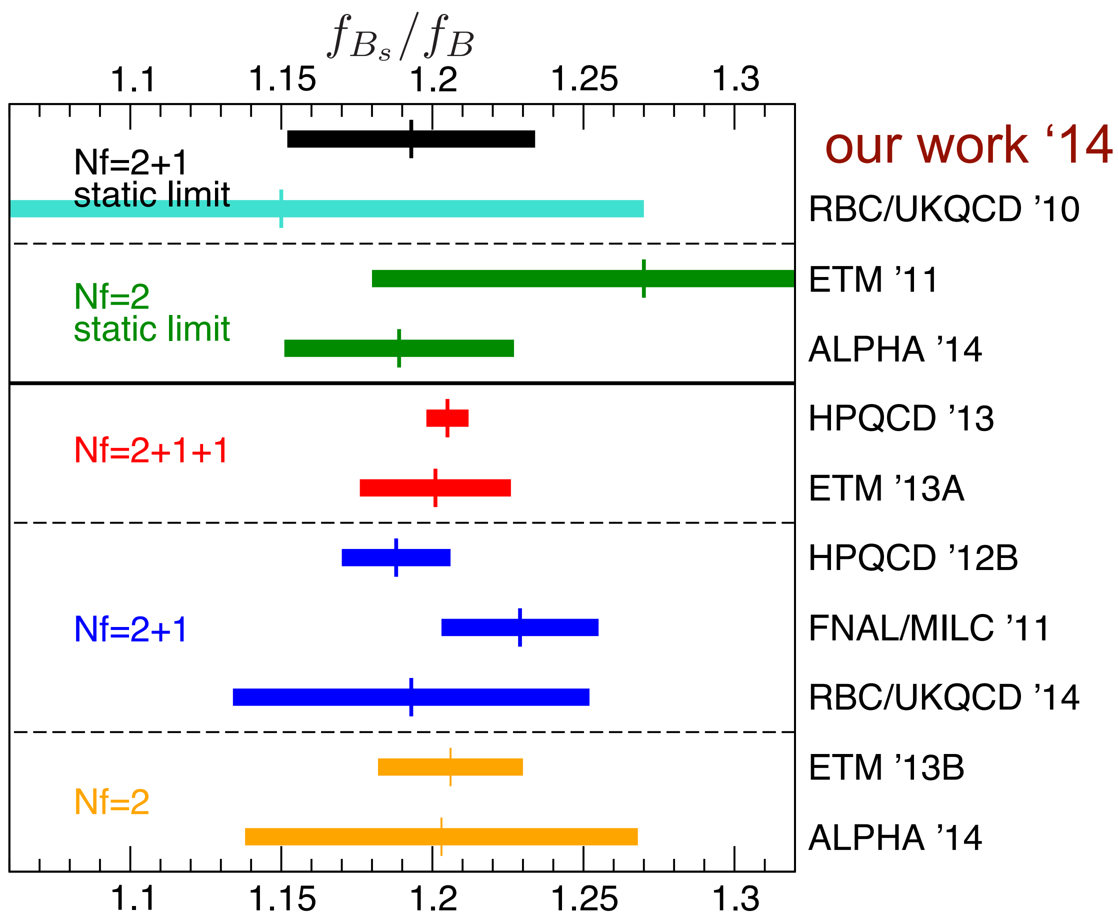


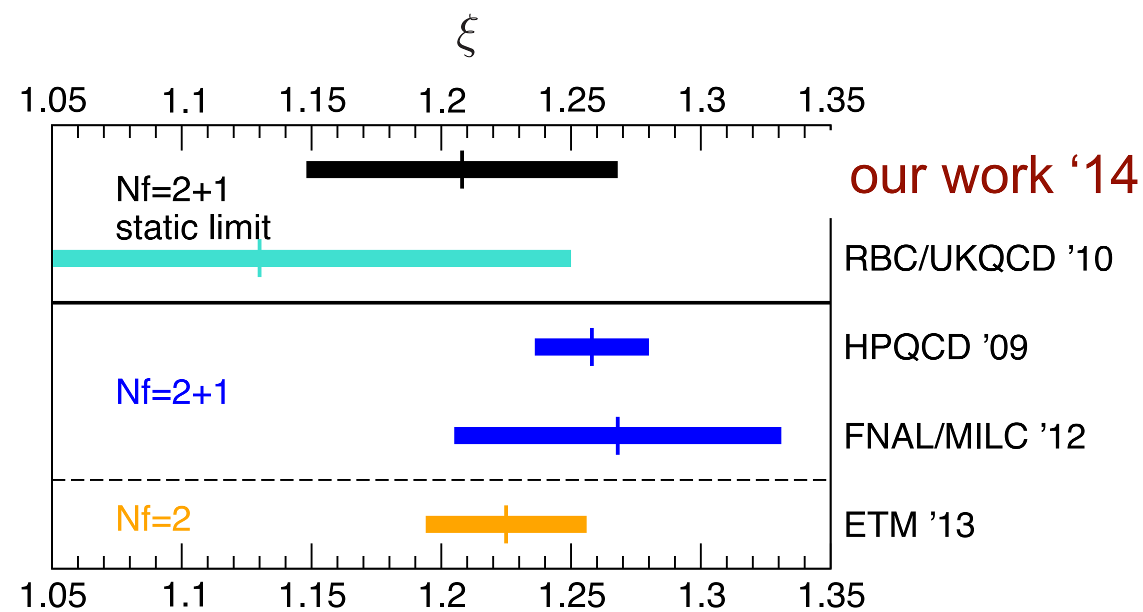
Current uncertainty is too large to see the deviation from physical b results.

