Massive photons: an infrared regularization scheme for lattice QCD+QED

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Motivation

- Isospin breaking effects on hadronic properties and processes in QCD arise from both electromagnetic interactions (quark charge differences) and quark mass differences
    - such effects can be understood nonperturbatively via LQCD
- Inclusion of electromagnetism in LQCD is straight-forward
    - electroquenched, and more recently dynamical QCD (via reweighing and direct simulation)
    - long-range nature of interactions result in power-like finite volume corrections; can be understood analytically via EFTs
- Finite volume extrapolations require simulations at multiple volumes; computationally costly… any better alternatives?

Introduction of a photon mass

- Introduction of a photon mass implies an additional extrapolation: $m_\gamma/m_\pi \to 0$
- If $m_\gamma L$ large, the volume dependence becomes exponentially suppressed; can trade power-law extrapolation in a/L for a power-law extrapolation in m_γ/m_π in this regime
  - better analytic control over the latter extrapolation?
  - does one gain anything from this trade-off (e.g., reliable extrapolations at smaller and cheaper volumes?)
- Finite m_γ might be useful for, e.g., charged particle scattering
- Exploratory study to test viability of the idea: compute simple quantities both ways and compare results/costs
  - e. g., look at hadron mass splitting and mass differences

Introduction of a photon mass

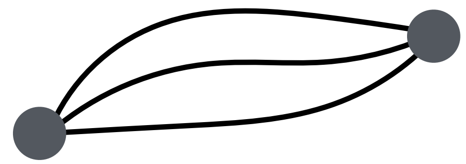
- Noncompact QED+QCD requires gauge-fixing and removal of zero-mode (otherwise path-integral ill-defined)
- Introduction of a photon mass term explicitly breaks gauge invariance (no need to gauge fix); zero-mode receives a mass…
  - but, charged correlators will vanish as $m_\gamma \to 0$
  - charged correlator signal/noise grows severely as $m_\gamma \to 0$
- Gauge fix first, then introduce mass term?
  - normally, although gauge fixing can prefer a direction (e.g., Coulomb) rotational invariance is still preserved
  - rotational invariance can be broken by photon mass term if introduced post-gauge fixing and gauge fixing not rot. inv.

Introduction of a photon mass

$$\mathcal{L}_\gamma = \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2\xi}(\partial_\mu A_\mu)^2 + \frac{1}{2}m_\gamma^2 A_\mu^2 \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- This work: choose Landau gauge (ξ=0)
  - preserves rotational invariance
  - complete gauge in Euclidean space; no flat directions in $m_\gamma \to 0$ limit except zero-mode
  - zero-mode(s) results in (very) mild signal/noise problem when $m_\gamma \to 0$
- Massive photons introduce a number of subtle issues, it seems, mainly associated with the zero mode

Massive zero-mode effects

$\mathcal{C}(\tau) =$  $\propto e^{iQ\frac{\tau}{V}\tilde{A}_0(0)}$ $\qquad V = L^3 T$

$$C(\tau) = \frac{1}{Z_0}\int \mathrm{d}\tilde{A}_0(0)\, e^{-\frac{m_\gamma^2}{2Vg^2}\tilde{A}_0^2(0)}\mathcal{C}(\tau) \;\propto e^{-Q^2 g^2 \tau^2/(2m_\gamma^2 V)}$$

$$C(\tau) \propto e^{-\tau M - x\tau^2} \qquad x = \frac{4\pi\alpha}{2m_\gamma^2 L^3 T}$$

- Leading A_0 zero-mode contribution to correlators can be seen from quark diagrams
  - every fermion line is accompanied by Wilson line
  - zero-mode part of Wilson line, when integrated over, gives rise to a quadratic time dependence in correlators

Photon mass and volume corrections

- Finite photon mass and volume corrections to hadron mass shifts can be computed using an effective field theory description for hadrons (mass M, charge Q)
  - EFT includes photon mass term and operators that are unconstrained by gauge invariance
  - corrections to mass shifts computed in an expansion in hadron's Compton wave-length
- Infrared-finite mass shifts (at infinite volume) given by:

$$\Delta M(\alpha, m_\gamma) = M(\alpha, m_\gamma) - M(\alpha, 0)$$

$$\Delta M^{LO} = -\frac{\alpha}{2} Q^2 m_\gamma \qquad \Delta M^{NLO} = \left(Ce^2 - \frac{\alpha}{4\pi} Q^2 \right) \frac{m_\gamma^2}{M} \qquad \Delta M^{NNLO} = \mathcal{O}\left(\frac{m_\gamma^3}{M^2} \right)$$

Photon mass and volume corrections

- Finite volume corrections to mass given by:

$$\delta_L M(\alpha, m_\gamma, L) = M(\alpha, m_\gamma, L) - M(\alpha, m_\gamma, \infty)$$

temporal zero-mode effects removed at level of correlator

$$\delta_L M^{LO} = 2\pi\alpha Q^2 m_\gamma \left[I_1(m_\gamma L) - \frac{1}{(m_\gamma L)^3} \right]$$

$$\delta_L M^{NLO} = \pi\alpha Q^2 \frac{m_\gamma^2}{M} \left[2 I_{1/2}(m_\gamma L) + I_{3/2}(m_\gamma L) \right]$$

$$I_n(z) = \frac{1}{2^{n+\frac{1}{2}} \pi^{\frac{3}{2}} \Gamma(n)} \sum_{\nu \neq \mathbf{0}} \frac{K_{\frac{3}{2}-n}(z|\nu|)}{(z|\nu|)^{\frac{3}{2}-n}}$$

Exponential fall-off for large z

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- When $m_\gamma L \ll 1$, formulas decompose into a sum of three contributions, associated with 1) zero-modes, 2) known power-like ($m_\gamma=0$) volume corrections, and 3) finite function of $m_\gamma L$

Numerical simulations

- Performed electroquenched calculations using modified version of the CHROMA software suite
- Studied nucleon and kaon mass shifts/splittings as a function of L and m_γ
  - shifts/splitting were extracted from *correlated* effective mass differences (or correlator ratios) $\Delta M^{AB}_{\text{eff}}(\tau) = M^{A}_{\text{eff}}(\tau) - M^{B}_{\text{eff}}(\tau)$
  - +/- e averaging over correlators to cancel O(e) stat. errors
  - mass splitting extrapolated to $m_\gamma/m_\pi \to 0$ and $a/L \to 0$

$$\Delta M(\alpha, L, m_\gamma) = \Delta M(\alpha) + \sum_{k=0}^{K} \Delta M^{N^k LO}(\alpha, m_\gamma) + \sum_{k=0}^{K_L} \delta_L M^{N^k LO}(\alpha, m_\gamma, L)$$

