Lattice 2015

Charged particles in QED with C^* boundary conditions (part 1)

Agostino Patella (speaker)
CERN & Plymouth University

Biagio Lucini
Swansea University

Alberto Ramos
CERN

Nazario Tantalo
Università di Roma Tor Vergata

Introduction

- Electromagnetic corrections to hadronic quantities

- Local formulation of QED in finite volume

- An old idea...
  Polley, Boundaries for SU(3)(C) x U(1)-el lattice gauge theory with a chemical potential, Z. Phys. C59, 1993

- C^* boundary conditions provide a framework to describe a certain class of electrically-charged states in a rigorous way. This class is wide enough to include most of the spectroscopic applications.

No charge in a periodic box

Gauss's law forbids a net electric charge in a periodic box

$$\partial_k E_k(x) = \rho(x) \quad \Rightarrow \quad Q = \int d^3x \; \rho(t, \mathbf{x}) = \int d^3x \; \partial_k E_k(t, \mathbf{x}) = 0$$

No charge in a periodic box
Quantum ElectroDynamics

- Generator of gauge transformations (operators in the Schrödinger picture)

$$\hat{G}(\mathbf{x}) = \partial_k \hat{E}_k(\mathbf{x}) - \hat{\rho}(\mathbf{x})$$

- Physical states are invariant under *local gauge transformations* (i.e. gauge transformations that are continuously connected to the identity)

$$\hat{G}(\mathbf{x})|\psi\rangle = 0$$

- Electric-charge operator is the generator of global gauge transformations

$$\hat{Q} = \int d^3x \ \hat{\rho}(\mathbf{x}) = \int d^3x \ \hat{G}(\mathbf{x})$$

- Since global gauge transformations are continuously connected to the identity, physical states have zero charge.

No charge in a periodic box

Gauge-fixed Quantum ElectroDynamics

- After gauge-fixing, the action is still invariant under global gauge transformations (electric charge is conserved). One might think to define the electron mass by looking at the two-point function

$$\langle \psi(x)\bar{\psi}(0)\rangle$$

- Large gauge transformations survive gauge fixing ($n \in \mathbb{Z}^4$)

$$\psi(x) \to e^{\sum_\rho \frac{2\pi \imath x_\rho n_\rho}{L_\rho}} \psi(x)$$

$$A_\mu(x) \to A_\mu(x) + \frac{2\pi n_\mu}{L_\mu}$$

- The two-point function is not invariant

$$\langle \psi(x)\bar{\psi}(0)\rangle = 0 \quad \text{if } x \neq 0$$

Local finite-volume QED

How do we deal with the zero modes?

- QED_{TL} enforces the constraint $\tilde{A}_\mu(0) = 0$
- QED_L enforces the constraint $\tilde{A}_\mu(p_0, \mathbf{0})$
- infinite volume QED + finite volume QCD
- QED with massive photon (Mike Endres, last Tuesday)

Why do we need another approach? We are looking for a local formulation at finite volume. (QED with massive photon is local.)

Local finite-volume QED

What does it mean?

- Microcausality

$$[A(t,\mathbf{x}), B(t,\mathbf{y})] = 0 \quad \text{for } \mathbf{x} \neq \mathbf{y}$$

- Equations of motion are local differential equations: time evolution of fields in x is determined only by the value of fields in an arbitrarily small neighbourhood of x.

- Local action

$$Z = \int_{\text{b.c.'s}} e^{-S} \ , \quad S = \int d^4x \ \mathcal{L}(x)$$

Local finite-volume QED

Why?

Locality guarantees, e.g.

- Renormalizability by power counting
- Volume-independence of renormalization constants
- Operator product expansion
- Effective-theory description of long-distance physics
- Symanzik improvement program
- ...

Does it mean that other formulations do not enjoy these properties? No! They might well do, but they should be not given for granted and proved case by case.

Finite-volume extrapolation $$m_\pi(L) = m_\pi + \frac{v_1}{L} + \frac{v_2}{L^2} + \dots$$

Lattice spacing extrapolation $$m_\pi(a) = m_\pi + c_1 a^2 + c_2 a^4 + \dots$$

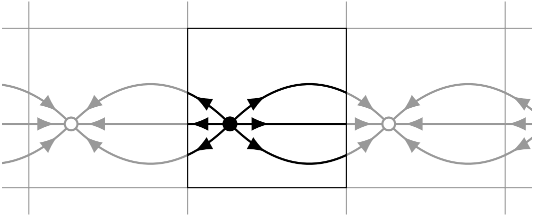
In a setup in which a description in terms of the Symanzik effective theory is not valid, the infinite volume limit must be taken first. A combined fit is not justified and might give the wrong result.