Summary of our previous work

Comparison (ratios)

[PRD91 (2015) 11, 114505]



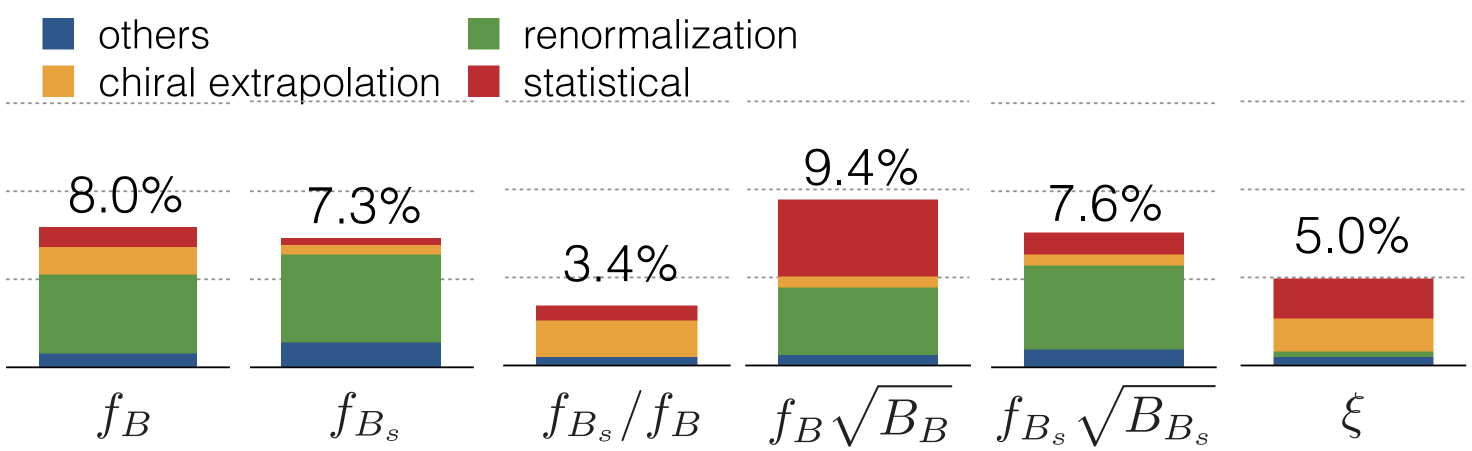


Deviation from other approach is small for ratios.

Summary of our previous work

Error budget

	f_B	f_{B_s}	f_{B_s}/f_B	$f_B\sqrt{\hat{B}_B}$	$f_{B_s}\sqrt{\hat{B}_{B_s}}$	ξ	$\hat{B}_B$	$\hat{B}_{B_s}$	B_{B_s}/B_B
Statistics	2.99	1.81	1.65	6.34	3.12	3.36	9.80	4.93	5.80
Chiral/continuum extrapolation	3.54	1.98	2.66	2.55	2.13	3.08	14.84	7.15	3.66
Finite volume effect	0.82	0.0	1.00	0.76	0.00	1.07	0.15	0.0	0.16
Discretization	1.0	1.0	0.2	1.0	1.0	0.2	1.0	1.0	0.2
One-loop renormalization	6.0	6.0	0.0	6.0	6.0	1.2	6.0	6.0	1.2
$g_{B^*B\pi}$	0.24	0.00	0.35	0.14	0.00	0.25	0.20	0.00	0.22
Scale	1.82	1.85	0.04	1.84	1.86	0.05	0.04	0.05	0.02
Physical quark mass	0.05	0.01	0.06	0.06	0.19	0.20	0.03	0.00	0.02
Off-physical sea s quark mass	0.84	0.69	0.79	0.20	0.39	0.91	0.28	0.19	0.42
Fit-range	0.44	2.31	0.26	0.10	1.74	0.58	3.14	0.00	1.54
Total systematic error	7.38	7.09	3.00	6.90	6.94	3.66	16.34	9.39	4.18
Total error (incl. statistical)	7.96	7.32	3.42	9.37	7.61	4.97	19.05	10.61	7.15



Current challenge

▸ Improvements to be addressed

- All-Mode-Averaging (AMA)

  improved operator using lattice symmetry ⟶ good statistics

- Almost physical pion ensemble (Mobius domain-wall (RBC/UKQCD))

action	$1/a$ [Gev]	lattice	size [fm]	m_π [MeV]
MDWF + IW	1.75	$48^3 \times 96 \times 24$	5.5	138
MDWF + IW	2.31	$64^3 \times 128 \times 12$	5.5	139

