Large N meson propagators from twisted space-time reduced model

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In the last few years, we have developed the twisted space-time reduced model in the large N limit.

For pure gauge theory, this is the twisted Eguchi-Kawai model (TEK-model). We have succeeded in calculation the continuum string tension and running coupling constants in the large N limit.

It is quite straightforward to include adjoint fermions in this framework. We have succeeded in showing that two flavor theory is a conformal theory, and have calculated mass anomalous dimension at infrared fixed point.

In this talk, we propose a new method to include fundamental fermion in the twisted space-time reduced model, and calculate meson propagators in the large N limit.

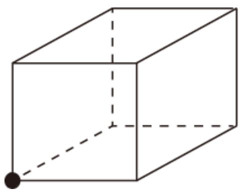
This is a quite challenging problem, because

Twist is the property of SU(N)/Z(N).
How can we introduce quarks in the fundamental representation in our framework?

Meson propagators are space-time extended object.
How can we introduce space-time extended object in the space-time reduced model?

We first review the known properties of the large N twisted reduced model of pure gauge theory, i. e. the TEK model.

- Twisted Eguchi-Kawai model



EK model can be viewed as the usual Wilson gauge theory having only one site with the periodic boundary conditions.

We introduce twisted boundary conditions in $SU(N)$, $N = L^2$ theory

$$S_{TEK} = bN \sum_{\mu \neq \nu = 1}^{d} Tr\left(z_{\nu\mu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \right)$$

$$Z_{\mu\nu} = \exp\left(k \frac{2\pi i}{L} \right), \quad Z_{\nu\mu}^* = -Z_{\mu\nu}, \qquad \mu < \nu$$

k , L : co-prime, k/L fixed as we go $N = L^2 \to \infty$

This model is related to ordinary SU(N) lattice theory on a lattice with space-time volume $V = L^4$ up to $O(1/N^2)$ corrections.

The number of degree of freedom of SU(N) matrix is $N^2 = L^4$

The vacuum configuration $U_\mu^{(0)} \equiv \Gamma_\mu$ of this model satisfy

$$z_{\nu\mu}\Gamma_\mu\Gamma_\nu\Gamma_\mu^\dagger\Gamma_\nu^\dagger = 1 \qquad \left[S_{TEK} = bN\sum \mathrm{Tr}\left(z_{\nu\mu}U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right) \right]$$

Γ_μ are four $L^2 \times L^2$ matrices with

$$\Gamma_\mu\Gamma_\nu = z_{\mu\nu}\Gamma_\nu\Gamma_\mu$$

Γ_μ play a central role in the construction of reduced model.

We can construct Γ_μ from two $L \times L$ matrices P_L and Q_L

$$P_L = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \cdot & & \\ & & & \cdot & 1 \\ 1 & & & & 0 \end{pmatrix} \quad , \quad Q_L = \begin{pmatrix} 1 & & & & \\ & z & & & \\ & & z^2 & & \\ & & & \cdot & \\ & & & & z^{\hat{L}-1} \end{pmatrix} \quad , \quad z = \exp\left(k \frac{2\pi i}{L} \right)$$

$$P_L Q_L = z Q_L P_L$$

$P_L, Q_L :\ L \times L$ matrices

$$\Gamma_0 = Q_L \otimes Q_L$$

$$\Gamma_1 = Q_L \otimes P_L Q_L$$

$$\Gamma_2 = Q_L \otimes P_L$$

$$\Gamma_3 = P_L \otimes I_L$$

$\Gamma_\mu :\ N \times N$ matrices with $N = L^2$

$$\Gamma_\nu \Gamma_\mu = z \Gamma_\mu \Gamma_\nu \ , \quad \mu < \nu$$

We start from the SU(N) lattice gauge theory $(N = L^2)$
with the Wilson fermion in the fundamental representation,

$$S = \sum_{n,\mu\nu} \mathrm{Tr}\left[V_\mu(n) V_\nu(n+\mu) V_\mu^\dagger(n+\nu) V_\nu^\dagger(n) \right]$$
$$+ \sum_n \bar{\Psi}(n)\Psi(n)$$
$$- \kappa \sum_{n,\mu} \left[\bar{\Psi}(n)(1-\gamma_\mu) V_\mu(n) \Psi(n+\mu) + \bar{\Psi}(n)(1+\gamma_\mu) V_\mu^\dagger(n) \Psi(n-\mu) \right]$$

on a lattice with the volume $V = L^3 \times \ell_0 L, \quad \ell_0$: positive integer

For the gauge part, the twisted space-time reduced model is obtained by the following reduction

$$V_\mu(n) = \Gamma(n) U_\mu \Gamma_\mu^\dagger \Gamma^\dagger(n)$$

U_μ are four SU(N) matrices

Here $\Gamma(n) = \Gamma_0^{n_0}\Gamma_1^{n_1}\Gamma_2^{n_2}\Gamma_3^{n_3}$ with Γ_μ the four twist eaters

$$\Gamma_\mu \Gamma_\nu = z_{\mu\nu} \Gamma_\nu \Gamma_\mu, \quad z_{\mu\nu} = \mathrm{e}^{2\pi i k/L}, \quad \mu < \nu$$

k and L are coprime

In fact, with this substitution we have

$$\mathrm{Tr}\left[V_\mu(n) V_\nu(n+\mu) V_\mu^\dagger(n+\nu) V_\nu^\dagger(n) \right] = z_{\nu\mu} \mathrm{Tr}\left(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \right)$$

This is the standard form of the TEK model

For the fermion part, we can not use the reduction prescription since fermion is in the fundamental representation (for adjoint fermion we can use reduction trick).