- α not renormalized, throughout we use α ~1/137

QCD Ensembles

- Dynamical SU(3) flavor symmetric QCD configs
  - obtained from W&M/JLAB
    - subset of those used in [arXiv:1301.5790] (NPLQCD)
  - isotropic lattices (tadpole-improved Luscher-Weisz gauge action and clover fermion action)
  - single lattice spacing: a ~ 0.145 fm
  - three lattice volumes: 3.48 fm, 4.64 fm and 6.96 fm corresponding to L/a = 24, 32 and 48, respectively
  - 800 MeV pions/kaons and 1.6 GeV nucleons: chosen so all appreciable vol. dependence is associated with QED effects
- Ensemble sizes: 956 (L/a=24) , 515 (L/a=32) and 342 (L/a=48)

QED Ensembles

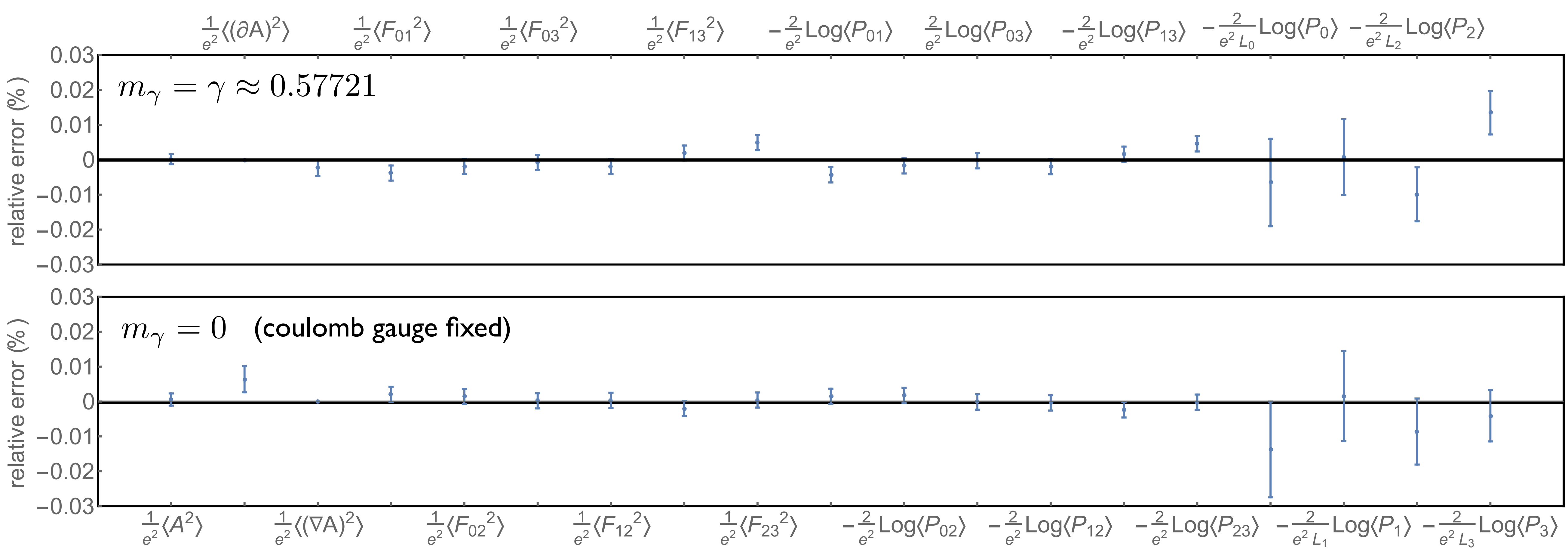
- Two sets of QED ensembles generated using noncompact formalism:
  - Coulomb gauge fixed, with zero-mode removed
  - Massive QED in Landau gauge
    - mass term introduced after gauge fixing
    - m_γ/m_π = 1/14, 1/7, 1/4, 1/3, 5/12, 1/2, 7/12, 1
    - zero mode *not* removed
- Performed numerical checks of U(1) code
  - compared analytic calculation of observables with high precision simulations

Pure U(1) gauge theory numerical checks

$13 \times 11 \times 7 \times 5$ lattice $\qquad N_{\text{conf}} = 1M \qquad e = \pi/3$

