JÜLICH FORSCHUNGSZENTRUM

Pure SU(3) Topological Susceptibility at Finite Temperature with the Wilson Flow

July 15, 2015 | Simon Mages | Szabolcs Borsányi Zoltán Fodor Sándor D. Katz
Dániel Nógrádi Kálmán K. Szabó

Outline

Motivation

- Missing part of Standard Model: Dark Matter
- Candidates: MACHOs, WIMPs, axions, ALPs, ...
- Peccei-Quinn mechanism for strong CP problem byproduct: axions
- PQ axion dynamics depend on non-perturbative QCD potential at finite temperature for CP restoration

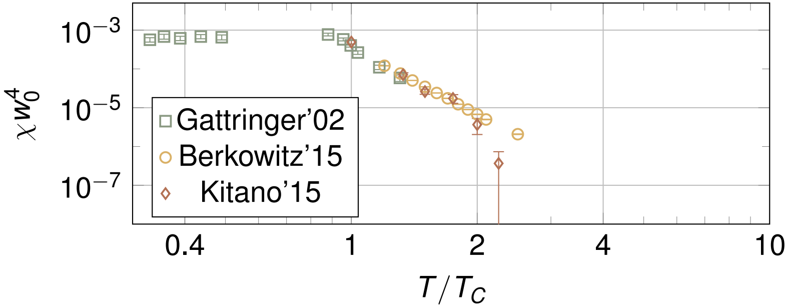
Use Lattice QCD to constrain Dark Matter:

$$\text{Lattice QCD} \Rightarrow \chi(T) = f_a^2 m_a(T)^2 \Rightarrow \text{Dark Matter}$$

Member of the Helmholtz-Association

Previous lattice studies

- [Alles:1996nm,Gattringer:2002mr] etc. 1st gen results
- [Berkowitz:2015aua] large volume/statistics up to $2.5\,T_c$
- [Kitano:2015fla] reweighting to get $Q \neq 0$



Topological Charge

Integral

$$Q = \int_{\mathcal{M}} \mathrm{d}^4 x q(x)$$

over the topological charge density

$$q(x) = \frac{1}{4\pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}(x) F_{\rho\sigma}(x)$$

- discretized in finite volume on $\mathcal{M} = \mathbb{T}^4$
- sectors with different Q separated by infinite action barrier in continuum
- problem for ergodicity of MC algorithms with small "step" size in field space

Topological Susceptibility

Integral of *qq* correlator

$$\chi = \int_{\mathcal{M}} \mathrm{d}x \langle q(0) q(x) \rangle$$

With global translation symmetry on $\mathcal{M} = \mathbb{T}^4$

$$\chi = \frac{1}{V_4} \langle Q^2 \rangle$$

- measurement must sample sectors with $Q \neq 0$
- difficult for simulations on $\mathcal{M} = \mathbb{T}^4$

Member of the Helmholtz-Association

Renormalization of χ

χ has additive and multiplicative renormalization [Alles:1997nu]

$$\chi^R = Z\chi + \chi_0$$

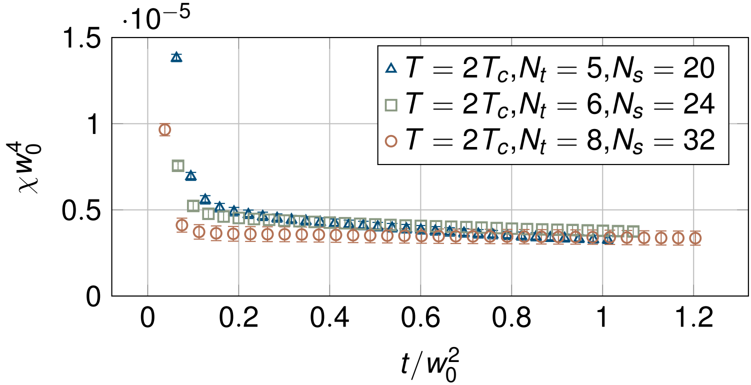
- cooling makes $Z \to 1$ and $\chi_0 \to 0$