Instead, we make the following change of variable

$$\Psi(n) = \Gamma(n)\chi(n), \quad \chi \text{ still has } n \text{ dependence!}$$

Together with $V_\mu(n) = \Gamma(n) U_\mu \Gamma_\mu^\dagger \Gamma^\dagger(n)$ for gauge field, the fermion action becomes

$$\sum_n \bar{\chi}(n)\chi(n) - \kappa \sum_{n,m,\mu} [\, \bar{\chi}(n)(1-\gamma_\mu) U_\mu \tilde{\Gamma}_\mu \chi(\mathrm{m}) + \bar{\chi}(n)(1+\gamma_\mu) U_\mu^\dagger \tilde{\Gamma}_\mu^\dagger \chi(\mathrm{m}) \,]$$

Here

$$\tilde{\Gamma}_\mu(n,m) = \Gamma_\mu^\dagger \Gamma^\dagger(n) \Gamma(\mathrm{m}) \delta(\mathrm{n}+\mu, \mathrm{m})$$

are four $V \times V$ matrices with $V = \ell_0 \times L^4$ the lattice volume.

This Wilson-Dirac matrix $\tilde{\Gamma}_{\mu}(n,m)$ is too large to take inverse.
However, we can significantly reduce the size of the matrix as follows.

Only $\tilde{\Gamma}_{0}(n,m)$ depends on n_0 . Its form is $\delta(n_0+1, m_0)$. Then we can diagonalize it in monentum space with elements $\exp(i\rho_0)$

Let define $L^3 \times L^3$ matrices $\hat{\Gamma}_{\mu}(n,m)$ by removing time components from $\tilde{\Gamma}_{\mu}(n,m)$. $\hat{\Gamma}_{\mu}(n,m)$ depends only on spatial coordinates and satisfies

$$\hat{\Gamma}_{\mu}\hat{\Gamma}_{\nu} = z^{*}_{\mu\nu}\hat{\Gamma}_{\nu}\hat{\Gamma}_{\mu}$$

$L^2 \times L^2$ matrices Γ^{*}_{μ} satisfy the same algebra and we can show

$$\hat{\Gamma}_{\mu} = \Omega \begin{pmatrix} z_{1,\mu}\Gamma^{*}_{\mu} & 0 & 0 & 0 \\ 0 & z_{2,\mu}\Gamma^{*}_{\mu} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & z_{L,\mu}\Gamma^{*}_{\mu} \end{pmatrix} \Omega^{\dagger}$$

Meson propagators do not depend on Ω and $z_{i,\mu}$!

Finally, point-point meson correlator in γ_A and γ_B channel with zero spatial momentum is given by

$$C_{AB}(t) = \sum_{\rho_0} \exp(i\rho_0 t)\,\mathrm{Tr}\left[\gamma_A D^{-1}(\rho_0)\gamma_B D^{-1}(0)\right]$$

with

$$D(\rho_0) = 1 - \kappa \sum_{\mu=1}^{4}\left[(1-\gamma_\mu)\tilde{U}_\mu \Gamma_\mu^* + (1+\gamma_\mu)\tilde{U}_\mu^\dagger \Gamma_\mu^t\right]$$

$$\tilde{U}_{\mu=0} = \exp(i\rho_0)U_{\mu=0}, \quad \tilde{U}_{\mu=1,2,3} = U_{\mu=1,2,3}$$

$$\rho_0 = \frac{2\pi m}{\ell_0 L}; \quad 0 \le m \le \ell_0 L - 1$$

$D(\rho_0)$ are $4L^4 \times 4L^4$ matrix. We have to invert $\ell_0 L$ such matrices.

We notice that we start from $4\ell_0 L^6 \times 4\ell_0 L^6$ matrix, which can not be inverted using presently available computer!

To test our new formula, we made following simulations.

- one value of gauge group:

$$N = 289 = L^2 = 17^2$$

- two values of inverse ‘tHooft coupling

$$b = 1/g_0^2 N = 0.36, 0.37$$

- seven values of κ for each b

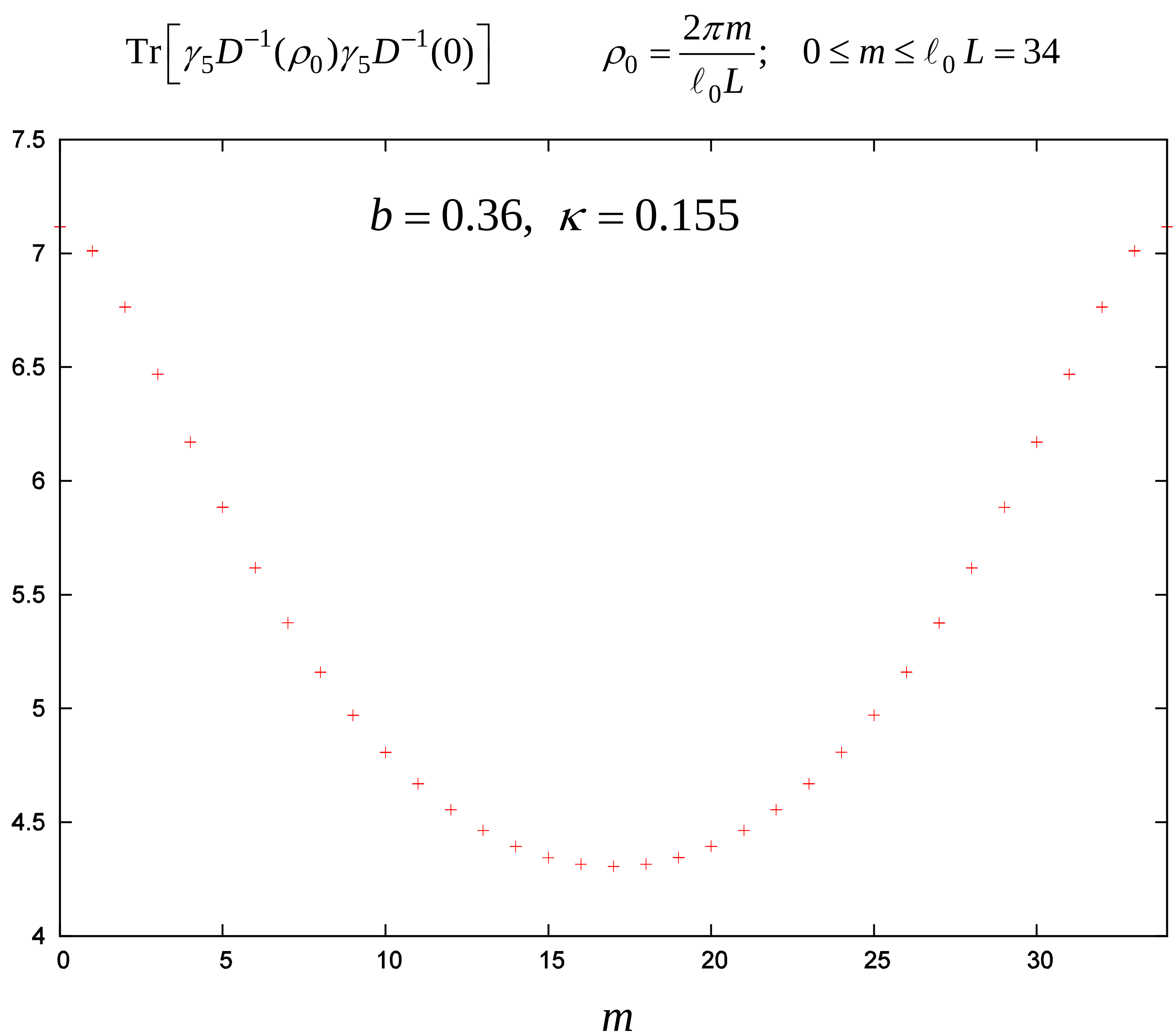
$$\kappa = 0.153,\ 0.154,\ 0.155,\ 0.156,\ 0.157,\ 0.158,\ 0.1585\ \ (b = 0.36)$$

$$\kappa = 0.150,\ 0.151,\ 0.152,\ 0.153,\ 0.154,\ 0.155,\ 0.1555\ \ (b = 0.37)$$

- two values of $\ell_0 = 1, 2$

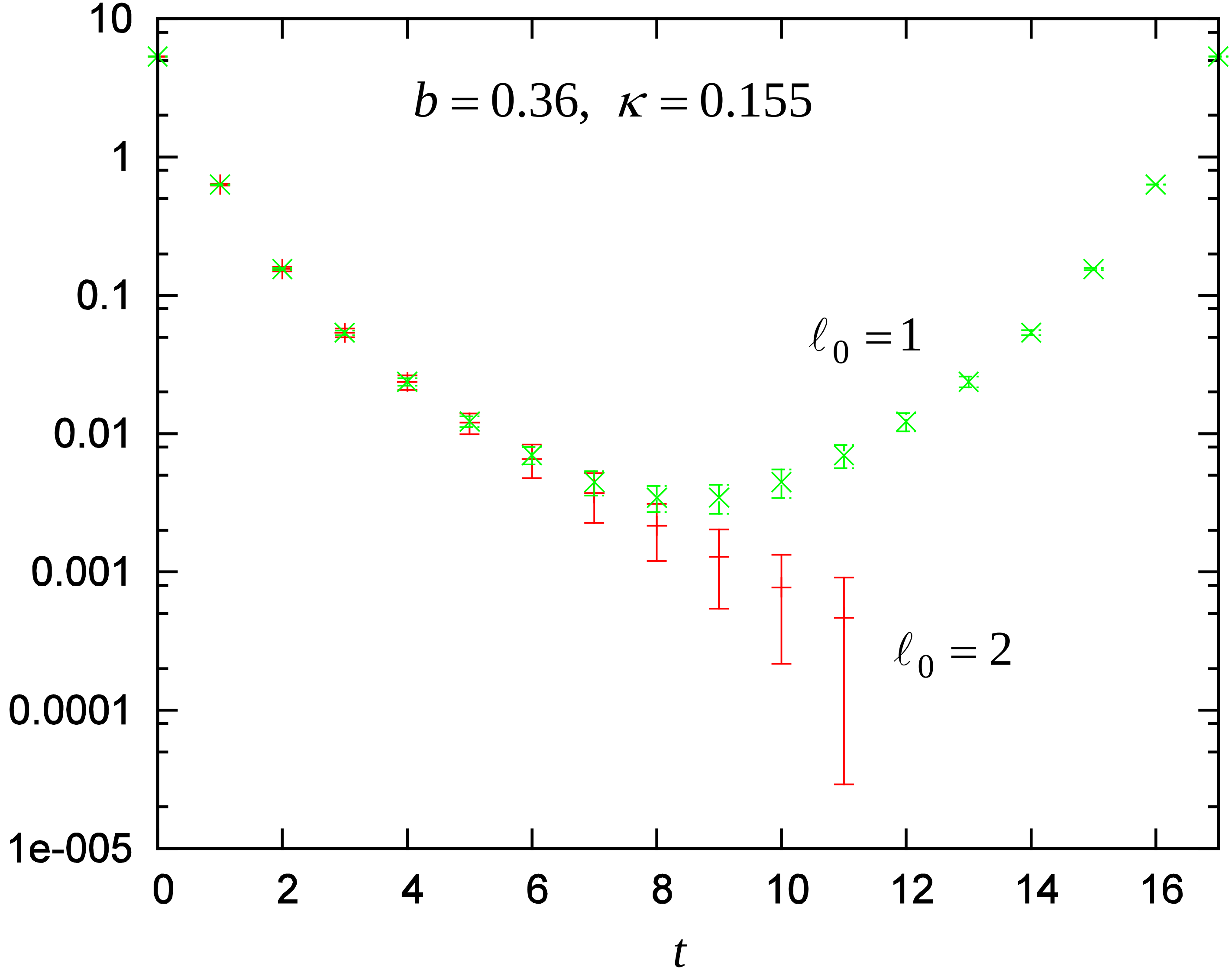
For each parameter set, we calculate meson propagator with 500 configurations. Each configuration is separated by 1000 MC sweeps.