∂ = 4-divergence $\qquad P_{\mu\nu}$ = plaquette

∇ = 3-divergence $\qquad P_{\mu}$ = Polyakov loop



Valence quark mass tuning

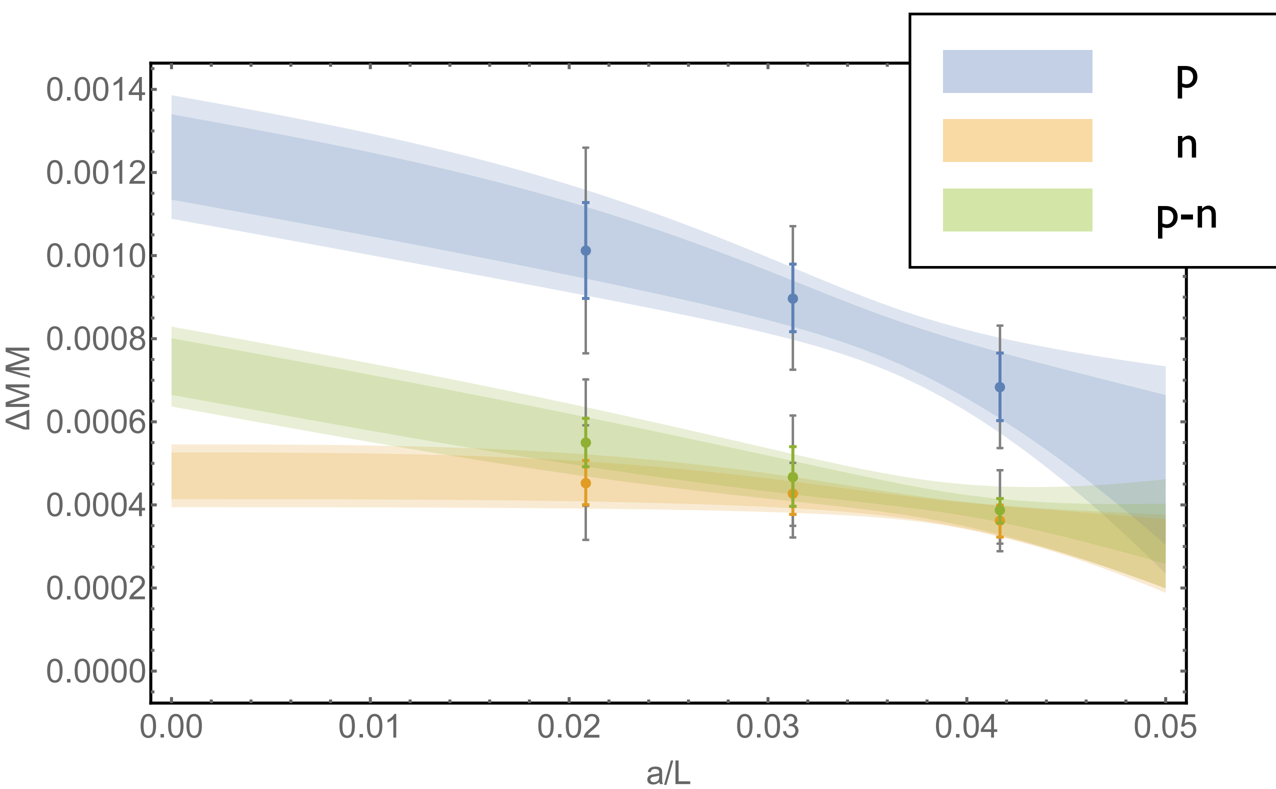
- Additive quark mass renormalization due to electromagnetism
- Tuned valence quark masses such that: $(m_{qq} - m_K)/m_K \lesssim \mathcal{O}(10^{-3})$
  - m_{qq} is the *connected* part of $q\bar{q}$ correlator
  - $m_\pi = m_K$ is the pion/kaon mass at α=0
  - underlying assumption is that electromagnetic corrections to neutral mesons are small compared to charged mesons
- Mistuning effects on mass splittings can be estimates from chiral perturbation theory (smallest m_γ/largest mistuned value)
  - Kaon: conservatively ~10%
  - Nucleon: conservatively ~25%

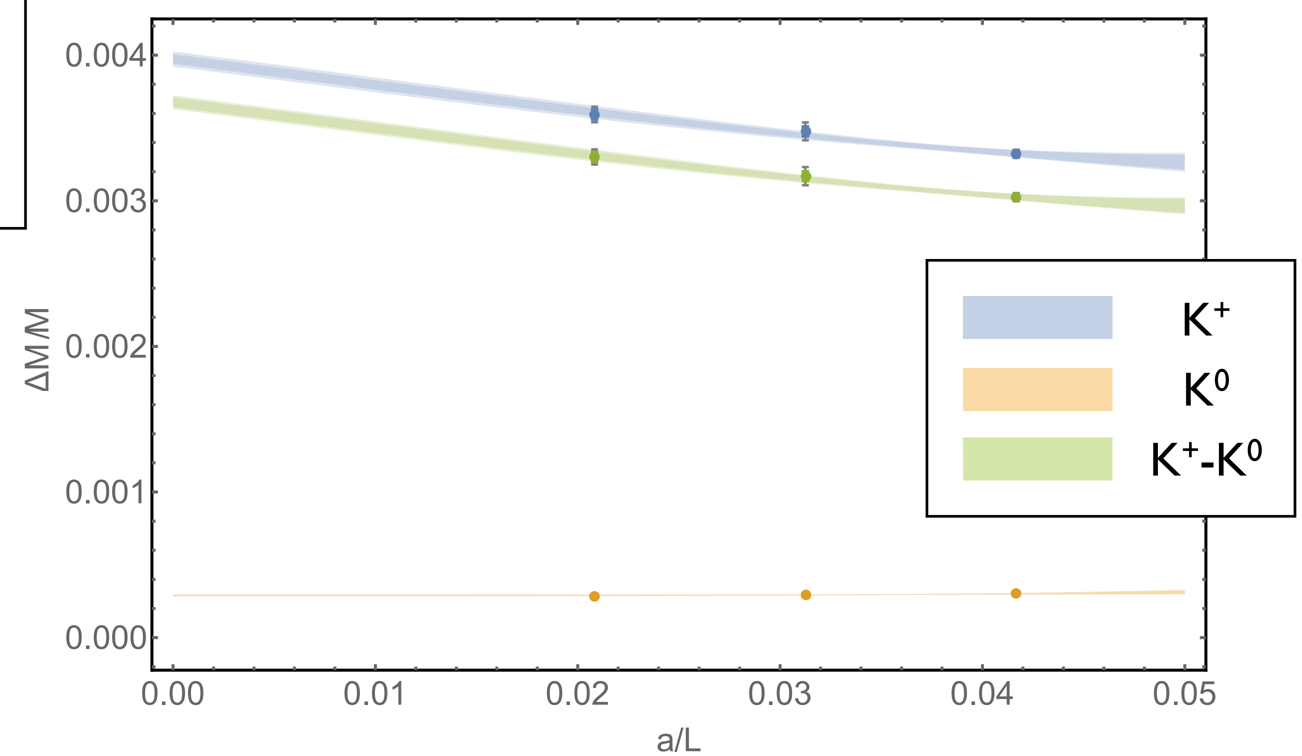
Massless case: a/L extrapolations

$$\Delta M(\alpha, L) = \Delta M(\alpha) + \frac{\alpha Q^2 c_1}{2L}\left(1 + \frac{2}{L}\right) + d\frac{1}{L^{4-f}} \qquad c_1 = -2.83729\cdots$$

