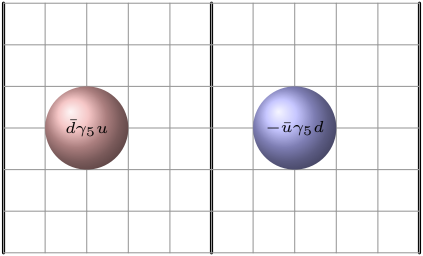
Nazario Tantalo
Rome University & INFN "Tor Vergata",
nazario.tantalo@roma2.infn.it

Charged particles in QED with $C^\star$ boundary conditions II

in collaboration with B.Lucini, A.Patella (previous talk) and A.Ramos

Kobe, 16-07-2015

- flavour symmetries in QED$_C$
- flavour symmetries in QCD+QED$_C$
- finite volume effects on the masses of charged hadrons
- outlooks



$$\phi(x+\hat{L}) = \phi^C(x)$$

the photon field is anti-periodic and has *no zero modes* (**A.Patella talk**)

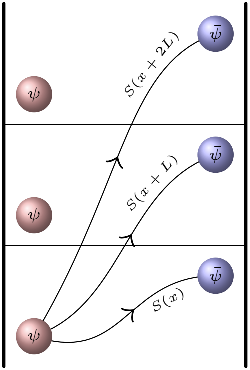
$$A_\mu(x+\hat{L}) = -A_\mu(x)$$

- $C^\star$ boundary conditions violate flavour and, consequently, electric charge conservation
- the pattern of flavour violation can be easily understood by means of propagators

- with periodic boundary conditions flavour is conserved and we have

$$\langle \psi(x)\bar{\psi}(0)\rangle = \sum_{\mathbf{n}} S(x+\mathbf{n}L)$$

$$\langle \psi(x)\psi^T(0)\rangle = 0$$

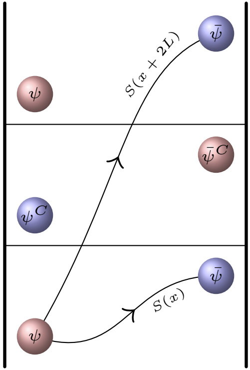


- $\text{C}^\star$ boundary conditions violate flavour and, consequently, electric charge conservation
- the pattern of flavour violation can be easily understood by means of propagators

- with $\text{C}^\star$ boundary conditions no flavour violation arises when matter particles travel around the world an even number of times

$$\langle \psi(x)\bar{\psi}(0)\rangle = \sum_{\langle \mathbf{n}\rangle=0} S(x+\mathbf{n}L)$$

$$\langle \mathbf{n}\rangle = (n_1+n_2+n_3) \mod 2$$



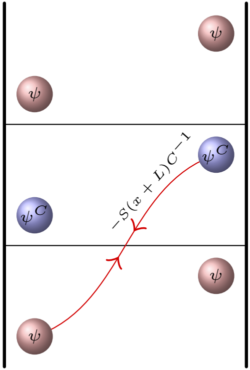
- $C^\star$ boundary conditions violate flavour and, consequently, electric charge conservation
- the pattern of flavour violation can be easily understood by means of propagators

- while flavour is violated of two units when matter particles cross the boundary an odd number of times

$$\langle \psi(x)\psi^T(0)\rangle = -\sum_{\langle \mathbf{n}\rangle=1} S(x+\mathbf{n}L)C^{-1} \simeq \left(\frac{m}{L}\right)^{\frac{3}{2}} e^{-mL}$$

$$\langle \mathbf{n}\rangle = (n_1+n_2+n_3) \mod 2$$

- if matter particles are massive this is an *exponentially vanishing* finite volume effect



in the theory with N_f flavours, by calling F_f the generator of the f-th $\mathrm{U}(1)$ flavour symmetry, we have that:

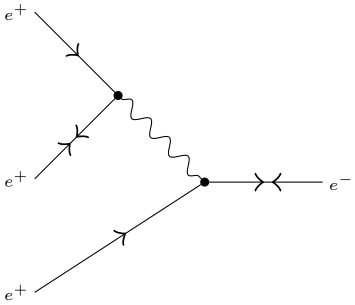
- F_f is broken by exponentially suppressed finite volume effects
- $(-1)^{F_f}$ is conserved
- the electric charge is a linear combination of the flavour symmetry generators