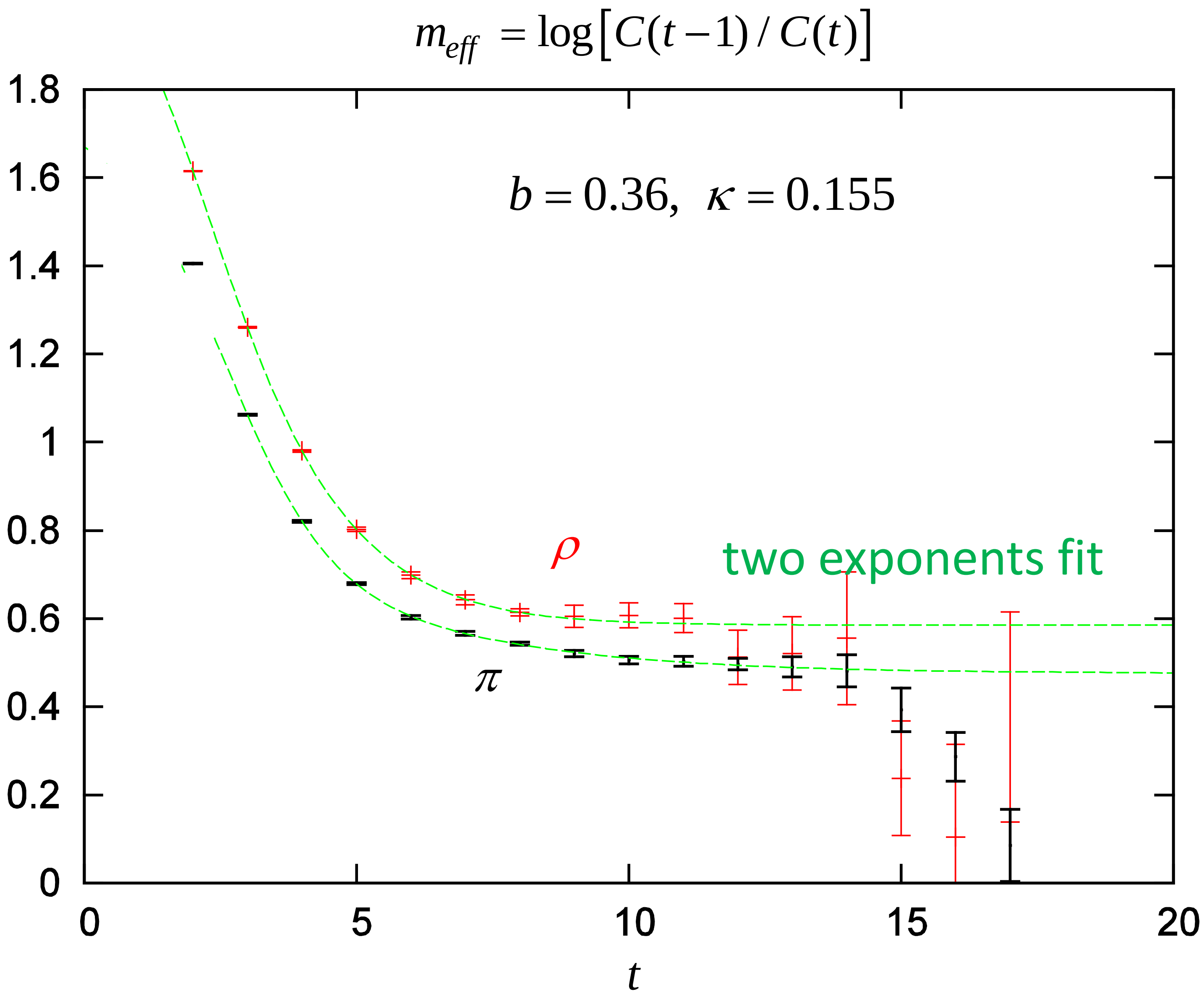
Results shown below are all preliminary.