- Non-perturbative renormalization

  We are trying NPR in position space. [P.Korcyl, July 14, Tue]

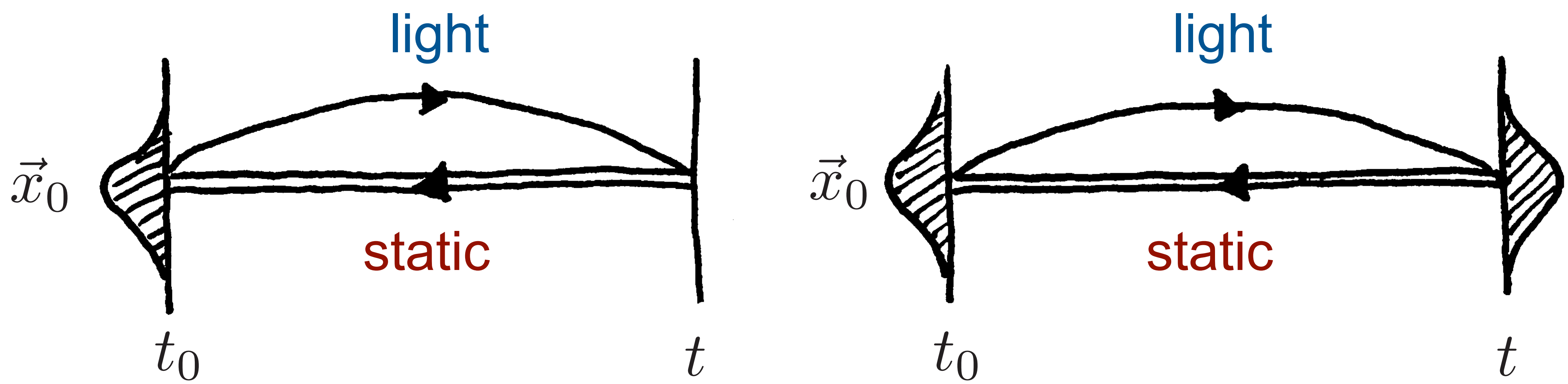
Simulation setup

- Standard static action with link smearing
  - Link smearing to reduce 1/a power divergence.
    - HYP1 [Hasenfratz and Knechtli, 2001]
    - HYP2 [Della Morte et al.(ALPHA), 2004]
- (Mobius) Domain-wall light quark action
- RBC-UKQCD Nf=2+1 ensembles

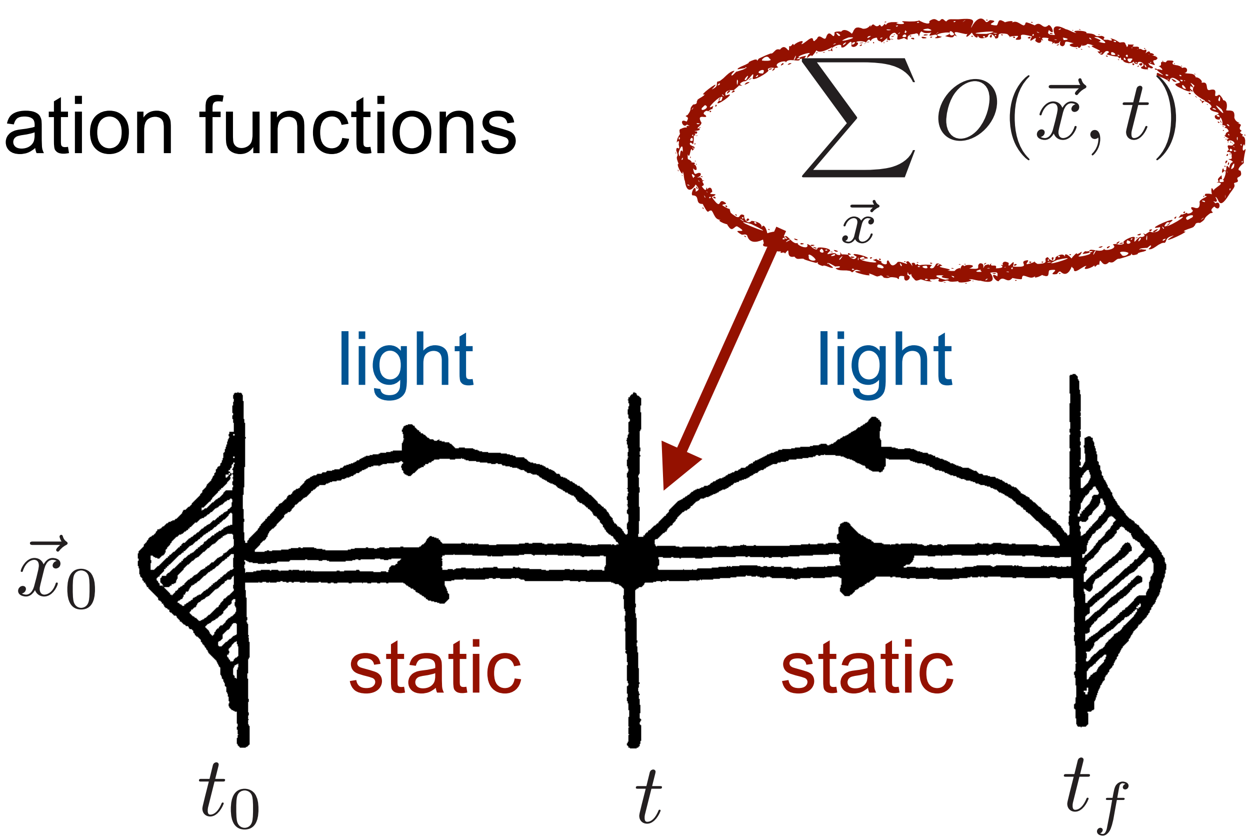
gluon	fermion	$L^3 \times T \times L_s$	am_l	am_h	$am_{\rm res}$	m_π [MeV]	size [fm]
Iwasaki	DWF	$24^3 \times 64 \times 16$	0.01	0.04	0.00308	420	2.7
$\beta = 2.13$	DWF	$24^3 \times 64 \times 16$	0.005	0.04	0.00308	330	2.7
$(a^{-1} \sim 1.75$ GeV$)$	MDWF	$48^3 \times 96 \times 24$	0.00078	0.0362	0.000614	138	5.5
Iwasaki	DWF	$32^3 \times 64 \times 16$	0.008	0.03	0.000664	420	2.6
$\beta = 2.25$	DWF	$32^3 \times 64 \times 16$	0.006	0.03	0.000664	370	2.6
$(a^{-1} \sim 2.31$ GeV$)$	DWF	$32^3 \times 64 \times 16$	0.004	0.03	0.000664	310	2.6
	MDWF	$64^3 \times 128 \times 12$	0.000678	0.02661	0.000314	139	5.5

