Proposal for a Quantum Simulation of the $CP(2)$ Model on an Optical Lattice

Concept : Simulations employing a **quantum system** (analog quantum computing) could solve the sign problem of complex actions; the phase factor is included.

Long-term goal : explore **QCD phase diagram** at finite baryon density and vacuum angle θ.

Here : proposal for the quantum simulation of the **2d $CP(2)$ model**, with **ultra-cold Alkaline Earth Atoms (AEAs) trapped in an optical lattice.**

W. Bietenholz[a], M. Dalmonte[b], W. Evans[c], U. Gerber[a],
C. Laflamme[b], H. Mejía-Díaz[a], U.-J. Wiese[c], P. Zoller[b]

[a] Universidad Nacional Autónoma de México, Mexico

[b] Universität Innsbruck, Austria, [c] Universität Bern, Switzerland

$CP(N-1)$ Models

2d $CP(N-1)$ models, as toy models for QCD [1]

$$S[\vec{z}] = \int d^2x \; \partial_\mu \vec{z}^\dagger \cdot \partial_\mu \vec{z} \; , \qquad \vec{z}(x) \in \mathbb{C}^N, \; |\vec{z}(x)| \equiv 1$$

or Hermitian projection matrices $P = |\vec{z}\rangle\langle\vec{z}|, \;\; \mathrm{Tr}\, P = 1, \;\; P = P^2 = P^\dagger$

- asymptotic freedom
- topological sectors
- dynamically generated mass gap

- $V = \infty$: SSB $SU(N) \to U(N-1)$
  $\Rightarrow \quad 2(N-1)$ Nambu-Goldstone bosons
- $V = L \times L', \quad L \gg L'$: $\qquad \xi \propto e^{\text{const.}\, L'}$

For increasing L' : $\xi \gg L'$, dimensional reduction

D-theory formulation of a $CP(N-1)$ model

$SU(N)$ quantum spins on a "ladder"

n coupled spin chains, plus β-direction

$$\hat{H} = -J \sum_{\langle xy \rangle} \sum_{a=1}^{N^2-1} T_x^a T_y^{a\,*} \qquad (J > 0: \text{ anti-ferromagnet}) \tag{1}$$

T_x^a ($T_y^{a\,*}$): spin operators in (anti-)fundamental representation of $SU(N)$

$[T_x^a, T_y^b] = \mathrm{i}\delta_{xy} f_{abc} T_x^c$

$N = 3$ or 4, $T \to 0$, $V \to \infty$ SSB down to $U(N-1)$

NGBs in $SU(N)/U(N-1) = CP(N-1)$

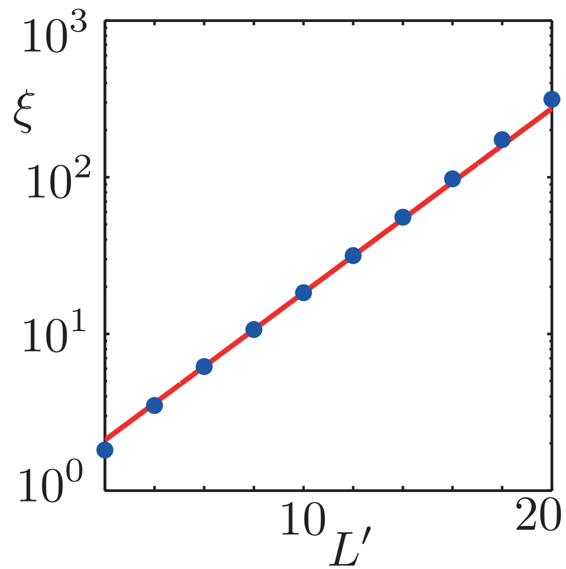
$CP(2)$ *or* $CP(3)$ *model as low energy eff. theory* [2]

$CP(2)$ model

$$S[P] = \frac{1}{g^2} \int d^2x \ \mathrm{Tr}\Big[\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P\Big] - \mathrm{i}\,\theta\, Q[P]$$

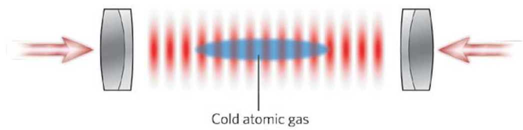
(c: spin wave velocity). For $L \gg L'$: quasi-NGBs, $\theta = L'\pi$

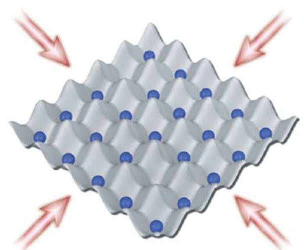
Simulations with loop cluster algorithm, $L = 1500$, reveals dim. reduction



Experimental Setup

Hamiltonian (1) can be implemented by ultra-cold Alkalin Earth Atoms (AEAs), trapped in an optical lattice





AEAs at $T \sim 10^{-9}$ K, in electronic ground state

Intrinsic $SU(N)$ symmetry, $N \leq 2I + 1$, I nuclear spin, can be realized up to $N = 10$ [3].