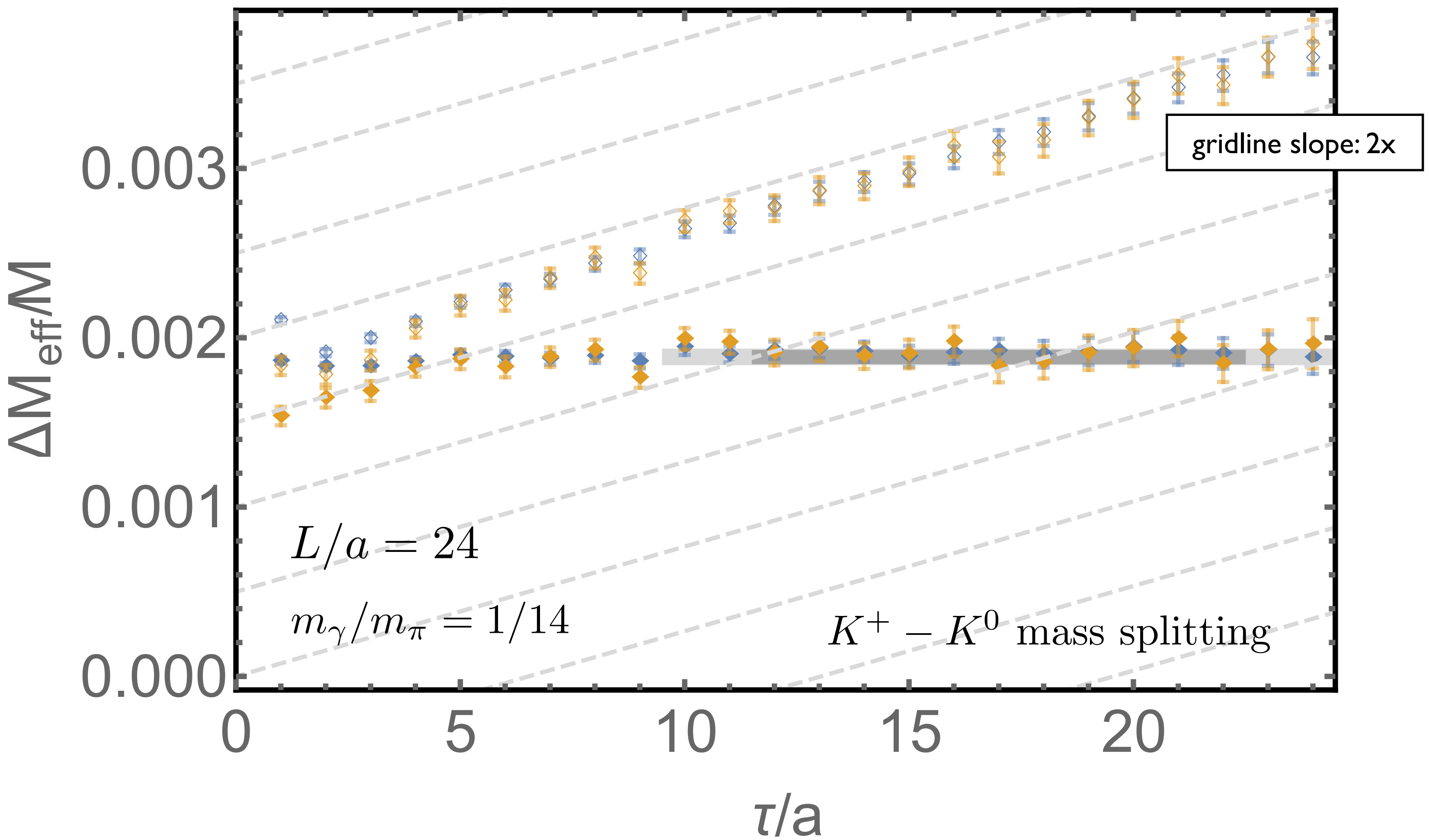
fit parameters

$$f = 0 \text{ (mesons) or } 1 \text{ (baryons)}$$





Massive case: zero mode effects



$$C(\tau) \propto e^{-\tau M - x\tau^2}$$

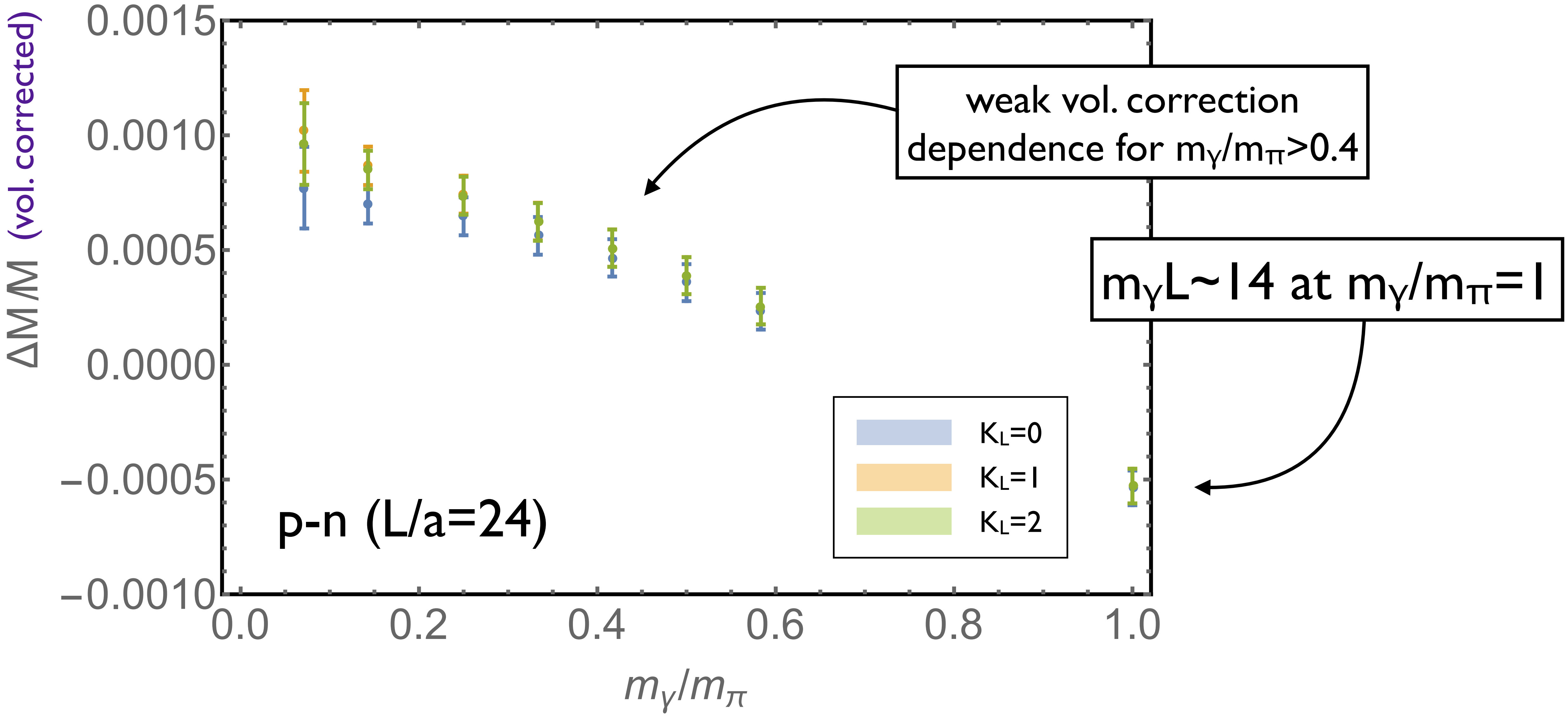
$$x = \frac{4\pi\alpha}{2m_\gamma^2 L^3 T}$$