Simulation setup

- Correlators
  - 2PT correlation functions



  - 3PT correlation functions



Simulation setup

Measurements with AMA

- Approximation for 24c, 32c : sloppy CG with DWF or MDWF
- Approximation for 48c : sloppy CG with zMobius (ls=10)

[C. Jung, July 14, Tue]
[G. McGlynn, July 14, Tue]

	β	am_l/am_h	am_q	# of conf	exact src	sloppy src	# of eig	sloppy iter
24c	2.13	0.005/0.04	0.005	40	4	128	320	100
			0.034	40	4	64	N/A	250
			0.04	40	4	64	N/A	250
		0.010/0.04	0.010	40	2	128	320	100
			0.034	40	2	64	N/A	250
			0.04	40	2	64	N/A	250
32c	2.25	0.004/0.03	0.004	40	4	64	130	125
			0.027	40	1	64	N/A	250
			0.03	40	4	64	130	150
		0.006/0.03	0.006	39	1	64	130	130
			0.027	39	1	64	N/A	250
			0.03	39	1	64	N/A	250
		0.008/0.03	0.008	32	1	64	130	130
			0.027	32	1	64	N/A	250
			0.03	32	1	64	N/A	250

	β	am_l/am_h	am_q	# of conf	exact src	sloppy src	# of eig	sloppy iter
48c	2.13	0.00078/0.0362	0.00078					
			0.03580	20 ($\rightarrow$ 40)	2	108	N/A	350
64c	2.25	0.000678/0.02661	0.000678					
			0.02539					

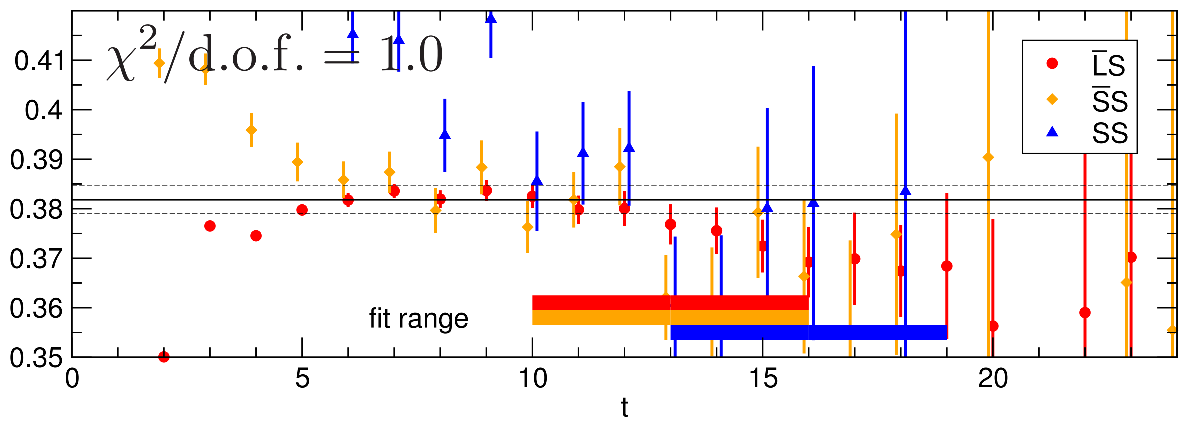
Preliminary results

Correlator data (32c)