C^* boundary conditions

$$A_\mu(x + L\mathbf{k}) = -A_\mu(x) \qquad \psi(x + L\mathbf{k}) = C^{-1}\bar{\psi}^T(x) \qquad \bar{\psi}(x + L\mathbf{k}) = -\psi^T(x)C$$

- No zero modes for the gauge fields
- No linear gauge transformations
- Classicly Gauss's law does not forbid charged states

$$Q(t) = \int d^3x \; \rho(t, \mathbf{x}) = \int d^3x \; \partial_k E_k(t, \mathbf{x}) \neq 0$$



C* boundary conditions

$$A_\mu(x + L\mathbf{k}) = -A_\mu(x) \qquad \psi(x + L\mathbf{k}) = C^{-1}\bar{\psi}^T(x) \qquad \bar{\psi}(x + L\mathbf{k}) = -\psi^T(x)C$$

- Admissible gauge transformations must leave b.c.'s unchanged

$$\Omega(x) = \pm e^{\imath\alpha(x)} \;, \qquad \alpha(x + L\mathbf{k}) = -\alpha(x)$$

- The group of continuous gauge transformations is disconnected $\mathcal{G} = \mathcal{G}_0 \times \mathbb{Z}_2$.
- The subgroup $\mathcal{G}_0$ generated by *local gauge transformations* is connected to the identity and does not contain any nontrivial global gauge transformation.
- Only two global gauge transformations $\Omega = \pm 1$ leave the b.c.'s unchanged.

Electrically-charged states

The group of continuous gauge transformations is disconnected $\mathcal{G} = \mathcal{G}_0 \times \mathbb{Z}_2$.

- Charge conservation is *partially* violated by the boundary conditions.
  - Q is not conserved but $(-1)^Q$ is.
  - Charge violation is a finite volume effect.
  - Even though finite-volume effects are generally power-like, charge-violation effects are exponentially suppressed.
  - Charge violation is not a problem in practice, see Tantalo's talk.

- Electrically-charged physical states:
  - invariant under local gauge transformations
  - but not under global gauge transformations, $(-1)^Q = -1$

Electrically-charged states

Dirac interpolating operator:

$$\Psi(t, \mathbf{x}) = e^{-\imath \int d^3y \; \Phi(\mathbf{y}-\mathbf{x}) \partial_k A_k(t,\mathbf{y})} \psi(t, \mathbf{x})$$

where $\Phi(\mathbf{x})$ is the electric potential of a unit charge in a box with C^* b.c.'s

$$\partial_k \partial_k \Phi(\mathbf{x}) = \delta^3(\mathbf{x})$$
$$\Phi(\mathbf{x} + L\mathbf{k}) = -\Phi(\mathbf{x})$$

Nontrivial fact: *such an electric potential exists!*

- $\Psi(t, \mathbf{x})$ is invariant under local gauge transformations
- $\Psi(t, \mathbf{x})$ is not invariant under global gauge transformations (i.e. it is electrically charged)

Electrically-charged states

Dirac interpolating operator:

$$\Psi(t,\mathbf{x}) = e^{-\imath \int d^3y \ \Phi(\mathbf{y}-\mathbf{x})\partial_k A_k(t,\mathbf{y})} \psi(t,\mathbf{x})$$

where $\Phi(\mathbf{x})$ is the electric potential of a unit charge in a box with C^* b.c.'s

$$\partial_k \partial_k \Phi(\mathbf{x}) = \delta^3(\mathbf{x})$$
$$\Phi(\mathbf{x} + L\mathbf{k}) = -\Phi(\mathbf{x})$$

Nontrivial fact: *such an electric potential exists!*

- $\Psi(t,\mathbf{x})|0\rangle$ is invariant under local gauge transformations
- $\Psi(t,\mathbf{x})|0\rangle$ is not invariant under global gauge transformations (i.e. it is electrically charged)

The electron mass is defined in a gauge-invariant fashion:

$$\langle \Psi(t,\mathbf{x})\bar{\Psi}(0) \rangle \simeq A(\mathbf{x}) e^{-tm}$$

Checkpoint

- QED with C^* boundary conditions is a possible local formulation of QED in finite volume.
- C^* boundary conditions provide a framework to describe a certain class of electrically-charged states in a rigorous and gauge-invariant way.
- C^* boundary conditions partially break charge conservation... see next talk.