$$m_{\text{eff,exp}}(\tau) = -\frac{1}{a}\log\frac{C(\tau+a)}{C(\tau)} + 2x\tau + x\Delta\tau$$

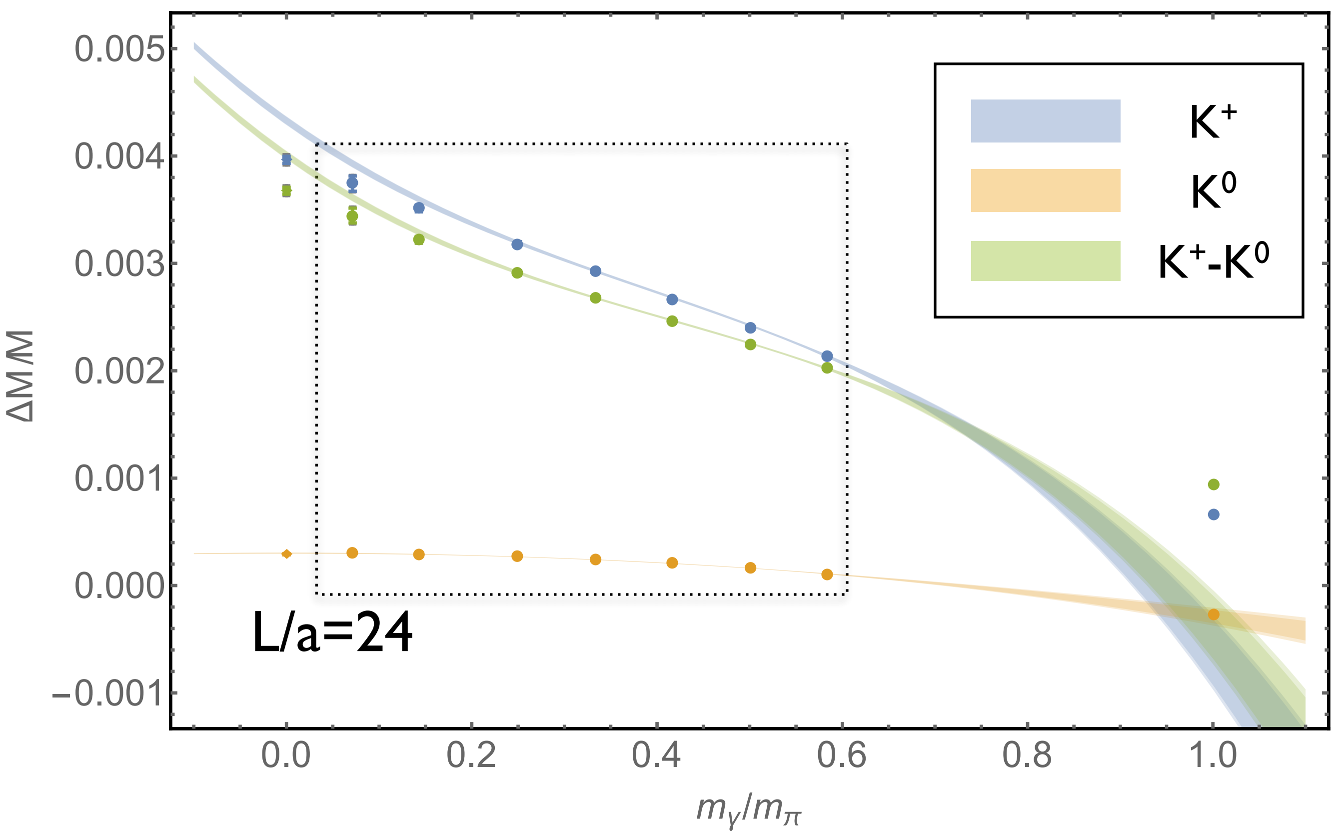
$$m_{\text{eff,cosh}}(\tau) = \frac{1}{a}\operatorname{arccos}\left[\frac{e^{(a-\beta+2\tau)xa}C(\tau+a) + e^{(a+\beta-2\tau)xa}C(\tau-a)}{2C(\tau)}\right] - xT$$