Re-write $\hat{H}$ in terms of fermionic bilinears, composed of $\hat{d}_x^\dagger$, $\hat{d}_x$:

$$T_x^a = \hat{d}_{xm}^\dagger \lambda_{mm'}^a \hat{d}_{xm'} \ , \quad -T_x^{a\,*} = -\hat{d}_{xm}^\dagger \lambda_{mm'}^{a\,*} \hat{d}_{xm'}$$

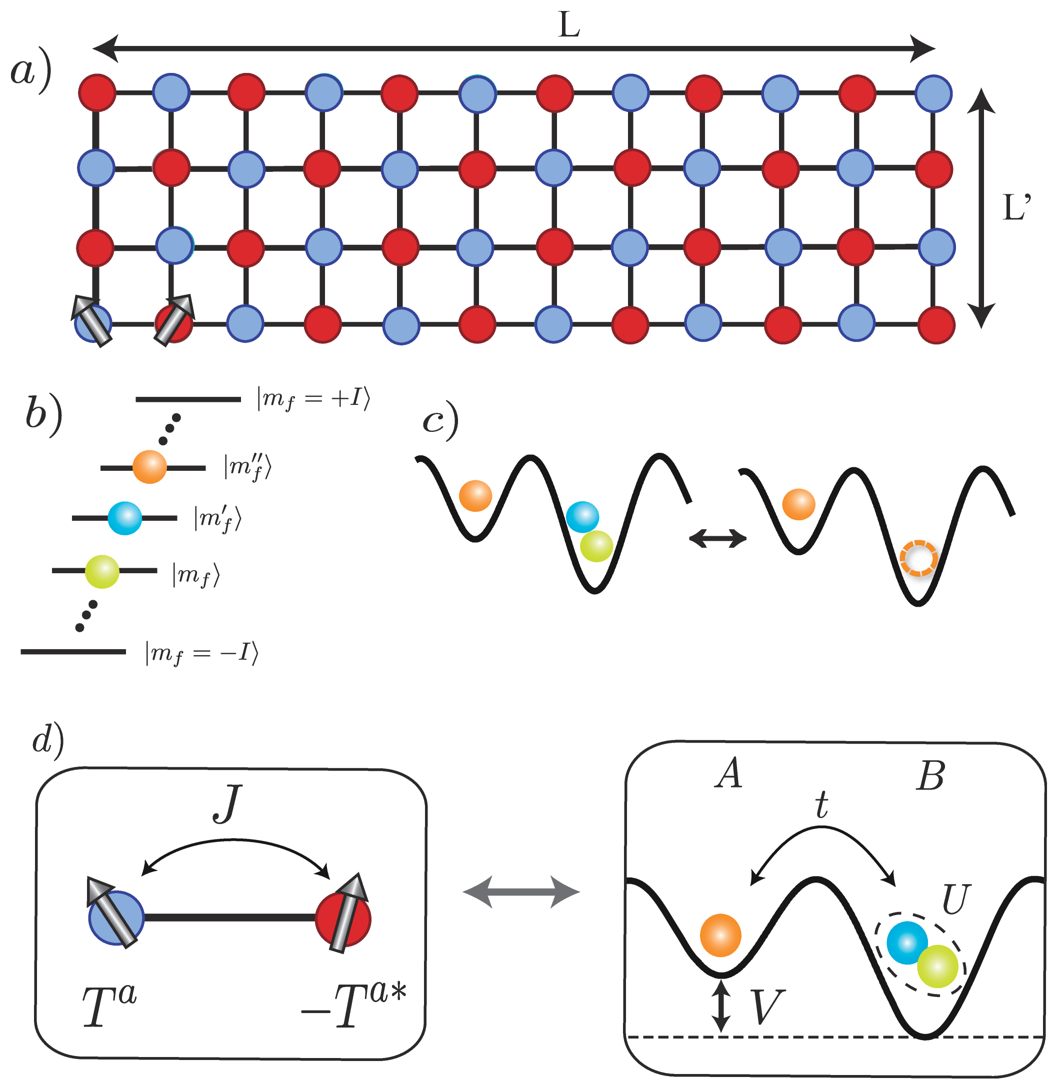
λ^a: generalised Gell-Mann matrices, Tr $[\lambda^a, \lambda^b] = \delta^{ab}$, $m, \ m' = 1 \ldots N$: state labels