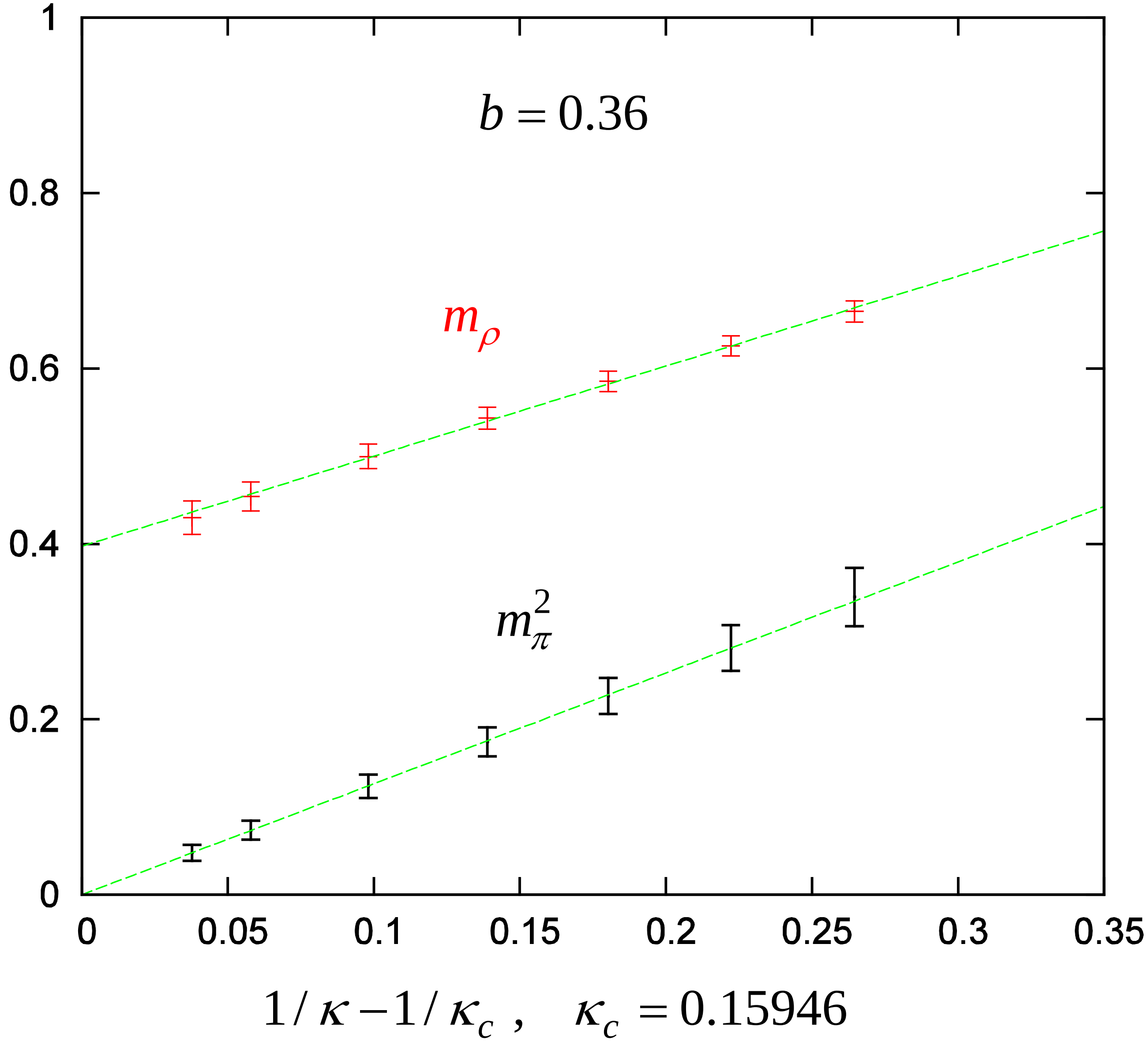
$\mathrm{Tr}\left[\gamma_5 D^{-1}(\rho_0)\gamma_5 D^{-1}(0)\right]$
$\rho_0 = \frac{2\pi m}{\ell_0 L}; \quad 0 \le m \le \ell_0 L = 34$
$b = 0.36, \ \kappa = 0.155$
7.5
7
6.5
6
5.5
5
4.5
4
0
5
10
15
20
25
30
m

$$C_{\pi\pi}(t) = \sum_{\rho_0} \exp(i\rho_0 t)\ \mathrm{Tr}\left[\gamma_5 D^{-1}(\rho_0)\gamma_5 D^{-1}(0)\right]$$