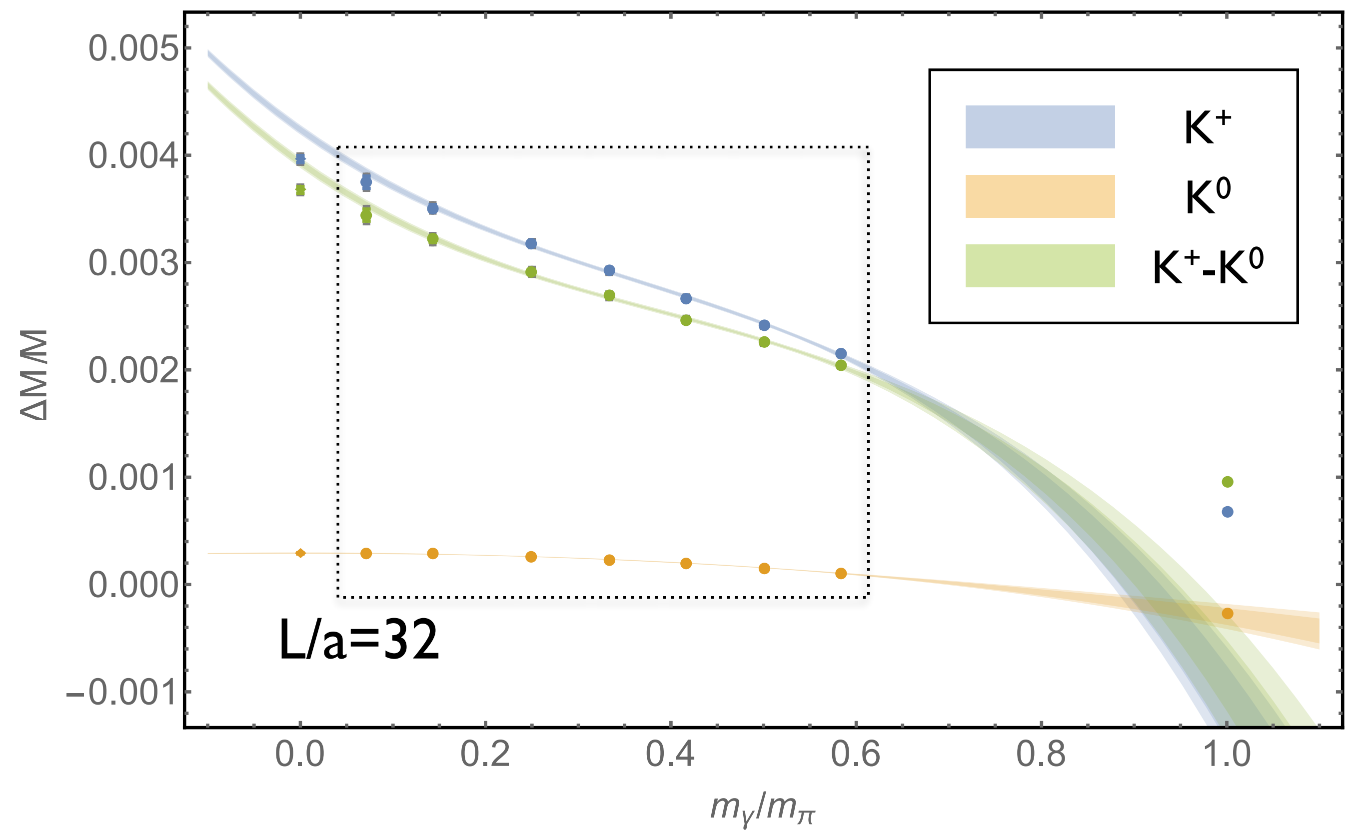
Massive case: finite volume effects

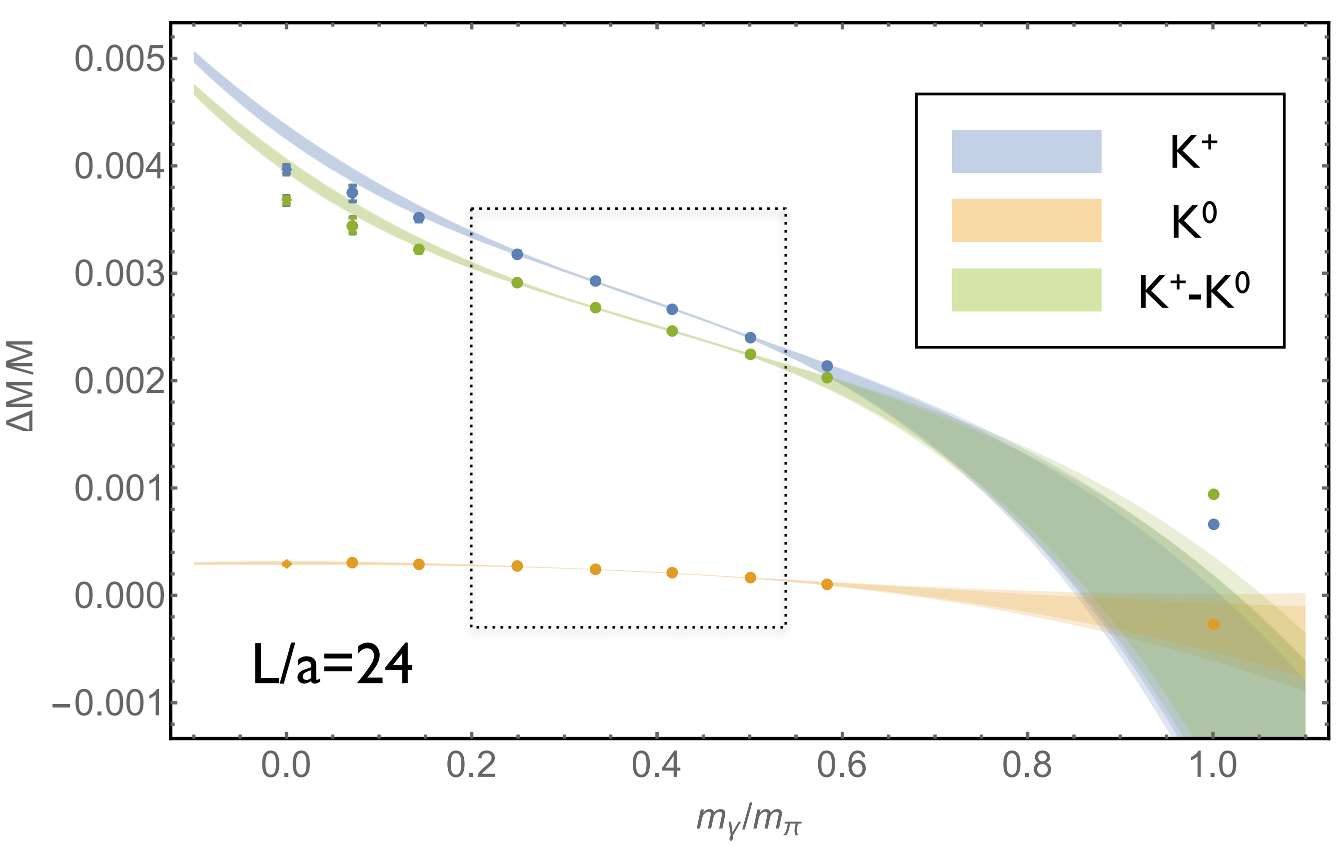
$$\underline{\Delta M(\alpha, L, m_\gamma) - \sum_{k=0}^{K_L} \delta_L M^{N^k LO}(\alpha, m_\gamma, L)} = \Delta M(\alpha) + \sum_{k=0}^{K} \Delta M^{N^k LO}(\alpha, m_\gamma)$$