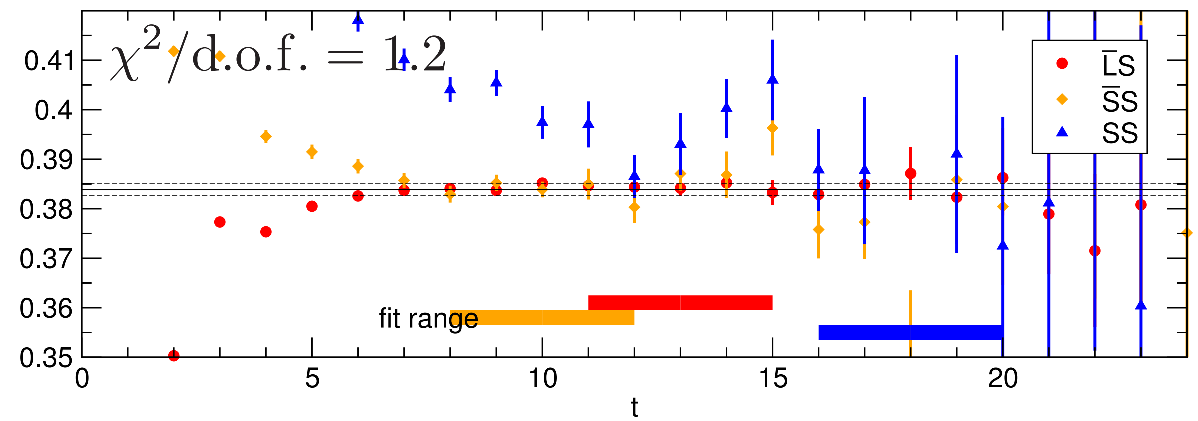
32c, ml/ms=0.004/0.03, mq=0.004, HYP2

2PT

(previous)

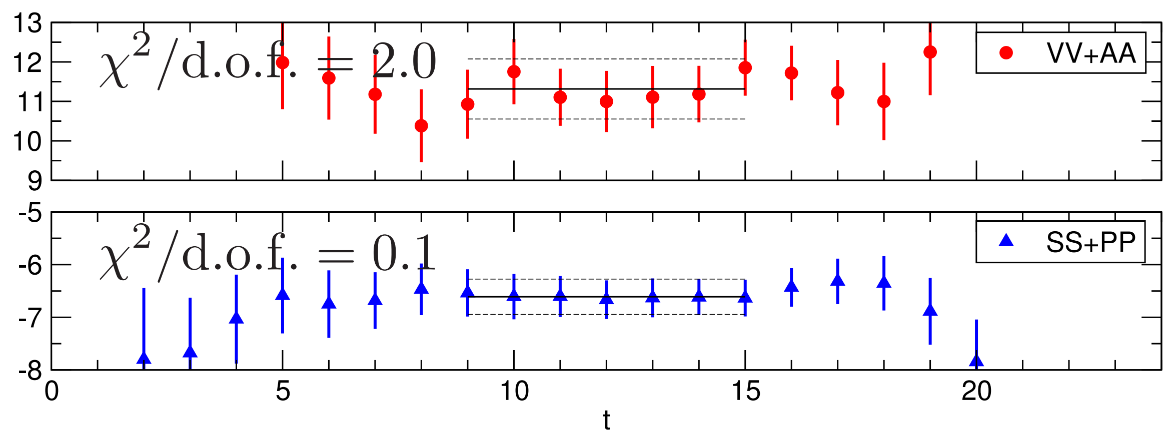


(AMA)

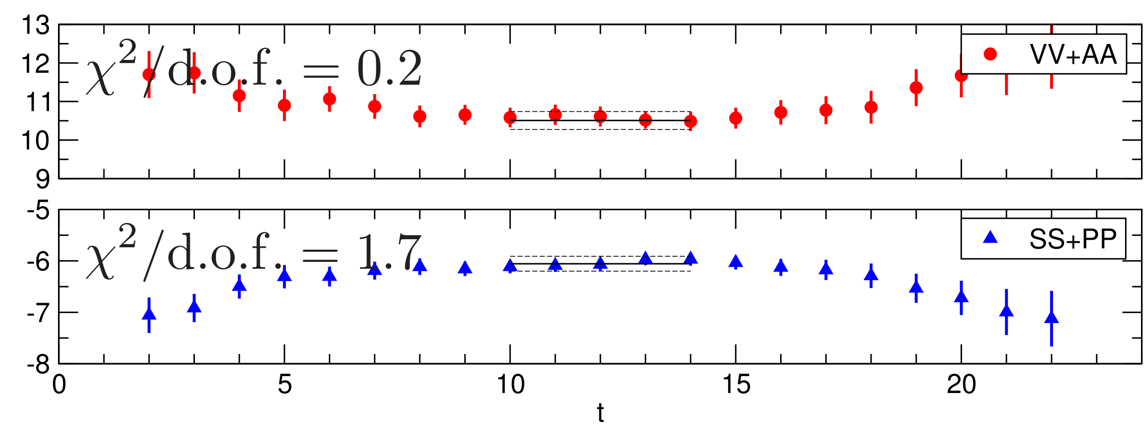


3PT

(previous)