$\chi(t)$ at finite flow time is already renormalized [Luscher:2010iy]

- sufficient to perform a continuum limit at flow time fixed in physical units, e.g. $t = w_0^2$
- choice of t impacts size of lattice artefacts

Flow dependence of $\chi(t)$

- $\chi(t)$ has weak dependence on choice of t
- we choose $t = w_0^2 \approx (0.176\text{ fm})^2$



Lattice Setup

Pure SU(3)

- Symanzik improved gauge action: $O(a^2)$ errors
- gluonic $q(x)$ from clover field strength tensor $F_{\mu\nu}$: $O(a^2)$ errors
- update sweep: 1 heatbath + 4 overrelaxation $\Rightarrow z = 1$

Parameters

- $0.1\ T_c \leq T \leq 4.0\ T_c$
- $n_t = 5, 6, 8$
- spatial volume fixed in physical units $L_{x,y} = 2/T_c$
- $L_z = 2L_{x,y}$ to enable subvolume analysis

Simulations on the QPACE machines in Wuppertal and Jülich

Subvolume Trick [Brower:2014bqa]

Possible solution

- discretization of Q is finite volume effect
- continuous Q_{sub} on finite subvolumes of $\mathbb{R}^4$ and $\mathbb{T}^4$
- calculate $\chi_{sub} = Q_{sub}^2 / V_{sub}$
- make infinite V_{sub} limit instead of infinite V_4 limit

Quenched and $T = 0$: large χ

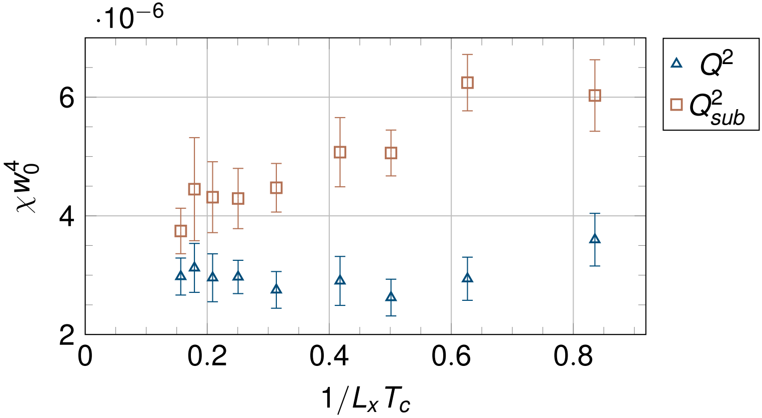
- plausible, works

Dynamic or $T \neq 0$: small χ

- finite volume errors are T independent
- errors larger than χ for reasonable volumes

Subvolume Trick - Finite Volume Errors

$T = 2T_c$, $N_t = 5$, $L_{sub} = L_z/2$ error scales like $1/L$



Subvolume Trick 2

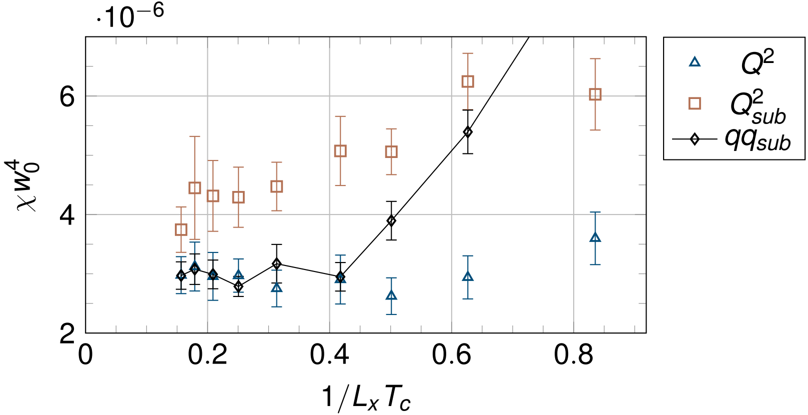
- step from $\chi = \int_{\mathcal{M}} \mathrm{d}x \langle q(0)q(x)\rangle$ to $\chi = \langle Q^2\rangle / V_4$ required translation invariance of $\mathcal{M} = \mathbb{T}^4$
- not valid for subvolume with boundary $\Rightarrow$ finite volume error
- large cancellations in integral of correlator $\Rightarrow$ large finite volume error