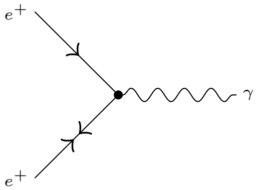
$$Q = \sum_{f}^{N_f} q_f F_f$$

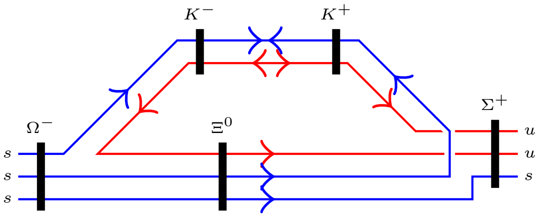
- and the rules

$$\Delta F_f = 0 \mod 2 \;, \quad \Delta Q = 0 \mod 2$$

imply that, for example, a single electron state can mix with a three-electron state but *not* with the vacuum







- in QCD+QED$_C$ we still have that each flavour generator is violated in units of 2,

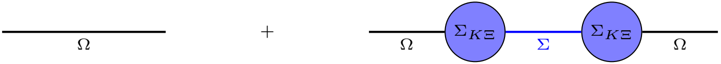
$$Q = \sum_f^{N_f} q_f F_f \ , \qquad F = \sum_f^{N_f} F_f \ , \qquad B = \frac{F}{3} \ , \qquad \Delta F_f = 0 \mod 2$$

- but, *if the box size is large enough*, only colourless particles can travel around the torus. it follows that

$$\Delta Q = 0 \mod 2 \ , \quad \Delta B = 0 \mod 2 \ , \quad \Delta F = 0 \mod 6$$

- pseudoscalar mesons (the pions, the kaons, D and B) cannot mix with lighter states and are therefore stable
- the proton cannot mix with states having $B = 0$ and it remains the lightest state with $B = 1$
- when mixings occur, as in the figure, the effect vanishes exponentially...

the Ω–Σ mixing can be quantified in the context of a "generic" effective theory of hadrons. the one-loop analysis is simple (the all-loop one a bit less. . .)



- the leading exponential contribution to the correlator is obtained when the Σ propagator goes on-shell, $p_\Sigma = (\pm i m_\Sigma, \mathbf{0})$, and the residue of the pole is proportional to the square of the self-energy

$$\begin{aligned} \Sigma_{K\Xi}(p_\Sigma) &= V_{\Omega-K-\Xi} \left\{ \sum_{\langle \mathbf{n} \rangle = 1} \int d^4x\, S_K(x + \mathbf{n}L)\, S_\Xi(-x) e^{i p_\Sigma x} \right\} V_{K-\Xi-\Sigma} + \cdots \\ &= \alpha\, e^{-m_K L} + \cdots \end{aligned}$$

$$C_{\Omega\Omega}(t) = \beta e^{-2 m_K L} \frac{e^{-m_\Sigma t}}{2 m_\Sigma} + Z_\Omega \frac{e^{-m_\Omega t}}{2 m_\Omega} + \cdots$$

- although to extract the Ω mass one has to take first the infinite volume limit of the correlator (effective mass)
- these flavour violating contributions should not represent an issue in practice!

$$e^{-2 m_K L} \simeq 5 \times 10^{-13}\,,$$

$$m_\pi L = 4$$

$$\frac{\Delta m(L)}{m} = \frac{e^2}{4\pi}\left\{\frac{q^2\xi(1)}{2mL} + \frac{q^2\xi(2)}{\pi(mL)^2} - \frac{1}{4\pi mL^4}\sum_{\ell=1}^{\infty}\frac{(-1)^{\ell}(2\ell)!}{\ell! L^{2(\ell-1)}}\mathcal{T}_{\ell}\xi(2+2\ell)\right\} + \ldots$$

let's analyze the structure of this expression:

$$\frac{\Delta m(L)}{m} = \frac{e^2}{4\pi}\left\{\frac{q^2\xi(1)}{2mL} + \frac{q^2\xi(2)}{\pi(mL)^2} - \frac{1}{4\pi mL^4}\sum_{\ell=1}^{\infty}\frac{(-1)^\ell(2\ell)!}{\ell!L^{2(\ell-1)}}\mathcal{T}_\ell\xi(2+2\ell)\right\} + \ldots$$

let's analyze the structure of this expression:

- the dependence upon the boundary conditions is contained *only* in the generalized zeta functions

	$1C^\star$	$2C^\star$	$3C^\star$
$\xi(1)$	-0.77438614142	-1.4803898065	-1.7475645946
$\xi(2)$	-0.30138022444	-1.8300453641	-2.5193561521
$\xi(4)$	0.68922257439	-2.1568872986	-3.8631638072

$$\xi(s) = \sum_{\mathbf{n}\neq\mathbf{0}}\frac{(-1)^{n_1+n_2+n_3}}{|\mathbf{n}|^s}$$

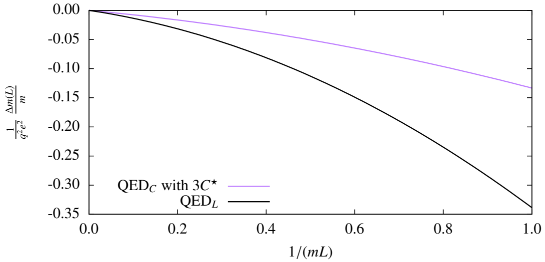
$$\frac{\Delta m(L)}{m} = \frac{e^2}{4\pi}\left\{\frac{q^2\xi(1)}{2mL} + \frac{q^2\xi(2)}{\pi(mL)^2} - \frac{1}{4\pi mL^4}\sum_{\ell=1}^{\infty}\frac{(-1)^\ell(2\ell)!}{\ell! L^{2(\ell-1)}}\mathcal{T}_\ell\,\xi(2+2\ell)\right\} + \ldots$$

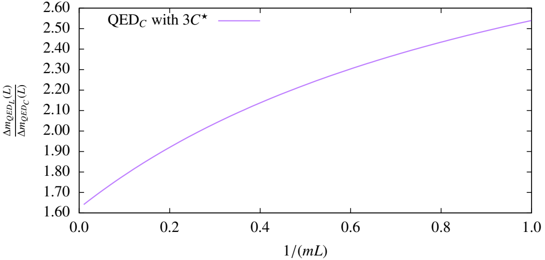
let's analyze the structure of this expression:

- the coefficients of the leading $1/L$ and $1/L^2$ terms are universal
- these are fixed by the Ward Identity (see also **Low 54, Gell-Mann and Goldberger 54**)
- and do not depend upon the internal structure nor on the spin of the hadron

the universality of the leading terms has already been noticed in the framework of $\mathrm{QED_L}$ and $\mathrm{QED_{TL}}$ by **BMW 14**

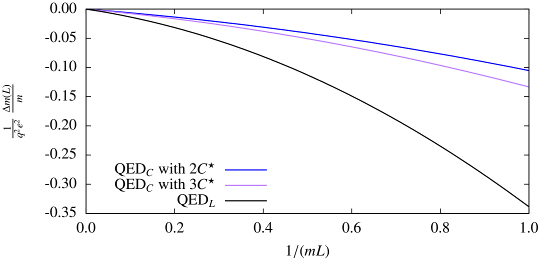
$$\left.\frac{\Delta m(L)}{m}\right|_{\mathrm{QED_L}} = \frac{e^2}{4\pi}\left\{-\frac{q^2\kappa}{2mL} - \frac{q^2\kappa}{(mL)^2} + \mathcal{O}\left(\frac{e^2}{L^3}\right)\right\}$$

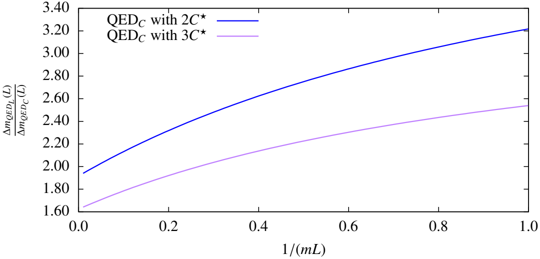




at $mL = 4$, universal finite volume effects are:

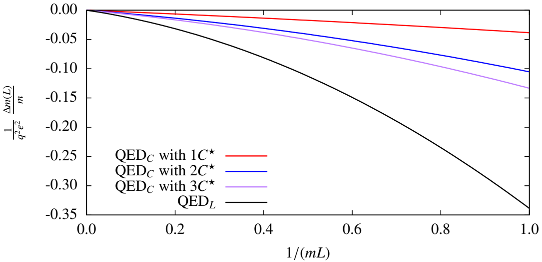
- 2 times smaller, $\mathrm{C}^\star$-BC along 3 directions

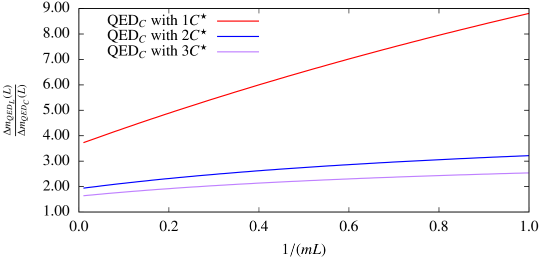




at $mL = 4$, universal finite volume effects are:

- 2 times smaller, $C^\star$-BC along 3 directions





at $mL = 4$, universal finite volume effects are:

- 2 times smaller, C*-BC along 3 directions
- 5 times smaller, C*-BC along 1 direction

$$\frac{\Delta m(L)}{m} = \frac{e^2}{4\pi}\left\{\frac{q^2\xi(1)}{2mL} + \frac{q^2\xi(2)}{\pi(mL)^2} - \frac{1}{4\pi m L^4}\sum_{\ell=1}^{\infty}\frac{(-1)^\ell(2\ell)!}{\ell! L^{2(\ell-1)}}\mathcal{T}_\ell\,\xi(2+2\ell)\right\} + \ldots$$

let's analyze the structure of this expression:

- spin and structure dependent terms start to contribute at $1/L^4$
- a part from the leading $1/L$ term, there are no inverse odd powers of L
- the structure dependent coefficients are related to physics

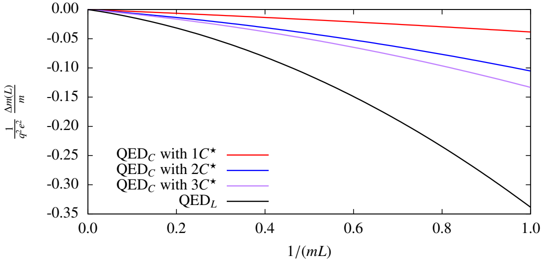
$$\mathcal{T}_\ell = \frac{d^\ell}{d(\mathbf{k}^2)^\ell}T_{\mu\mu}(i|\mathbf{k}|,\mathbf{k})\Big|_{|\mathbf{k}|=0}$$

the derivatives of the forward Compton scattering amplitude

spin and structure dependent terms start to contribute at $1/L^3$ in $\mathrm{QED_L}$ (**BMW 14**)

$$\frac{\Delta m(L)}{m}\Big|_{\mathrm{QED_L}} = \frac{e^2}{4\pi}\left\{-\frac{q^2\kappa}{2mL} - \frac{q^2\kappa}{(mL)^2} + \mathcal{O}\left(\frac{e^2}{L^3}\right)\right\}$$

- $C^\star$ boundary conditions allow to solve the problem of charged particles on a finite volume in a local field theory
- flavour and electric charge violating finite volume effects arise. . .
- but these are exponentially suppressed and should not represent an issue in practical applications
- the leading $1/L$ and $1/L^2$ finite volume corrections to the mass of a charged hadron are universal and much smaller than in $\mathrm{QED_L}$
- the structure dependent finite volume corrections to the mass of a charged hadron start to contribute at $1/L^4$ (vs. $1/L^3$ in $\mathrm{QED_L}$)



- the paper will be out soon: it will contain a detailed analysis of the symmetries and of the compact formulation of $\mathrm{QCD{+}QED_C}$
- we look forward to a numerical implementation!