(AMA)

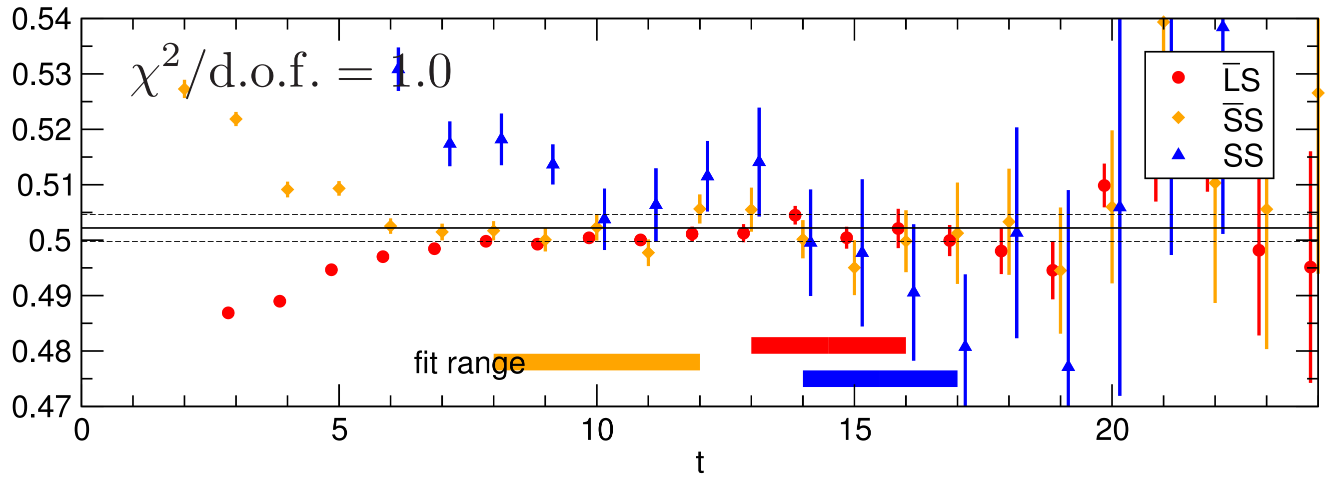


Preliminary results

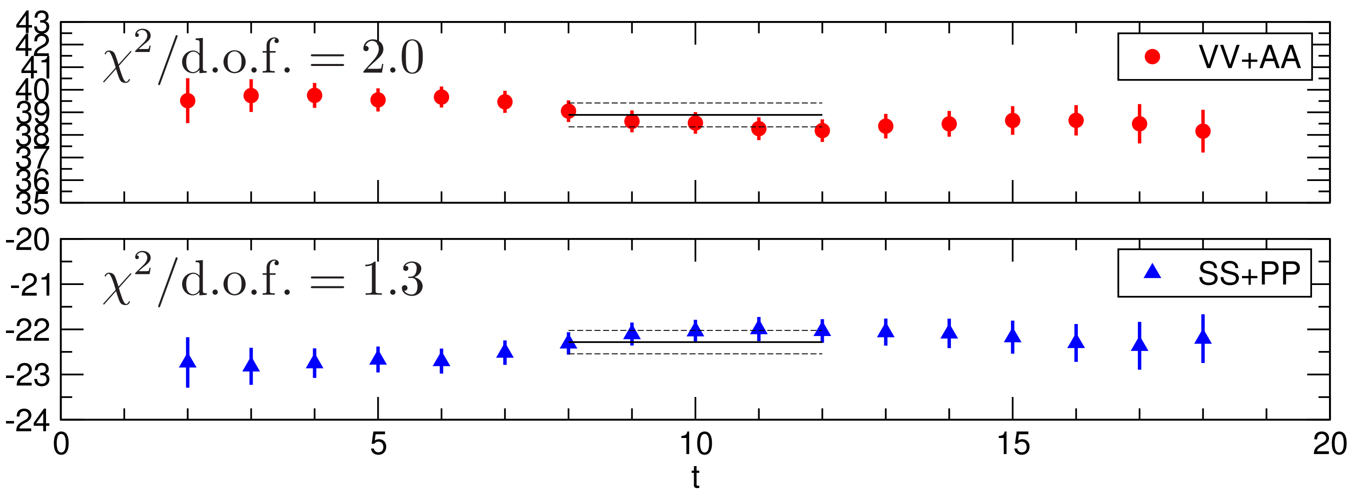
- Correlator data (48c)