Alternative:

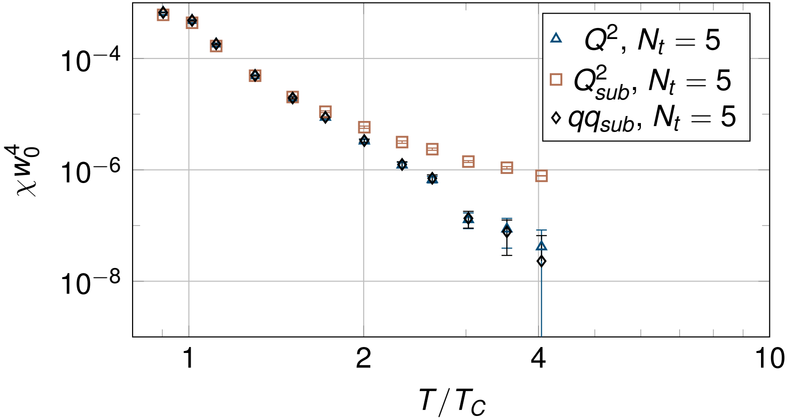
- introduce largest distance in $\chi = \int_{\mathcal{M}} \mathrm{d}x \langle q(0)q(x)\rangle$
- reduced finite volume errors
- reduced global/topological effects of discretization of Q

Subvolume Trick 2 - Finite Volume Errors

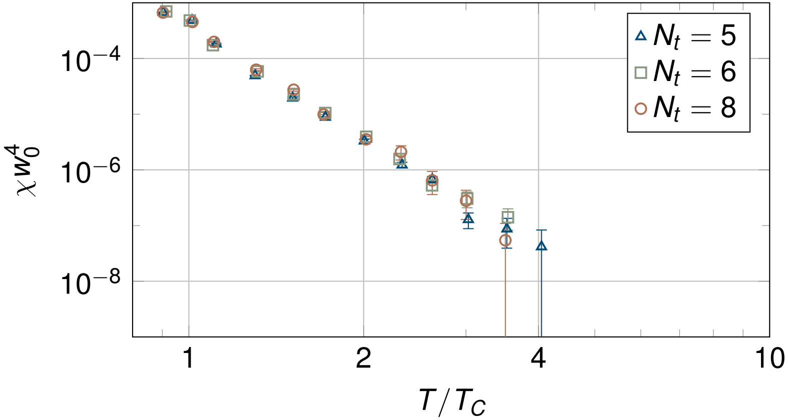
$T = 2T_c$, $N_t = 5$, $L_{sub} = L_z/2$ no $1/L$