$$\hat{H} = \hat{H}_t + \hat{H}_U$$

$$\hat{H}_t = -t \sum_{\langle xy \rangle, m} (\hat{c}_{xm}^\dagger \hat{c}_{ym} + \hat{c}_{ym}^\dagger \hat{c}_{xm})$$

$$\hat{H}_U = \frac{U}{2} \sum_x n_x(n_x - 1) + V \sum_{x \in A} n_x$$

$\hat{c}_{xm}$: annihilates nuclear spin level $m \in \{-I \ldots I\}$ at x, t : hopping parameter,
U: onsite interaction, $n_x = \sum_m \hat{c}_{xm}^\dagger \hat{c}_{xm}$ occupation number,
V : energy offset between staggered sub-lattices A, B


L
a)
L'
b)
$|m_f = +I\rangle$
$|m''_f\rangle$
$|m'_f\rangle$
$|m_f\rangle$
$|m_f = -I\rangle$
c)
d)
J
T^a
$-T^{a*}$
A
B
t
U
V

a) $SU(N)$ spins on a bipartite $L' \times L$ lattice, $L' \gg L$

b) AEA with $N \leq 2I+1$ hyperfine states; $\vec{B}$-field $\rightarrow$ Zeeman splitting

c) Sub-lattice A : $\hat{d}^\dagger = \hat{c}$, $SU(N)$ spin in fundamental rep.
Sub-lattice B : $\hat{d} = \hat{c}$, ... anti-fundamental rep.

$A \longleftrightarrow B$: particle $\longleftrightarrow$ hole transformation

d) Interaction $T^a_{x \in A} \leftrightarrow -T^{a\,*}_{y \in B}$

Initial state: sub-lattice A : 1 AEA / site $\widehat{=}$ 1 fermion
sub-lattice B : $N-1$ AEAs / site $\widehat{=}$ $N-1$ fermions $\widehat{=}$ 1 hole

Hopping parameter t $\longleftrightarrow$ coupling J, tuned by energy offset V

Strong coupling: $t \ll U,\ V$

$\approx$ eigenstates of $\hat{H}_U$, plus tunnelling due to $\hat{H}_t \mathrel{\widehat{=}} SU(N)$ exchange terms

To $O((t/U)^2)$ this reproduces $\hat{H}$, with [4]

$$J = \frac{t^2 U}{[-V + U(N-3)]\,[V - U(N-1)]}$$

Initial state preparation:

- Start with N AEAs/site, each level m occupied once (can be achieved by *optical pumping* [4])
- Split each site adiabatically into a double well ($\rightarrow A$, B sub-lattice formation), reduce barrier $\rightarrow$ **quantum dynamics according to $\hat{H}$** (realized already for *bosonic* alkaline atoms [5]).

Large L and L' up to ≈ 12 is feasible $\Rightarrow$ dim. reduction sets in, cf. Fig. 1. Experimentally ξ can be measured by Bragg spectroscopy [5].

Phase transition at $\theta = \pi$

L' odd $\;\Rightarrow\;$ $\theta = \pi$

1^{st} order phase transition with spontaneous C sym. breaking; numerical evidence in Ref. [6].

In experiment : C = shift in L-direction
$T_x^a \to -T_{x+\hat{1}}^{a\,*}$, $\quad -T_x^{a\,*} \to T_{x+\hat{1}}^a$ where x $(x+\hat{1})$ in sub-lattice A (B).

Order parameter for C sym. breaking

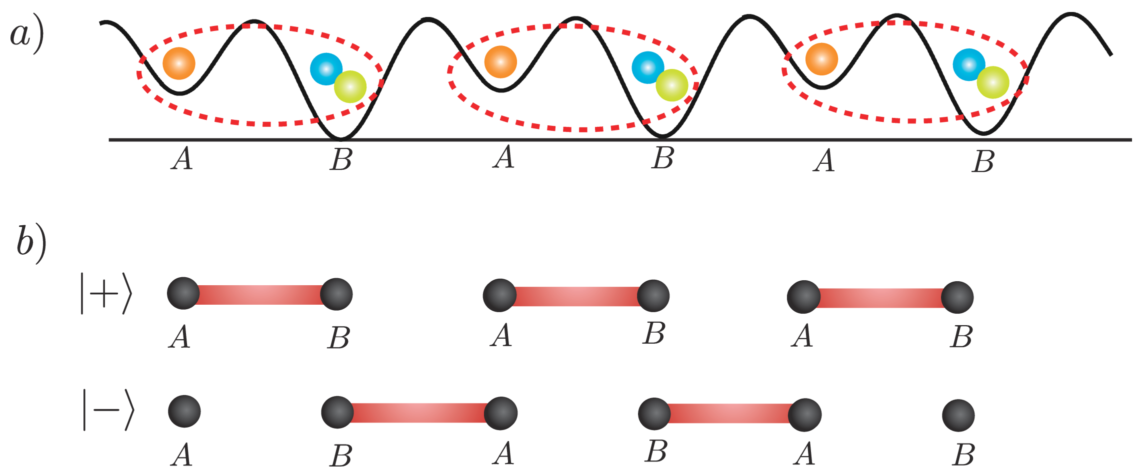
$$D = \sum_{x \in A} \left\langle T_x^a T_{x+\hat{1}}^{a\,*} - T_x^a T_{x-\hat{1}}^{a\,*} \right\rangle$$