48c, mu/ms=0.00078/0.0362, mq=0.0358, HYP2

2PT

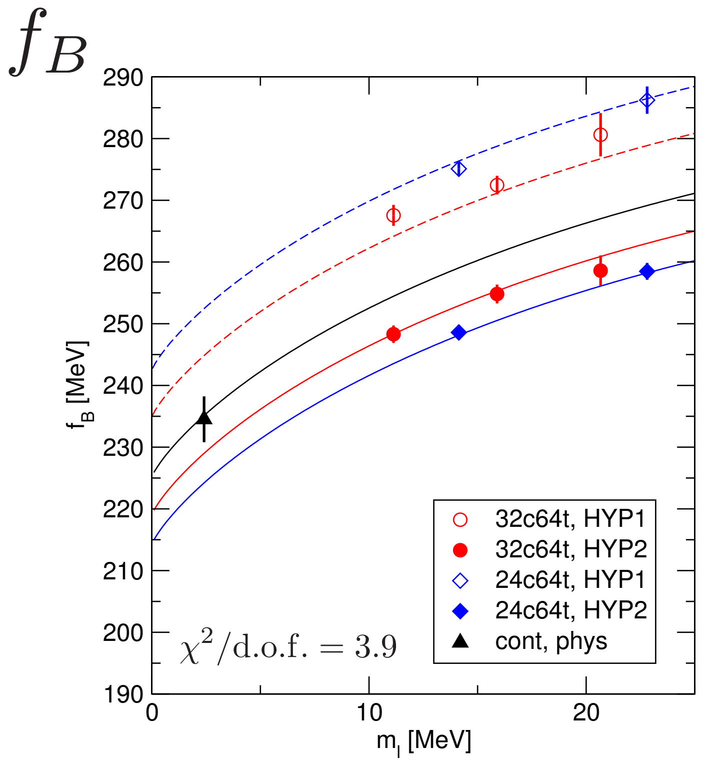


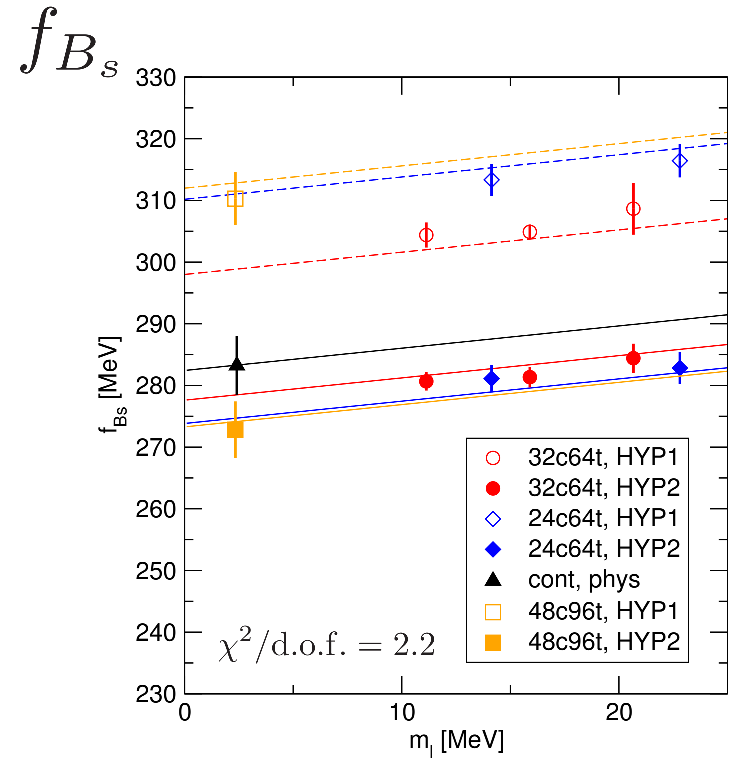
3PT

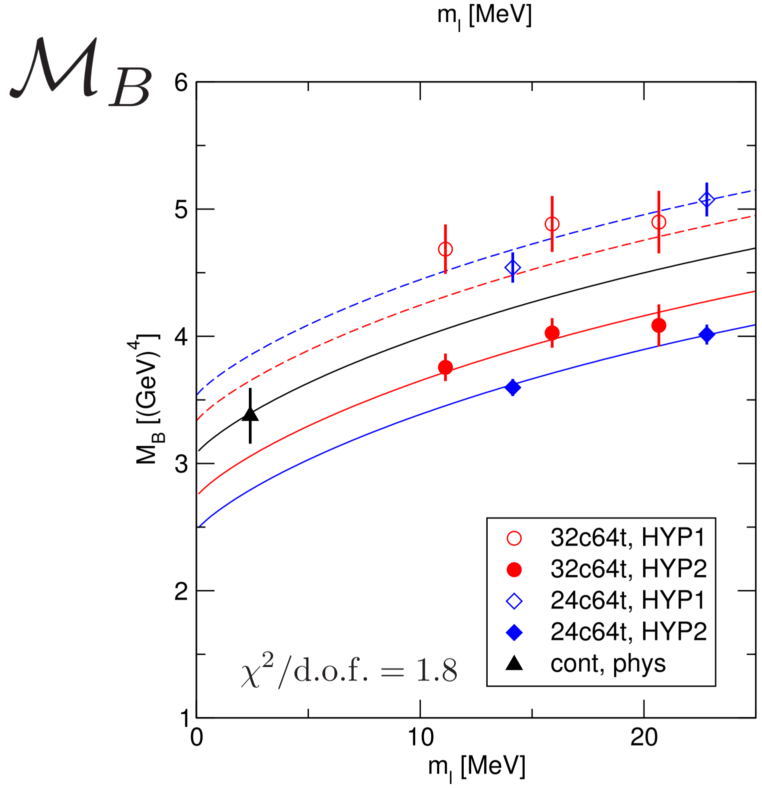


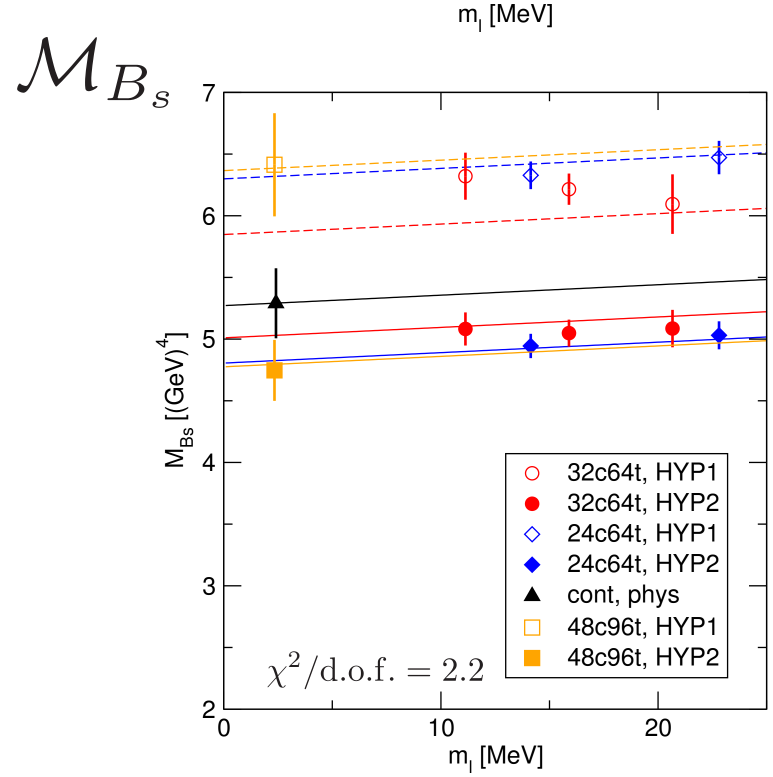
Preliminary results

- Chiral and continuum extrapolation









SU(2) HMChPT

HYP1 and HYP2 data are combined in the fitting assuming their continuum limits are common.

Preliminary results

Change from previous results

[PRD91 (2015) 11, 114505]

$$
\begin{aligned}
f_B &= 218.8(6.5)_{\rm stat}(16.1)_{\rm sys}\ \text{MeV}\\
f_{Bs} &= 263.5(4.8)_{\rm stat}(18.7)_{\rm sys}\ \text{MeV}\\
f_{Bs}/f_B &= 1.193(20)_{\rm stat}(36)_{\rm sys}\\
f_B\sqrt{\hat{B}_B} &= 240(15)_{\rm stat}(17)_{\rm sys}\ \text{MeV}\\
f_{Bs}\sqrt{\hat{B}_{Bs}} &= 290(09)_{\rm stat}(20)_{\rm sys}\ \text{MeV}\\
\xi &= 1.208(41)_{\rm stat}(44)_{\rm sys}
\end{aligned}
$$

⇨

AMA (Preliminary)

$$
\begin{aligned}
f_B &= 234.5(3.7)_{\rm stat}(?)_{\rm sys}\ \text{MeV}\\
f_{Bs} &= 283.2(4.8)_{\rm stat}(?)_{\rm sys}\ \text{MeV}\\
f_{Bs}/f_B &= 1.198(16)_{\rm stat}(?)_{\rm sys}\\
f_B\sqrt{\hat{B}_B} &= 262.1(8.5)_{\rm stat}(?)_{\rm sys}\ \text{MeV}\\