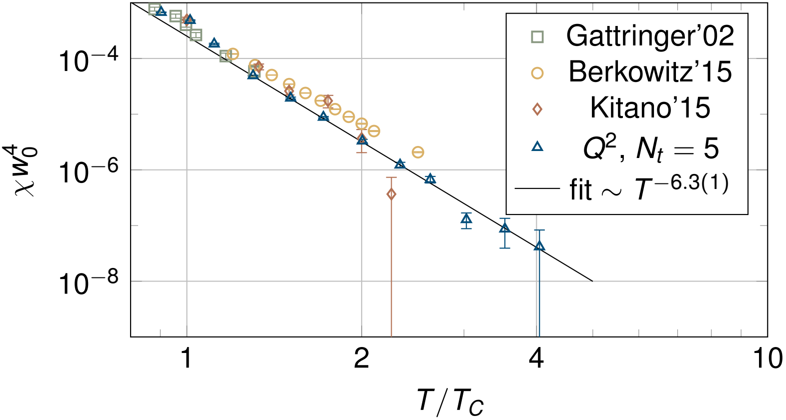
Results - Full Volume VS Sub Volume



Result - Lattice Spacing Dependence



Preliminary Results - Previous Studies



Discussion and Outlook

Technical:

- χ up to $T = 4T_c$
- $O(a^2)$ improved action and operator
- Gluonic charge at finite Wilson Flow time
- new subvolume method with less finite volume errors

Outlook:

- Continuum extrapolated results
- beat down quenched error with more statistics at $T = 4T_c$
- even higher T
- evaluate subvolume method at larger V/V_{sub}

Wilson Flow

flow $B_\mu(t,x)$ defined by

$$\frac{d}{dt}B_\mu = D_\nu G_{\nu\mu},$$
$$B_\mu|_{t=0} = A_\mu,$$

with field strength tensor G at finite t

Generated by stout smearing steps

$$\frac{d}{d\hat{t}}V_\mu = \mathrm{i}Q_\mu V_\mu,$$
$$V_\mu|_{t=0} = U_\mu,$$

$\rightarrow$ approximation by small stout smearing steps with $r^2_{smear} = 8t = 8N_{stout}\rho_{stout}$

Stout Smearing

damp UV fluctuations i.e. high energy part of the spectrum
generate ”fat” links by

$$
\begin{aligned}
U'_\mu =& e^{iQ_\mu} U_\mu, \\
Q_\mu =& \frac{i}{2}\left(\Omega^\dagger_\mu - \Omega_\mu - \frac{1}{3}\mathrm{tr}[\Omega^\dagger_\mu - \Omega_\mu]\right), \\
\Omega_\mu =& \left(\rho_{Stout} \sum_{\nu\neq\mu} C_{\mu\nu}\right) U^\dagger_\mu,
\end{aligned}
$$

with staples $C_{\mu\nu}$
effective smearing radius

$$r_{smear} = a\sqrt{8\rho_{Stout} N_{Stout}}.$$

- m. luscher, jhep **1008** (2010) 071 [arxiv:1006.4518 [hep-lat]].
- s. borsanyi, s. durr, z. fodor, c. hoelbling, s. d. katz, s. krieg, t. kurth and l. lellouch *et al.*, jhep **1209** (2012) 010 [arxiv:1203.4469 [hep-lat]].
- k. symanzik, nucl. phys. b **226** (1983) 187.
- h. baier, h. boettiger, m. drochner, n. eicker, u. fischer, z. fodor, a. frommer and c. gomez *et al.*, pos lat **2009** (2009) 001 [arxiv:0911.2174 [hep-lat]].
- s. borsanyi, g. endrodi, z. fodor, s. d. katz and k. k. szabo, jhep **1207** (2012) 056 [arxiv:1204.6184 [hep-lat]].
- B. Alles, M. D'Elia, A. Di Giacomo and R. Kirchner, Phys. Rev. D **58** (1998) 114506 [hep-lat/9711026].

- R. Kitano and N. Yamada, arXiv:1506.00370 [hep-ph].
- E. Berkowitz, M. I. Buchoff and E. Rinaldi, arXiv:1505.07455 [hep-ph].
- B. Alles, M. D'Elia and A. Di Giacomo, Nucl. Phys. B **494** (1997) 281 [Nucl. Phys. B **679** (2004) 397] [hep-lat/9605013].
- R. C. Brower *et al.* [LSD Collaboration], Phys. Rev. D **90** (2014) 1, 014503 [arXiv:1403.2761 [hep-lat]].