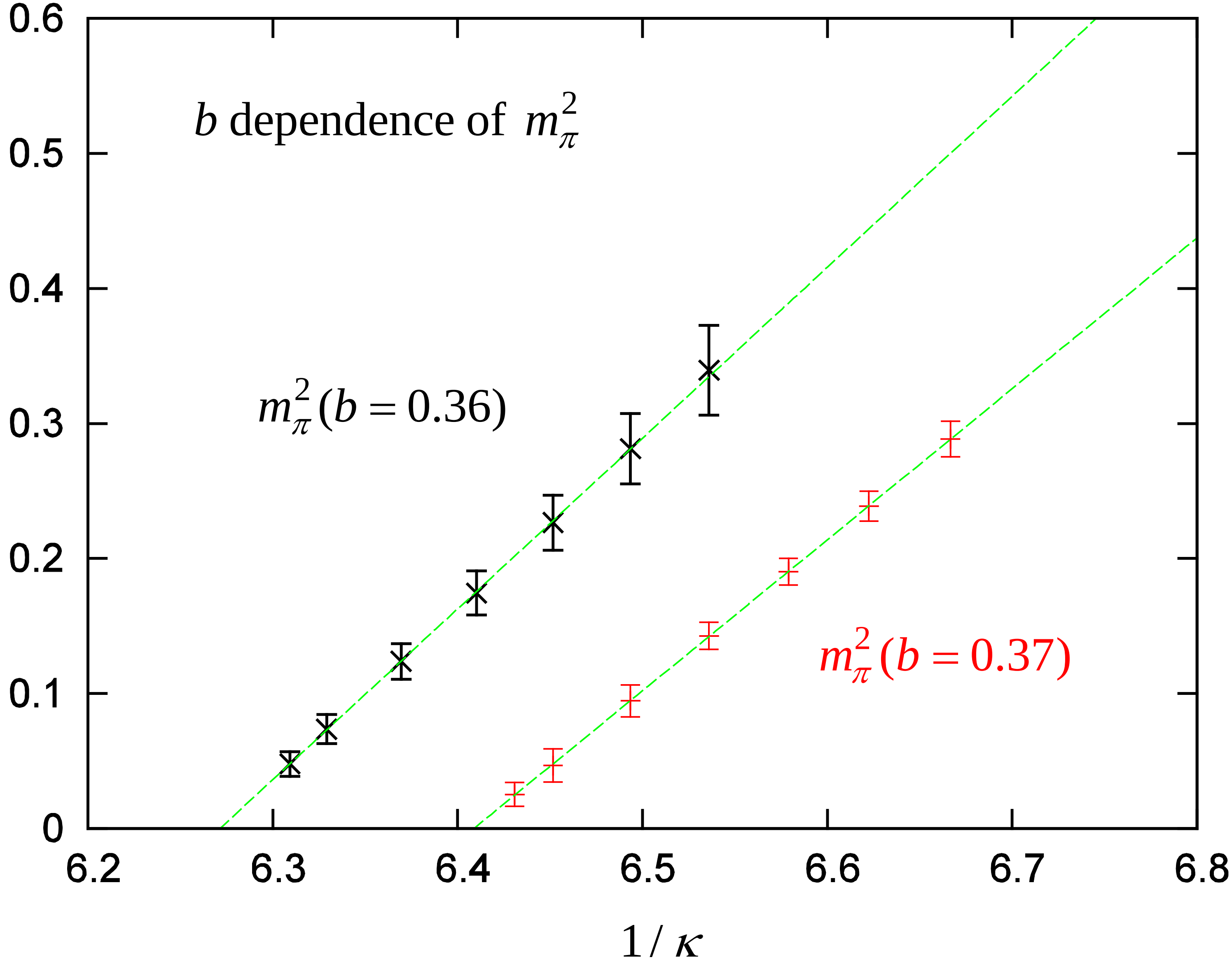




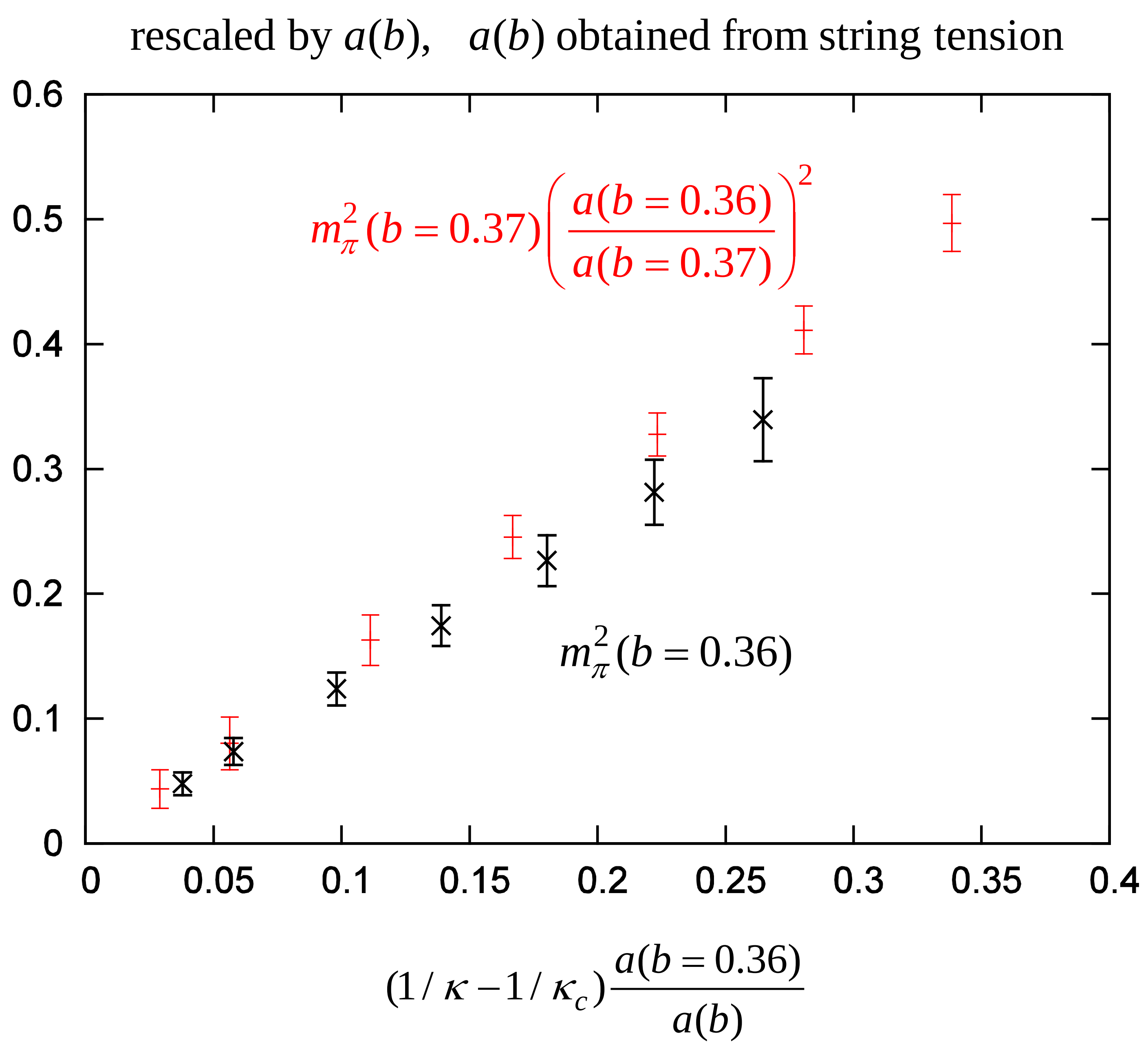
$m_{eff} = \log\left[C(t-1)/C(t)\right]$
$b = 0.36, \;\; \kappa = 0.155$
ρ
two exponents fit
π
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
t