Massive case: m_γ/m_π extrapolations for fixed $m_\pi L$

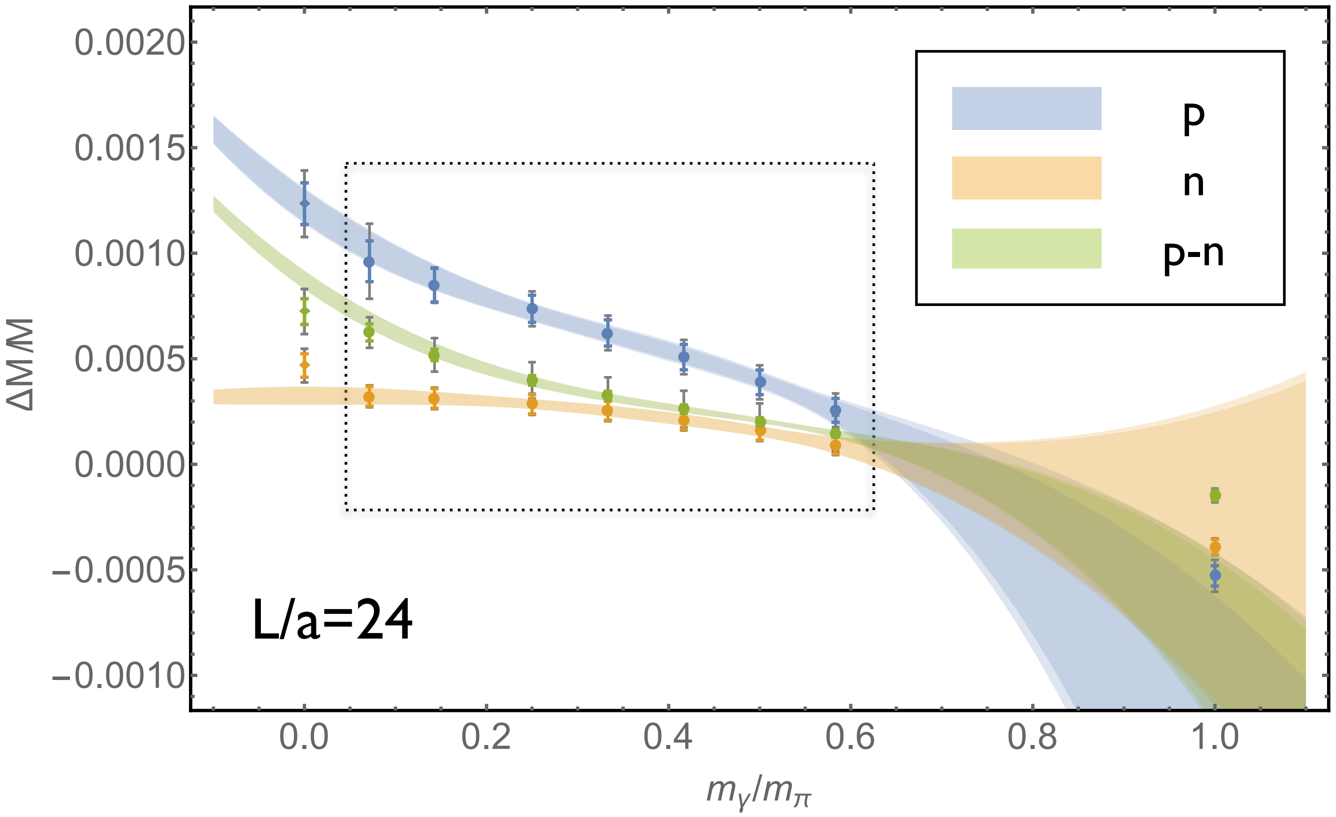


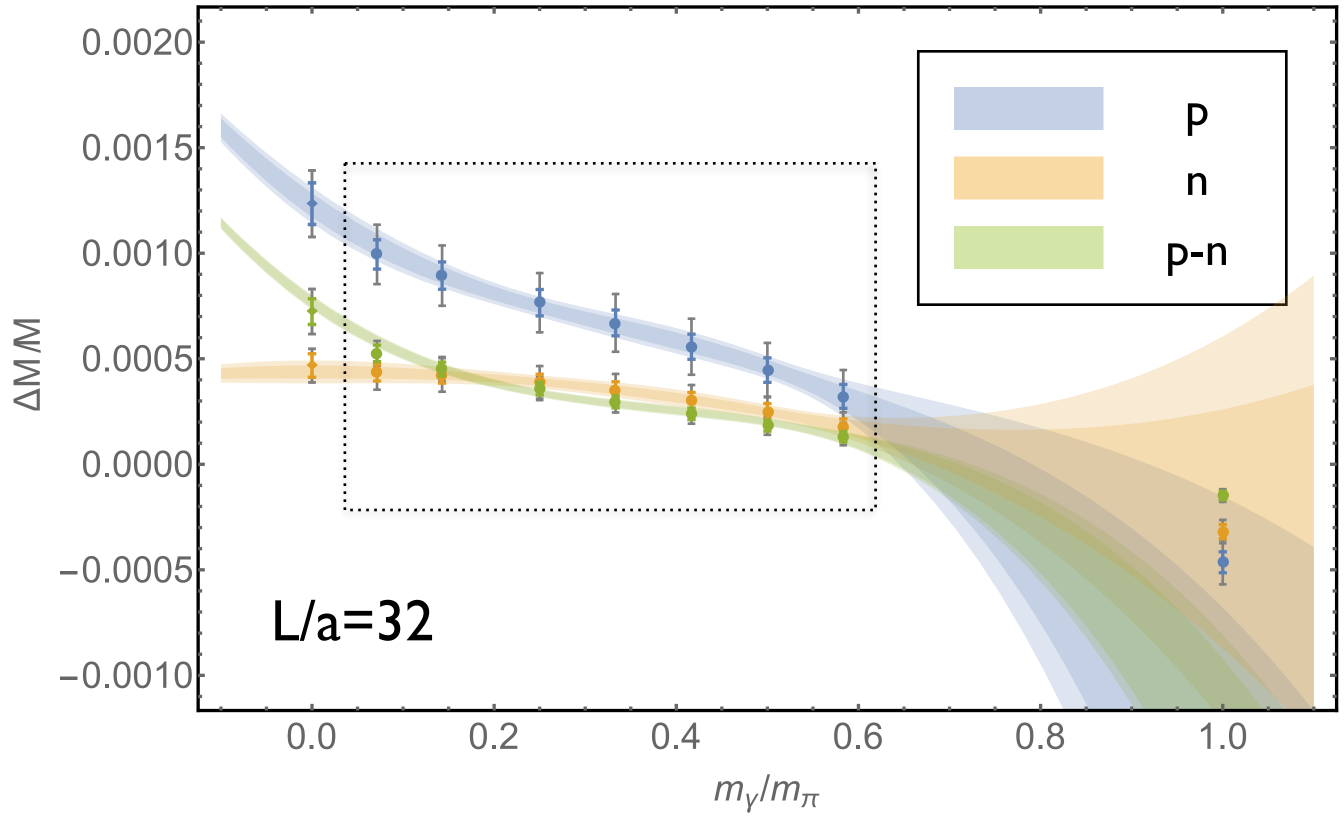


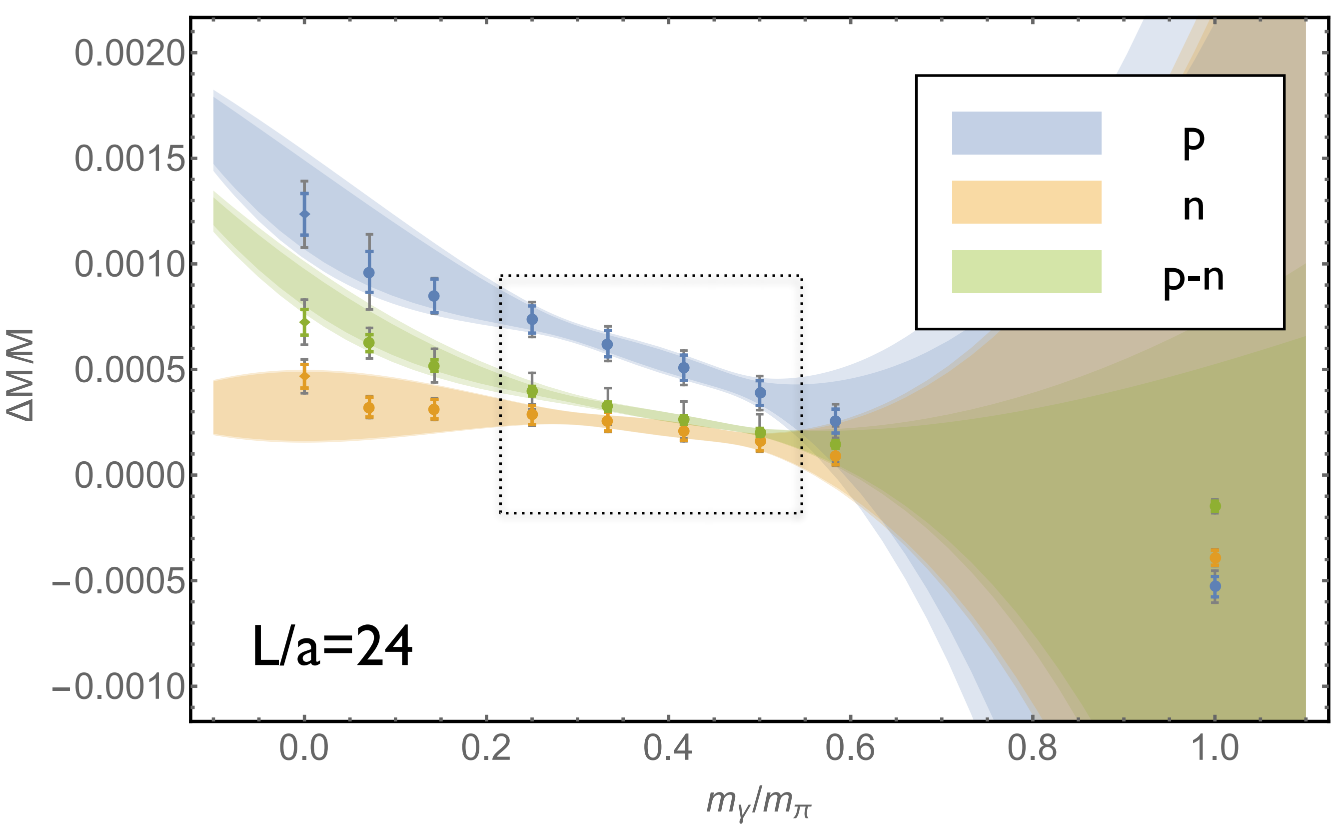


- m_γ extrapolations appear robust w.r.t. variation of fit range
- 48^3 extrapolation is consistent with 24^3 and 32^3
- fit formula forces disagreement at $m_\gamma=0$; possibly attributed to valence quark mass mistuning or discretization errors

Massive case: m_γ/m_π extrapolations for fixed $m_\pi L$







- m_γ extrapolations appear robust w.r.t. variation of fit range
- 48^3 extrapolation is consistent with 32^3, but seems low (2σ); note these ensemble has fewest trajectories
- 24^3 and 32^3 extrapolations are consistent with $m_\gamma=0$ results

Summary

- Most aspects to the analysis understood/resolved, e.g., removal of the zero-mode effects on correlators, accounting for zero-mode effects in mass shifts due to finite L, etc.

- Results are close to final:

  - mistuning of quark masses possibly plays a role in disagreement in kaon mass difference extrapolations, when this systematic is included in our uncertainties, extrapolated results agree

  - currently investigating whether mistuning systematic can be removed for better comparison

  - use a chirally symmetric fermion discretization or do an order of magnitude better tuning in the future

- Neglecting mistuning issues, for *equal* measurement cost, small volume massive QED appears to gives equal or smaller uncertainties on extrapolated values compared to conventional volume extrapolations; improvement is even greater after accounting for the overhead of generating L/a=32, 48 ensembles

Thank you for your attention!