detects dimerisation.

$\theta = 0$: C holds, $D = 0$

$\theta = \pi$: C broken, two degenerate ground states with $\pm D$:

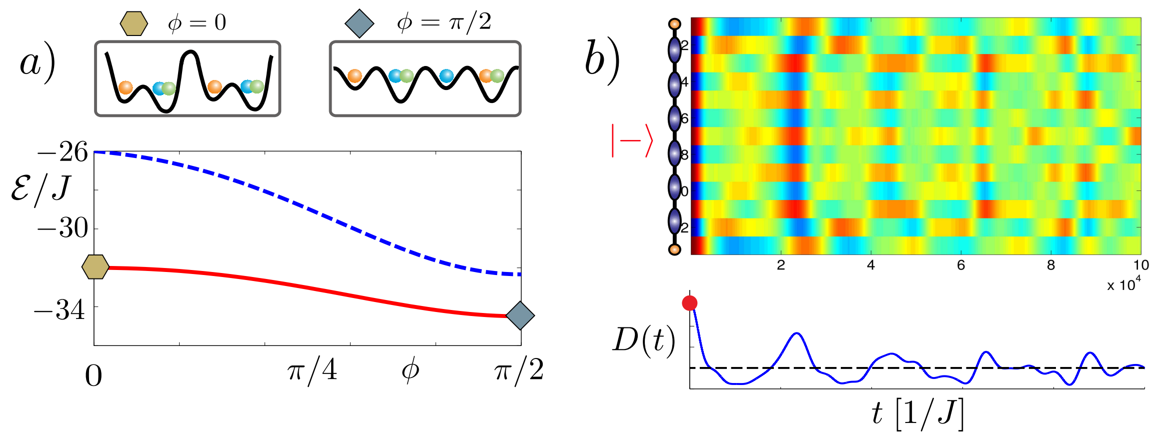
bonds between nearest neighbour sites, ambiguous $\rightarrow$ degeneracy $|+\rangle$, $|-\rangle$



For ultra-cold atoms: singlets that contribute to D can be measured by spin changing collisions [5, 7].

Proposal for the ground state preparation:

start from total dimerisation, turn on hopping parameter t adiabatically



a) E_0 and E_1 as functions of an adiabatic parameter ϕ

b) Decay of the false "vacuum" $|-\rangle$ (all singlet states, D maximal) as a function of real time, once the evolution is driven by $\hat{H}$.

Dynamics at $L = 14$, exactly computed by $\hat{H}$ diagonalisation; corresponds to real time evolution of false vacua in $CP(N-1)$ models.

Conclusions

Proposal for quantum simulation of $CP(N-1)$ models

by ultra-cold AEAs fixed in an optical lattice.

$SU(N)$ **symmetric spin interactions (**$N \leq 2I+1$**,** $I =$ **nuclear spin).**

Tools for ground state preparation and adiabatic modification exist

$\Rightarrow$ **observe** $e.g.$ **dynamics of C breaking** $\leftrightarrow$ **restoration.**

Access to cont. limit thanks to asymptotic freedom

$\rightarrow$ **to be confronted with numerical results in Fig. 1.**

References

[1] A. D'Adda, M. Lüscher, P. Di Vecchia, Nucl. Phys. B146 (1978) 63.

[2] K. Harada, N. Kawashima, M. Troyer, Phys. Rev. Lett. 90 (2003) 117203.

[3] A.V. Gorshkov *et al.*, Nature Phys. 6 (2010) 289.

[4] F. Scazza *et al.*, Nature Phys. 10 (2014) 779.

[5] S. Nascimbène *et al.*, Phys. Rev. Lett. 108 (2012) 205301.

[6] B.B. Beard, M. Pepe, S. Riederer, U.-J. Wiese, Phys. Rev. Lett. 94 (2005) 010603.

[7] B. Paredes, I. Bloch, Phys. Rev. A77 (2008) 023603.