1
0.8
0.6
0.4
0.2
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
$b = 0.36$
m_ρ
m_π^2
$1/\kappa - 1/\kappa_c$, $\kappa_c = 0.15946$

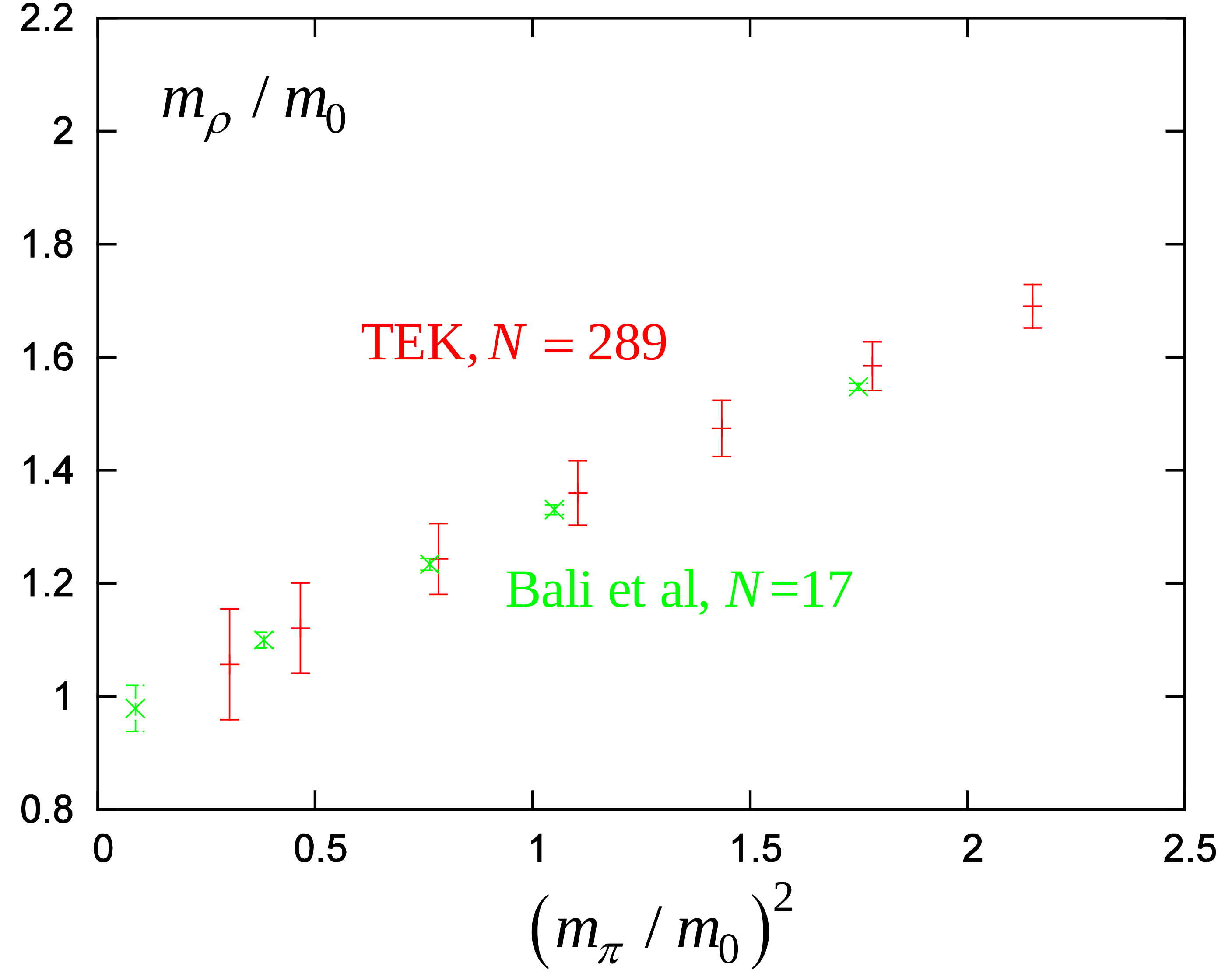


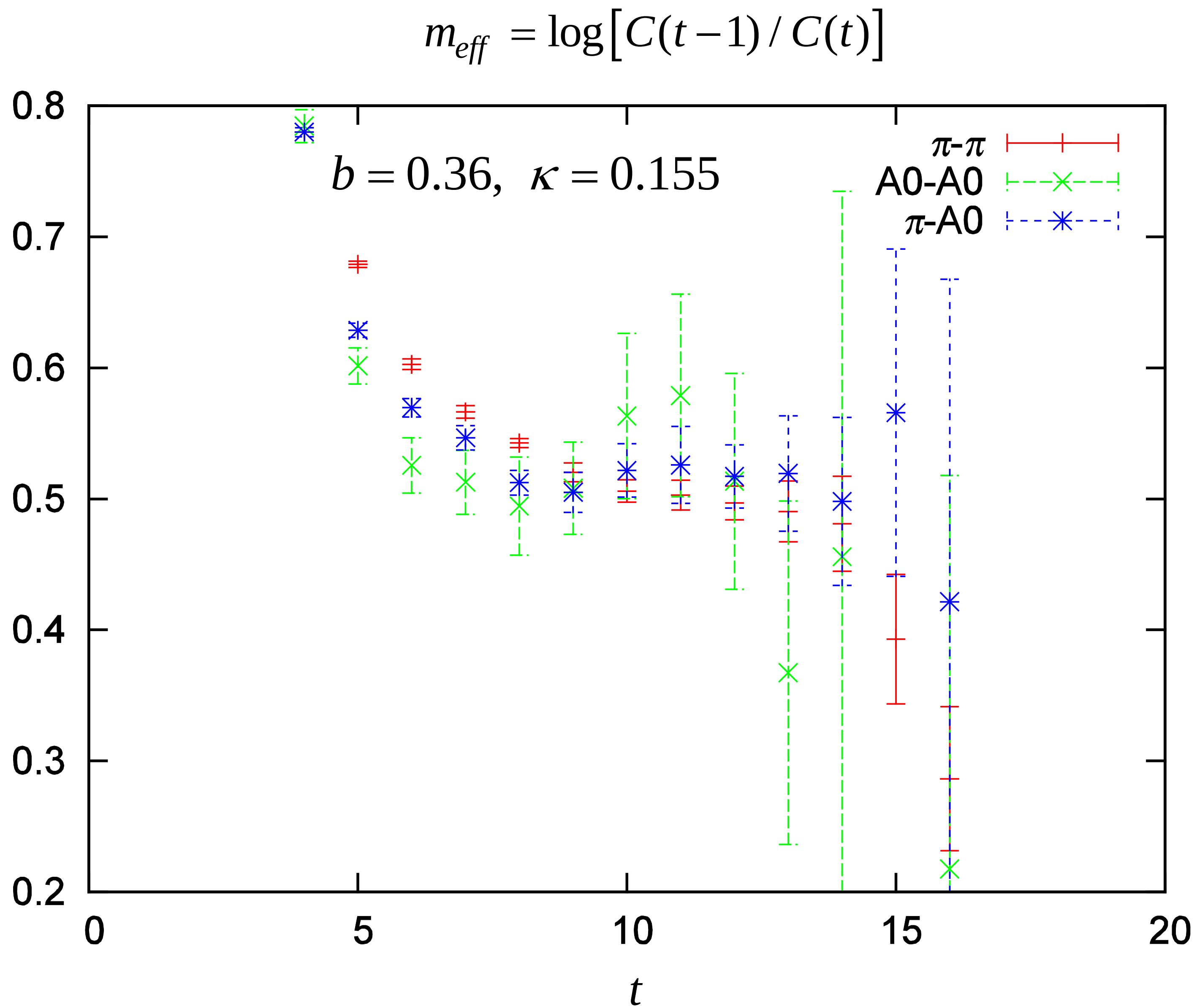
b dependence of m_π^2
$m_\pi^2(b=0.36)$
$m_\pi^2(b=0.37)$
0
0.1
0.2
0.3
0.4
0.5
0.6
6.2
6.3
6.4
6.5
6.6
6.7
6.8
$1/\kappa$



rescaled by $a(b)$, $a(b)$ obtained from string tension
$m_\pi^2(b=0.37)\left(\frac{a(b=0.36)}{a(b=0.37)}\right)^2$
$m_\pi^2(b=0.36)$
$(1/\kappa - 1/\kappa_c)\frac{a(b=0.36)}{a(b)}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.05
0.15
0.25
0.35

Edinburgh plot $m_0 = m_\rho(\kappa = \kappa_c)$





$m_{eff} = \log[C(t-1)/C(t)]$
$b = 0.36, \ \kappa = 0.155$
π-π
A0-A0
π-A0
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
5
10
15
20
t

Conclusion

In this talk, we have proposed a new method to calculate fundamental meson propagators in the large N limit using twisted space-time reduced model.

We have to invert $4L^4 \times 4L^4$ matrix $\ell_0 L$ times, which can be done using presently available supercomputer with $L \approx 20$.

Our preliminary calculations are quite promising.
However, we have to improve the method to make precise estimations of meson masses and decay constants, mainly because formula used in this talk are for point-point correlators.

Our next task is to study spatially extended meson operators and those smearing.