f_{Bs}\sqrt{\hat{B}_{Bs}} &= 323.1(8.5)_{\rm stat}(?)_{\rm sys}\ \text{MeV}\\
\xi &= 1.212(32)_{\rm stat}(?)_{\rm sys}
\end{aligned}
$$

- More statistics might be needed to reduce error.
- Large upward shift in non-ratio quantities:

partly originates from large move in lattice spacings
from [PRD 83, 074508(2011)] to [arXiv:1411.7017]

$$
\begin{aligned}
a_{24c}^{-1} &: \quad 1\sigma \text{ shift upward}\\
a_{32c}^{-1} &: \quad 1.8\sigma \text{ shift upward}
\end{aligned}
$$

Summary and outlook

- B meson decay constants and neutral B meson mixing matrix elements in the continuum limit are calculated with AMA. Statistical error is reduced somewhat from previous results. More statistics might be needed for large reduction.
- Bs quantities at physical pion are calculated with zMobius+AMA. The data seemingly be appeared in expected places so far. The statistics are now being increased.
- Bd quantities at physical pion are also planned to be calculated to remove chiral extrapolation uncertainty with zMobius+AMA.
- For non-ratio quantities, non-perturbative matching is important. We are trying it using position space NPR method.

[P.Korcyl, July 14, Tue]